Constrained Markovian MC for the initial state PDFs

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The long standing problem

- Markovian MC implementing the QCD/QED evolution equations is basic ingredient in all parton shower type MCs

Unconstrained Markovian, with evolution kernels from perturbative QED/QED, can only be used for FSR (inefficient for ISR).

For ISR the Backward Markovian of Sjostrand (Phys.Lett. 157B, 1985) is a widely adopted remedy.

Backward Markovian does not solve evolution eqs. It merely exploits their solutions coming from the external non-MC methods.

Is it possible to invent an efficient MC algorithm for constrained Markovian based on internal MC solutions of the evolution eqs?
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- **Is it possible to invent an efficient MC algorithm for constrained Markovian based on *internal* MC solutions of the evolution eqs?**
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  It agrees with the Markovian $\text{EvolMC}$ to within 0.2%.

- Next step: Prototyping, testing and documenting the entire family of constrained MC algorithms that we see...
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  Obvious candidate processes: ISR at ELCs, W/Z at LHC and DIS.
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Solutions class I and II

\[ \int dx_0 \, D(x_0) \int \prod_i dz_i P(z_i) \, H(sx_0 \prod_i z_i) \]

Solutions class I (more difficult because of \(\delta(\cdots)\)):
\[ \int dx \, dx_0 \, D(x_0) \, H(sx) \int \prod_i dz_i P(z_i) \delta(x - x_0 \prod_i z_i) \]

Solutions class II (only for QCD) NEW!:
\[ \int dx \, H(sx) \int \prod_i \frac{dz_i}{z_i} P(z_i) \, D(x / \prod_i z_i) \Theta(\prod z_i - x) \]
Replace $D(x_0) \rightarrow 1/x_0 = x \prod \frac{1}{z_i}$. Compensated by MC weight.

Must generate $P(z_i) = 2C_A \left( \frac{1}{z_i} + \frac{1}{1-z_i} \right)$
with the constraint $\prod_i z_i \geq x$. Not so trivial!
Solution by the multibranching method:

$\frac{1}{z} + \frac{1}{1-z} \quad = \quad \frac{1}{1-z} \quad + \quad \frac{1}{z}$
Multibranching in IIB

Using

\[ \frac{1}{z} + \frac{1}{1-z} \]

Leads to sum over branches:

\[ \sum \]

Contributions 1 = \( z \) and 1 = \( (1 - z) \) are combined and resummed separately.

Worst-case scenario (pure gluon bremsstrahlung) is now prototyped and tested.

Constrained Markovian MC for the initial state PDFs – p.6/8
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Leads to sum over branches:

\[ \sum_{z} \Theta \left( \frac{x}{x_0} - \prod z_i \right) \rightarrow \prod \Theta \left( \frac{x}{x_0} - z_i \right) \]

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Testing prototype IIB

Comparison of IIB solution with the Markovian MC EvolMC for pure gluonstrahlung. Two solutions and the ratio (lower plot).

Agreement to within 0.2%
Short term prospects

- More testing of IIB.
- Numerical test of solutions class I (several solutions found, under tests)
- Implementing transitions $Q \rightarrow G$ and $G \rightarrow Q$ (at least 2 methods found)
- Adding NLL corrections (looks rather trivial)

Most important:
NEW AVENUES are opened in the construction of the ISR PARTON SHOWER type MCs