Two-loop susy contributions to the anomalous magnetic moment of the muon

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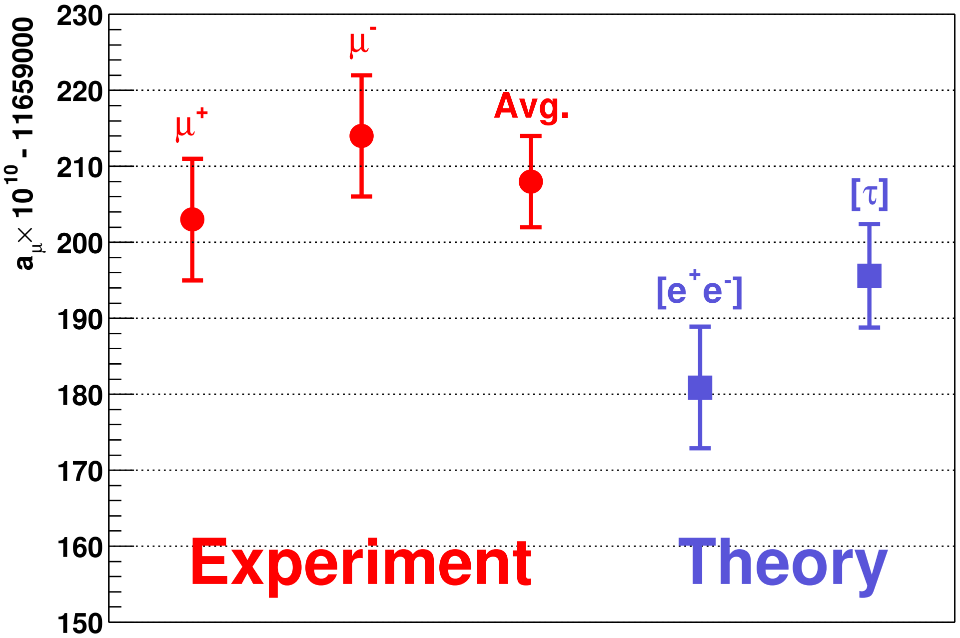
collaboration with S. Heinemeyer and G. Weiglein

- experimental and SM status
- 1-Loop susy contributions
- 2-loop sfermion corrections
- 2-loop chargino/neutralino corrections

Measurement of a_μ



Final experimental result

$$a_\mu^{\text{exp}} = (11\,659\,208 \pm 6) \times 10^{-10}$$

SM theory prediction

Source	contr. to $a_\mu[10^{-10}]$		
QED	11 658 470.6		*Kinoshita et al '99*
LO hadr.	696.3 ± 7.2	(e^+e^-)	*Davier et al '03*
	711.0 ± 5.8	(τ)	*Davier et al '03*
	694.8 ± 8.6	(e^+e^-)	*Jegerlehner et al '03*
	691.7 ± 6.1	(e^+e^-)	*Hagiwara et al '03*
LBL	8.0 ± 4.0		*Knecht, Nyffeler '02*
EW 1L	19		
EW 2L	-4		*Czarnecki et al '98*
exp. res.	6		

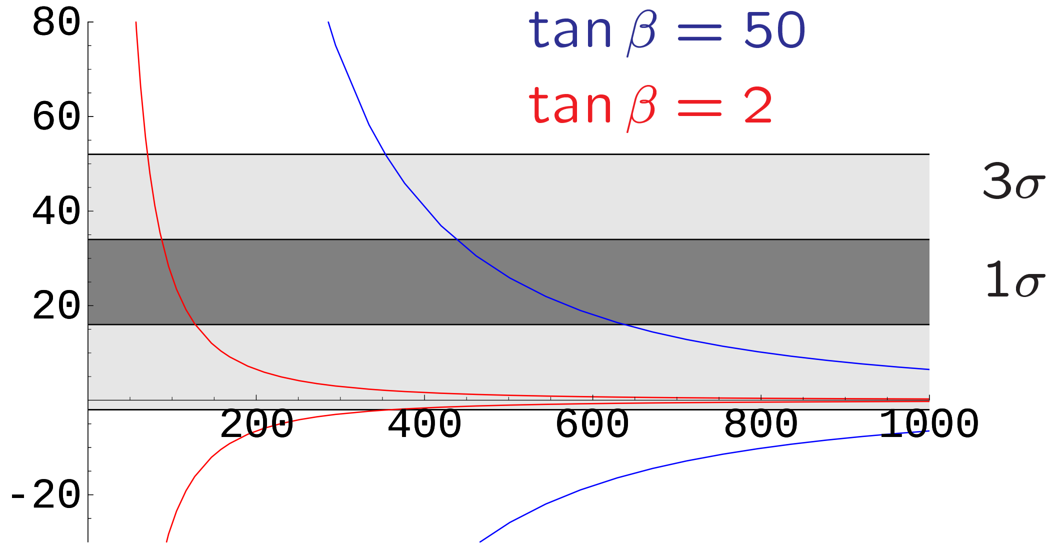
LBL new	old $+$ 5.6 $\pm$2.5		*Melnikov et al '03*
QED new	old $+$ 1.3		*Kinoshita, Nio '04*

SM prediction too low by

27 ± 10	2.7σ	*Davier, Eidelman, Höcker, Zhang '03*
32 ± 9	3.3σ	*Hagiwara, Martin, Nomura, Teubner '03*
25 ± 9	2.8σ	*new LBL, QED; HMNT '03*

a_μ in the MSSM

$$a_\mu^{\mathrm{SUSY}} \approx 13 \times 10^{-10} \tan\beta \; \mathrm{sign}(\mu) \left(\frac{100\mathrm{GeV}}{M_{\mathrm{SUSY}}}\right)^2$$



a_μ in the MSSM

Compare:

- deviation $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25 \pm 9) \times 10^{-10}$
- susy $a_\mu^{1L,SUSY} = (-100 \ldots + 100) \times 10^{-10}$

$\Rightarrow$strong restrictions on SUSY parameter space

preferred range: $m_{\tilde{\mu}}, m_{\tilde{\chi}} = 100 \ldots 600$ GeV

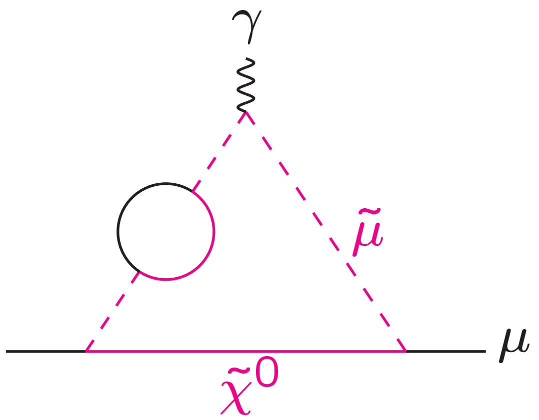
$\longrightarrow$ interesting for LC

knowledge of a_μ to $\approx 1 \times 10^{-10}$ necessary

$\Rightarrow$ Calculate SUSY 2-Loop contributions

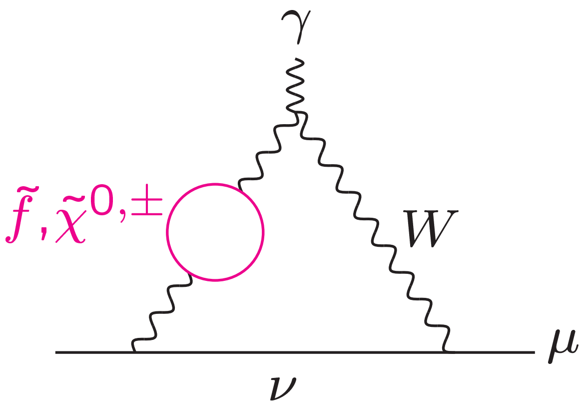
2-Loop contributions

diagrams involving a 1-Loop SUSY diagram



- QED-logs: $\frac{4\alpha}{\pi}\log\frac{m_\mu}{M_{\text{SUSY}}} \times a_\mu^{\text{1L,SUSY}} \approx -9\%$

SM/2HDM 1-Loop diagram + SUSY insertion



- could be large even if 1-Loop contribution is small
  (e.g. heavy smuons, charginos but light stops)

$\longrightarrow$ our calculation: $\tilde{f}$ and $\tilde{\chi}^{0,\pm}$ insertions

2-Loop contributions

Basic packages: *FeynArts, TwoCalc*

Starting with 3-point function

$$\Gamma_{\mu\bar{\mu}\gamma}(q) = \Gamma_{\mu\bar{\mu}\gamma}(0) + q^\mu \partial_{q^\mu} \left(\Gamma_{\mu\bar{\mu}\gamma}\right)_{q=0} + \ldots$$

$\rightarrow$ 2-point integrals

$\rightarrow$ high powers of propagators and numerators

Task: calculate 2-loop 2-point integrals with

- $p^2 = m_\mu^2$
- internal masses: m_μ, $M_{W,Z}, M_H$, $m_{\tilde{f}}$, $m_{\tilde{\chi}}$
- high powers of propagators and numerators

Steps:

- large mass expansion in $\frac{m_\mu}{M_{\text{heavy}}}$
- reduction to master integrals using partial integration

Mathematica-Bug

Bug in `Series`-command:

```
Series[expr, {epsilon,0,2}]
```

can "forget" terms in complicated expressions.

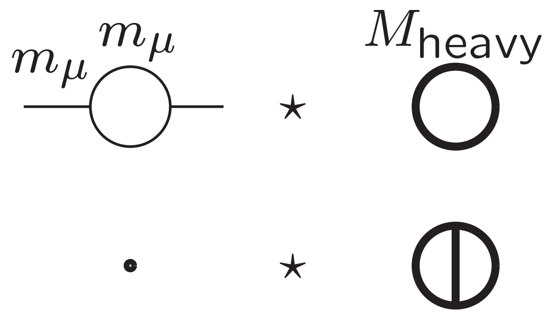
2-Loop contributions

Large mass expansion in $\frac{m_\mu}{M_{\text{heavy}}}$

$$\Gamma \to \sum_{\gamma \supset \text{heavy subgraph}} \Gamma/\gamma \quad \star \quad \mathcal{T}(\gamma)$$

either

or



Reduction of numerators/powers of denominators

- Numerators: Passarino/Veltman
- Denominators: Integration by parts

Result:

- A_0, B_0, 2-loop vacuum master integral
- Coefficients: rational functions in masses, couplings

2-Loop contributions

Discussion of results

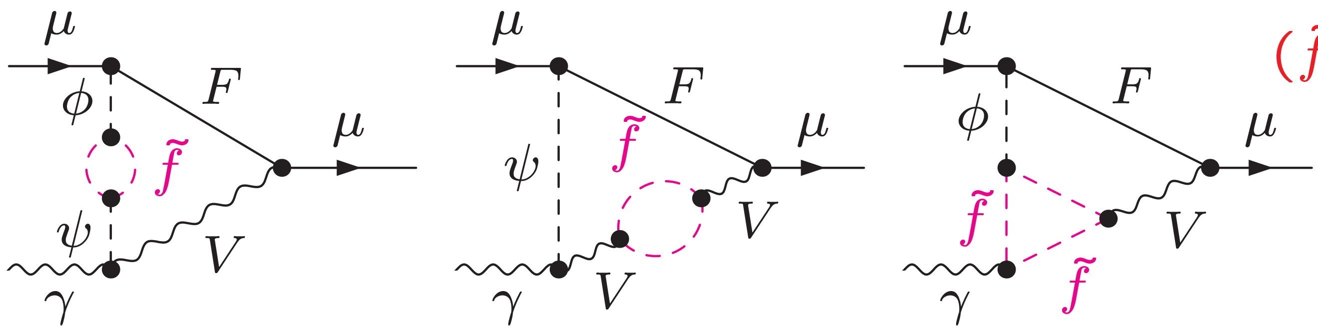
Sfermion-loop contributions: μ, $\tan\beta$; $m_{\tilde{f}}$, A_f

- complicated dependence
- experimental constraints very important
- e.g. μ=coupling, $\mu \nearrow$: $a_\mu^{\tilde{f}} \nearrow$

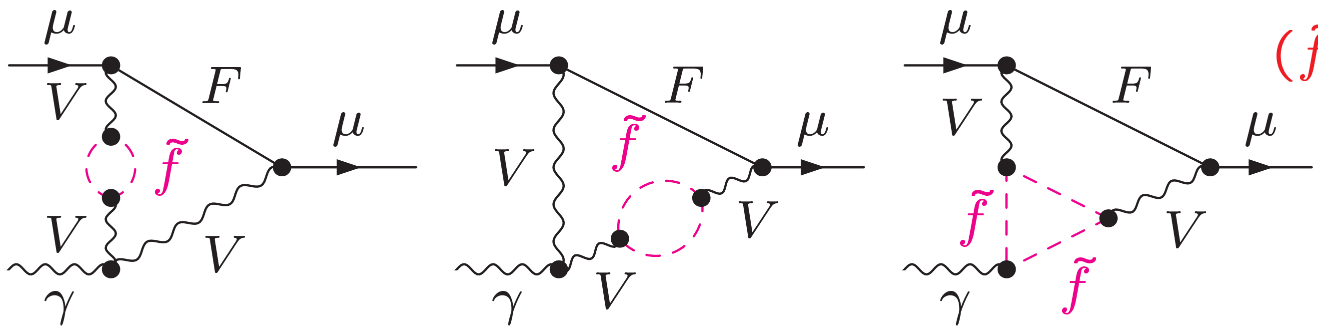
Chargino/Neutralino contributions: μ, $\tan\beta$; M_2

- straightforward dependence
- e.g. μ=mass, $\mu \nearrow$: $a_\mu^{\tilde{\chi}} \searrow$

2-Loop $\tilde{f}$-loop contributions



$(\tilde{f}V\phi)$



$(\tilde{f}VV)$

2-Loop $\tilde{f}$-loop contributions

Partial results exist: *[Arhrib/Baek, Chen/Geng '01]*

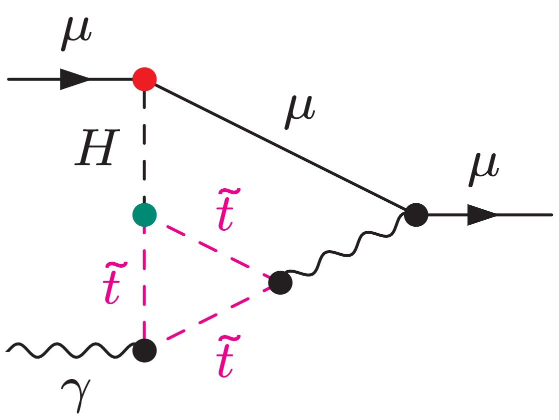
- contributions up to 20×10^{-10} possible, but disagreement by factor 4

Two Questions:

- How large are the complete contributions?
- Are such large contributions (μ = 3TeV!) really possible? (Experimental constraints!)

2-Loop $\tilde{f}$-loop contributions

Example:



$\to m_\mu \ \tan\beta$

$\to \mu\, m_t$

$\Rightarrow$ Enhancement: $a_\mu \sim \tan\beta\, \frac{\mu\, m_t}{M_H\, m_{\tilde{t}}}$

$\Rightarrow$ can be large even if SUSY 1L is small !
(other particles in the loops)

$$a_\mu \approx -13 \times 10^{-10} \left(\frac{\tan\beta}{50}\right) \left(\frac{m_t}{M_H}\right) \left(\frac{\mu}{20\ m_{\tilde{t}}}\right) \ \mathrm{sgn}(A_t)$$

- ratio $\left(\frac{\mu}{m_{\tilde{t}}}\right)$ relevant
- for $\left(\frac{\mu}{m_{\tilde{t}}}\right) > 20$, $a_\mu = 20 \times 10^{-10}$ possible

2-Loop $\tilde{f}$-loop contributions

Experimental constraints: $\left(\frac{\mu}{m_{\tilde{t}}}\right)$ strongly restricted

lightest Higgs mass: $M_h >$ 114GeV

- universality for $\tilde{t}$, $\tilde{b}$ mass parameters

$$\Rightarrow \left(\frac{\mu}{m_{\tilde{t}}}\right) < 2.5, \qquad |a_\mu| < 5 \times 10^{-10}$$

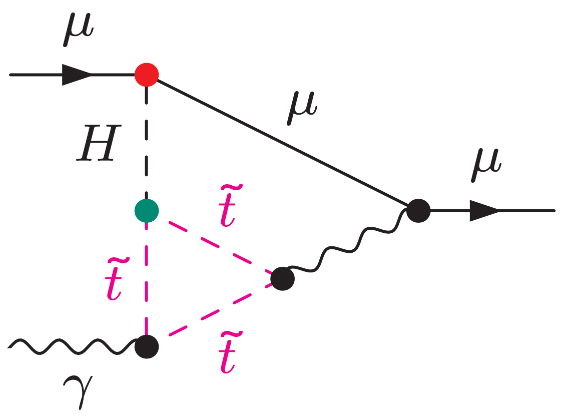
- $M_{\tilde{b}_R} \gg M_{\tilde{t}_R}$

$$\Rightarrow \left(\frac{\mu}{m_{\tilde{t}}}\right), \qquad |a_\mu| \text{ can be larger}$$

B-decays $b \to s\gamma$, $B_s \to \mu^+\mu^-$: less striking

2-Loop $\tilde{f}$-loop contributions

Largest possible contribution:



$\rightarrow m_\mu \tan\beta$

$\rightarrow \mu\, m_t$

Enhancement: μ, $\tan\beta$ / Suppression: $\frac{1}{M_H}$, $\frac{1}{m_{\tilde{t}}}$

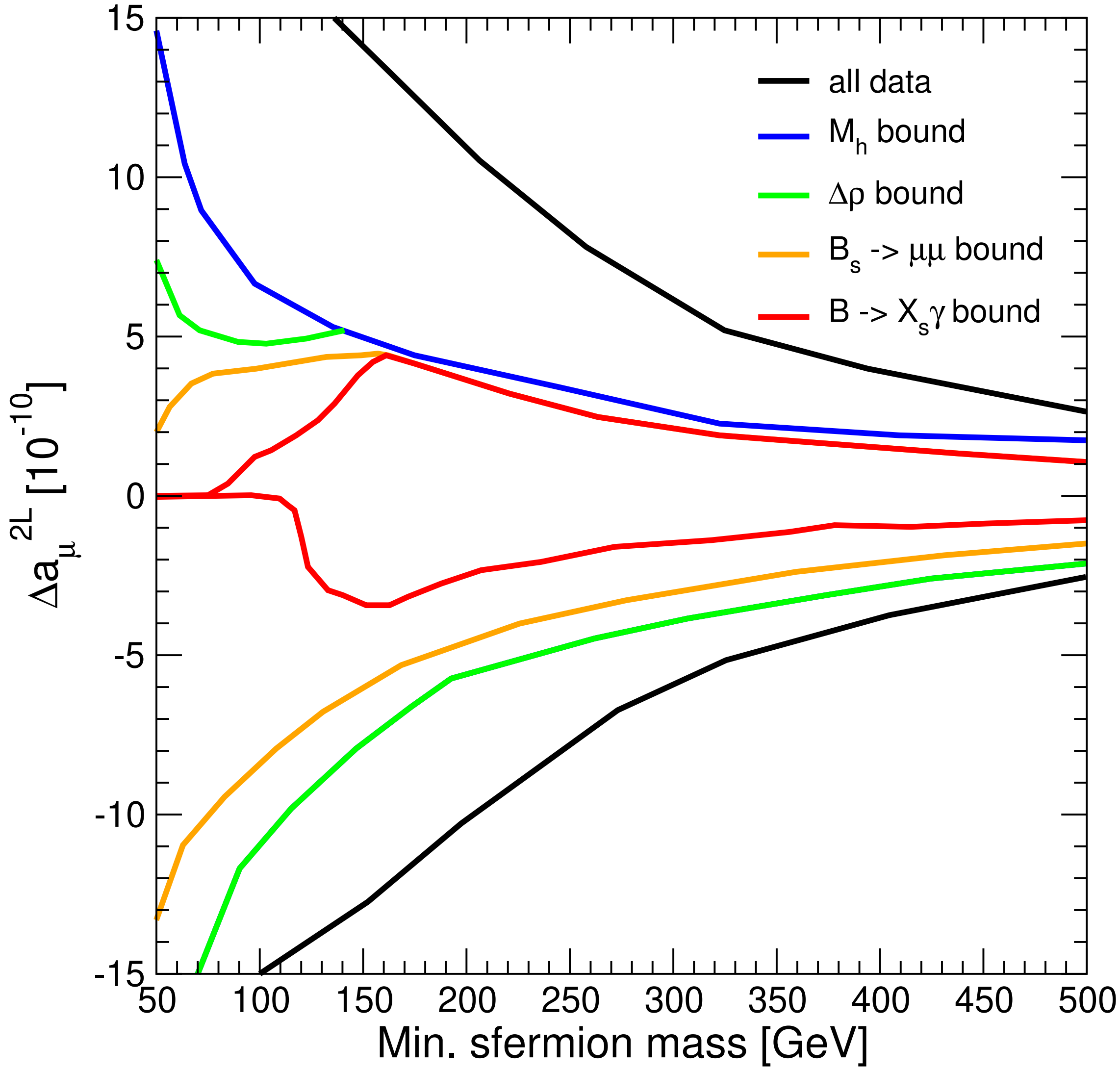
Maximised contribution: $a_\mu \approx \left(\frac{\mu}{m_{\tilde{t}}}\right) \times 10^{-10}$

(deviation $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25 \pm 9) \times 10^{-10}$)

However, for maximising parameters:

$$\left(\frac{\mu}{m_{\tilde{t}}}\right) > 2.5 \Rightarrow M_h \text{ too small}$$

2-Loop $\tilde{f}$-loop contributions



[Heinemeyer, DS, Weiglein '03]

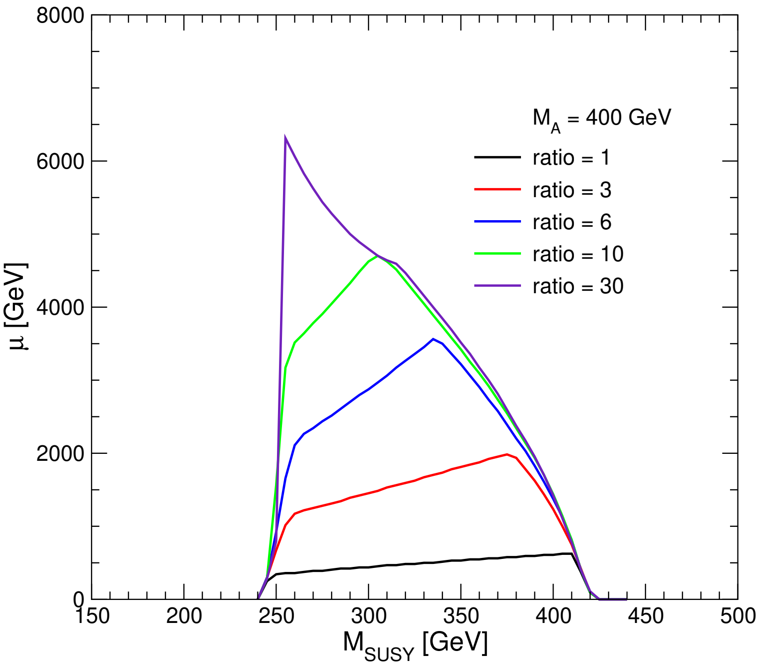
$$\text{No exp. constraints}: \quad |a_\mu^{2L,\tilde{f}}| \leq 20 \times 10^{-10}$$

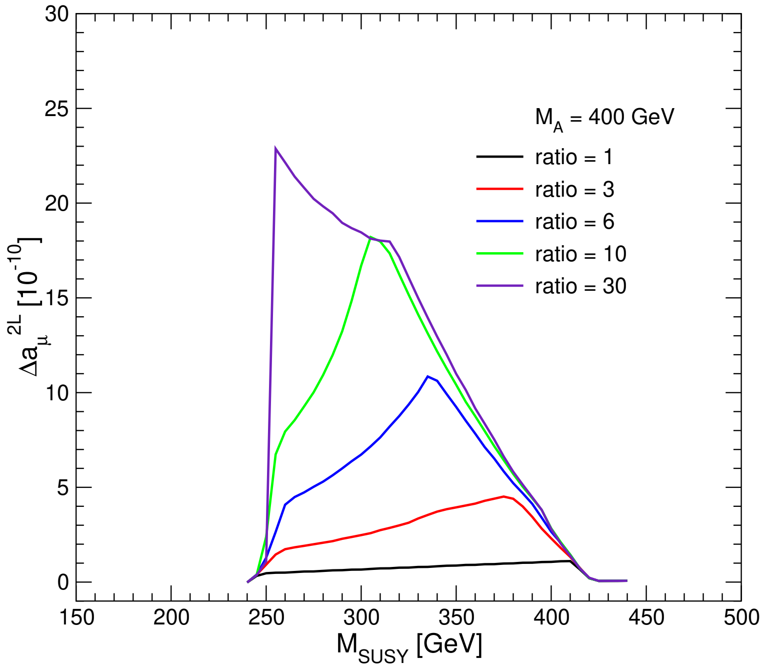
$$\text{With exp. constraints}: \quad |a_\mu^{2L,\tilde{f}}| \leq 5 \times 10^{-10}$$

2-Loop $\tilde{f}$-loop contributions

Results for non-universality: $M_{\tilde{t}_R} \neq M_{\tilde{b}_R}$

$$M_{\mathrm{SUSY}} = M_Q = M_U = M_L \neq M_D = M_E$$
$$M_A = 400\mathrm{GeV},\ m_{\tilde{t}_1} = 150\mathrm{GeV}$$

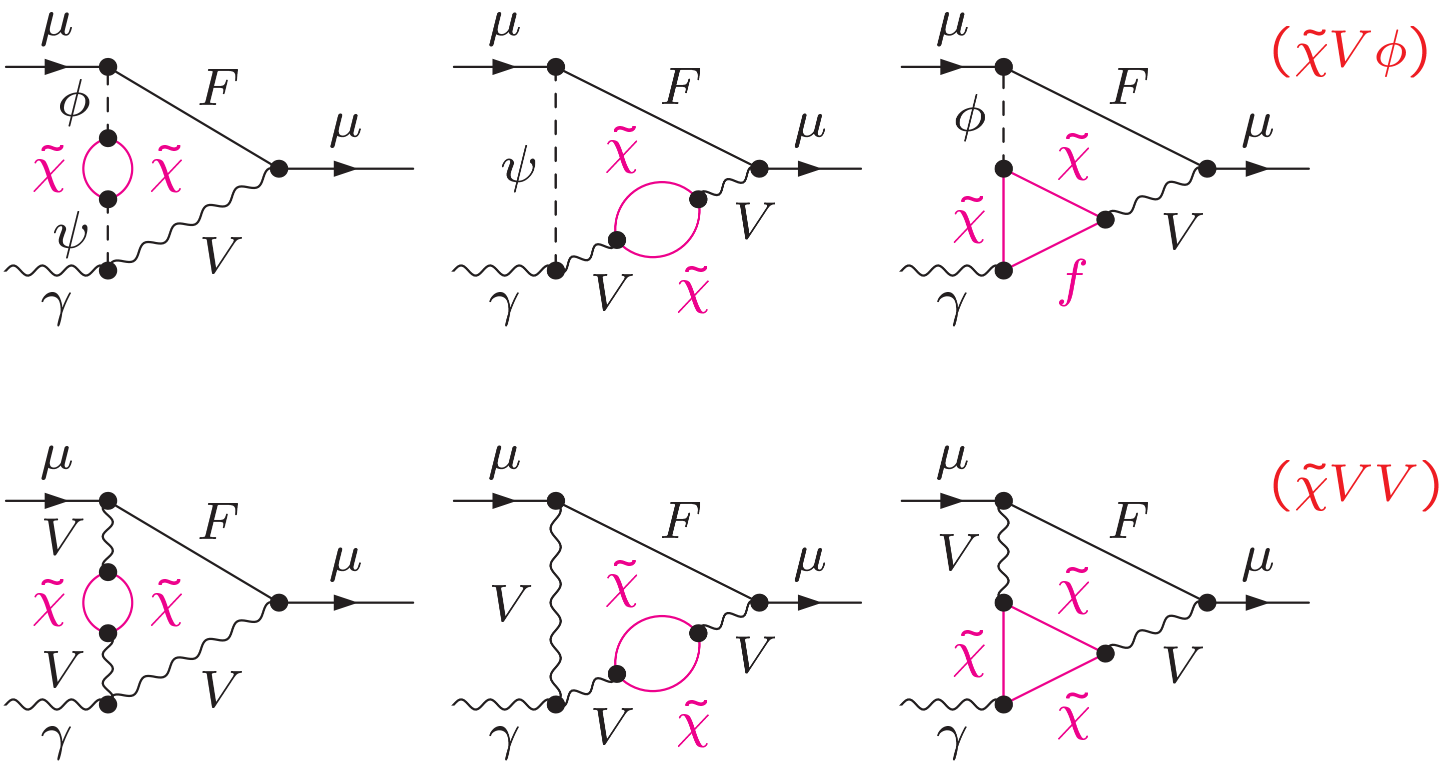




$\mu > 5\mathrm{TeV}$ possible $\Rightarrow a_\mu^{2\mathrm{L}} > 20 \times 10^{-10}$ possible

if $M_U : M_D \approx 1 \ldots 5$: no large enhancement

2-Loop Chargino/Neutralino contributions



[Heinemeyer, DS, Weiglein '04]

2-Loop Chargino/Neutralino contributions

Parameters:

- χ-masses: μ, M_2; Higgs-mass M_A
- $\tan\beta$

Approximate result ($\mu = M_2 = M_A$):

$$a_\mu^\chi \approx 13 \times 10^{-10} \left(\frac{\tan\beta}{50}\right) \text{sign}(\mu) \left(\frac{100\text{GeV}}{\text{M}_{\text{SUSY}}}\right)^2$$

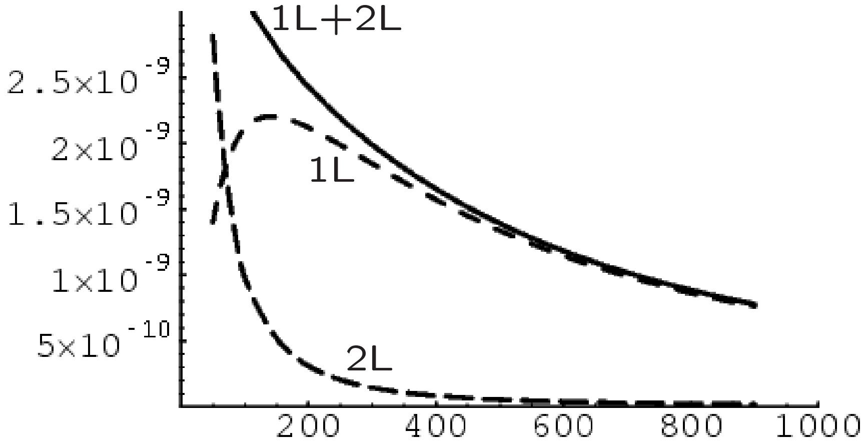
Large results $> 10 \times 10^{-10}$ possible

considerable modification of the one-loop result (depending on $m_{\tilde{\mu}}$)

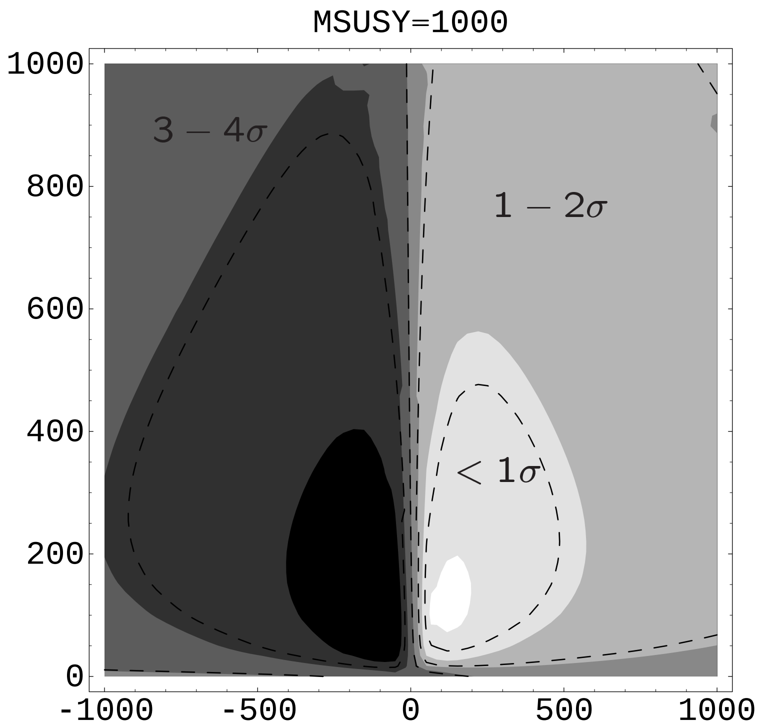
Compare 1-Loop and 2-Loop

$\tan\beta = 50$, $m_{\tilde{\mu},\tilde{\nu}} = 1\,\mathrm{TeV}$:

Plot for $\mu = M_2 = M_A$:



Plot in μ–M_2–plane:



1-Loop result, corresponding bounds are modified

Conclusions

$$\begin{aligned} a_\mu^{\text{exp}} - a_\mu^{\text{SM}} &= (25 \pm 9) \times 10^{-10} \\ a_\mu^{\text{SUSY}} &= (-100 \ldots + 100) \times 10^{-10} \end{aligned}$$

SUSY could easily explain the deviation
(large $\tan\beta$, $M_{SUSY} \approx 500$GeV)
$\rightarrow$ strong restrictions on SUSY parameters

2-Loop sfermion corrections much smaller than previously estimated

- mainly M_h-bound $\Rightarrow \left(\frac{\mu}{m_{\tilde{t}}}\right)$ restricted
  $\Rightarrow a_\mu^{2L} < 5 \times 10^{-10}$

2-Loop chargino/neutralino corrections also important

- $\approx 5 \times 10^{-10}$ possible
- complementary for $\mu \nearrow$:
  $\tilde{f}$-corrections $\nearrow$ but Cha/Neu-corrections $\searrow$
- necessary for deriving correct bounds