Status of NNLO 3-jet calculations

Stefan Weinzierl

Max-Planck-Institut für Physik München

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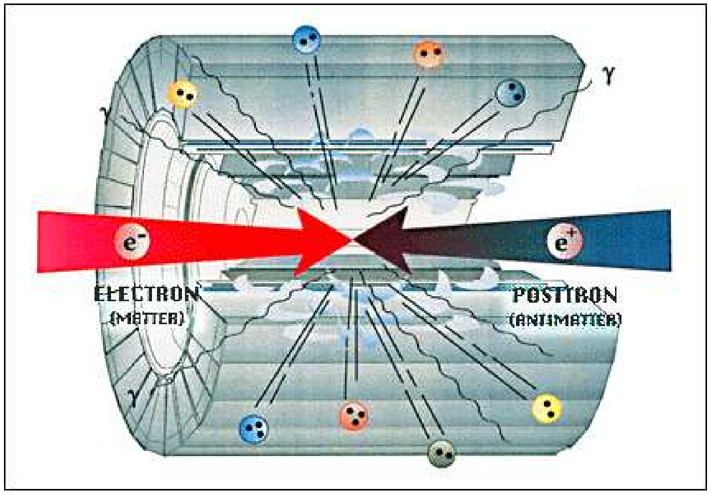
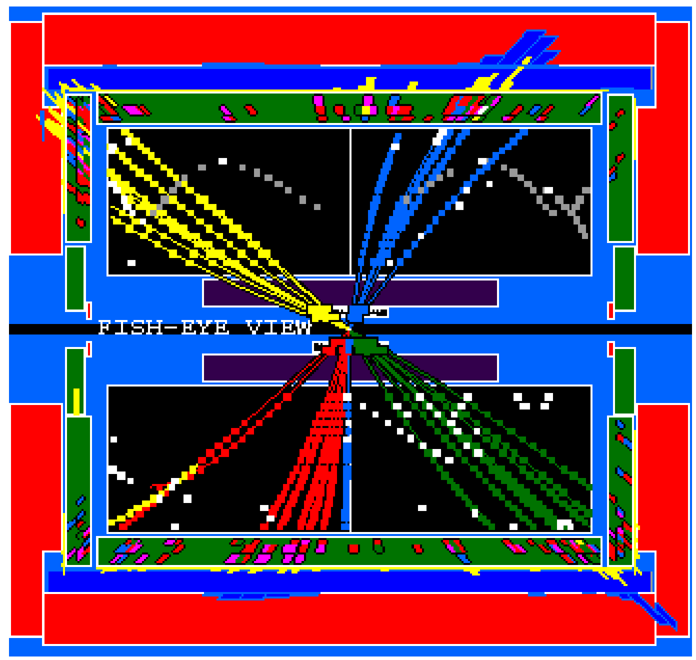
2. **Two-loop amplitudes**
3. **Cancellation of infrared divergences**
4. **Outlook**

What is observed: Jet physics



A schematic view of electron-positron annihilation.

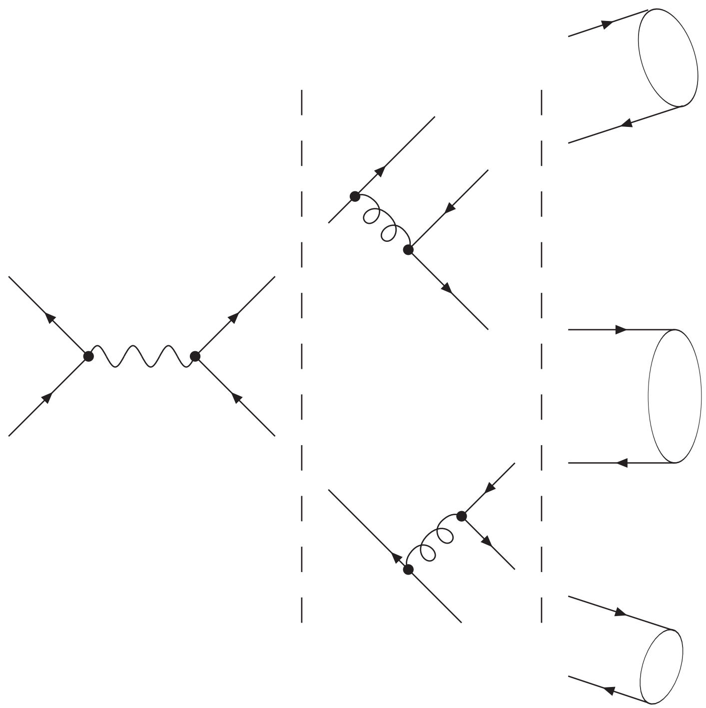
A four-jet event from the Aleph experiment at LEP:



Jets: A bunch of particles moving in the same direction

A rough description by event generators

Event generators rely on a three-stage process:



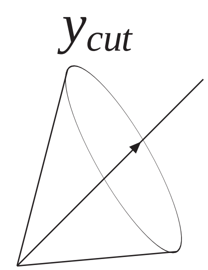
Showering and hadronization depends on approximations and/or models.

For infrared-safe observables we can do better !

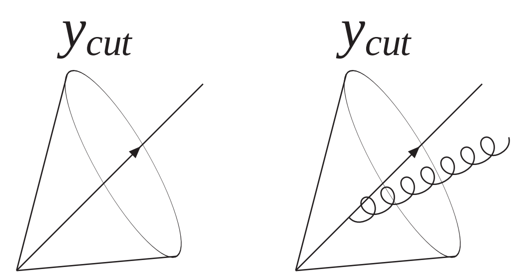
Modeling of jets:

In a perturbative calculation jets are modeled by only a few partons. This improves with the order to which the calculation is done.

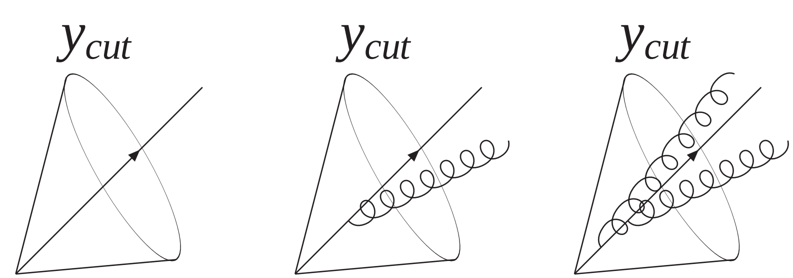
At leading order:



At next-to-leading order:



At next-to-next-to-leading order:



Infrared-safe observables and event shapes

Observables which do not depend on long-distance behaviour, are called infrared-safe observables and can reliably be calculated in perturbation theory.

In particular, it is required that they do not change value, if infinitessimal soft or collinear particles are added.

$$O_{n+l}(p_1, \ldots, p_{n+l}) \quad \rightarrow \quad O_n(p'_1, \ldots, p'_n),$$

Infrared-safe event shape observables, like thrust, jet broadening or aplanarity reveal more information about an event then the total cross section alone.

Example: Thrust

$$T \quad = \quad \text{max}_{\hat{n}} \frac{\sum\limits_i |\vec{p}_i \cdot \hat{n}|}{\sum\limits_i |\vec{p}_i|}$$

The need for NNLO calculations

Hunting for the Higgs and other yet-to-be-discovered particles requires a better knowledge of the theoretical cross section.

The strong coupling constant α_s is one fundamental parameter of the theory and its precise value affects the magnitude of many (background) processes.

The next generation of colliders will increase the experimental precision. This has to be matched by an improvement in the accuracy of theoretical predictions.

Theoretical predictions are calculated as a power expansion in the coupling. Higher precision is reached by including the next higher term in the perturbative expansion.

What is necessary:

NNLO calculations

Perturbative NNLO calculations

The experimental needs are numerical programs which yield predictions for a wide range of observables.

Fully differential NNLO programs for processes like

- Bhabha scattering
- $pp \longrightarrow 2$ jets
- $e^+e^- \longrightarrow 3$ jets

which allow the calculation of any infrared safe observable.

$e^+e^- \longrightarrow 3$ **jets**

- Measurement of α_s using data of $e^+e^- \longrightarrow 3$ jets at the Z-peak. A NNLO calculation is expected to reduce the theoretical error in the extraction of α_s down to 1%.

- A NNLO calculation models the jet structure more accurately and should allow to improve the knowledge on the interplay between perturbative QCD and power corrections.

Higher orders and power corrections

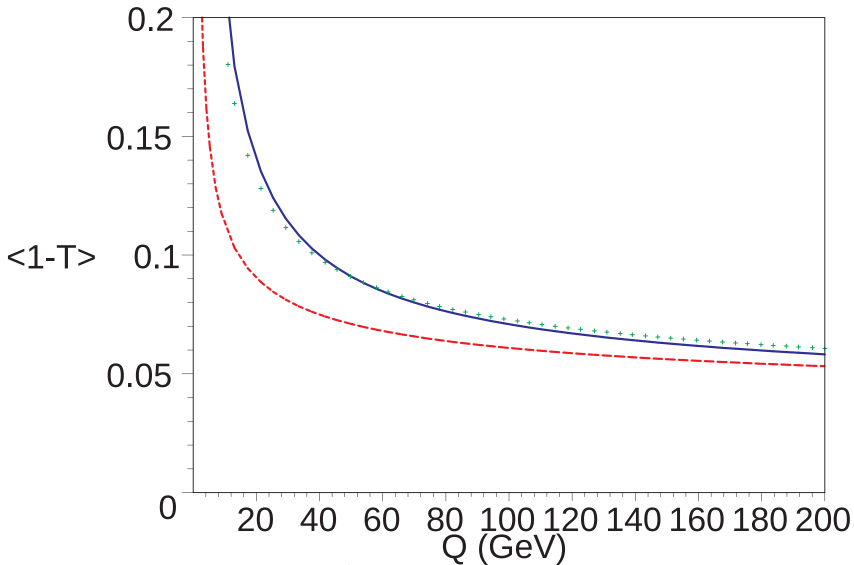
Currently: Experimental data = NLO prediction + power corrections

Higher orders may reduce the size of the power corrections needed to fit the data

(N. Glover)

Example: Thrust

$$\langle 1 - T \rangle \approx 0.33\alpha_s + 1.0\alpha_s^2 + A_3\alpha_s^3 + \frac{\lambda}{Q}$$



Red curve: $\lambda = 0$ GeV, $A_3 = 0$;

Blue curve: $\lambda = 1$ GeV, $A_3 = 0$;

Green curve: $\lambda = 0.5$ GeV, $A_3 = 3$;

Data is not good enough to discriminate between the functional forms $\frac{\lambda}{Q}$ and $\frac{1}{\ln^n\left(\frac{Q}{\Lambda}\right)}$.

Theoretical uncertainties

The theoretical prediction should be independent of μ_r.

The change due to varying the scale is formally of higher order:

$$\frac{\partial}{\partial \ln \mu_r^2} \sum_{n=0}^{N} \alpha_s^n(\mu_r) A_n(\mu_r) \quad = \quad \mathcal{O}\left(\alpha_s^{N+1}\right).$$

Variation produces only copies of the lower order terms, e.g.

$$\begin{aligned} f_3\left(y, \frac{\mu_r^2}{s}\right) \quad = \quad & \frac{\alpha_s(\mu_r)}{2\pi} A_1(y) + \left(\frac{\alpha_s(\mu_r)}{2\pi}\right)^2 \left(A_2(y) + \frac{1}{2}\beta_0 \ln\left(\frac{\mu_r^2}{s}\right) A_1(y)\right) \\ & + \left(\frac{\alpha_s(\mu_r)}{2\pi}\right)^3 \left(A_3(y) + \beta_0 \ln\left(\frac{\mu_r^2}{s}\right) A_2(y) + \frac{1}{4}\left(\beta_0^2 \ln\left(\frac{\mu_r^2}{s}\right)^2 + \beta_1 \ln\left(\frac{\mu_r^2}{s}\right)\right) A_1(y)\right) \end{aligned}$$

for a 3-jet observable.

μ_r variation is only an estimate of higher order terms.

A large variation means that predictable higher order terms are large - but does not say anything about the new terms.

Necessary ingredients for a NNLO calculation

- Calculation of the (two-loop) amplitudes.

  Requires: Two-loop integrals and tensor reduction.

- Cancellation of IR divergences has to be done before any Monte Carlo integration.

  Requires: Extension of the subtraction or slicing method to NNLO.

- The final numerical computer program.

  Requires: Stable and efficient numerical methods.

The amplitudes for $e^+e^- \longrightarrow 3$ jets at NNLO

A NNLO calculation of $e^+e^- \longrightarrow 3$ jets requires the following amplitudes:

- Born amplitudes for $e^+e^- \longrightarrow 5$ jets:

  F. Berends, W. Giele and H. Kuijf.

- One-loop amplitudes for $e^+e^- \longrightarrow 4$ jets:

  Z. Bern, L. Dixon, D.A. Kosower and S.W.;

  J. Campbell, N. Glover and D. Miller.

- Two-loop amplitudes for $e^+e^- \longrightarrow 3$ jets:

  L. Garland, T. Gehrmann, N. Glover, A. Koukoutsakis and E. Remiddi;

  S. Moch, P. Uwer and S.W.

The calculation of two-loop integrals

- The first double box integrals with the help of Mellin-Barnes formula,

  Smirnov '99, Tausk '99.

- More refined techniques:

  - Differential equations and integration-by-parts, Gehrmann, Remiddi '00.
  - Nested sums, Moch, Uwer, S.W. '01.
  - Reduction algorithm, Tarasov '96, Laporta '01.

- Calculation of two-loop amplitudes

  - Bhabha, Bern, Dixon, Ghinculov '01.
  - $pp \longrightarrow$ 2 jets, Anastasiou, Glover, Oleari, Tejeda-Yeomans '01;

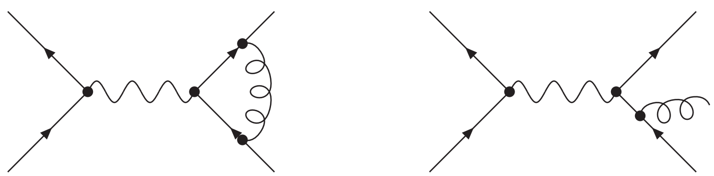
    Bern, De Freitas, Dixon, Ghinculov, Wong '01.
  - $e^+e^- \longrightarrow$ 3 jets, L. Garland, T. Gehrmann, N. Glover, A. Koukoutsakis and E. Remiddi;

    S. Moch, P. Uwer and S.W.

Infrared divergences and the Kinoshita-Lee-Nauenberg theorem

In addition to ultraviolet divergences, loop integrals can have infrared divergences.

For each IR divergence there is a corresponding divergence with the opposite sign in the real emission amplitude, when particles becomes soft or collinear (e.g. unresolved).



The Kinoshita-Lee-Nauenberg theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.

General methods at NLO

Fully differential NLO Monte Carlo programs need a general method to handle the cancelation of infrared divergencies.

- Phase space slicing
  - e^+e^-: W. Giele and N. Glover, (1992)
  - initial hadrons: W. Giele, N. Glover and D.A. Kosower, (1993)
  - massive partons, fragmentation: S. Keller and E. Laenen, (1999)
- Subtraction method
  - residue approach: S. Frixione, Z. Kunzst and A. Signer, (1995)
  - dipole formalism: S. Catani and M. Seymour, (1996)
  - massive partons: L. Phaf and S.W. (2001),
    S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

The subtraction method at NLO

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

$$\begin{aligned}\sigma^{NLO} &= \int_{n+1} d\sigma^R + \int_n d\sigma^V \\ &= \int_{n+1} \left(d\sigma^R - d\sigma^A\right) + \int_n \left(d\sigma^V + \int_1 d\sigma^A\right)\end{aligned}$$

The approximation $d\sigma^A$ has to fulfill the following requirements:

- $d\sigma^A$ must be a proper approximation of $d\sigma^R$ such as to have the same pointwise singular behaviour in D dimensions as $d\sigma^R$ itself. Thus, $d\sigma^A$ acts as a local counterterm for $d\sigma^R$ and one can safely perform the limit $\varepsilon \to 0$.

- Analytic integrability in D dimensions over the one-parton subspace leading to soft and collinear divergences.

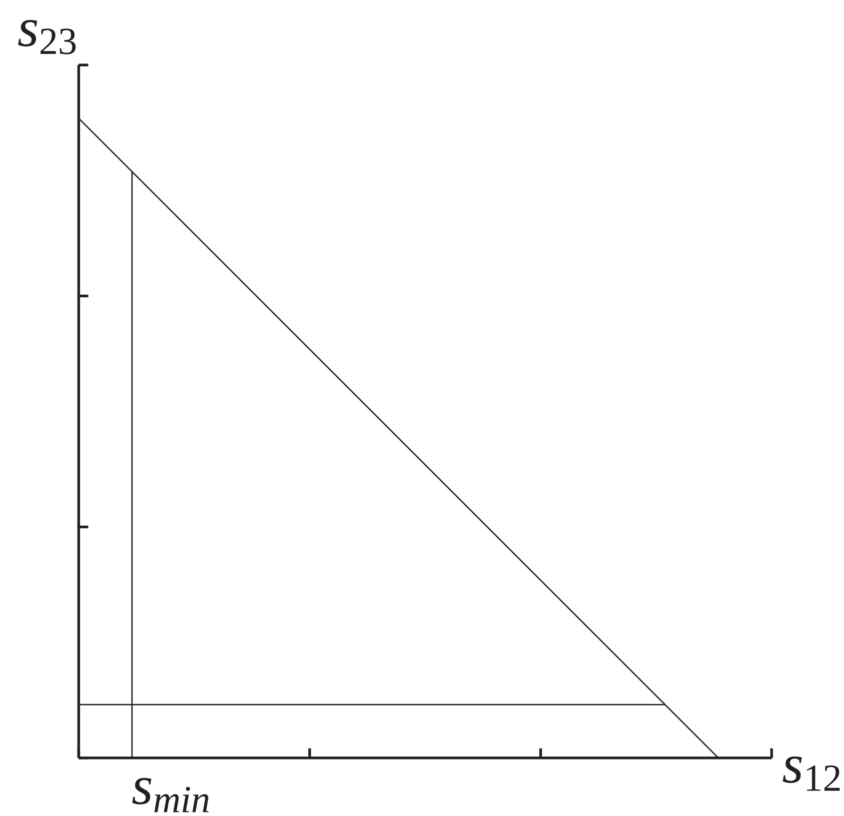
An example: $e^+e^- \longrightarrow$ 2 jets at NLO

The matrix element squared for $\gamma^* \to qg\bar{q}$:

$$M_3 = 8(1-\varepsilon)\left[2\frac{s_{123}^2}{s_{12}s_{23}} - 2\frac{s_{123}}{s_{12}} - 2\frac{s_{123}}{s_{23}} + (1-\varepsilon)\frac{s_{23}}{s_{12}} + (1-\varepsilon)\frac{s_{12}}{s_{23}} - 2\varepsilon\right]$$

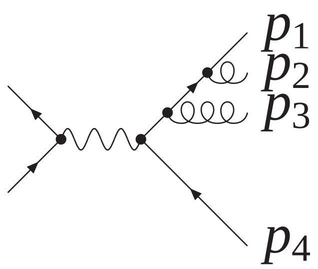
The subtraction terms:

$$\mathcal{D}_{12,3} + \mathcal{D}_{32,1} = 8(1-\varepsilon)$$
$$\left[2\frac{s_{123}^2}{s_{12}(s_{12}+s_{23})} - 2\frac{s_{123}}{s_{12}} + (1-\varepsilon)\frac{s_{23}}{s_{12}}\right]$$
$$+\left[2\frac{s_{123}^2}{s_{23}(s_{12}+s_{23})} - 2\frac{s_{123}}{s_{23}} + (1-\varepsilon)\frac{s_{12}}{s_{23}}\right]$$



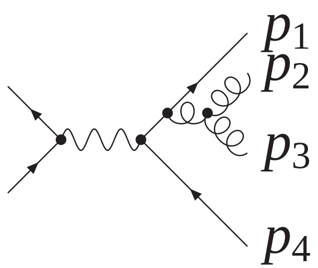
An example involving double unresolved configurations

The leading-colour contributions to $e^+e^- \to qgg\bar{q}$.



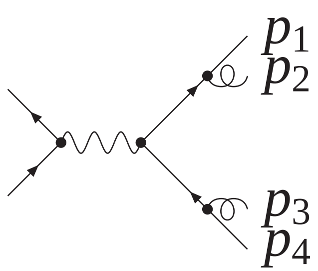
Double unresolved configurations:

- Two pairs of separately collinear particles
- Three particles collinear
- Two particles collinear and a third soft particle
- Two soft particles
- Coplanar degeneracy



Single unresolved configurations:

- Two collinear particles
- One soft particle



The subtraction method at NNLO

- Singular behaviour
  - Factorization of tree amplitudes in double unresolved limits, Berends, Giele, Cambell, Glover, Catani, Grazzini, Del Duca, Frizzo, Maltoni, Kosower '99
  - Factorization of one-loop amplitudes in single unresolved limits, Bern, Del Duca, Kilgore, Schmidt, Kosower, Uwer, Catani, Grazzini, '99
- Interpolation and construction of subtraction terms, Kosower '03, S.W. '03, Kilgore '04
- Integration, either analytically or by sector decomposition, S.W. '03, Anastasiou, Melnikov, Petriello '03, Gehrmann-De Ridder, Gehrmann, Heinrich '03, Binoth, Heinrich '04, Gehrmann-De Ridder, Gehrmann, Glover '04
- Applications:
  - $pp \rightarrow W$, Anastasiou, Dixon, Melnikov, Petriello '03,
  - $e^+e^- \rightarrow$ 2 jets, Anastasiou, Melnikov, Petriello '04,

Outlook

NNLO needed to reduce the theoretical uncertainty.

- Calculation of two-loop amplitudes: Nested sums provide an efficient way to calculate two-loop integrals.

- Cancellation of IR divergences: Extension of the subtraction method to NNLO.
  - Construction of the subtraction terms
    $\longrightarrow$ Subtracted matrix elements can be integrated numerically !
  - Integration of the subtraction terms for one-loop amplitudes with one unresolved parton.
  - To be done: Analytic integration over the unresolved phase space for double unresolved terms.

- To be done: The final numerical computer program.