Neutral Higgs Boson Production and CP Violation at the LC

Kang Young Lee

(KAIST, Korea)

in collaboration with

C.S. Kim and Chaehyun Yu (Yonsei Univ., Korea)

Introduction

- **Higgs Boson in the Standard Model**

  - **Only one neutral scalar exists**

  - **Remnant of the electroweak symmetry breaking**

  - **SM Higgs boson production at the LC**

$$
\begin{array}{ccc}
 & e^-e^+ \rightarrow ZH, & \\
e^-e^+ \rightarrow W^-W^+H, & & e^-e^+ \rightarrow ZZH, \\
 & e^-e^+ \rightarrow t^-t^+H, &
\end{array}
$$

  - **Higgs boson branching ratios**

$$
\begin{aligned}
H &\rightarrow b\bar{b}, \\
H &\rightarrow W^-W^+ \\
H &\rightarrow ZZ \\
H &\rightarrow t\bar{t},
\end{aligned}
$$

- **2 Higgs Doublet Extension**

  - **The simplest extension of Higgs sector**

  - **3 neutral Higgs boson + 1 pair of charged Higgs boson**

  - **Preserves** $\rho \equiv m_W/(m_Z \cos\theta_W) = 1$ **up to finite radiative correction**

  - **Dangerous Higgs-mediated flavour-changing neutral interactions exist at tree-level in general**

    $\rightarrow$ **Models of 3 type in Yukawa couplings have been suggested**

  - **type I : Only one Higgs doublet couples to the fermions**

  - **type II : One Higgs doublet couples only to up-type quarks and the other Higgs doublet couples only to down-type quarks. This model arises in the MSSM**

    $\leftarrow$ **by imposing a discrete symmetry**

  - **type III : Tree level Higgs-mediated FCNC are present and suppressed**

    $\rightarrow$ **Spontaneous and explicit CP violation in the Higgs sector are possible.**

Two Higgs Doublet Model with CP Violation

- **Analysis on the Higgs potential**

  - **The Higgs potential**

$$
\begin{aligned}
V &= \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&\quad + \frac{1}{2}[\lambda_5(\phi_1^\dagger\phi_2)^2 + H.c.] + [\lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + H.c.] \\
&\quad - m_{11}^2((\phi_1^\dagger\phi_1) - m_{22}^2((\phi_2^\dagger\phi_2) - [m_{12}^2((\phi_1^\dagger\phi_2) + H.c.]
\end{aligned}
$$

  - **NFC : Z_2 symmetry imposed,**

    $\phi_1 \to -\phi_1, \qquad \phi_2 \to \phi_2.$

    $\to \lambda_6 = \lambda_7 = m_{12}^2 = 0$

    $\to$ **Tree level FCNC and CP violation are absent.**

  - **Soft violation : allow $m_{12}^2 \neq 0$**

– **The minimum of the potential**

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix},$$

parametrized by

$$\tan\beta = \frac{v_2}{v_1}, \qquad v^2 = v_1^2 + v_2^2$$

– **By minimizing the potential**

$$\mathrm{Im}(m_{12}^2 e^{i\xi}) = v_1 v_2 \mathrm{Im}(\lambda_5 e^{2i\xi})$$

– **The global transform** $\phi_i \to e^{i\varphi_i}$ **with the rephasing;**

$$\lambda_5 \to \lambda_5 e^{-2i(\varphi_2-\varphi_1)}, \qquad m_{12}^2 \to m_{12}^2\ e^{-i(\varphi_2-\varphi_1)},$$

$$\xi \to \xi + \varphi_2 - \varphi_1,$$

with $\lambda_i, i = 1, 2, 3, 4$ **and** $m_{11,22}^2$ **invariant.**

$\to$ **We can choose** $\xi = 0$

indicating no spontaneous CP violation but wholly explicit CP violation.

- **Neutral Higgs bosons**

  - **The neutral states are defined by**

$$
\begin{aligned}
G^0 &= \sqrt{2}(\mathrm{Im}\ \phi_1^0 \cos\beta + \mathrm{Im}\ \phi_2^0 \sin\beta), \\
A^0 &= \sqrt{2}(-\mathrm{Im}\ \phi_1^0 \sin\beta + \mathrm{Im}\ \phi_2^0 \cos\beta), \\
\varphi_1 &= \sqrt{2}\mathrm{Re}\ \phi_1^0, \\
\varphi_2 &= \sqrt{2}\mathrm{Re}\ \phi_2^0.
\end{aligned}
$$

  - **The mass matrix of neutral Higgs bosons**

$$
\mathcal{M}^2 = \begin{pmatrix} \mathcal{M}_{11}^2 & \mathcal{M}_{12}^2 & -\frac{1}{2}\mathrm{Im}(\lambda_5)\sin\beta \\ \mathcal{M}_{21}^2 & \mathcal{M}_{22}^2 & -\frac{1}{2}\mathrm{Im}(\lambda_5)\cos\beta \\ -\frac{1}{2}\mathrm{Im}(\lambda_5)\sin\beta & -\frac{1}{2}\mathrm{Im}(\lambda_5)\cos\beta & \mathcal{M}_{33}^2 \end{pmatrix} v^2
$$

where

$$
\begin{aligned}
\mathcal{M}_{11}^2 &= R\sin^2\beta + \lambda_1\cos^2\beta, \\
\mathcal{M}_{22}^2 &= R\cos^2\beta + \lambda_2\sin^2\beta, \\
\mathcal{M}_{12}^2 &= (\lambda_3 + \lambda_4 + \mathrm{Re}\lambda_5 - R)\,\frac{\sin 2\beta}{2}, \\
\mathcal{M}_{33}^2 &= R - \mathrm{Re}\lambda_5,
\end{aligned}
$$

with

$$
R = \frac{\mathrm{Re}(m_{12}^2)}{v_1 v_2}
$$

– **Diagonalization of the mass matrix**

$$\mathcal{M}_d^2 = \mathcal{R}\mathcal{M}^2\mathcal{R}^\dagger,$$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \mathcal{R} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ A \end{pmatrix}$$

– **Parametrizaton of the rotation matrix**

$$\begin{aligned} \mathcal{R} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_c & s_c \\ 0 & -s_c & c_c \end{pmatrix} \begin{pmatrix} c_b & 0 & s_b \\ 0 & 1 & 0 \\ -s_b & 0 & c_b \end{pmatrix} \begin{pmatrix} -s_a & c_a & 0 \\ c_a & s_a & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -c_b s_a & c_a c_b & s_b \\ c_a c_c + s_a s_b s_c & s_a c_c - c_a s_b s_c & c_b s_c \\ -c_a s_c + s_a s_b c_c & -s_a s_c - c_a s_b c_c & c_b c_c \end{pmatrix}, \end{aligned}$$

where $s_{a,b,c} = \sin\theta_{a,b,c}$ **and** $c_{a,b,c} = \cos\theta_{a,b,c}$.

– **The CP-odd state** A **is mixed with CP-even states** φ_1, φ_2

$\rightarrow$ **manifest CP violation in the neutral Higgs sector.**

- **Neutral Higgs Boson Production $e^+e^- \to Zh_i$ and $e^+e^- \to h_ih_j$**

  - **Generalized h_iZZ vertices**

$$\begin{aligned} h_1ZZ &\sim \sin(\beta-\alpha)\cos\alpha_b, \\ h_2ZZ &\sim \cos(\beta-\alpha)\cos\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\sin\alpha_c, \\ h_3ZZ &\sim -\cos(\beta-\alpha)\sin\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\cos\alpha_c. \end{aligned}$$

  - **The cross sections for $e^+e^- \to h_iZ$ processes**

$$\sigma(e^+e^- \to h_iZ) = \frac{f_i^2\pi\alpha^2\lambda^{1/2}(\lambda+12sm_Z^2)\left[1+(1-4\sin^2\theta_W)^2\right]}{192s^2\sin^4\theta_W\cos^4\theta_W(s-m_Z^2)^2}$$

where where f_i are the h_iZZ coupling given above

$$\lambda = \lambda(s, m_h^2, m_Z^2)$$

with

$$\lambda(a,b,c) = (a+b-c)^2 - 4ab$$

  - **CP violating coupling**

$$\mathcal{L} = \frac{gm_Z}{2\cos\theta_W}\frac{\eta}{4}\epsilon_{\mu\nu\alpha\beta}Z^{\mu\nu}Z^{\alpha\beta}$$

induces the CP violation in this process.

$\to$ suppressed by loop

– **Generalized Zh_ih_j vertices**

$$\begin{aligned} Zh_1h_3 &\sim \cos(\beta-\alpha)\cos\alpha_c - \sin(\beta-\alpha)\sin\alpha_b\sin\alpha_c, \\ Zh_2h_3 &\sim -\sin(\beta-\alpha)\cos\alpha_b, \\ Zh_1h_2 &\sim \cos(\beta-\alpha)\sin\alpha_c + \sin(\beta-\alpha)\sin\alpha_b\cos\alpha_c. \end{aligned}$$

– **The cross sections for $e^+e^- \to h_ih_j$ processes**

$$\begin{aligned} \sigma(e^+e^- \to h_ih_j) &= \frac{g^4}{196\pi\cos^2\theta_W} f_{ij}^2 \left(\frac{8\sin^4\theta_W - 4\sin^2\theta_W + 1}{\cos^2\theta_W}\right) \\ &\times \frac{\kappa^3}{\sqrt{s}\,[(s-m_Z^2)^2 + \Gamma_Z^2 m_Z^2]} \end{aligned}$$

where f_{ij} are the h_ih_jZ coupling given above and the kinematic factor

$$\kappa^2 = \frac{\lambda(s, m_{h_i}^2, m_{h_j}^2)}{4s}$$

– **Numerical constraints**

$$\begin{aligned} &: \text{ordering}, \quad m_1 < m_2 < m_3, \\ &: \text{perturbativity}, \quad \frac{\lambda}{4\pi} < 1 \end{aligned}$$

- **Discussion on a few limiting cases**

  - **If** $\theta_b = \theta_c = 0$ **:**
    $\rightarrow$ **CP conserving case**
    $\rightarrow$ $h_1, h_2 \sim$ **CP-even,** $h_3 \sim$ **CP-odd**

    $\sigma(e^+e^- \rightarrow Zh_3)$, $\sigma(e^+e^- \rightarrow h_1h_2)$ **are suppressed.**

  - **If** $\sin\theta_b \sim \sin\theta_c \sim 1$ **:**
    $\rightarrow$ $h_1 \sim$ **CP-odd,** $h_2, h_3 \sim$ **CP-even**

  - **If** $\sin\theta_b \sim 0$, $\sin\theta_c \sim 1$ **:**
    $\rightarrow$ $h_2 \sim$ **CP-odd,** $h_1, h_3 \sim$ **CP-even**

  - **If** $\sin\theta_c \sim 0$ **:**

$$\begin{aligned} \mathcal{M}^2_{13} &= s_a c_b s_b (m_3^2 - m_1^2), \\ \mathcal{M}^2_{23} &= -c_a c_b s_b (m_3^2 - m_1^2). \end{aligned}$$

    $\rightarrow$ $\tan\beta \approx -\tan\theta_a$,
    $\beta \approx -\theta$.

$$\text{Im } \lambda_5 = \sin 2\theta_b \frac{m_3^2 - m_1^2}{v^2}.$$

  - **Additionally** $\sin\theta_b \sim 1$ **:**
    $\rightarrow$ $h_1 \sim$ **CP-odd,** $h_2, h_3 \sim$ **CP-even**

but

$$\begin{aligned} h_2 ZZ &\sim \cos(\beta-\alpha), \\ h_3 ZZ &\sim -\sin(\beta-\alpha), \\ h_1 h_3 Z &\sim \cos(\beta-\alpha), \\ h_1 h_2 Z &\sim \sin(\beta-\alpha), \end{aligned}$$

h_2, h_3 **couplings are exchanged!**

$$\frac{g_{h_2 ZZ}}{g_{h_3 ZZ}} = \frac{1}{\tan(\beta-\alpha)}$$

while

$$\frac{g_{hZZ}}{g_{HZZ}} = \tan(\beta-\alpha)$$

in the CP conserving case.

Summary

- **The 2 Higgs doublet model with CP violation may enhance the $e^-e^+ \to Zh$ and $e^-e^+ \to h_i h_j$ cross sections compared with those of CP conserving case.**

- **In the limit of $\sin\theta_c \to 0$ and $\sin\theta_b \to 1$, the ratio of hZZ and HZZ couplings are reversed to that of the CP conserving case and the mixing angle $\alpha(=\theta_a)$ is close to $-\beta$.**

- **The neutral Higgs boson production has very sensitive behavior near the CP conserving case.**

- **The 2 Higgs doublet model with CP violation will be able to be tested at the LC through neutral Higgs boson production.**