ε'/ε: 5 YEARS AFTER THE DISCOVERY (THEORETICAL STATUS)

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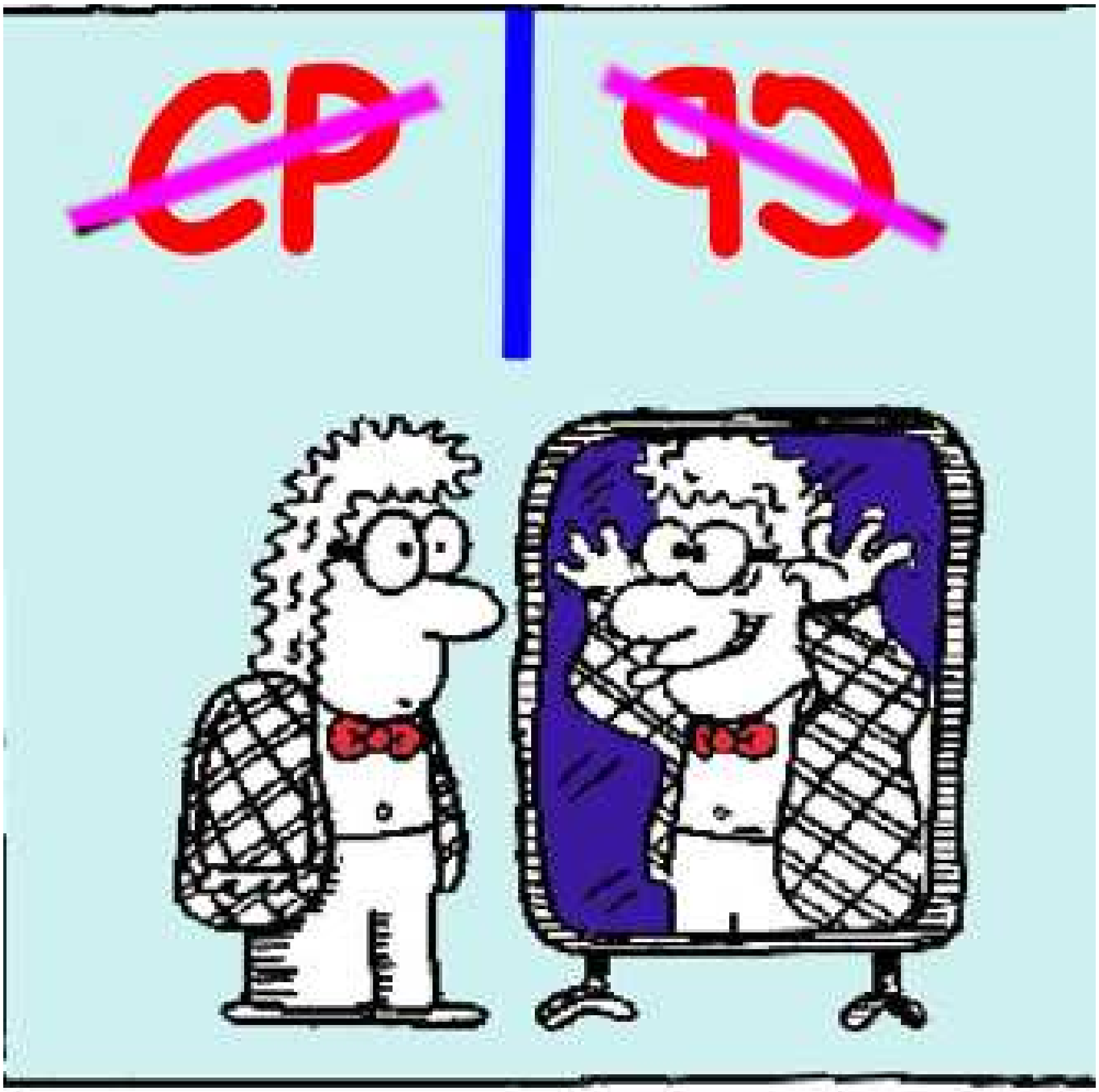

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PREHISTORY

The ε'/ε problem before 1999:

- **Experiment:**

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \begin{cases} (23 \pm 7) \times 10^{-4}\,, & \text{NA31, CERN, 1991} \\ (6 \pm 7) \times 10^{-4}\,, & \text{E731, FNAL, 1993} \end{cases}$$

- **Theory**: (Standard Model)

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \begin{cases} (34 \pm 26) \times 10^{-4}\,, & m_s = 125 \text{ MeV} \\ (52 \pm 48) \times 10^{-4}\,, & m_s = 100 \div 150 \text{ MeV} \\ & \text{Paschos et al., 1991} \\ (3.4 \pm 3.4) \times 10^{-4}\,, & m_s = 150 \pm 20 \text{ MeV} \\ (10.4 \pm 8.3) \times 10^{-4}\,, & m_s = 100 \pm 20 \text{ MeV} \\ & \text{Buras et al., 1996} \\ (4.6 \pm 3.0 \pm 0.4) \times 10^{-4}\,, & \text{Ciuchini et al., 1997} \\ (4 \pm 5) \times 10^{-4}\,, & \text{Bertolini et al., 1996} \\ (17^{+14}_{-10}) \times 10^{-4}\,, & 1998 \end{cases}$$

Year 1999: first results from NA48 and KTeV

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \begin{cases} (18.5 \pm 7.3) \times 10^{-4}\,, & \text{NA48, CERN, 1999} \\ (28.0 \pm 4.1) \times 10^{-4}\,, & \text{KTeV, FNAL, 1999} \end{cases}$$

Present status:

$$\mathrm{Re}\frac{\varepsilon'}{\varepsilon} = \begin{cases} (14.7 \pm 2.2) \times 10^{-4}\,, & \text{NA48, CERN, 2002} \\ (20.7 \pm 2.8) \times 10^{-4}\,, & \text{KTeV, FNAL, 2001} \end{cases}$$

$1/N_c$ approach [München group: A.Buras et al.]

- **Large N_c limit:** chiral-loop expansion of $\langle Q_i(M)\rangle$ is a series in $1/F_0^2 \sim 1/N_c$, which is in direct correspondance with SD expansion in terms of $\alpha_s \sim 1/N_c$

- **Matching** between LD and SD evoluations:

$$
\begin{aligned}
T_I(K \to 2\pi) &= \frac{G_F}{\sqrt{2}} \sum_{i=1}^{10} \langle (\pi\pi)_I | Q_i(M=\mu) | K \rangle C_i(\mu) \\
&= \frac{G_F}{\sqrt{2}} \langle \mathbf{Q}^T(\mu) \rangle_I \mathbf{C}(\mu), \quad I = 0, 2
\end{aligned}
$$

SD evolution: $\mathbf{C}(\mu) = \hat{U}(\mu, M_W)\mathbf{C}(M_W)$

LD evolution: $\langle \mathbf{Q}^T(\mu) \rangle = \langle \mathbf{Q}^T(0) \rangle \hat{U}(0, \mu)$

$$
< \pi\pi | Q_i(M) | K >= F_0(M^2) A_i \left[1 + \frac{\Pi_i(M^2)}{F_0^2(M^2)} + O(1/N_c^2) \right]
$$

- **Truncated chiral lagrangian** in large N_c limit:

$$
\mathcal{L}_{tr} = \frac{F_0^2}{4} \left[\langle D_\mu U D^\mu U^\dagger \rangle + r \langle m(U+U^\dagger) \rangle - \frac{r}{\Lambda_\chi^2} \langle m(D^2U + D^2U^\dagger) \rangle \right]
$$

$$\Downarrow$$

Chiral representation of currents and densities

- **B_i–factors:** $B_i^{(\Delta I)}(\mu) = \langle Q_i(\mu)\rangle_I / \langle Q_i\rangle_I^{(VSA)}$

- **(Current×current) operators:** $M = \mu \approx 600$ MeV

$$B_1^{(1/2)} = 5.2\,, \quad B_2^{(1/2)} = 2.2\,, \quad B_1^{(3/2)} = B_2^{(3/2)} = 0.55$$

- **(Density×density) operators:** (pengium diagrams):

$C_{6,8} \sim 0(1/N_c) \Rightarrow$ No chiral loops in leading $0(1/N_c)$

$$\begin{cases} C_{6,8} \sim [\alpha_s(\mu)]^{a_{6,8}}, & m_s^2(\mu) \sim [\alpha_s(\mu)]^{a_m} \\ a_{6,8} = a_m = 9/11\,, & \text{for } N_c \to \infty \end{cases}$$

$\Longrightarrow$ Evolution of $C_{6,8}(\mu)$ cancelled by the evolution of the running mass in $\langle Q_{6,8}(\mu)\rangle^{(VSA)} \sim m_s^{-2}(\mu)$

$$\Downarrow$$

$$B_6^{(1/2)} = B_8^{(3/2)} = 1$$

- **Problems:**

$\Longrightarrow$ $N_c = 3$ — not infinite

$\Longrightarrow$ $\delta_{0,2=0}$ — no unitarity

$\Longrightarrow$ No description of the $\Delta I = 1/2$ rule

$\Longrightarrow$ $M = \mu$ is only naive approximation: $\mu \geq 1$ GeV,
$M \leq 0.6$ GeV

$\Longrightarrow$ No amalytical matching between $\langle Q_i(M)\rangle$ and $C_i(\mu)$ (M^2-terms in $\langle Q_i(M)\rangle$)

$\Longrightarrow$ $\langle Q_i(M)\rangle$ are insensitive to the renorm. scheme

- **Semiphenomenological approach:**

$\Longrightarrow$ $\mu = m_c = 1.3$ GeV – penguins are decoupled from the CP conserving amplitudes in HV scheme

$\Longrightarrow$ $B_{1,2}$ are extracted from CP-conserv. $K \to 2\pi$ data

$$\Downarrow$$

$$B_1^{(1/2)} \approx 15\,, \quad B_2^{(1/2)} = 6.3 \pm 1.0\,, \quad B_1^{(3/2)} = B_2^{(3/2)} = 0.47$$

$$(\mu = m_c, \quad \Lambda_{\overline{MS}}^{(4)} = 325 \text{ MeV}, \quad \text{HV scheme})$$

In leading $0(1/N_c)$: $B_2^{(1/2)}(0.6 \text{ GeV}) \longrightarrow B_2^{(1/2)}(m_c) \approx 2.8$

- **ε'/ε analysis:**

$$\varepsilon'/\varepsilon = \text{Im}\lambda_t \cdot F_{\varepsilon'}\left(m_t, \Lambda_{\overline{MS}}^{(4)}, m_s, B_6^{(1/2)}, B_8^{(3/2)}, \Omega_{IB}\right)$$

- **Analysis 1996:** [*Buras et al.: PL B389 (1996) 749*]

$$B_6^{(1/2)} = 1.0{\pm}0.2\,, \quad B_8^{(3/2)} = 1.0{\pm}0.2\,, \quad \Omega_{IB} = 0.25{\pm}0.08$$

$$\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110) \text{ MeV} \Longleftarrow \alpha_s(M_Z) = 0.117 \pm 0.007$$

$$\Downarrow$$

$m_s(m_c)$ (MeV)	Scanning	Gaussian approach
	$0.86 \leq \text{Im}\,\lambda_t \cdot 10^4 \leq 1.71$	$\text{Im}\,\lambda_t = (1.29 \pm 0.22) \cdot 10^{-4}$
150 ± 20	$1.2 \leq \varepsilon'/\varepsilon \cdot 10^4 \leq 16$	$\varepsilon'/\varepsilon = (3.6 \pm 3.4) \cdot 10^{-4}$
100 ± 20	$0.0 \leq \varepsilon'/\varepsilon \cdot 10^4 \leq 43$	$\varepsilon'/\varepsilon = (10.4 \pm 8.3) \cdot 10^{-4}$

$m_s(m_c) = (100 \pm 20)$MeV $\Longleftarrow$ Lattice estimates (1997)

- **Analysis 1999:** *[Bosch et al.: NP B565 (2000) 3]*

$$F_{\varepsilon'} \approx \left[B_6^{(1/2)}(1-\Omega_{IB}) - 0.4\, B_8^{(3/2)} \left(\frac{m_t}{165\ \mathrm{GeV}}\right)^{2.5} \right] \times \left[\frac{110\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left(\frac{\Lambda_{\overline{MS}}^{(4)}}{340\ \mathrm{MeV}}\right)$$

$$B_6^{(1/2)} = 1.0 \pm 0.3\,, \quad B_8^{(3/2)} = 0.8 \pm 0.2\,, \quad (B_6^{(1/2)} > B_8^{(3/2)})$$

$$\Lambda_{\overline{MS}}^{(4)} = (340 \pm 50)\ \mathrm{MeV} \quad \Longleftarrow \quad \alpha_s(M_Z) = 0.119 \pm 0.003$$

More recent lattice results:

$$m_s(2\mathrm{GeV}) = (110 \pm 20)\ \mathrm{MeV} \Longrightarrow m_s(m_c) = (130 \pm 25)\ \mathrm{MeV}$$

$$\Downarrow$$

	Scanning	Gaussian approach
	$1.04 \le \mathrm{Im}\,\lambda_t \cdot 10^4 \le 1.63$	$\mathrm{Im}\,\lambda_t = (1.33 \pm 0.14) \cdot 10^{-4}$
NDR	$1.05 \le \varepsilon'/\varepsilon \cdot 10^4 \le 28.2$	$\varepsilon'/\varepsilon = (7.7^{+6.0}_{-3.5}) \cdot 10^{-4}$
HV	$0.26 \le \varepsilon'/\varepsilon \cdot 10^4 \le 22$	$\varepsilon'/\varepsilon = (5.2^{+4.6}_{-2.7}) \cdot 10^{-4}$

$$\Downarrow$$

ε'/ε in the SM can be made consistent with data only if all relevant parameters are simultaneously close to their extreme values. The values of ε'/ε in the ball park of NA31, NA48 and KTeV results are rather improbable

Extension beyond leading $0(1/N_c)$
Dortmund group: E.Paschos et al.

- **Analysis 1992:** [*J.Heinrich et al.: PL B279 (1992) 140*]

$\Longrightarrow$ Meson loops have been incorporated for the $1/N_c$ corrections to the matrix elements $\langle Q_{6,8}\rangle$

$\Longrightarrow$ Large enhancement of $\langle Q_6\rangle$:

$$B_6^{(1/2)} \approx 2.4 \pm 0.2, \quad B_8^{(3/2)} \approx 1$$

$$\Downarrow$$

$$\boxed{\begin{array}{ll} 7.6 \le \varepsilon'/\varepsilon \cdot 10^4 \le 60 \;\;, & m_s(1\ \mathrm{GeV}) = 125\ \mathrm{MeV} \\ 4.5 \le \varepsilon'/\varepsilon \cdot 10^4 \le 100\,, & m_s(1\ \mathrm{GeV}) = 125 \pm 25\ \mathrm{MeV} \end{array}}$$

- **Extended $1/N_c$ expansion:** [*Hambye et al., 1998*]

$\Longrightarrow$ Twofold expansion in external momenta and $1/N_c$

$\Longrightarrow$ New identification of virtual momenta in meson loops: two currents (densities) are considered to be connected through the colour-singlet-boson exchange

$\Longrightarrow$ Explicit classification of the diagrammes into factorizable and non-factorizable

$\Longrightarrow$ Two different UV cutoffs:
λ_c – factorizable (strong), dissappear in renormaliz.
Λ_c – non-factorizable (weak), for matching ($\Lambda_c = \mu$)

- **Chiral lagrangian** including p^4-terms: (large N_c)

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{F_0^2}{4}\Big[\langle D_\mu U^\dagger D^\mu U\rangle + r\langle mU^\dagger + Um^\dagger\rangle\Big] \\ &+ rL_5\langle D_\mu U^\dagger D^\mu U(m^\dagger U + U^\dagger m)\rangle \\ &+ r^2L_8\langle m^\dagger U m^\dagger U + mU^\dagger mU^\dagger\rangle + r^2H_2\langle m^\dagger m\rangle\end{aligned}$$

with $r = -2 < \hat{q}q > /F_0^2$

- **B-factors:** (no meson loops for $\langle Q_{6,8}\rangle_I$)

$\langle Q_{1,2}\rangle_{0,2} : 0(p^2 + p^4 + p^2/N_c)$ $\Longrightarrow$ Λ_c^2 terms

$$\left.\begin{array}{ll}\langle Q_8\rangle_{0,2} : & 0(p^0 + p^0/N_c + p^2 + p^2/N_c) \\ \langle Q_6\rangle_0 : & 0(p^0/N_c + p^2 + p^2/N_c)\end{array}\right\} \Longrightarrow \text{no } \Lambda_c^2 \text{ terms}$$

For matching $\Lambda_c = (0.6 \leftrightarrow 0.9)$ GeV:

$$B_1^{(1/2)} = 8.2 \leftrightarrow 14.2\,, \quad B_2^{(1/2)} = 2.9 \leftrightarrow 9.6$$

$\Longrightarrow$ a_0 in agreement with data $\oplus$

$$B_1^{(3/2)} = B_2^{(3/2)} = 0.11 \leftrightarrow -0.61$$

$\Longrightarrow$ a_2 is problematic $\ominus$

$$B_6^{(1/2)} = 1.10 \leftrightarrow 0.72\,, \quad B_8^{(3/2)} = 0.64 \leftrightarrow 0.42$$

$\Longrightarrow$ $B_6^{(1/2)}/B_8^{(3/2)} \approx 1.7$

- **Meson loops for $\langle Q_6\rangle_0$:**

$\Longrightarrow$ Large positive contr. of $0(p^2/N_c)$ due to Λ_c^2 terms

$$B_6^{(1/2)} = 1.5 \leftrightarrow 1.62$$

- ε'/ε **analysis:** [*T.Hambye et al.: NP B564 (2000) 391*]

$\Longrightarrow$ Input parameters:

$$\Omega_{IB} = 0.25 \pm 0.10\,, \quad m_s(1\ \mathrm{GeV}) = (150 \pm 25)\ \mathrm{MeV}$$

$$245\ \mathrm{MeV} \le \Lambda^{(4)}_{\overline{MS}} \le 405\ \mathrm{MeV} \Longleftarrow \alpha_s(M_z) = 0.118 \pm 0.005$$

$\Longrightarrow$ Wilson coefficient in LO, NDR, HV

$\Longrightarrow$ $B^{(3/2)}_{1,2}$ to be fixed from a_2 data

$\Longrightarrow$ $B^{(1/2)}_{1,2}$, $B^{(1/2)}_5$, $B^{(3/2)}_8$ from calculations of $\langle Q_i \rangle_I$

$\Downarrow$

	Λ_c (MeV) $(0.6 \leftrightarrow 0.9)$	$\delta_{0,2} = 0$	Phenomenolog. $\delta_{0,2}$
A	Central	$4.2 \le \varepsilon'/\varepsilon \cdot 10^4 \le 9.5$	$5.8 \le \varepsilon'/\varepsilon \cdot 10^4 \le 12.9$
	Scanning	$1.5 \le \varepsilon'/\varepsilon \cdot 10^4 \le 23.2$	$1.9 \le \varepsilon'/\varepsilon \cdot 10^4 \le 31.6$
B	Central	$7.0 \le \varepsilon'/\varepsilon \cdot 10^4 \le 19.4$	$9.3 \le \varepsilon'/\varepsilon \cdot 10^4 \le 24.7$
	Scanning	$2.2 \le \varepsilon'/\varepsilon \cdot 10^4 \le 49.8$	$2.8 \le \varepsilon'/\varepsilon \cdot 10^4 \le 63.9$

A: $\quad \langle Q_6 \rangle = 0(p^0/N_c + p^2)$

B: $\quad \langle Q_6 \rangle = 0(p^0/N_c + p^2 + p^2/N_c)$

$\Downarrow$

$$\boxed{2 \le \varepsilon'/\varepsilon \cdot 10^4 \le 64}$$

Functional cutoff momentum
Y.L.Wu, Beijing *[PR D64 (2001) 016001]*

$\Longrightarrow$ Functional cutoff momentum $M \equiv M(\mu)$,

$\Longrightarrow$ Keep the leading M^2 terms in $\langle Q_i(M(\mu))\rangle_I$

$\Longrightarrow$ Analitical matching on the scale $M(\mu = \Lambda_\chi \approx 1\text{GeV})$

$$\frac{4M^2(\mu)}{\Lambda_F^2} \approx \frac{3\alpha_s(\mu)}{4\pi} + \frac{3\alpha_s(\mu)}{4\pi} \ln\left(\frac{\mu^2}{\mu_0^2}\right)$$

$$\Lambda_F \sim \Lambda_\chi \sim 1 \text{ GeV}\,, \quad \mu_0 \approx \Lambda_F \sqrt{3\alpha_s(\mu_0)/(16\pi)}$$

$$\Downarrow$$

$$\boxed{\text{Only logarithmic } \mu\text{-dependence in } \langle Q_i(M(\mu))\rangle_I}$$

$$M(\mu = \Lambda_\chi \approx 1 \text{ GeV}) \approx 0.71^{+0.11}_{-0.10}$$

$$m_s(\mu = \Lambda_\chi) = 196 \text{ MeV}\,, \quad \mu_0 \approx (453 \pm 70 \text{ MeV})$$

- ***B*-factors:**

$$B_1^{(1/2)} = 6.0^{+2.0}_{-1.3}\,, \quad B_2^{(1/2)} = 2.6^{+0.4}_{-0.3}\,, \quad B_1^{(3/2)} = 0.55^{+0.09}_{-0.06}$$

$$B_6^{(1/2)} = 3.2^{+0.7}_{-0.4}\,, \quad B_8^{(3/2)} = 1$$

- **ε'/ε analysis:**

$$\Omega_{IB} = 0.16\,, \qquad 0.7 \le \text{Im}\lambda_t \cdot 10^4 \le 1.4$$

$$\Downarrow$$

$$\boxed{13.8^{+7.7}_{-4.5} \le \varepsilon'/\varepsilon \cdot 10^4 \le 33.6^{+17.6}_{-11.1}}$$

χQM [Trieste group: S.Bertolini et al.]

- **Basic ideas:**

$\Longrightarrow$ ENJL: $\mathcal{L}_{\chi QM}^{int} = -M(\bar{q}_R U q_L + \bar{q}_L U^+ q_R)$

$\Longrightarrow$ Quark loops: integration of the constituent quarks

$$Q_i \to \sum_\alpha G_\alpha(Q_i) B_\alpha \quad \text{(bosonic operators)}$$

with $G_\alpha(Q_i)$ depending on M, $\langle\bar{q}q\rangle$, $\langle\alpha_S GG/\pi\rangle$ and γ_5-scheme (no scale dependence)

$\Longrightarrow$ Meson loops: μ_{LD} in dim. reg. to be matched with

$$\mathcal{L}_{\chi QM}(\Delta S = 1) = \sum_{i,\alpha} C_i(\mu_{SD}) G_\alpha(Q_i) B_\alpha$$

- **Analysis 1996:** *[S.Bertolini et al.: NP, B476 (1996) 225]*

$\Longrightarrow$ $\Delta I = 1/2$ rule: $\langle\alpha_S GG/\pi\rangle = (376 \pm 47 \text{ MeV})^4$

$M = 220$ MeV, $\langle\bar{q}q\rangle = -(240 \pm 40 \text{ MeV})^3$

$\Longrightarrow$ Matching scale: $\mu = (0.6 \leftrightarrow 1.0)$ GeV

$B_1^{(1/2)}$	$B_2^{(1/2)}$	$B_1^{(3/2)} = B_2^{(3/2)}$	$B_6^{(1/2)}$	$B_8^{(3/2)}$
$10.6 \leftrightarrow 11.1$	$2.8 \leftrightarrow 3.0$	$0.52 \leftrightarrow 0.55$	$1.8 \leftrightarrow 0.61$	$2.3 \leftrightarrow 1.4$

$\Longrightarrow$ $0.75 \le \text{Im}\,\lambda_t \cdot 10^4 \le 1.9\,, \quad \Omega_{IB} = 0.25 \pm 0.05$

$$\Downarrow$$

$$\boxed{-50 \le \varepsilon'/\varepsilon \cdot 10^4 \le 14}$$

- **Analysis 1998:** [*Bertolini et al.: NP B514 (1998) 93*]

$\Longrightarrow$ m_q terms to be added to $\langle Q_{7,8}\rangle$

$\Longrightarrow$ Complete p^4-calculations of $< Q_i >$

$\Longrightarrow$ $\Delta I = 1/2$ rule: $\langle \alpha_s GG/\pi \rangle = (334 \pm 4 \text{ MeV})^4$

$$M = 200^{+5}_{-3} \text{ MeV}, \quad \langle \bar{q}q \rangle = -(240^{+30}_{-10} \text{ MeV})^3$$

$B_1^{(1/2)}$	$B_2^{(1/2)}$	$B_1^{(3/2)} = B_2^{(3/2)}$	$B_6^{(1/2)}$	$B_8^{(3/2)}$
9.5	2.9	0.41	1.6 ± 0.3	0.92 ± 0.02

$\Downarrow$

$$\boxed{\varepsilon'/\varepsilon = 17^{+14}_{-10} \cdot 10^{-4}}$$

- **Analysis 2000:** [*Bertolini et al.: PR D63 (2001) 056009*]

$\Longrightarrow$ $\delta_0 \approx 20^\circ$, $\delta_2 \approx -12^\circ$ in $0(p^4)$ of χQM

$\Longrightarrow$ Phenom. inclusion of $\delta_{0,2}$: $\mathrm{Re}\langle Q_i \rangle_I \times (\cos\delta_I)^{-1}$

$\Longrightarrow$ $\Delta I = 1/2$ rule: $\langle \alpha_s GG/\pi \rangle = (330 \pm 5 \text{ MeV})^4$

$$M = 195^{+25}_{-15} \text{ MeV}, \quad \langle \bar{q}q \rangle = -(235 \pm 25 \text{ MeV})^3$$

$B_1^{(1/2)}$	$B_2^{(1/2)}$	$B_1^{(3/2)} = B_2^{(3/2)}$	$B_6^{(1/2)}$	$B_8^{(3/2)}$
9.3	2.8	0.42	$1.9^{+0.8}_{-0.5}$	0.89

$\Longrightarrow$ $\mathrm{Im}\,\lambda_t = (1.14 \pm 0.11) \cdot 10^{-4}\,, \quad \Omega_{IB} = 0.25 \pm 0.10$

$\Downarrow$

$$\boxed{\varepsilon'/\varepsilon = (22 \pm 8) \cdot 10^{-4}}$$

Lattice QCD [Rome group: M.Ciuchini et al.]

- **Basic ideas:**

$\Longrightarrow$ $\langle Q_i \rangle_I$ from the first principles $\Leftarrow$ Dependence on the renorm. scale μ and γ_5-scheme under full control

$\Longrightarrow$ Matching at $\mu \sim a^{-1} \geq 2$ GeV $\Leftarrow$ $C_i(\mu)$ is calculated with high precision

$\Longrightarrow$ χPT relations to obtain phisical $\langle \pi\pi | Q_i | K \rangle$ by using lattice values for $\langle 0 | Q_i | K \rangle$ and $\langle \pi | Q_i | K \rangle$

- **Analysis 1995/7:** *[M.Chiuchini et al.: ZP, C68 (1995) 239; NP (Proc. Suppl.), B59 (1997) 141]*

$\Longrightarrow$ B-factors from lattice before 1997: ($\mu = 2$ GeV)

$B_{5,6}^{(1/2)}$	$B_{7,8}^{(3/2)}$	$B_9^{(3/2)}$	$B_{3,4}^{(1/2)}$	$B_{7,8,9}^{(3/2)}$
1.0 ± 0.2	1.0 ± 0.2	0.62 ± 0.10	$1 \div 6$	1

$\Longrightarrow$ $\Lambda_{QCD}^{(5)} = (230 \pm 80)$ MeV

$$m_s(2\ \mathrm{GeV}) = (128 \pm 18)\ \mathrm{MeV}\,, \quad \Omega_{IB} = 0.25 \pm 0.10$$

$$\Downarrow$$

$$\varepsilon'/\varepsilon = \begin{cases} (3.1 \pm 2.5) \cdot 10^{-4} & (1995) \\ (4.6 \pm 3.2) \cdot 10^{-4} & (1997) \end{cases}$$

- **Analysis 2000:** [*M.Chiuchini et al.: NP, B573 (2000) 201*]

$\Longrightarrow$ Large systematic uncertainty in lattice $\langle Q_{5,6}\rangle_0$:

$$B_6^{(1/2)} = \begin{cases} (0.67 \pm 0.04 \pm 0.05) \cdot 10^{-4}\,, & \text{quenched} \\ (0.76 \pm 0.03 \pm 0.05) \cdot 10^{-4}\,, & n_f = 2 \end{cases}$$

(staggered fermions) [*D.Pekurovsky and G.Kilcup (1998)*]

$\Longrightarrow$ B-factors: (HV)

$B_5^{(1/2)}$	$B_6^{(1/2)}$	$B_7^{(3/2)}$	$B_8^{(3/2)}$	$B_9^{(3/2)}$	$B_{3,4}^{(1/2)}$
$1/3 \pm 1/3$	1 ± 1	0.89 ± 0.30	0.93 ± 0.18	0.63 ± 0.09	3.5 ± 2.5

$\Downarrow$

$$\boxed{\varepsilon'/\varepsilon = (3.6^{+6.7}_{-6.3} \pm 0.5) \cdot 10^{-4}}$$

- **Recent surprising news from lattice:**

$\Longrightarrow$ Considerable improvement of $\Delta I = 1/2$ rule $\oplus$

$\Longrightarrow$ Negative values of $\langle Q_6\rangle_0$

$\Downarrow$

Staggered fermions: [*D.Pekurovsky and G.Kilcup (2001)*]

$$\varepsilon'/\varepsilon = (-38.6 \pm 2.1(stat) \pm 9.1(sys)) \cdot 10^{-4}$$

Domain wall fermions:

$$\varepsilon'/\varepsilon = \begin{cases} (-7.7 \pm 2.0) \cdot 10^{-4} & \text{[\textit{J.Noaki et al (2003)}]} \\ (-4.0 \pm 2.3) \cdot 10^{-4} & \text{[\textit{T.Blum et al (2003)}]} \end{cases}$$

OTHER APPROACHES

- **Strong enhancement of ε'/ε by FSI:**

[*E.Pallante, A.Pich: PRL. 84 (2000) 2568*]

$$\varepsilon'/\varepsilon \sim \left[R_0 B_6^{(1/2)} \left(1 - \frac{R_2}{R_0} \Omega_{IB} \right) - 0.4\, R_2 B_8^{(3/2)} \right]$$

$$\left\{ B_i \sim 1, \quad \Omega_{IB} \sim 0.25 \right\} \quad \Longrightarrow \quad \varepsilon'/\varepsilon \sim 7 \times 10^{-4}$$

$$\left\{ R_0 \sim 1.55, \quad R_2 \sim 0.92 \right\} \quad \Longrightarrow \quad \varepsilon'/\varepsilon \sim 19 \times 10^{-4}$$

$$\Downarrow$$

$$\boxed{\varepsilon'/\varepsilon = (17 \pm 9) \times 10^{-4}}$$

- **Isospin breaking effects in $K \to 2\pi$:**

$\Longrightarrow$ $(\pi^0 - \eta - \eta')$ mixing in the χPT:

$\Omega_{IB} = 0.25 \pm 0.10$ "standard" value

[*A.Buras, J.-M.Gerard: PL. B192 (1987) 156*]

$\Omega_{IB} = 0.16 \pm 0.03$ $\pi^0\eta$-mixing at $O(p^2 + p^4)$

[*G.Ecker et al.: PL. B477 (2000) 88*]

$\Longrightarrow$ Full NLO corrections in the χPT:

$\Omega_{IB} \approx 0.08 \pm 0.05$

[*C.E.Wolfe, K.Maltman: PR. D63 (2000) 014008*]

$\Omega_{IB} \approx 0.060 \pm 0.077$ EM & EMW correct.

[*V.Cirigliano et al.: PRL. D91 (2003) 162001*]

- **Improved determination of $\langle Q_{7,8}\rangle$:** ($\mu = 2$ GeV)

$\Longrightarrow$ Dispersive approach based on the QCD sum rules:

$$B_8^{(3/2)} = 1.49 \pm 0.32 \quad (\mathrm{NDR})$$

[S.Narison: NP. B593 (2001) 3]

$$B_8^{(3/2)} = 1.76 \pm 0.48 \quad (\mathrm{NDR})\,; \qquad 1.95 \pm 0.49 \quad (\mathrm{HV})$$

[V.Ciligliano et al.: PL. B555 (2003) 71]

$$B_8^{(3/2)} = 1.27 \pm 0.96 \quad (\mathrm{NDR})\,; \qquad 1.38 \pm 0.96 \quad (\mathrm{HV})$$

[J.Bijnens et al.: JHEP. 0110 (2001) 009]

$\Longrightarrow$ Large-N_c approach:

$$B_8^{(3/2)} = 2.49 \pm 0.71 \quad (\mathrm{NDR})\,; \qquad 2.68 \pm 0.84 \quad (\mathrm{HV})$$

[M.Knecht et al.: PL. B508 (2001) 117]

- **X-boson method + ENJL:** ($\mu = 2$ GeV)

$$B_6^{(1/2)} = 2.5 \pm 0.4\,, \qquad B_8^{(3/2)} = 1.35 \pm 0.20$$

$$\Omega_{IB} = 0.16 \pm 0.03 \quad + \quad \text{FSI corrections}$$

$$\Downarrow$$

$$\boxed{\varepsilon'/\varepsilon = (34 \pm 18) \times 10^{-4}}$$

[J.Bijnens, J.Prades: JHEP 0006 (2000) 035]

- **Instanton induced six-quark interaction:**

$\Longrightarrow$ Instanton 't Hooft determinantial interaction leads to large enhancement (by factor 2) of the $\Delta I = 1/2$ amplitude a_0

Bosonized chiral lagrangians up to $O(p^6)$ [Dubna]

- **Effective weak lagrangian at quark level:**

$$\mathcal{L}_w^q(|\Delta S| = 1) = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_i \widetilde{C}_i^{QCD}(\mu)\, \mathcal{O}_i$$

$\Rightarrow \mathcal{O}_i \;\propto\; \bar{q}_L \gamma_\mu q_L \times \bar{q}_L \gamma^\mu q_L \quad (i = 1, ..., 4)$ (non-penguins)

$\Rightarrow \mathcal{O}_i \;\propto\; \bar{q}_L q_R \times \bar{q}_R q_L \quad (i = 5, ..., 8)$ (penguins)

- **Bosonization of quarks:**

$\Longrightarrow$ Effective lagrangian of strong interaction from the quark determinant of the ENJL:

$$\mathcal{L}_s(S, P, V_\mu, A_\mu) = \mathcal{L}_s^{(p^2)} + \mathcal{L}_s^{(p^4)} + \mathcal{L}_s^{(p^6)} + \ldots$$

$\Longrightarrow$ Effective lagrangian for nonleptonic kaon decays in terms of bosonised (meson) currents and densities:

$\mathcal{L}_w(J_\mu \times J_\mu) = \mathcal{L}_w^{(p^2)} + \mathcal{L}_w^{(p^4)} + \mathcal{L}_w^{(p^6)} + \ldots$ (non-penguins)

$\mathcal{L}_w(J \times J) = \mathcal{L}_w^{(p^0)} + \mathcal{L}_w^{(p^2)} + \mathcal{L}_w^{(p^4)} + \mathcal{L}_w^{(p^6)} + \ldots$ (penguins)

- **$\widetilde{B}_i$-factors:**

$$\mathcal{L}_w^q(|\Delta S| = 1) = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_{i=1}^{8} \widetilde{C}_i^{phen}\, \mathcal{O}_i$$

$$\widetilde{\mathbf{C}}^{phen} \equiv \hat{U}(\mu = 0, M_W)\widetilde{\mathbf{C}}(M_W)$$

$$\Downarrow$$

$$\widetilde{C}_i^{phen} = \widetilde{C}_i^{QCD}(\mu)\widetilde{B}_i(\mu)$$

- **Computation of $K \to 2\pi$ amplitudes:**

$\Longrightarrow$ Up to $O(p^6)$: tree, one-, and two-loop diagrams

$\Longrightarrow$ Superpropagator regularization (UV divergencies)

$$\begin{array}{rcl} \mu & \longrightarrow & \mu_{SP} = 4\pi F_0 \approx 1\text{GeV} \\ (\mathcal{C} - 1/\varepsilon) & \longrightarrow & C_{SP} = -1 + 4\mathcal{C} + \beta\pi \end{array}$$

$\Longrightarrow$ Isotopic symmetry breaking:

$\pi^0 - \eta - \eta^{'}$ mixing

π^0-, $\pi^{\pm}$-mass splitting in kinematics

- **Theoretical and phenomenological input:**

$\widetilde{C}_i^{QCD}(\mu) \Leftarrow$ QCD calculations ($\mu = 1\,\text{GeV}$, $m_t = 170\,\text{GeV}$)

[A.Buras et al., 1996]

$\text{Im}\lambda_t \Leftarrow$ Indirect CP-violation and $B^0 - \bar{B}^0$ mixing

[A.Buras et al., 1996, 1999]

$L_i \Leftarrow$ Low–energy meson processes at $O(p^4)$ of ChPT

[J.Bijnens et al., 1995]

$Q_i \Leftarrow$ Bosonization of NJL model at $O(p^6)$

[A.Bel'kov et al., 1996]

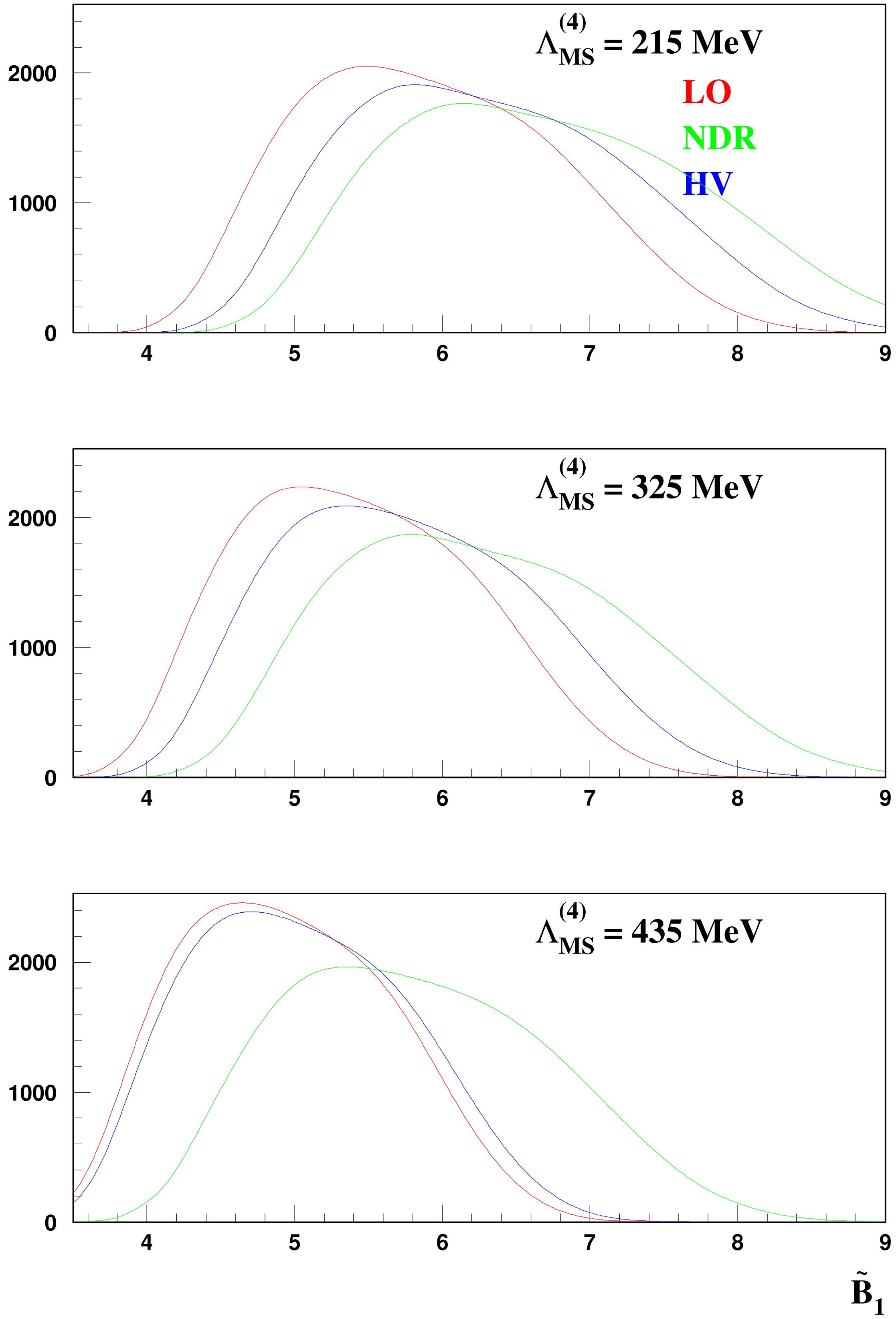
- **Experimental input:**

$$A_0^{(exp)}, \quad A_2^{(exp)}, \quad \text{Re}\,(\varepsilon^{'}/\varepsilon)^{(world\ average)}$$

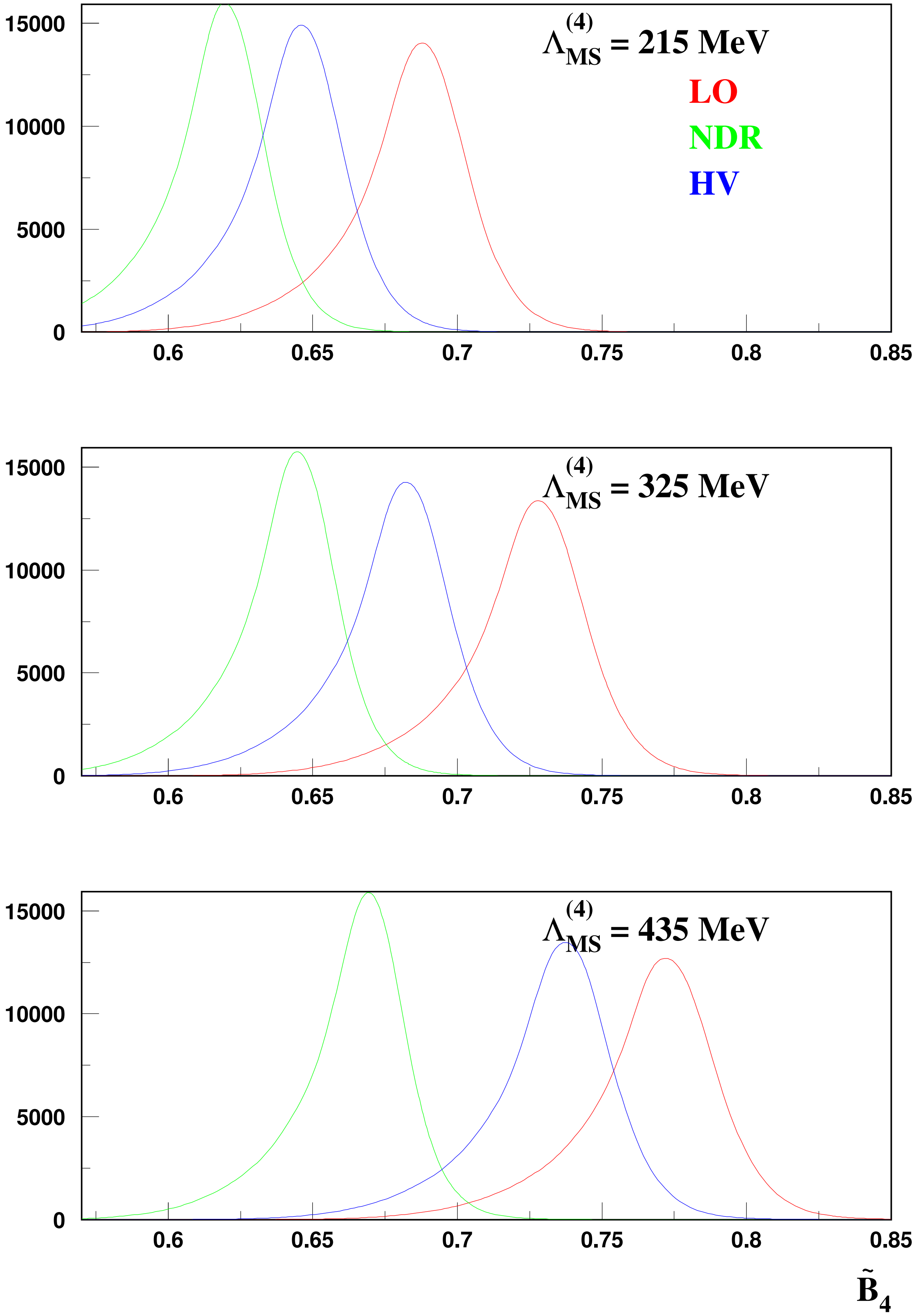
- **Output from analysis:** $\widetilde{B}_1$, $\widetilde{B}_4$, $\widetilde{B}_5$ (for fixed $\widetilde{B}_8$)

- “Gaussian” method of variation of input parameters:

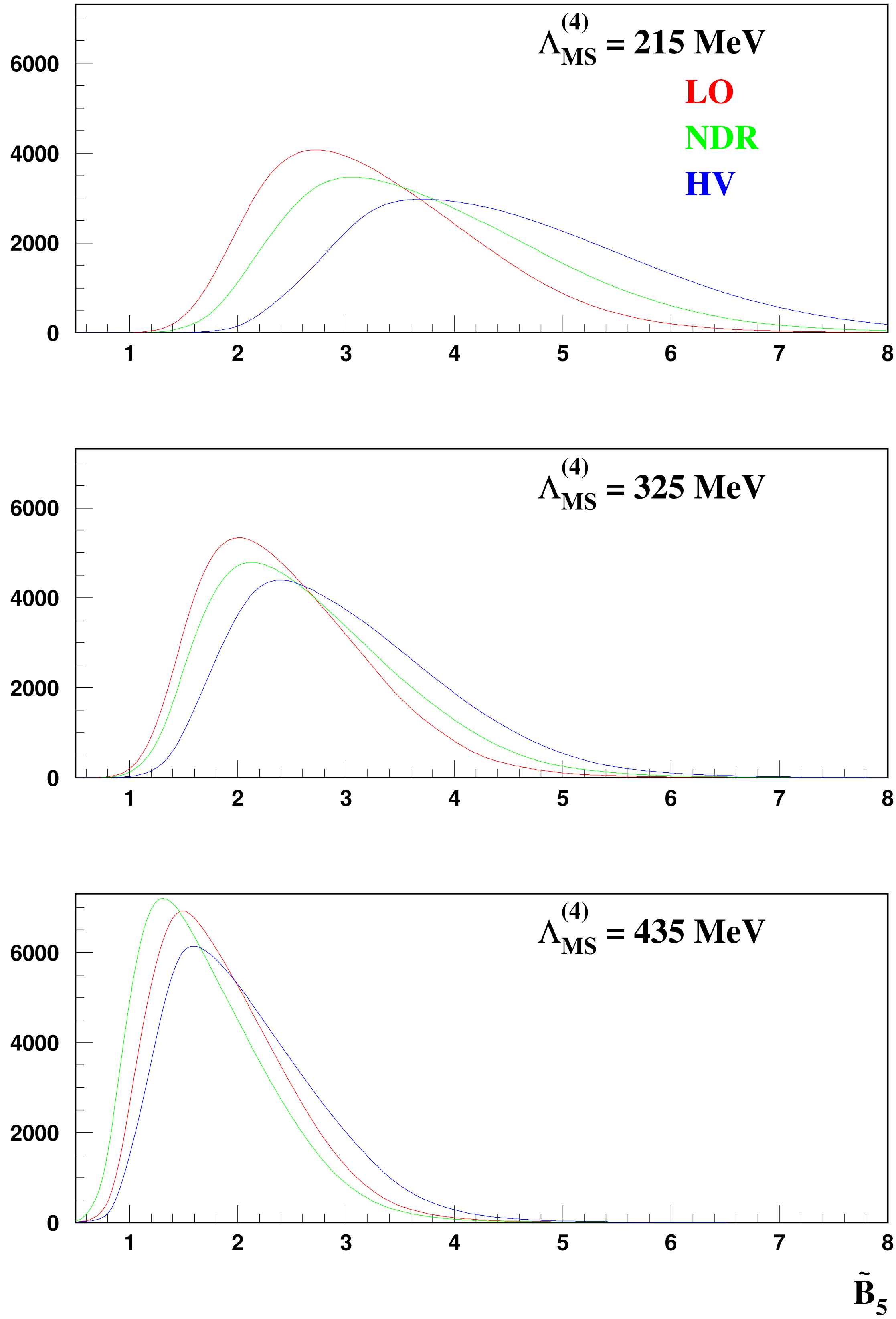
$\Longrightarrow$ Probability density distributions for $\widetilde{B}_1$



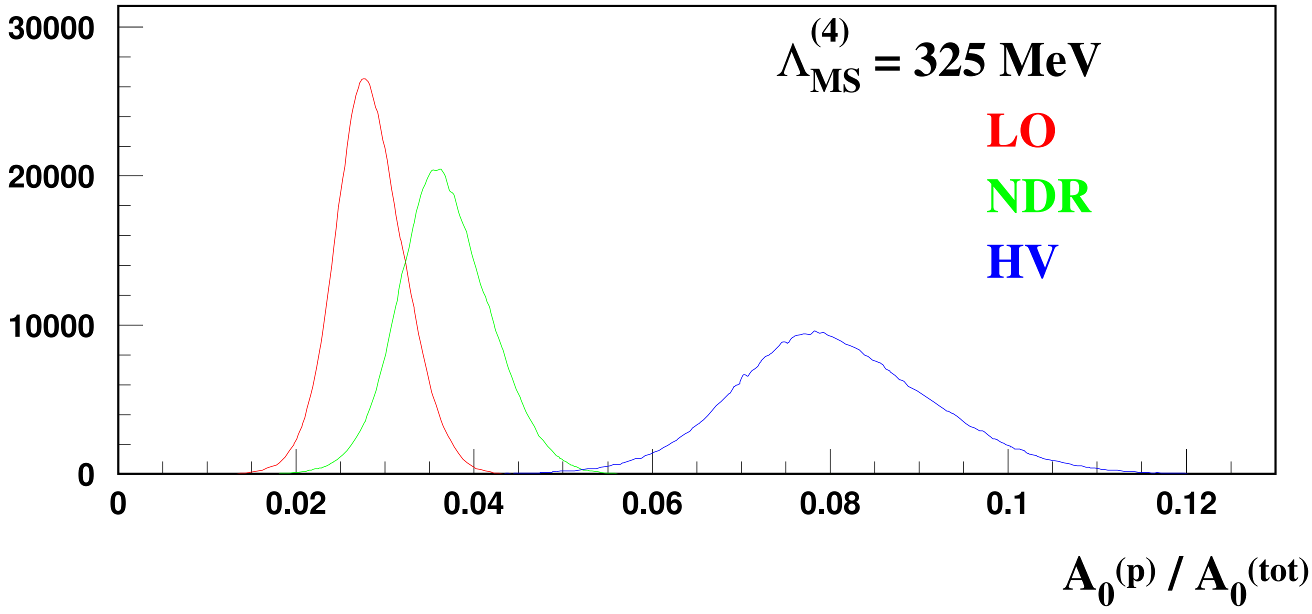
$\Longrightarrow$ Probability density distributions for $\widetilde{B}_4$



$\Longrightarrow$ Probability density distributions for $\widetilde{B}_5$



$\Longrightarrow$ Probability density distributions for the relative contribution of the penguin operators to the $\Delta I = 1/2$ amplitude A_0



$\Downarrow$

- The results of analysis are very sensitive to final-state interactions, isotopic-symmetry-breaking effects and other refinements of the model calculations
- The fact that phenomenological values of $\widetilde{B}_i$ $(i = 1, 4, 5)$ considerably differ from unity may be a hint that the long-distance contributions are still not completely understood
- The contribution of non-penguin operators to the $\Delta I = 1/2$ amplitude dominates

DIRECT CP VIOLATION IN $K \to 3\pi$ DECAYS

- Slope parameters of the $K \to 3\pi$ Dalitz plot:

$$|T(K \to 3\pi)|^2 \propto 1 + gY + hY^2 + kX^2 + \ldots$$

where X, Y are Dalitz variables:

$$X = \frac{s_{K\pi_2} - s_{K\pi_1}}{m_\pi^2}\,, \qquad Y = \frac{s_{K\pi_3} - s_0}{m_\pi^2}\,,$$

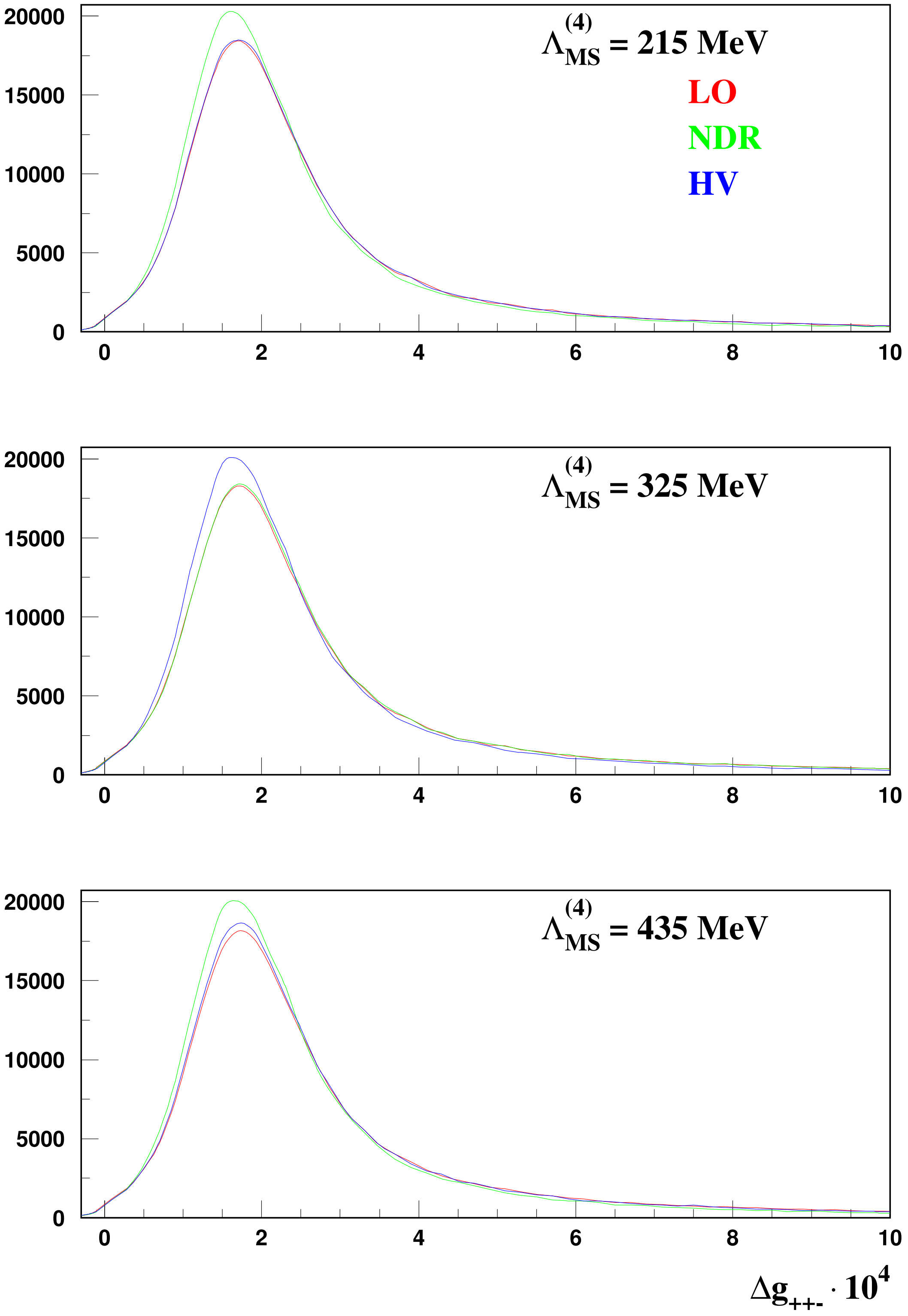
$$s_0 = \frac{1}{3}(s_1 + s_2 + s_3) = \frac{m_K^2}{3} + m_\pi^2$$

- Direct CP violation $\Longrightarrow$ charge asymmetry Δg:

$$\Delta g(K^\pm \to 3\pi) = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}$$

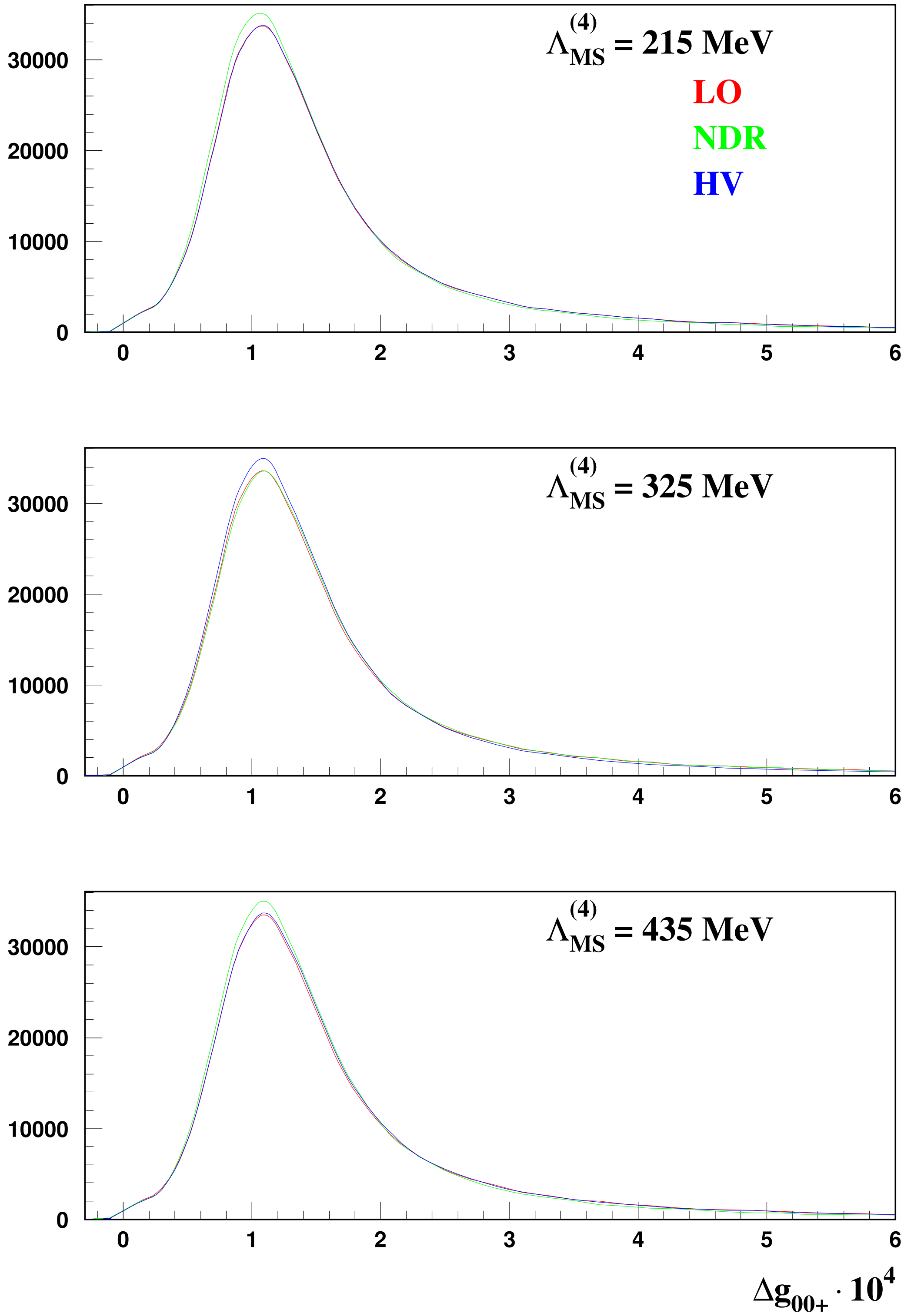
- Dynamical mechanism of enhancement of the charge asymmetry Δg by h. o. contributions in the effective chiral lagrangian approach [A.Bel'kov et al., 1989]:
$\Longrightarrow$ difference in the momentum power counting behavior between penguin and non–penguin contributions to the isotopic amplitudes of $K \to 3\pi$ decays, which appears in high orders of Chiral Theory with Meson Loops

- Probability density distributions for $\Delta g(K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp})$:



$$CL = 68\% \; : \qquad 1.1 \; \leq \; \Delta g_{++-} \cdot 10^4 \; \leq \; 4.7$$

- Probability density distributions for $\Delta g(K^\pm \to \pi^0\pi^0\pi^\pm):$



$CL = 68\%$: $\quad 0.7 \le \Delta g_{++-} \cdot 10^4 \le 2.7$

CONCLUSION

Alvaro De Rujula's view of the relationship between experiment and theory:

"Experimentalists were using QCD in much the same way as a drunkard uses a lamp-post: for support rather than illumination"
CERN Courier, Vol.19 (1979) 244

MESSAGE FROM THE THEORY STATUS ANALYSIS

- The theory is not able now to estimate quantitatively the observable effects of direct CP-violation even to within an order of magnitude
- Why? – the long-distance contributions dominate but are still beyond theoretical capability
- We still need the selfconsistent theoretical approach which should provide at the same time:
  - universal scheme for full matching of SD and LD contributions to the total amplitudes
  - consistent calculation of all higher order corrections including FSI and IB effects
  - selfconsistent description of CP-conserving experimental values: decay widths ($\Delta I = 1/2$ rule), slope parameters, FSI phases
- If there is an opportunity to reach the level 10^{-4} of sensitivity for Δg at the NA48 set up – it is already an important complete argument to carry out the experiment

You ought to do it –
because your facility is able to do it!