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HERA - LHC Workshop
DESY, Hamburg, Germany

“AGK cutting rules in perturbative QCD”

1. On Regge poles and Regge cuts.
2. The AGK cutting rules.
3. Switching on colour.

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Regge Theory:

Good phenomenological description of high energy particle physics.

Major challenge:
Based on assumptions: do they hold for the fundamental microscopic theory: QCD?

Basics:

Simple meson exchange violates unitarity.
Single exchange in t–channel with spin J.
At large s and fixed t:

$$A(s,t) \sim s^J$$

Optical theorem: $\sigma_{\rm tot} \sim s^{2J-2}$
If exchanged spin $J > 1$ violates Froissart bound.
Multiplying the problem = solution:
Exchange of family of resonances = Reggeon

$$A(s,t) \sim s^{\alpha(t)}$$

Preserve the Froissart bound if $\alpha(0) < 1$.

How does this work:

General properties of S-matrix:

$$a + b \to c + d$$

- Lorentz invariance:

$s = (p_a + p_b)^2 \quad t = (p_a - p_c)^2$

Scattering amplitude $= A\,(s, t)$

- Unitarity: $SS^\dagger = S^\dagger S \; = \mathbb{I}$

Optical Theorem : $\sigma_{\text{total}} = \frac{1}{s} \text{Im} A_{\text{elastic}}\,(s, t = 0)$

- Analyticity:

Polynomial

$$A_{ab \to cd}\,(s, t) = \sum_{l=0}^{\infty}\,(2l + 1)\,a_l\,(t)\,P_l\left(1 + 2\frac{s}{t}\right)$$

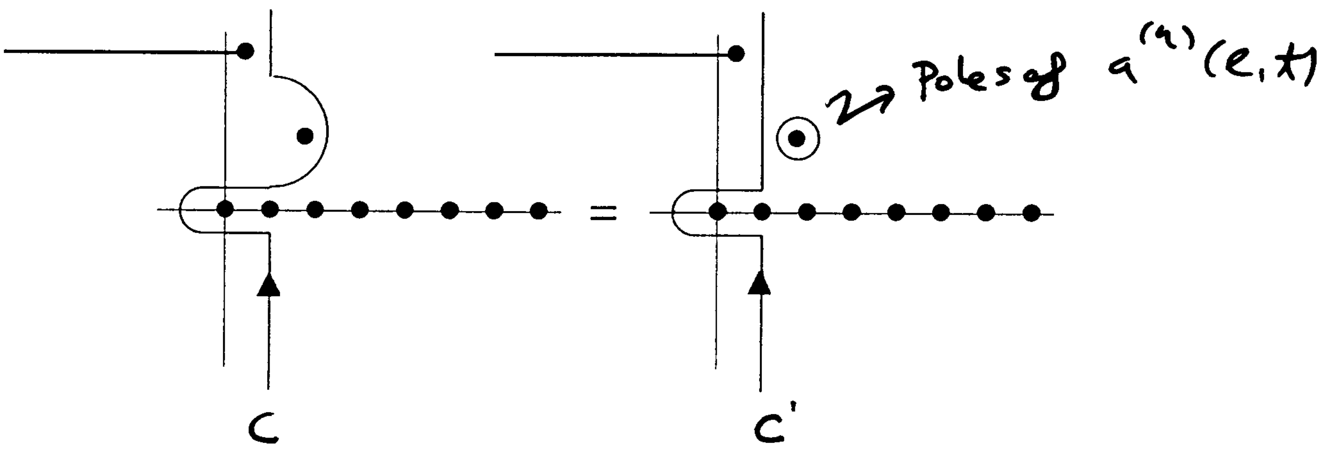
Partial Wave expansion

Introduce t-channel complex angular momentum l

$$A(s,l) \sim \int_C dl \sum_{\eta=\pm 1} \frac{\left(\eta + e^{-i\pi l}\right)}{2} \frac{(2l+1)a^{(\eta)}(l,t)}{\sin \pi l} P\left(l, 1 + 2\frac{s}{t}\right)$$

Even/odd-signature partial wave functions $a^{(\eta)}(l,t)$.

Pole with largest real part dominates at high energies ...

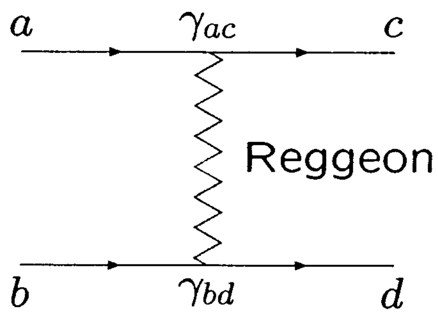


In the limit $s \gg |t|$ we have

$$P\left(l, 1 + 2\frac{s}{t}\right) \to \frac{\Gamma(2l+1)}{\Gamma(l+1)} \left(\frac{s}{2t}\right)^l$$

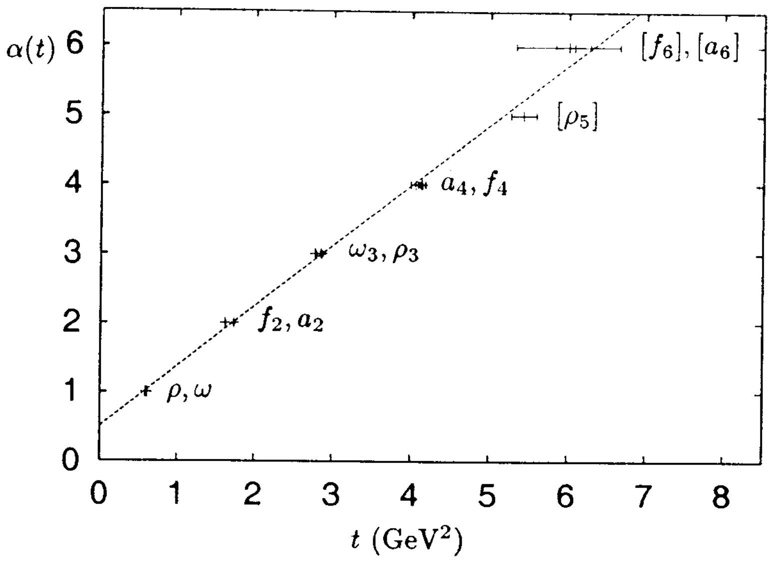
and we can write

$$A(s,t) \to \frac{\left(\eta + e^{-i\pi\alpha(t)}\right)}{2\sin\pi\alpha(t)} \frac{\gamma_{ac}(t)\gamma_{bd}(t)}{\Gamma\left(\alpha(t)\right)} s^{\alpha(t)}$$



When $\alpha(t)$ is a positive integer the amplitude has a pole corresponding to a t-channel exchange of a resonance of spin α.

$\alpha(t)$ "trajectories" are seen in experiments ...



$\alpha(t)$
0
1
2
3
4
5
6
0 1 2 3 4 5 6 7 8
t (GeV^2)
ρ, ω
f_2, a_2
ω_3, ρ_3
a_4, f_4
$[\rho_5]$
$[f_6], [a_6]$

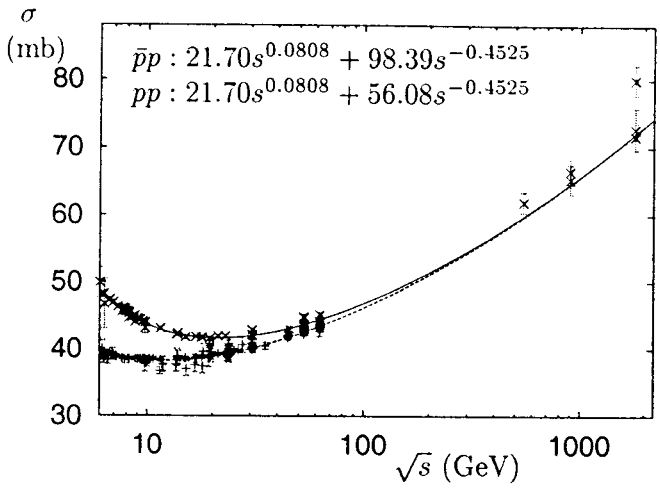
From the optical theorem we can derive

$$\sigma_{\text{total}} \sim s^{\alpha(0)-1}$$

The Pomeranchuk theorem states that if there is charge exchange σ should decrease with s.

All known meson trajectories have $\alpha(0) < 1$.

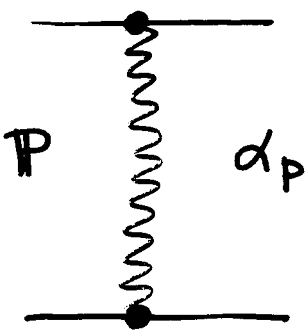
BUT experimentally we see that
there is a rise in pp and $p\bar{p}$ σ_{total} ...


σ
(mb)
p̄p : 21.70s^0.0808 + 98.39s^−0.4525
pp : 21.70s^0.0808 + 56.08s^−0.4525
80
70
60
50
40
30
10
100
1000
√s (GeV)

So the mechanism responsible for this rise must be an exchange with the quantum numbers of the vacuum.

Pomeron exchange.

Its trajectory populated by glueballs?.



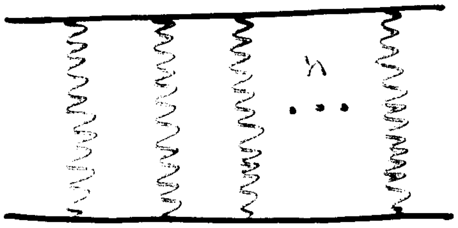
$$\alpha_{P}(t) = \alpha_{P}(0) + \alpha'_{P} \cdot t$$

$$\alpha_{P}(0) > 1$$

Violates Unitarity

'Power-like'

$$A(s,t) \sim s^{\alpha} \qquad a(\ell,t) \sim \frac{1}{\ell - \alpha}$$



Regge Cuts:

Other singularities appearing in the l–plane when several Reggeons are exchanged.

They restore s–channel unitarity.

If n Pomerons with trajectory $\alpha_P = 1 + \alpha'_P t$ are exchanged the amplitude goes like

$$A(s,t) \sim \frac{s^{\alpha_c(t)}}{\ln^{n-1} s} \qquad a(l,t) \sim \ln(l-\alpha)$$

with $\alpha_c(t) = n\,(\alpha_P(0) - 1) + 1 + \frac{\alpha'_P}{n} t$.

At larger $|t|$ the multipomeron exchange is more important.

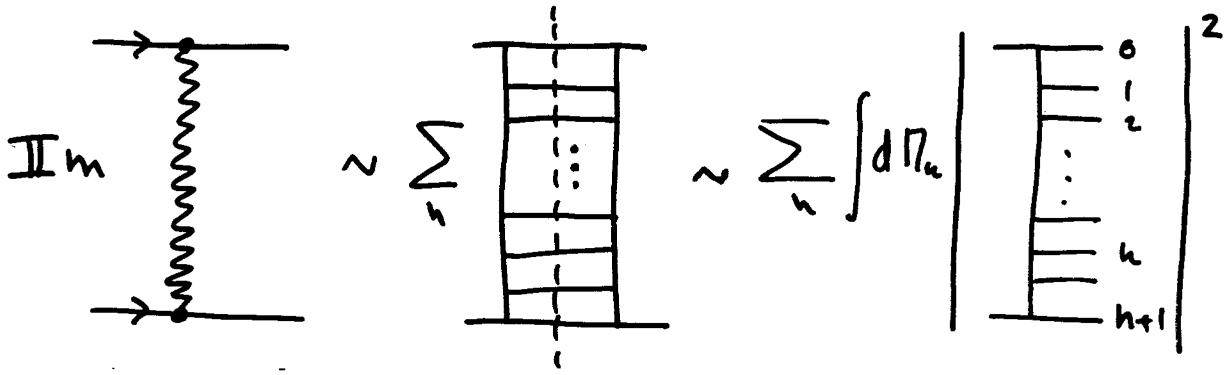
There is destructive interference between one and two Pomeron exchange.

Two Pomeron cut gives negative contribution to cross section:

$$\sigma_{\text{tot}} \sim A s^{\alpha_P(0)-1} - B \frac{s^{2(\alpha_P(0)-1)}}{\ln s}$$

s–channel picture of one Reggeon:

The unitarity cut of a Regge pole diagram: Signals in Final States:



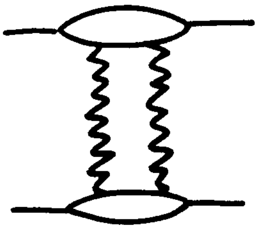
• Short Range correlations in rapidity. The correlation function exponentially decreases with rapidity difference:

$$\frac{d\sigma}{\sigma^{\text{in}} dy_1 dy_2} - \frac{d\sigma}{\sigma^{\text{in}} dy_1} \frac{d\sigma}{\sigma^{\text{in}} dy_2} \sim e^{-\lambda(y_1 - y_2)}$$

• Multiplicities:

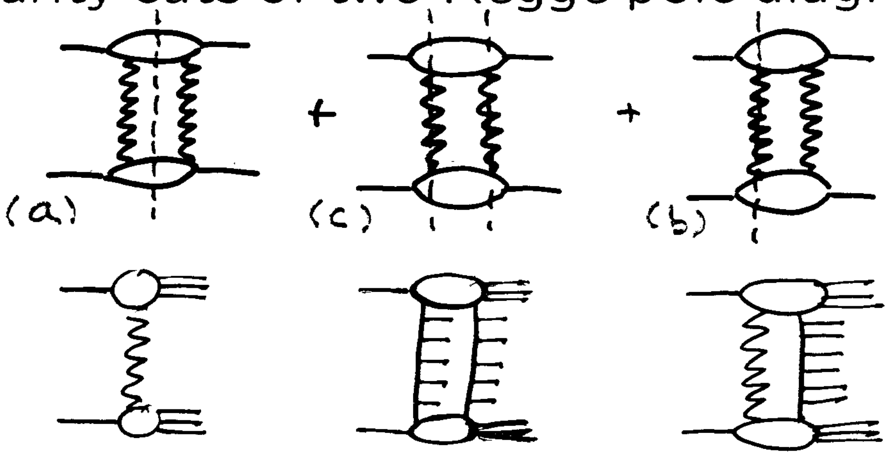
$$\langle n \rangle \sim \ln s$$

following a Poisson distribution.

$T_{2\to 2} \sim$ 

s–channel picture of two Reggeons:

The unitarity cuts of two Regge pole diagrams:

$\mathrm{Im}\, T_{2\to 2} \sim$ 

+ cc

+1   +2   −4

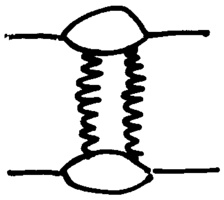
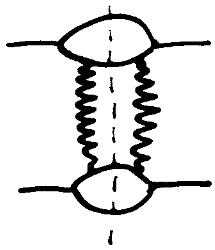
Abramovsky, Gribov, Kanchely (1974): Relative contributions to the total cross section

'DIFFRACTIVE'   'ABSORPTIVE'   'DOUBLE CUT'

$$\sigma_{\mathrm{a}} = -1\sigma_{\mathrm{tot}}^{PP},\ \sigma_{\mathrm{b}} = 4\sigma_{\mathrm{tot}}^{PP},\ \sigma_{\mathrm{c}} = -2\sigma_{\mathrm{tot}}^{PP}$$

$\sigma_{\mathrm{tot}}^{PP} < 0$ two Pomeron contribution to σ_{tot}.

Simple relation between DIFFRACTION and sum of all contributions.

$\sum_{\text{cuts}}$ () $= -$ 

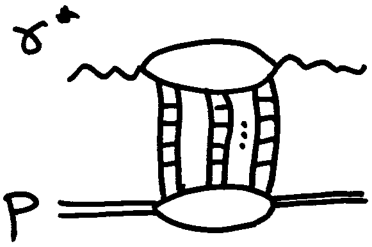
$+1+2-4 \quad = \quad -1$

Where and how to use them?:

- **HERA:**

Saturation at low x and small Q^2?
Can we find a more direct evidence?
All models so far are too inclusive: F_2
"sum over multiple exchanges"

$$\sigma^{\gamma^* p} \sim \sum_n$$

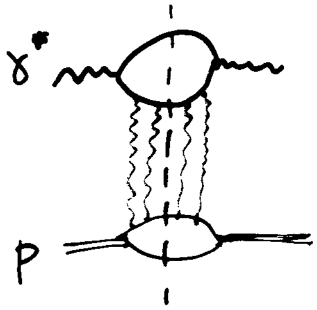


Understand multi parton correlations:

Main Goal:
"Open" these saturation models
Investigate Final State properties (Monte Carlo particle production)

HERA → use them at → LHC

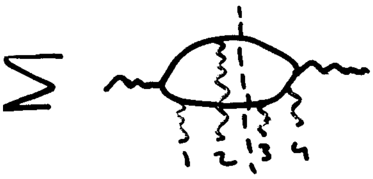
How?:
Use AGK for QCD ladders...



2. Switching on colour

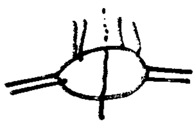
Investigate two ladder exchange in DIS:

- Coupling to Photon within pQCD



Singlet

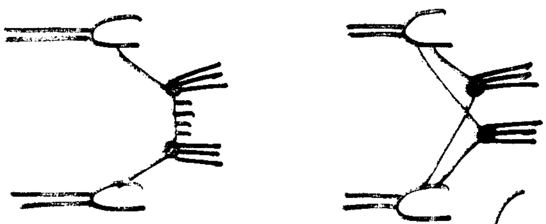
- Pieces symmetric in (1234) Colour + Momenta.
  ↳ Satisfies AGK for 4 gluons.
- Antisymmetric parts under study...

- Proton side Assume 4 gluon correlator has the same symmetry structure as the pQCD part.
  Relative weight of  to 
  within a model (GBW..) fixed by global fit to F_2.

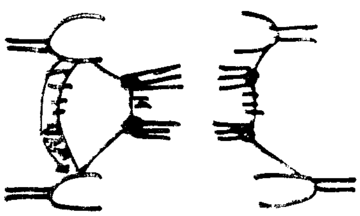
- Transport results to the LHC: prediction for multiple scattering.

  Multiple jet pair production:

e.g.



(AGK related to e.g.:



Conclusions:

- Use our knowledge of the AGK cutting rules in QCD (the colour degree of freedom not straightforward).

- More exclusive test of saturation at HERA. For this we need a Monte Carlo in agreement with AGK cutting rules.

- Transport what has been learnt to the LHC, multiple interactions.