Non-Singlet QCD Analysis of the Structure Function F_2 in 3–Loops

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- **Moments - Comparison: QCD and Lattice**
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In Collaboration with
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Motivation

- We aim at parameterizations of the parton densities and their fully correlated 1σ error bands which are directly applicable to determine 'experimental' errors for other observables in hard scattering processes.

- Having the recently completed NNLO anomalous dimensions available $^{(*)}$ a consistent 3–loop analysis of the unpolarized deep inelastic Structure Functions can be carried out.

$\Longrightarrow$ **Precision Tests of QCD**

- The present Analysis concentrates on the Non–Singlet evolution only to firstly obtain an accurate as possible picture for the valence quark distributions.

- The aim of the analysis is to determine $\alpha_s(Q^2)$ and the valence quark distributions with Correlated Errors to 3–loop accuracy.

- An NNLO QCD analysis of the structure function $F_2(x, Q^2)$ may allow to reduce the theoretical error in determining $\alpha_s(Q^2)$ to at least the level of the experimental error.

$^{(*)}$ **Ref.:** S.Moch,J.A.M.Vermaseren and A.Vogt, hep-ph/0403192.

Motivation, cont'd

- Comparison of QCD analysis results with results from recent lattice simulations concerning both QCD parameters and low order moments has shown astonishing agreement in the polarized case:

[**Ref.:** J.Blümlein and H. Böttcher, Nucl. Phys. **B636** (2002) 225]

Δf	n	QCD	Lattice	
		moment at $Q^2 = 4\,GeV^2$	QCDSF	LHPC/ SESAM
Δu_v	1	0.926 fixed	0.889(29)	0.860(69)
	2	0.163 ± 0.014	$0.198(8)$	$0.242(22)$
	3	0.055 ± 0.006	0.041(9)	0.116(42)
Δd_v	1	-0.341 fixed	-0.236(27)	-0.171(43)
	2	-0.047 ± 0.021	$-0.048(3)$	$-0.029(13)$
	3	-0.015 ± 0.009	-0.028(2)	0.001(25)

- With the steadily improving Lattice Simulations a Comparison of the results of the present analysis and the measurement of the lowest Moments for unpolarized valence quark densities may become feasible soon.

QCD Analysis Formalism

- In Mellin-N space the non-singlet parts of the structure function $F_2(N, Q^2)$ are written as

$$F_2^{\pm,\mathrm{V}}(N, Q^2) = \left[1 + C_1(N)a + C_2(N)a^2\right] f^{\pm,\mathrm{V}}(N, Q^2)\,,$$

where $f^{\pm,\mathrm{V}}(N, Q^2)$ stand for the non-singlet (NS) quark combinations.

- The quark combinations to be considered are

$$\begin{aligned} \Delta^{\pm} &= (u \pm \bar{u}) - (d \pm \bar{d}) \\ V &= (u - \bar{u}) + (d - \bar{d}) \end{aligned}$$

and the non-singlet parts of F_2 are then given by :

$$\begin{aligned} F_2^{NS} &\propto \frac{1}{3}\Delta^{+} \propto \frac{1}{3}(u_v - d_v - 2(\bar{d} - \bar{u})) \\ F_2^{p,V} &\propto \frac{5}{18}V + \frac{1}{6}\Delta^{-} \propto \frac{4}{9}u_v + \frac{1}{9}d_v \\ F_2^{d,V} &\propto \frac{5}{18}V \propto \frac{1}{2}(\frac{5}{9}u_v + \frac{5}{9}d_v)\,. \end{aligned}$$

QCD Analysis Formalism cont'd

- Solving the evolution equation for $F_2^{\pm,\mathrm{V}}$ up to 3-Loops gives

$$
\begin{aligned}
F_2^{\pm,\mathrm{V}}(Q^2) \quad = \quad & F_2^{\pm,\mathrm{V}}(Q_0^2)\left(\frac{a}{a_0}\right)^{-P_0/\beta_0} \\
& \left\{1 - \frac{1}{\beta_0}(a-a_0)\left[P_1^{\pm} - \frac{\beta_1}{\beta_0}P_0 - C_1\beta_0\right]\right. \\
& -\frac{1}{2\beta_0}\left(a^2 - a_0^2\right)\left[P_2^{\pm,\mathrm{V}} - \frac{\beta_1}{\beta_0}P_1^{\pm}\right. \\
& \left. +\left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)P_0 - 2C_2\beta_0 - C_1\beta_1 + C_1^2\beta_0\right] \\
& \left. +\frac{1}{2\beta_0^2}(a-a_0)^2\left(P_1^{\pm} - \frac{\beta_1}{\beta_0}P_0 - C_1\beta_0\right)^2\right\},
\end{aligned}
$$

where C_i are the Wilson coefficients $^{(*)}$, P_i the splitting functions, and $a = \alpha_s/4\pi$ with $a_0 = a(Q_0^2)$.

$^{(*)}$ C_2: W.L. van Neerven and A.Vogt, Nucl.Phys.**B568**(2000)263.

- The evolution of the parton densities u_v, d_v, and $\bar{d} - \bar{u}$ goes without the Wilson coefficient terms.

Parameterization of the Non-Singlet Part

- Choice of the parameterization of the parton densities at the input scale of $Q_0^2 = 4\ GeV^2$:

$$xu_v(x, Q_0^2) = A_{u_v} x^{a_u} (1-x)^{b_u} (1 - 1.108 x^{\frac{1}{2}} + 26.283 x)$$

$$xd_v(x, Q_0^2) = A_{d_v} x^{a_d} (1-x)^{b_d} (1 + 0.895 x^{\frac{1}{2}} + 18.179 x)$$

and as adopted from MRST: $\Longrightarrow$ figure

[**Ref.:** Eur.Phys.J.**C23**(2002)73]

$$x(\bar{d} - \bar{u})(x, Q_0^2) = 1.195 x^{1.24} (1-x)^{9.10} (1 + 14.05 x - 45.52 x^2)$$

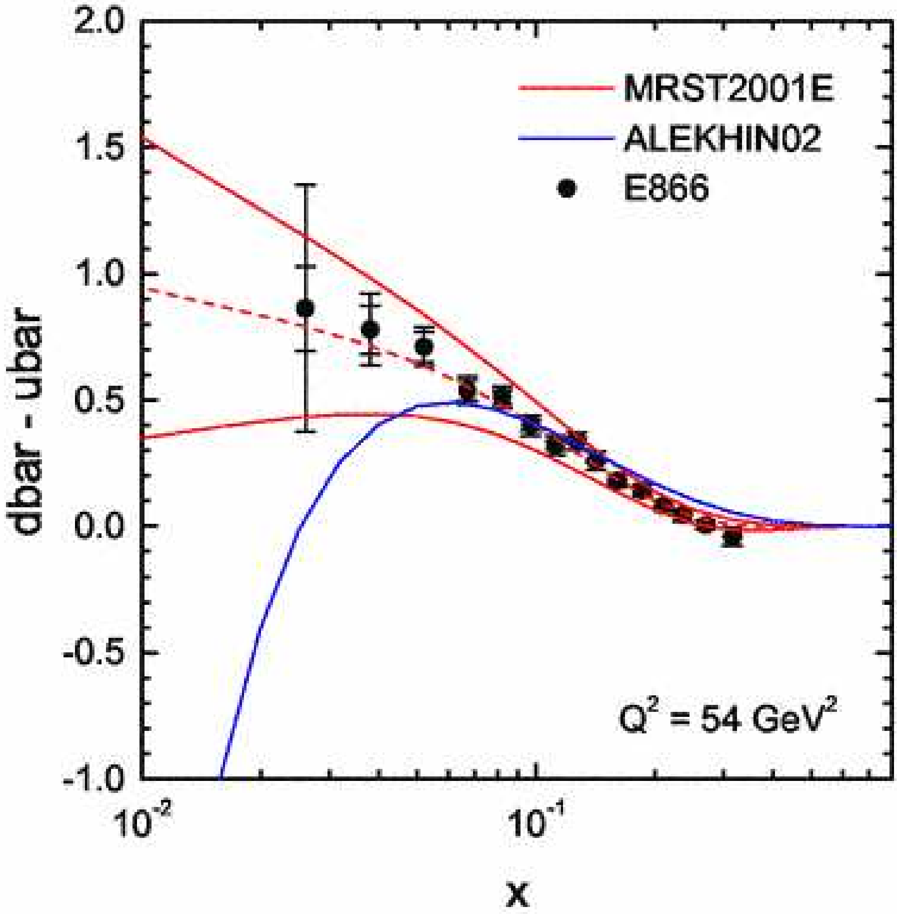
- The normalization constants A_{u_v} and A_{d_v} are fixed by the conservation of the number of valence quarks:

$$\int_0^1 u_v(x) dx = 2\,, \ \int_0^1 d_v(x) dx = 1.$$

$\Longrightarrow$ Finally 4 parameters are to be determined in the fit:

u_v: a_u , b_u , d_v: a_d , b_d

and in addition Λ_{QCD}.



- evidence for
  - $\bar{d} > \bar{u}$
  - $x(\bar{d} - \bar{u}) \to 0$

  as $x \to 0$?

- HERA3 (ep and ed DIS at small x) could provide an interesting measurement!

Ref.: J. Stirling, 'Phenomenology of Parton Distributions', Talk given at KITP Conference: Collider Physics, Santa Barbara, USA, January - April, 2004.

The F_2 Data Handling

- R **correction**:

  BCDMS(R_{QCD}) $\Longrightarrow$ BCDMS(R_{1998})

  [**Ref**: E143 Collaboration, K.Abe et al., Phys.Lett.**B452**(1999)194.]

- **Deuteron Data correction**:

  Fermi motion and offshell correction

  [**Ref**: W.Melnitchouk and A.W.Thomas, Phys.Lett.**B377**(1996)11.]

- **Kinematic cuts**:

$$0.3 < x < 1.0 \text{ for } F_2^p \text{ and } F_2^d$$

$$0.0 < x < 0.3 \text{ for } F_2^{ns} = 2(F_2^p - F_2^d)$$

$$4.0 < Q^2 < 30000\, GeV^2,\ W^2 > 12.5\, GeV^2$$

- **Normalization uncertainties**:
  We allow for a Relative Normalization Shift between the different data sets within the normalization uncertainties quoted by the experiments or assumed accordingly (fitted and then fixed).

The World Data on F_2

$Experiment$	x	Q^2, GeV^2	F_2	$Norm$
BCDMS (100)	0.35 – 0.75	11.75 – 75.00	51	1.016
BCDMS (120)	0.35 – 0.75	13.25 – 75.00	59	1.009
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	1.012
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	1.014
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	1.003
SLAC (comb)	0.30 – 0.62	7.30 – 21.39	57	1.017
H1 (hQ2)	0.40 – 0.65	200 – 30000	26	1.018
ZEUS (hQ2)	0.40 – 0.65	650 – 30000	15	1.001
$proton$			322	
BCDMS (120)	0.35 – 0.75	13.25 – 99.00	59	0.987
BCDMS (200)	0.35 – 0.75	32.50 – 137.50	50	0.985
BCDMS (280)	0.35 – 0.75	43.00 – 230.00	49	0.987
NMC (comb)	0.35 – 0.50	7.00 – 65.00	15	0.980
SLAC (comb)	0.30 – 0.62	10.00 – 21.40	59	0.980
$deuteron$			232	
BCDMS (120)	0.070 – 0.275	8.75 – 43.00	36	1.000
BCDMS (200)	0.070 – 0.275	17.00 – 75.00	29	1.000
BCDMS (280)	0.100 – 0.275	32.50 – 115.50	27	1.000
NMC (comb)	0.013 – 0.275	4.50 – 65.00	88	1.000
SLAC (comb)	0.153 – 0.293	4.18 – 5.50	28	1.000
$non - singlet$			208	
$total$			762	

- **CUTS**: $0.3 < x < 1.0$ for F_2^p and F_2^d
  $0.0 < x < 0.3$ for $F_2^{ns} = 2(F_2^p - F_2^d)$
  $4.0 < Q^2 < 30000\, GeV^2$, $W^2 > 12.5\, GeV^2$

- **Data points without BCDMS:** 303

Error Treatment

⇒ Problem: Systematic errors are known to be partly correlated.

- Experimental Errors in this analysis:
  For all data sets we used the simplest procedure by adding the statistical and total systematic errors in quadrature. Only fits with a Positive Definite Covariance Matrix are accepted to be able to

  $\Longrightarrow$ calculate the Fully Correlated 1σ Error Bands by Gaussian error propagation.

- χ^2 Expression:

$$\chi^2 = \sum_{i=1}^{n^{exp}} \left[\frac{(N_i - 1)^2}{(\Delta N_i)^2} + \sum_{j=1}^{n^{data}} \frac{(N_i F_{2,j}^{data} - F_{2,j}^{theor})^2}{(\Delta F_{2,j}^{data})^2} \right]$$

Fully Correlated Error Calculation

- The fully correlated 1σ error for the parton density f_q as given by Gaussian error propagation is

$$\sigma(f_q(x)^2 = \sum_{i,j=1}^{n_p} \left(\frac{\partial f_q}{\partial p_i} \frac{\partial f_q}{\partial p_j} \right) \mathrm{cov}(p_i, p_j) \ ,$$

  where the $\partial f_q / \partial p_i$ are the derivatives of f_q w.r.t. the parameters p_i and the $\mathrm{cov}(p_i, p_j)$ are the elements of the covariance matrix as determined in the fit.

- The derivatives $\partial f_q / \partial p_i$ at the input scale Q_0^2 can be calculated analytically. Their values at Q^2 are given by evolution.

- The derivatives evolved in $\mathrm{MELLIN\text{-}N}$ space are transformed back to x–space and can then be used according to the error propagation formula above.

$\Longrightarrow$ As an example the derivative of $f(x, a, b)$ w.r.t. parameter a in $\mathrm{MELLIN–N}$ space reads:

Derivatives in MELLIN-N space

The general form of the derivative of the MELLIN moment w.r.t. parameter a for complex values of N is

$$\frac{\partial \mathbf{M}[f(x,a,b)](N)}{\partial a} = A\frac{\partial \overline{\mathbf{M}}}{\partial a} + \overline{\mathbf{M}}\frac{\partial A}{\partial a},$$

with $\overline{\mathbf{M}} = \mathbf{M}/A$ and A the normalization constant.

$$\begin{aligned}
\frac{\partial \overline{\mathbf{M}}}{\partial a} &= \{[\Psi(a-1+N) - \Psi(a+N+b)] \\
&\quad + \gamma\frac{a-1+N}{a+N+b}(\Psi(a+N) - \Psi(a+N+b+1))\} \\
&\quad B(a-1+N, b+1) \\
&\quad + \rho[\Psi(a-\frac{1}{2}+N) - \Psi(a+\frac{1}{2}+N+b)] \\
&\quad B(a-\frac{1}{2}+N, b+1)
\end{aligned}$$

$$\frac{\partial A}{\partial a} = -AZ_a/X_a = -CZ_a/X_a^2$$

$$\begin{aligned}
Z_a &= [\Psi(a) - \Psi(a+b+1)]B(a,b+1) + \rho[\Psi(a+1/2) \\
&\quad -\Psi(a+\frac{1}{2}+b+1)]B(a+\frac{1}{2}, b+1) + \gamma \\
&\quad \times[\Psi(a+1) - \Psi(a+1+b+1)]B(a+1,b+1)
\end{aligned}$$

$$X_a = B(a,b+1) + \rho B(a+\frac{1}{2}, b+1) + \gamma B(a+1,b+1)$$

with C the number of valence quarks, i.e. **2** for $\mathbf{u_v}$ and **1** for $\mathbf{d_v}$.

Fit Results

- Parameter values at the input scale $Q_0^2 = 4.0\,GeV^2$

$$xq_i(x, Q_0^2) = A_i x^{a_i}(1-x)^{b_i}(1 + \rho_i x^{\frac{1}{2}} + \gamma_i x)$$

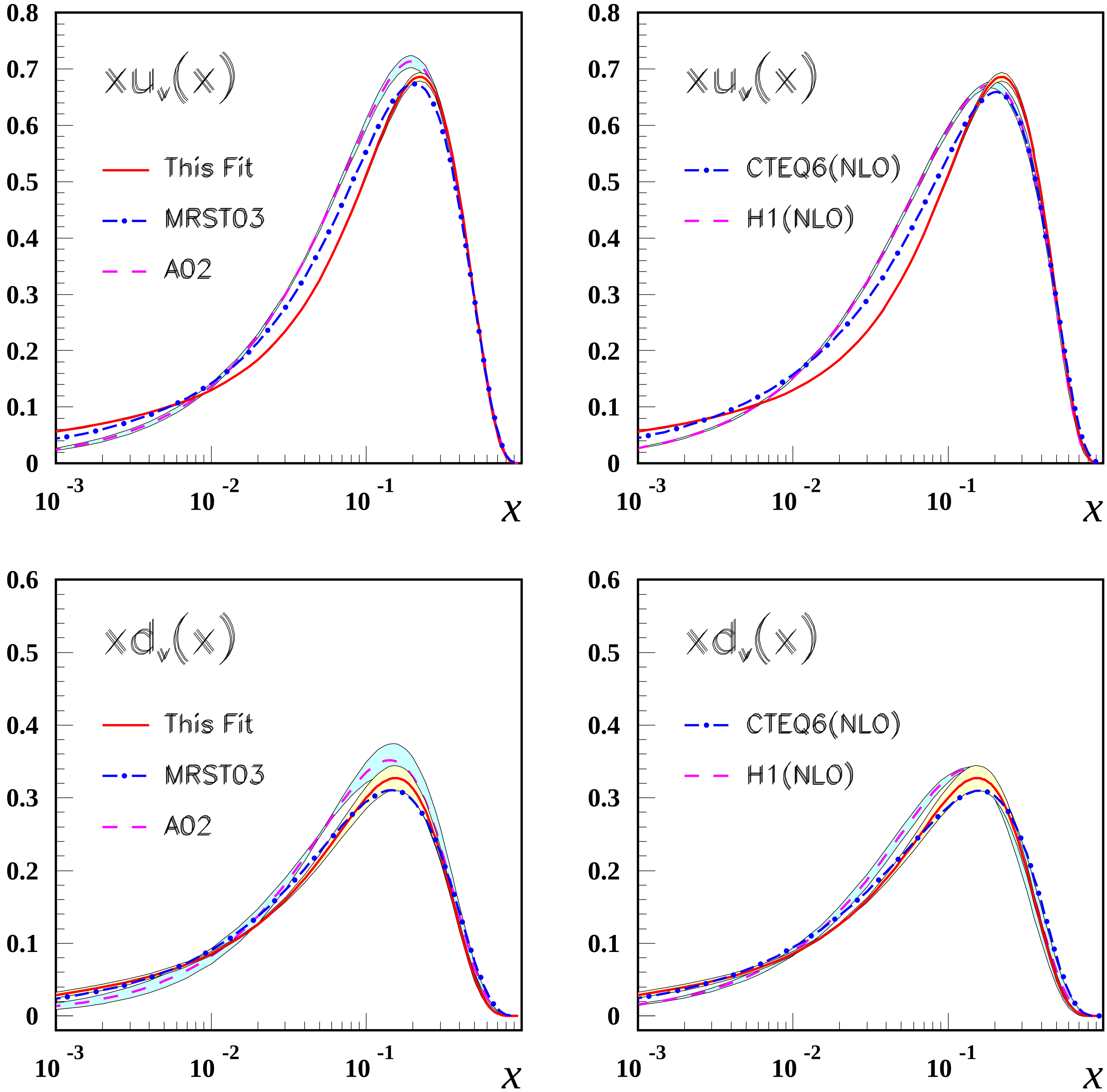
u_v	a	0.314 $\pm$ 0.007
	b	4.199 $\pm$ 0.032
	ρ	-1.108
	γ	26.283
d_v	a	0.413 $\pm$ 0.047
	b	6.196 $\pm$ 0.332
	ρ	0.895
	γ	18.179
$\Lambda_{QCD}^{(4)}$		227 $\pm$ 30 MeV
χ^2/ndf $= 652/757 = 0.86$		

- Covariance Matrix at the input scale $Q_0^2 = 4.0\,GeV^2$

	$\Lambda_{QCD}^{(4)}$	a_{uv}	b_{uv}	a_{dv}	b_{dv}
$\Lambda_{QCD}^{(4)}$	**9.27E-4**				
a_{uv}	1.09E-4	**5.52E-5**			
b_{uv}	-1.13E-4	1.63E-4	**9.94E-4**		
a_{dv}	2.23E-4	-1.40E-5	-4.85E-4	**2.18E-3**	
b_{dv}	1.25E-3	2.53E-4	-2.83E-3	1.43E-2	**1.11E-1**

Valence Parton Densities at $Q_0^2 = 4.0\,GeV^2$

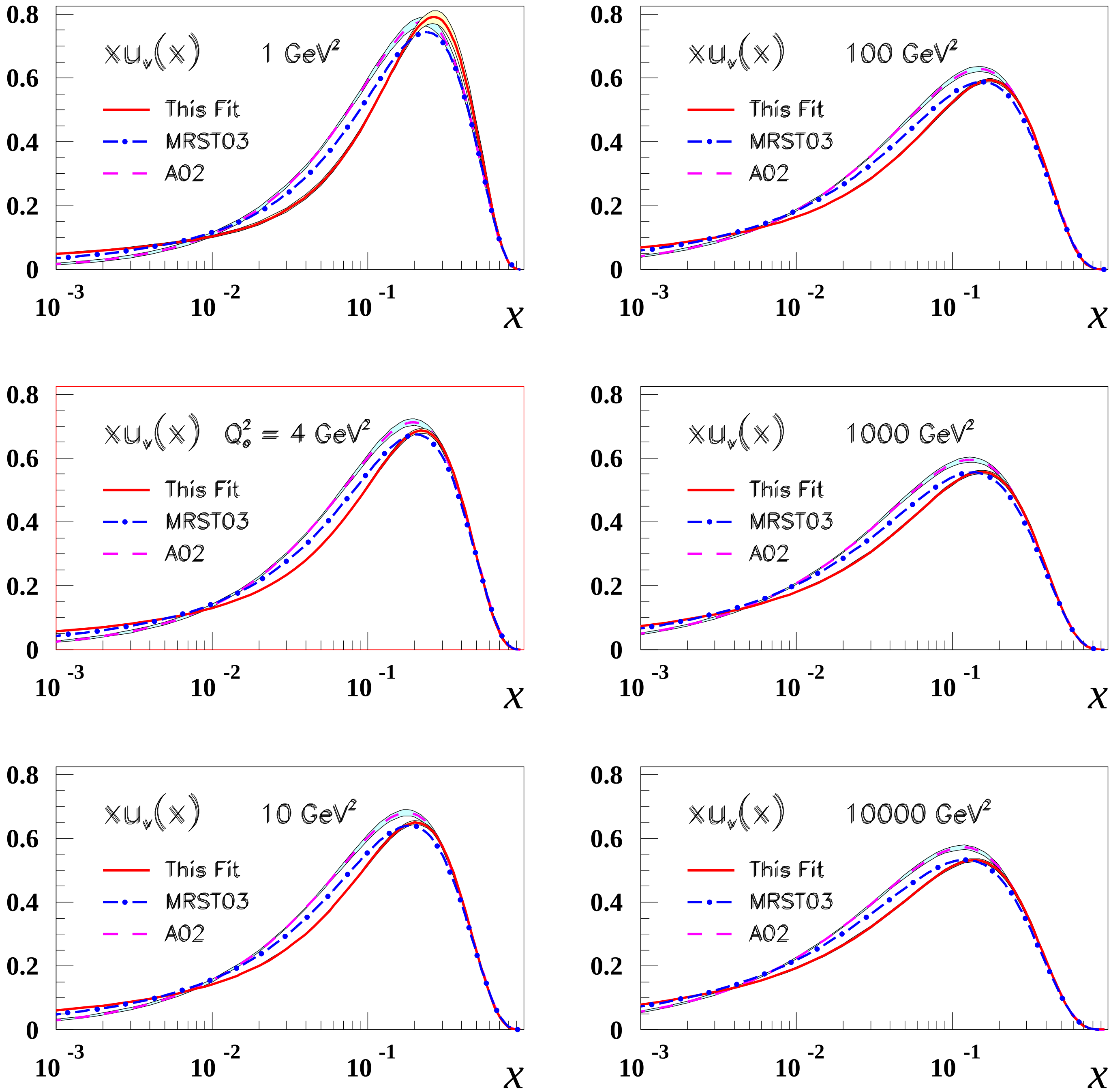
- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ error as given by Gaussian error propagation.

Evolution of the Parton Density $xu_v(x)$

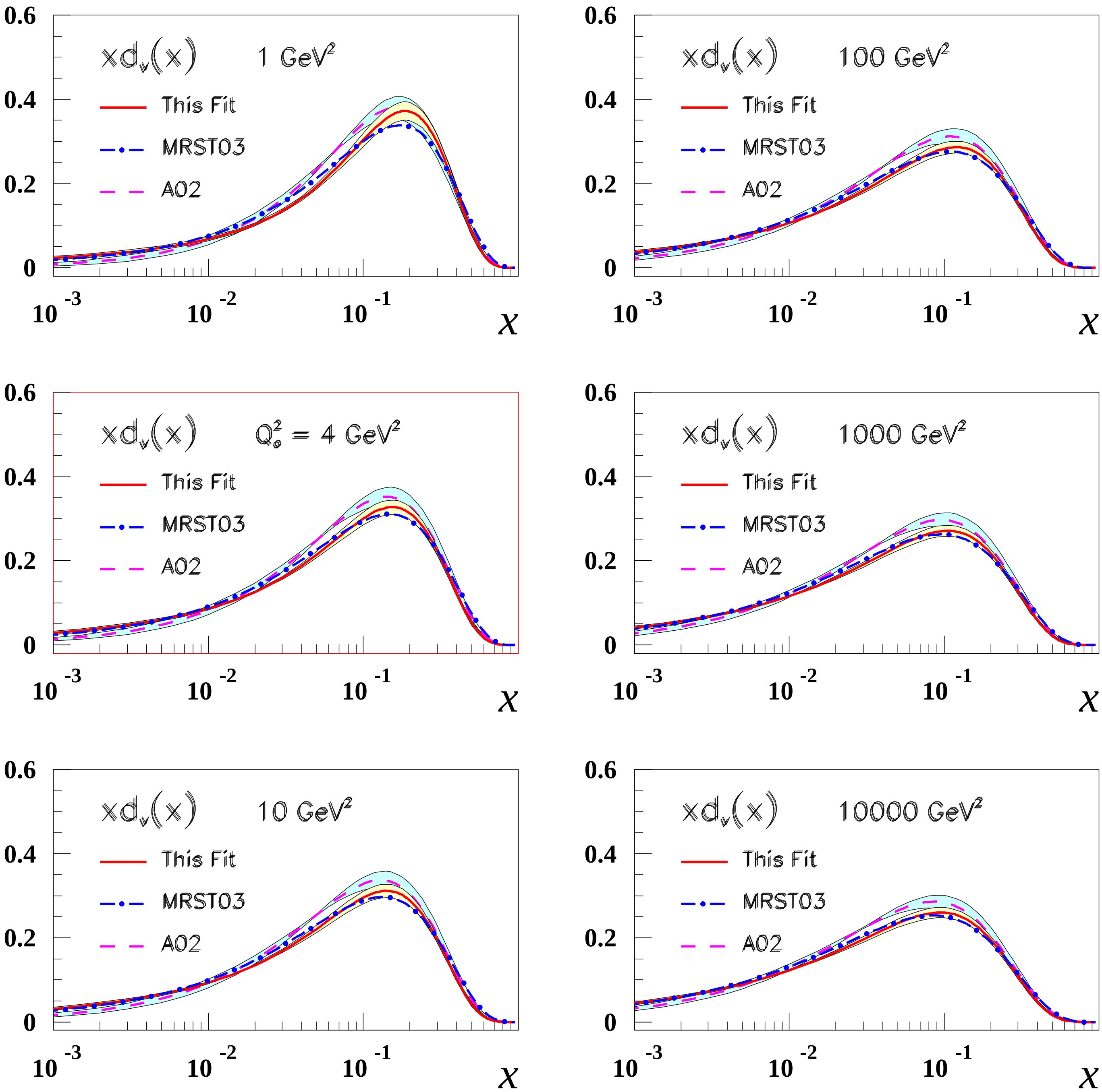
- 4+1 parameter non-singlet fit to F_2 data:



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

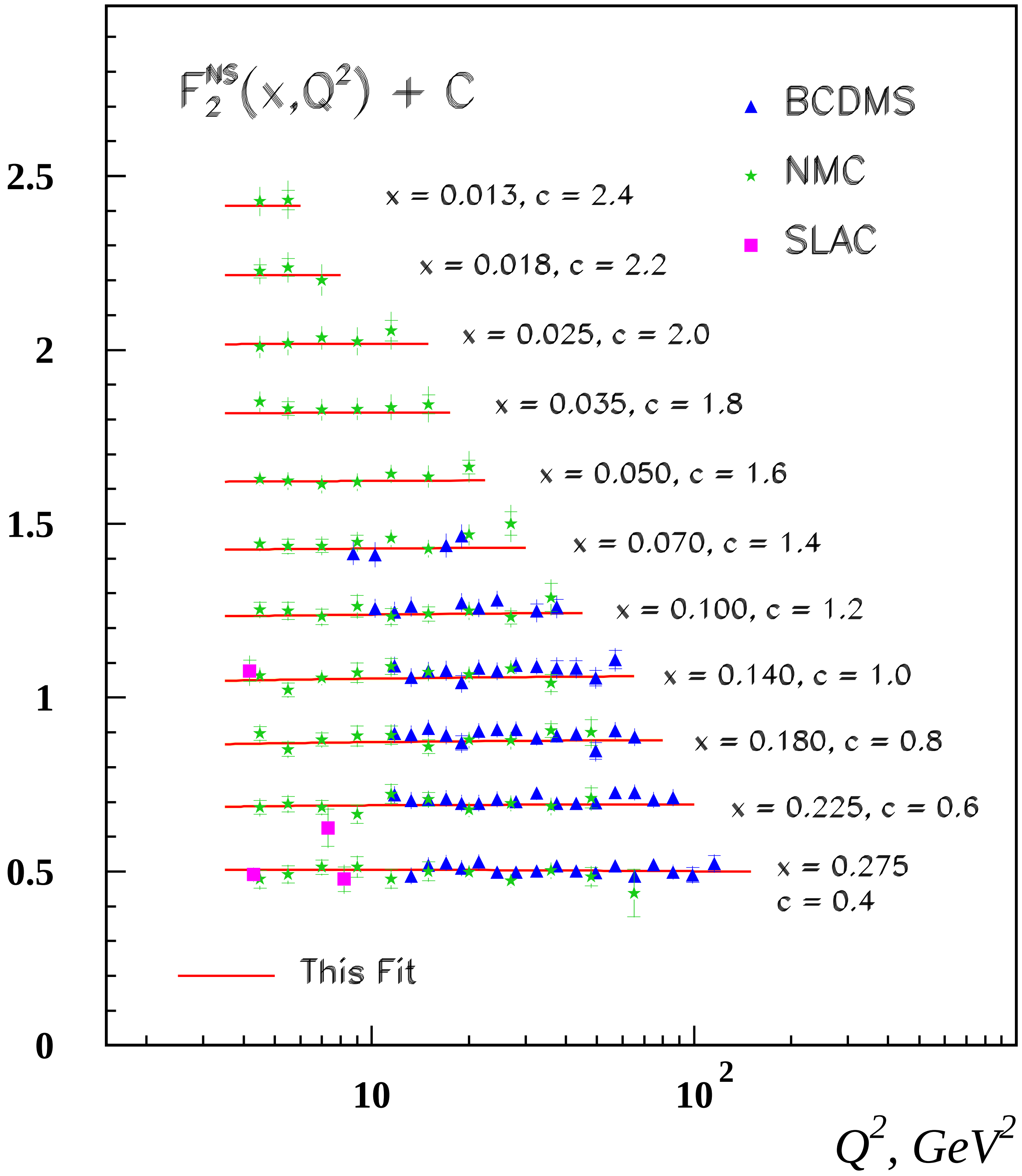
Evolution of the Parton Density $xd_v(x)$

- 4+1 parameter non-singlet fit to F_2 data:



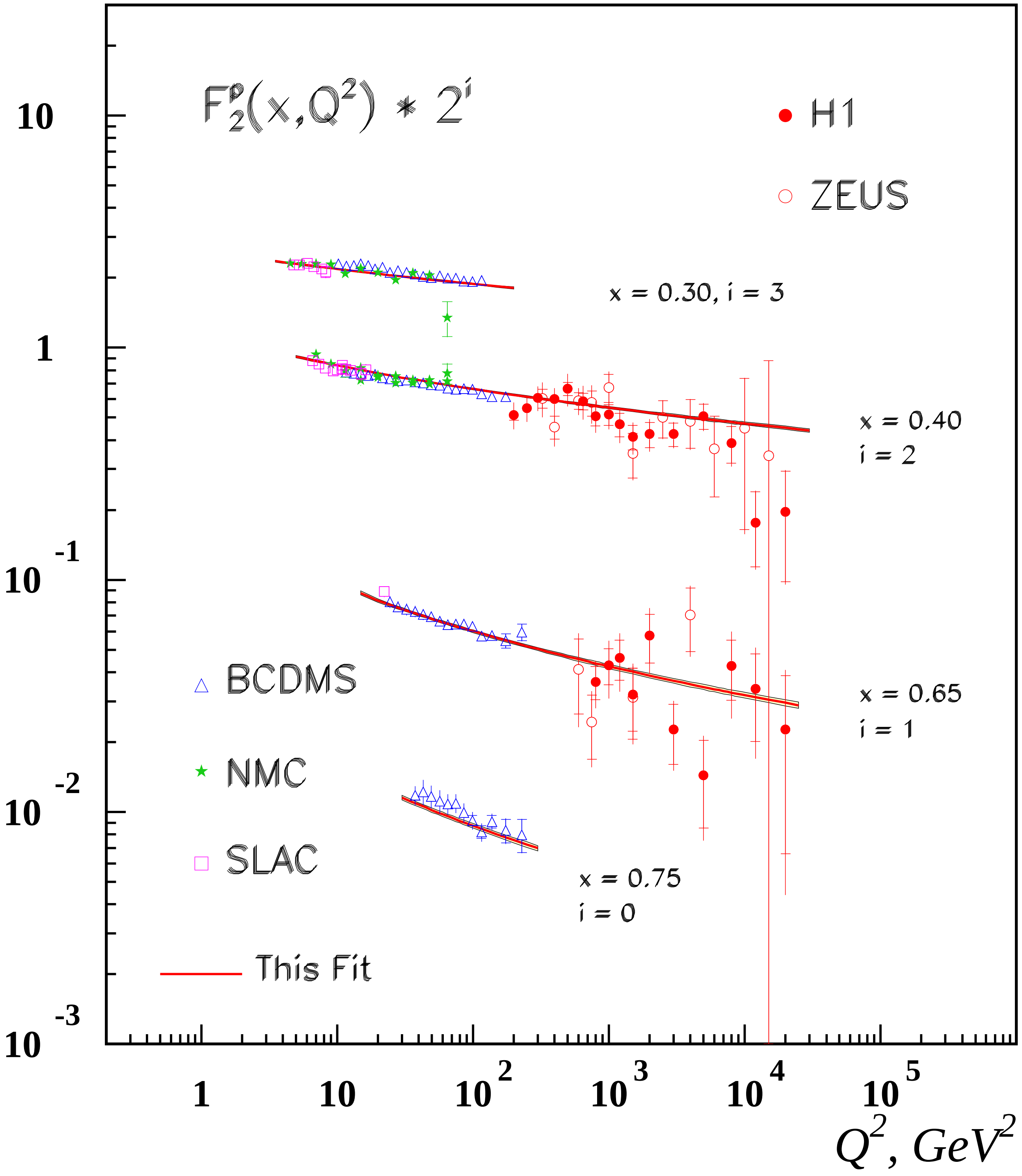
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^{NS}(x)$ **versus** Q^2



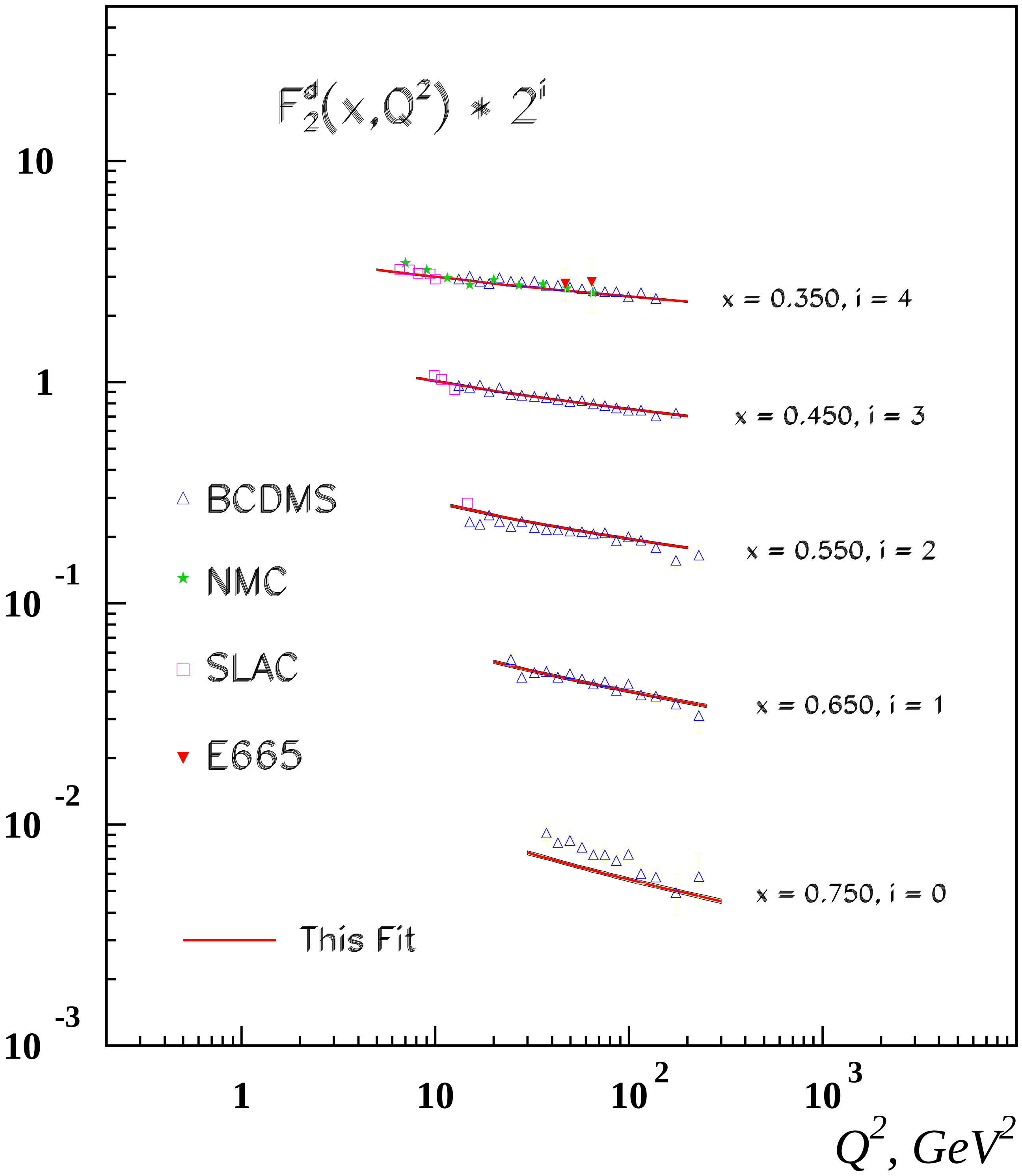
⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^p(x)$ versus Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$F_2^d(x)$ **versus** Q^2



⇒ Yellow error band: Fully correlated 1σ Gaussian error propagation through the evolution equation.

$\Lambda_{QCD}^{(4)} \Rightarrow \alpha_s(M_Z^2)$

	$\Lambda_{QCD}^{(4)}$, MeV	$\alpha_s(M_Z^2)$
This Fit	227 ± 30	0.1135 $^{+0.0023}_{-0.0026}$ (expt)

$\Rightarrow$ $\Lambda_{QCD}^{(4)}$ stable against a variation of the Q^2-cut on the data $(4\,,7\,,10\,GeV^2)$.

$\Rightarrow$ latest world average: 0.1182 ± 0.0027

Ref.: S.Bethke, LL2004, Zinnowitz, April 25-30, 2004

Comparison of $\alpha_s(M_Z^2)$

$\Longrightarrow$ This Fit: $\alpha_s(M_Z^2) = 0.1135 \begin{array}{c} +0.0023 \\ -0.0026 \end{array}$ (expt)

- Comparison with other QCD analyses (significant sea and gluon contributions and correlations):

	$\alpha_s(M_Z^2)$	expt	theory	model	Ref.
NLO					
CTEQ6	0.1165	± 0.0065			[1]
MRST03	0.1165	± 0.0020	± 0.0030		[2]
A02	0.1171	± 0.0015	± 0.0033		[3]
ZEUS	0.1166	± 0.0049		± 0.0018	[4]
H1	0.1150	± 0.0017	± 0.0050	$\begin{array}{c} +0.0009 \\ -0.0005 \end{array}$	[5]
BCDMS	0.110	± 0.006			[6]
BB (pol)	0.113	± 0.004	$\begin{array}{c} +0.009 \\ -0.006 \end{array}$		[7]
NNLO					
MRST03	0.1153	± 0.0020	± 0.0030		[2]
A02	0.1143	± 0.0014	± 0.0009		[3]
SY01(ep)	0.1166	± 0.0013			[8]
SY01(νN)	0.1153	± 0.0063			[8]

[1]: CTEQ Coll.: J.Pumplin et al., JHEP 0207:012 (2002). [2]: MRST Coll.: A.D.Martin et al., hep-ph/0307262. [3]: S.Alekhin, hep-ph/0211096. [4]: ZEUS Coll.: S.Chekanov et al., Phys.Rev.**D67** (2003) 012007. [5]: H1 Coll.: C.Adloff et al., Eur.Phys. **C21** (2001) 33. [6]: BCDMS Coll.: A.C.Benvenuti et al., Phys.Lett. **237** (1990) 592. [7]: J.Blümlein and H.Böttcher, Nucl.Phys. **B636** (2002) 225. [8]: J.Santiago and F.J.Yndurain, Nucl.Phys. **B611** (2001) 447.

Comparison of Moments at $Q^2 = 4.0\,GeV^2$

f	n	This Fit	MRST03	A02
u_v	2	0.289 ± 0.003	0.289	0.304
	3	0.085 ± 0.001	0.084	0.087
	4	0.0324 ± 0.0004	0.032	0.033
d_v	2	0.109 ± 0.004	0.113	0.120
	3	0.025 ± 0.001	0.028	0.028
	4	0.0076 ± 0.0004	0.010	0.010
$u_v - d_v$	2	0.180 ± 0.005	0.176	0.184
	3	0.060 ± 0.001	0.056	0.059
	4	0.0248 ± 0.0006	0.023	0.024

f	n	QCD	Lattice
		This Fit	QCDSF
$u_v - d_v$	2	0.180 ± 0.005	$0.191 \pm 0.012^{*)}$

$$\Longrightarrow \Gamma_f(Q^2) = \int_0^1 x^{n-1} f(x, Q^2) dx$$

Lattice simulation: Scale $\mu^2 = 1/a^2 \sim 4\,GeV^2$.

$*)$ G.Schierholz, private communication.

Summary

- A Non–Singlet QCD Analysis of the Structure Function $F_2(x, Q^2)$ based on the Non–Singlet World Data has been performed to 3–loops.

- The value for $\alpha_s(M_Z^2)$ was determined to be:

$$\alpha_s(M_Z^2)|_{\rm NS} = 0.1135 \begin{array}{c} +0.0023 \\ -0.0026 \end{array} \text{(expt)} \, ,$$

  compatible with results from other QCD analyses and yet with the world average.

- Correlated Errors on all Fit–Parameters were determined and their propagation through the Evolution Equations was performed analytically.

- Analyses by other groups are mainly based on combined Singlet and Non–singlet fits and are therefore subject to correlations between the Sea and Gluon parameters.

- We calculated the low moments of the Distributions $u_v, d_v, u_v - d_v$ with correlated Errors which may be compared to upcoming Lattice Measurements.