Extended BK equation

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1. Motivation
2. Extended Balitzkij-Kovchegov
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4. Talk based on:
   - M.Kimber, J.Kwiecinski, A. Martin
     Phys. Lett. B508(2001) 58-64
   - K.Kutak, J.Kwiecinski
     Eur.Phys.J.C29(2003) 521
   - K.Kutak, A.M.Stasto to be published

1.Motivations

Reliable description of proceses towards the region of very small values of x and large k^2

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- Constrains on parton distributions implied by HERA data.
- BFKL dynamics with subleading corrections.
- Complete DGLAP evolution i.e. large k^2.
- Non-linear screening effects built into the evolution equations.

2.Extended Balitzkij-Kovchegov(BK) equation

Nonlinear evolution equation for unintegrated gluon distribution(linked with the dipole cross-cections):

$$f(x,k^2)=\tilde{f}^{(0)}(x,k^2)+K^1\otimes f-K^2\otimes f^2 \qquad (1)$$

where

$$\tilde{f}^{(0)}(x,k^2)=\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dzP_{gg}(z)\frac{x}{z}g(\frac{x}{z},k_0^2)$$

$$K^1\otimes f=2N_c\frac{\alpha_s(k^2)}{2\pi}k^2\int_x^1\frac{dz}{z}\int_{k_0^2}\frac{dk'^2}{k'^2}$$

$$\left\{\frac{f\left(\frac{x}{z},k'^2\right)\Theta\left(\frac{k^2}{z}-k'^2\right)-f\left(\frac{x}{z},k^2\right)}{|k'^2-k^2|}+\frac{f\left(\frac{x}{z},k^2\right)}{[4k'^4+k^4]^{\frac{1}{2}}}\right\}$$

$$+\frac{\alpha_s(k^2)}{2\pi}\int_x^1\frac{dz}{z}\left[(zP_{gg}(z)-2N_c)\int_{k_0^2}^{k^2}\frac{dk'^2}{k'^2}f\left(\frac{x}{z},k'^2\right)\right.$$

$$K^2\otimes f^2=\left(1-k^2\frac{d}{dk^2}\right)^2\frac{k^2}{R^2}$$

$$\int_x^1\frac{dz}{z}\left[\int_{k^2}^{\infty}\frac{dk'^2}{k'^4}\alpha_s(k'^2)ln\left(\frac{k'^2}{k^2}\right)f(z,k'^2)\right]^2$$

Extended because in linear part we include:

- kinematic constraint on gluon emission $k'^2 < \frac{k^2}{z}$(dominant subleading BFKL effects)
- inclusion of DGLAP evolution

$\downarrow$

How to solve BK equation?

BK equation $\rightarrow$ effective equation

$$\Theta(k^2/z - k'^2) \rightarrow \Theta(k^2 - k'^2) + \left(\frac{k^2}{k'^2}\right)^{\omega_{eff}} \Theta(k'^2 - k^2) \quad (2)$$

where $\omega_{eff} = 0.2$

$$\int_x^1 \frac{dz}{z}[zP_{gg}(z) - 2N_c] f\left(\frac{x}{z}, k'^2\right) \rightarrow \bar{P}_{gg}(\omega = 0) f(x, k'^2) \quad (3)$$

where $\bar{P}_{gg}(\omega)$ is a moment function of $zP_{gg}(z) - 2N_c$, i.e.

$$\bar{P}_{gg}(\omega) = \int_0^1 \frac{dz}{z} z^{\omega}[(zP_{gg}(z) - 2N_c] \quad (4)$$

These approximations $\rightarrow$ nonlinear BK becomes evolution equation in $ln 1/x$.

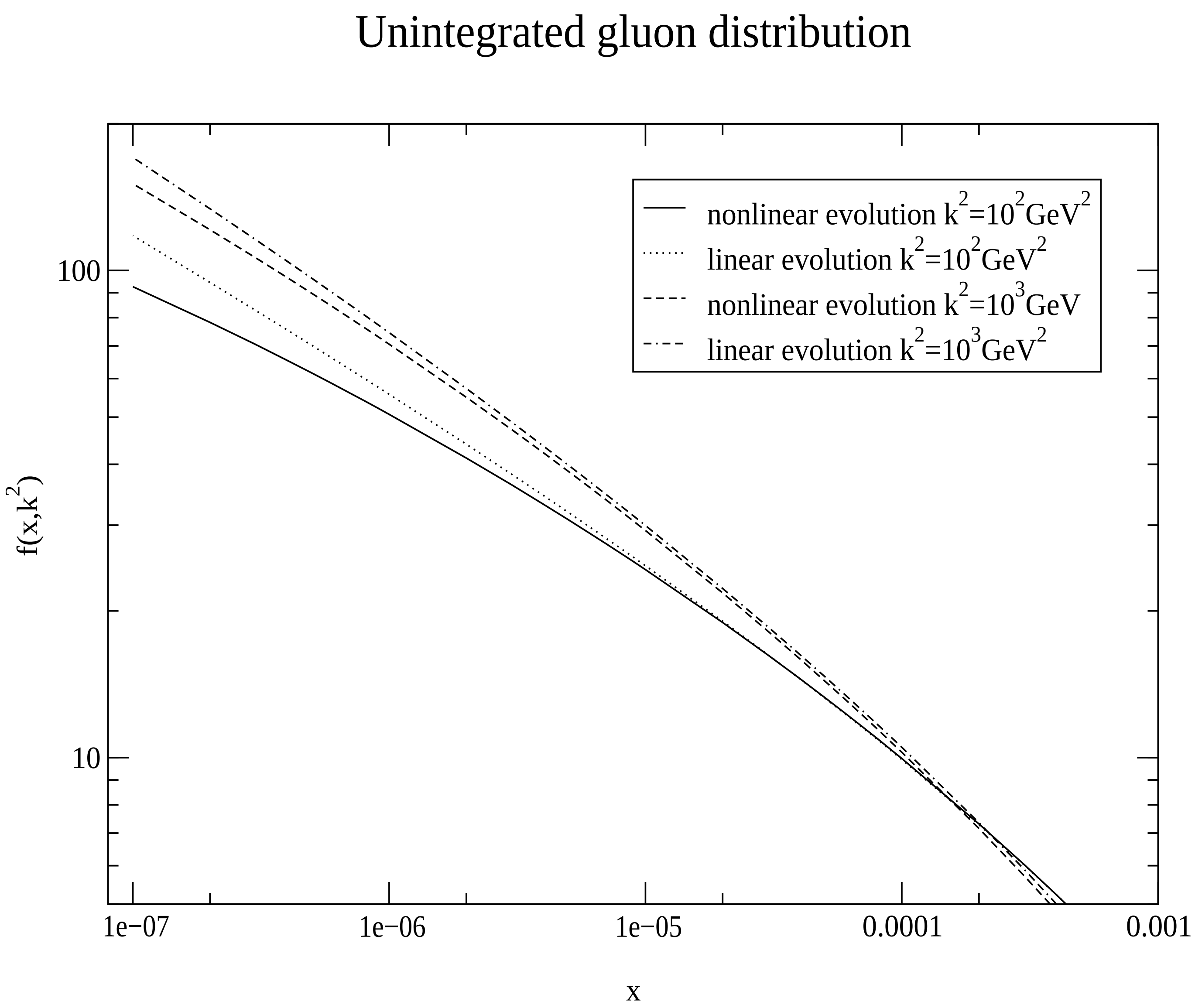

Figure 1: Linear equation vs nonlinear

Origin of suppresion of $f(x, k^2)$

- At very small x the **linear** (DGLAP or BFKL) evolution generates strong increase of the gluon distributions for $x \to 0$ which eventually violates unitarity.
- Non-linear screening corrections tame the growth at small x.
- Emergence of the **saturation scale** $Q_s^2(x) \sim x^{-\lambda}$
- Non-linear effects in the unintegrated gluon distributions are small (eventually negligible) for $k^2 > Q_s^2(x)$.
- Non-linear effects are very strong $\to$ **saturation** for $k^2 < Q_s^2(x)$
- For $k^2 << Q_s^2(x)$, $f << f^{linear}$.

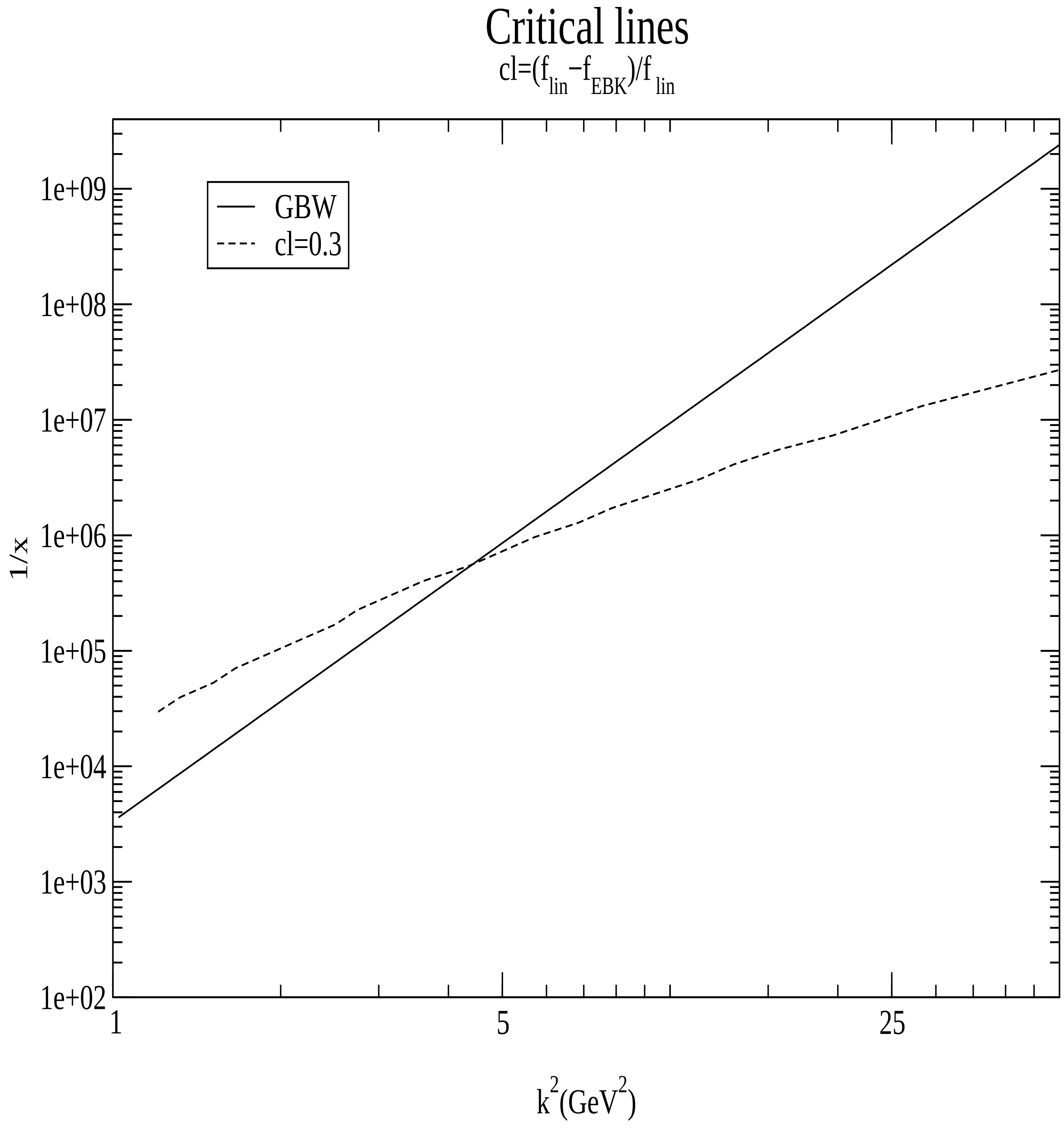

Figure 2: Critical line

4. Conclusions

Presented equation allows to study prcesses for very small x and large k^2 and takes into account nonlinear effects.

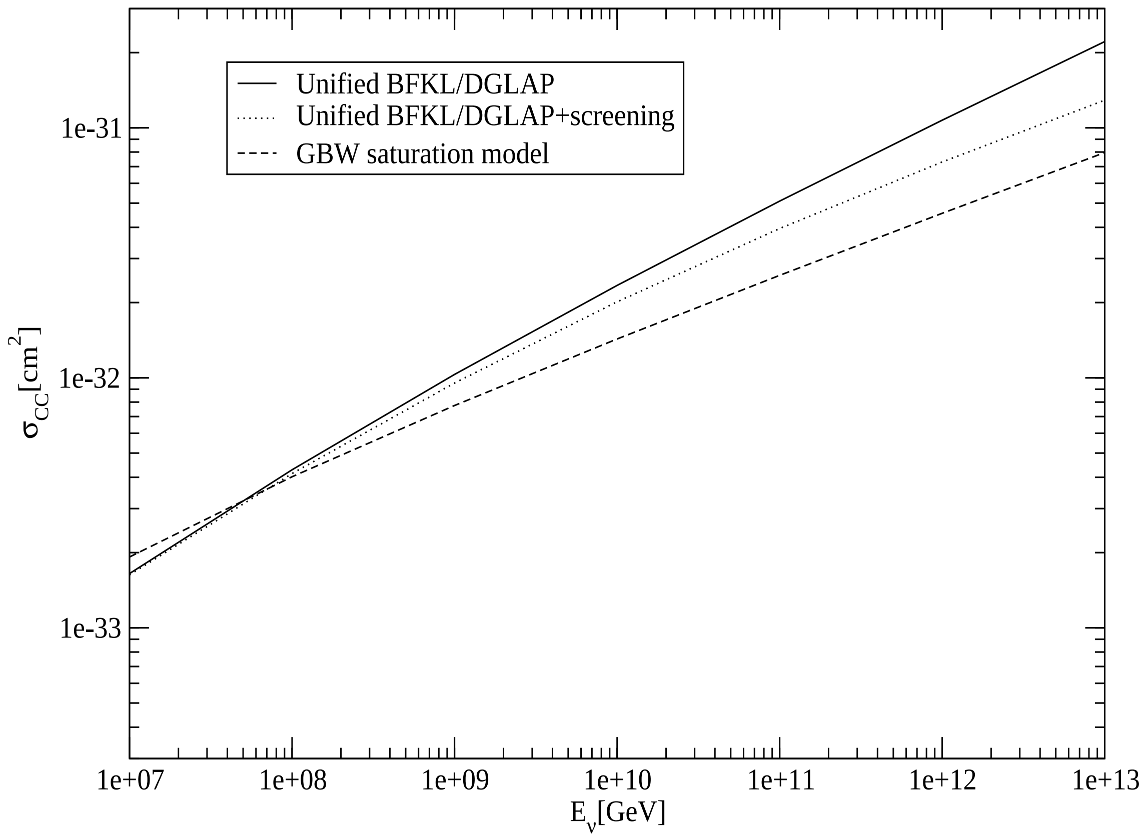

Figure 3: Ultrahigh energy neutrino-nucleon cross sections