Low x Evolution beyond the Balitsky - Kovchegov Equation
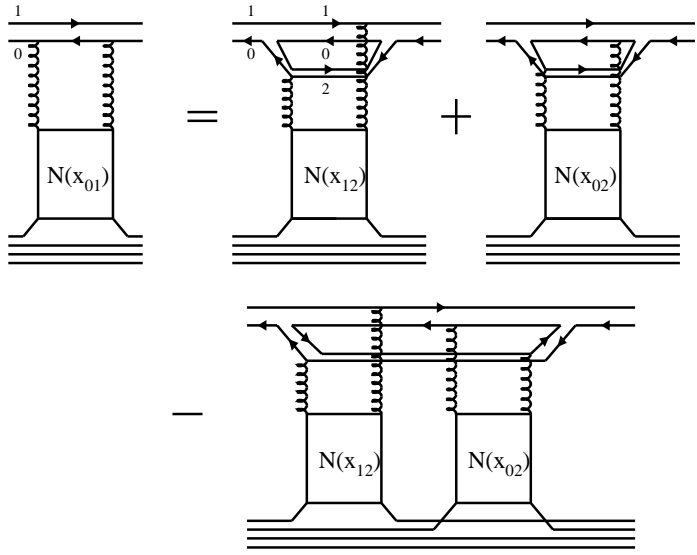
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DESY/Theory

Balitsky-Kovchegov Non-linear Evolution

GLR (81)
Mueller & Qiu (86)
....
Balitsky (95)
Kovchegov (99)
Braun (2000)
Iancu, Leonidov & McLerran (2000)

$$\frac{dN\,(\,x_{01}\,,\,y\,;\,b\,)}{dy} \;=\; -\,\frac{2\,C_F\,\alpha_S}{\pi}\,N\,(\,x_{01}\,,\,y\,;\,b\,)\,\ln\frac{x_{01}^2}{\rho^2}\;+$$

$$\frac{C_F\,\alpha_S}{\pi^2}\;\times\;\int_\rho\,d^2\,x_2\,\frac{x_{01}^2}{x_{02}^2\,x_{12}^2}\;\times$$

$$(\,2\,N\,(\,x_{02}\,,\,y\,;\,b\,)\;-\;N\,(\,x_{02}\,,\,y\,;\,b\,)\,N\,(\,x_{12}\,,\,y\,;\,b\,)\,)$$

Approximations: LLog $1/x$; $N_c \to \infty$; α_s - const.; b - large.

$$\frac{d\,N}{d\,y}\;=\;\alpha_s\;Ker\,\otimes\,(\,N\;-\;N\,N\,)\,. \qquad\qquad BKE$$

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BKE = LO BFKL + Unitarization (non-linearity)

Theoretical successes

- LO BFKL evolution
- s-channel unitarity of partial waves (fixed impact parameter)
- Gluon saturation
- Infrared diffusion problem

Phenomenological successes

- F_2 DIS data
  E. Gotsman, E. Levin, M.L, U. Maor (2002)
  E. Iancu, S. Munier and K. Itakura (2003)
  J/Ψ production
  E. Gotsman, E. Levin, M.L, U. Maor and E. Naftali (2003)
- Saturation scale $Q_s(x)$ and geometrical scaling
  M. Braun and N. Armesto (2001);
  M.L. (2001);
  K. Golec-Biernat, L. Motyka and A. Stasto (2001)
- Diffractive Dissociation
  E. Levin and M.L. (2001)

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• Approximations

- LO BFKL: soft gluon emission approx. (fixed α_s)

- Large N_c - dipole picture - mean field theory without correlations

- No target correlations

Problems

- Violates the t-channel unitarity (Pomeron loops)

- Violates the s-channel unitarity at intermediate steps of the evolution
  A. Mueller and A. Shoshi (2004)

- Violates the Froissart bound

- Needs high order corrections including running of α_s

- NLO BFKL (α_s corrections)

  D.N. Triantafyllopoulos (2002) NLO BFKL + saturation boundary;

  I. Balitsky and A. Belitsky (2001) α_s suppressed cubic term:

$$\frac{d\,N}{d\,y} = \alpha_s\,Ker \otimes (N - N\,N) - \alpha_s^2\,\tilde{Ker} \otimes N\,N\,N.$$

G. Chachamis, M.L., A. Sabio-Vera (2004?) BKE + rapidity veto

$$\frac{d\,N(y)}{d\,y} = \alpha_s\,Ker \otimes (N(y-\eta) - N(y-\eta)\,N(y-\eta))$$

- N_c corrections (JIMWLK equation = Balitsky's chain)

$$N \equiv \langle W \rangle_{target}$$

$$\frac{d\langle W \rangle}{d\,y} = \alpha_s\,Ker \otimes (\langle W \rangle - \langle W\,W \rangle). \qquad Bal.\ 1$$

$$\frac{d\langle W\,W \rangle}{d\,y} = \alpha_s\,Ker \otimes (\langle W\,W \rangle - \langle W\,W\,W\,W \rangle). \quad Bal.\ 2$$

Kovchegov: $\langle W\,W \rangle = \langle W \rangle\langle W \rangle = N\,N$; $N_c \to \infty$
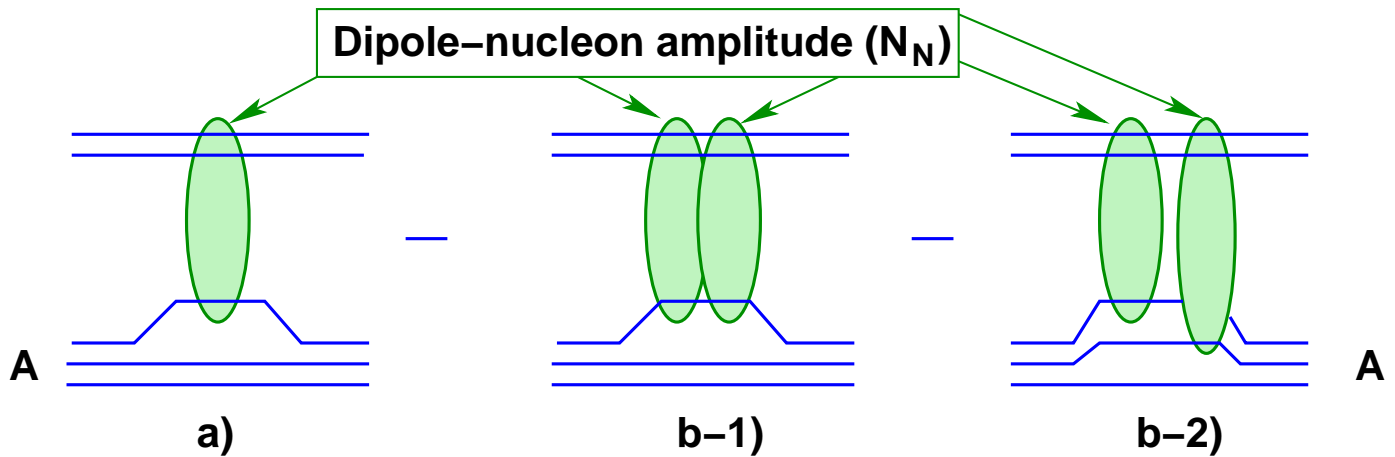
K. Rummukainen and H. Weigert (2003) numerical solution of JIMWLK;

J. Bartels, L. Lipatov, and G.P. Vacca (2004)

N_c correction to triple Pomeron vertex:

$$\frac{d\,N}{d\,y} = \alpha_s\,Ker \otimes (N - N\,N - \frac{1}{N_c^2}\,n)$$

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• Target correlations for proton and realistic (not dense) nuclei
E. Levin and M.L. (2003) Linear functional differential equation

Dipole–nucleon amplitude (N_N)

A   A

a)   b–1)   b–2)

Model dependent modification of the BKE:

$$\frac{d\,N_A}{d\,y} = \alpha_s\,Ker\,\otimes\,(\,N_A\,-\,\frac{1+\kappa}{\kappa}\,N_A\,N_A\,);$$

$$\kappa\;=\;\pi\,R_p^2\,S_A(b)\;\propto\;A^{1/3}$$

• Pomeron loops
E. Iancu and A. Mueller (2003); M. Kozlov and E. Levin (2004)
cannot be formulated as an equation.

$$N(Y)\;=\;1\,-\,e^{-\,c\,(Y-Y_0)^2}\,;\quad Y\,\rightarrow\,\infty\qquad BKE$$

$$N(Y)\;=\;1\,-\,e^{-\,1/2\,c\,(Y-Y_0)^2}\,;\quad Y\,\rightarrow\,\infty\qquad PL$$
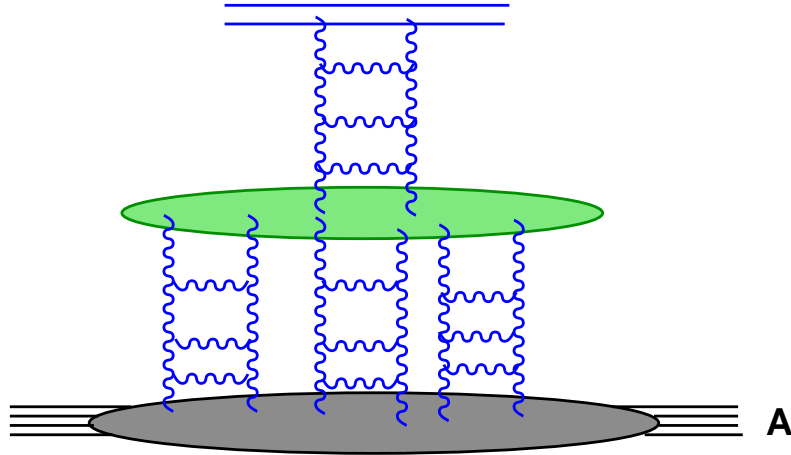
$$c\;=\;2\,\bar{\alpha}_s$$

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- **Multi Pomeron vertices**

I. Balitsky and A. Belitsky (2001) α_s suppressed cubic term;

E. Levin and M. Lublinsky (2003, 2004?)
eikonal resummation of all multi-pomeron vertices - modified BKE

$$\frac{dN}{dy} = (1 - N)\,\alpha_s\,Ker \otimes (N - N\,N).$$

BKE: $N \ll 1$; $N \sim N^2$

- Impact parameter dependence

- Dipole picture vs. traditional diagramatics

- **Simple** effective Reggeon field theory in QCD

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