Effects of energy-momentum conservation in the dipole formalism of Mueller and the BK eq.

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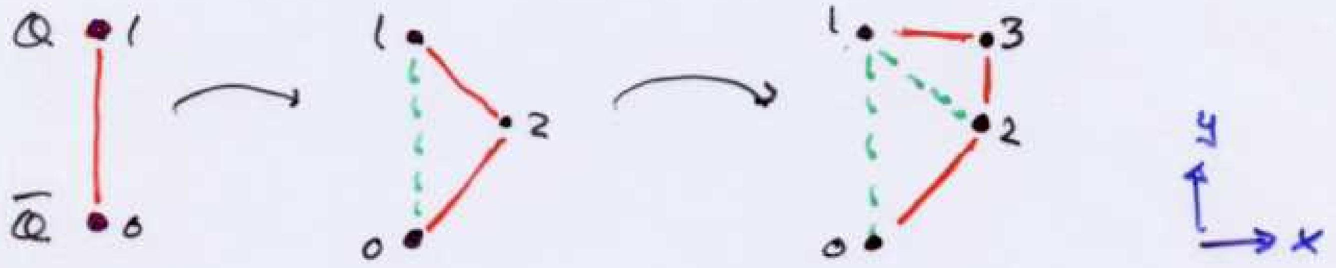
(presented by Gösta Gustafson)

$$\gamma^* \to \underset{\substack{\uparrow \\ \text{colour} \\ \text{dipde}}}{Q\bar{Q}} \to Q\underset{\substack{\uparrow \\ \text{2 dipoles}}}{g}\bar{Q} \to Qgg\bar{Q}$$



$$\frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi}\frac{d^2\bar{r}_2\, r_{01}^2}{r_{02}^2\, r_{12}^2} \cdot \underset{\uparrow}{Sud}$$

$$\exp\left[-\frac{\bar{\alpha}}{2\pi}\int dy \underset{\substack{\uparrow \\ \text{diverges if no cutoff}}}{\int d^2 r_2} \frac{r_{01}^2}{r_{02}^2\, r_{12}^2}\right]$$

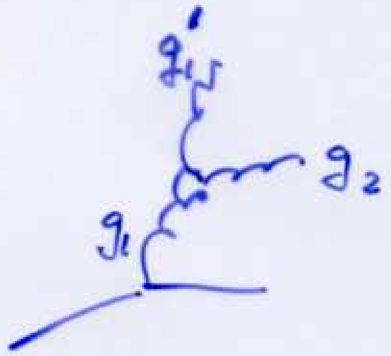
$r_{02} > \rho$
$r_{12} > \rho$

ρ small ⇒ very many dipoles with small r

$\bar{r}$ well localized (r small) $\Rightarrow$ $k_\perp$ large

$$k_\perp \sim \frac{1}{r}$$

} possible to separate $\Rightarrow$ g_1 gets a recoil



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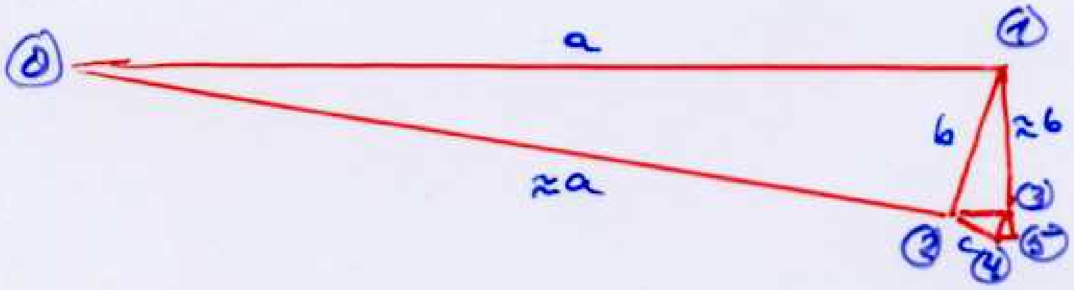
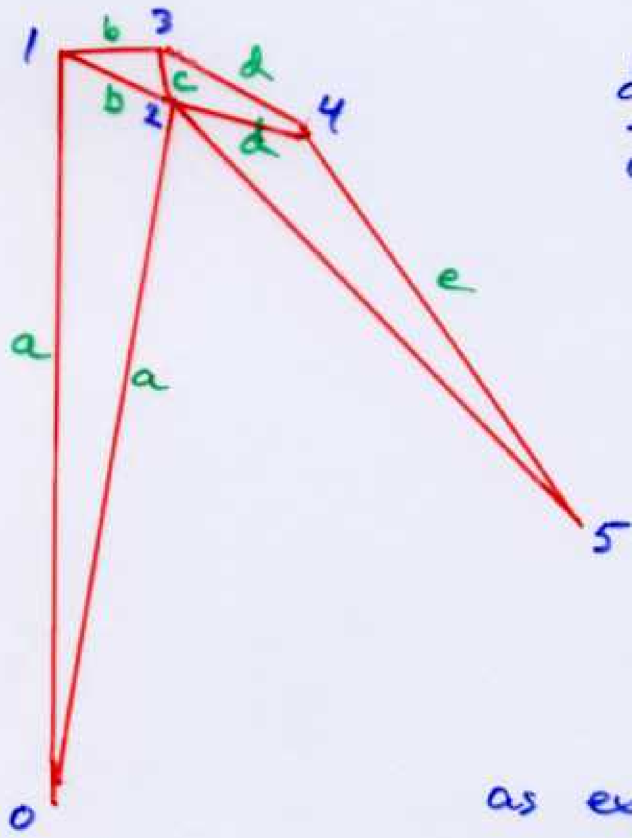
DGLAP chain : $k_\perp$ increasing



$$\frac{d^2 r_2\, a^2}{a^2 b^2} \cdot \frac{d^2 r_3 \cdot b^2}{b^2 c^2} \cdot \frac{d^2 r_4 \cdot c^2}{c^2 d^2} \cdot \ldots$$

$$= \pi \frac{d^2 r_i}{r_i^2} \sim \pi \frac{d^2 k_{\perp i}}{k_{\perp i}^2}$$

$k_\perp$ increases to $k_{\perp\,\max}$, and then decreases



$$\frac{d^2r_2 \cdot a^2}{a^2\, b^2} \cdot \frac{d^2 r_3\, b^2}{b^2\, \cancel{c^2}} \cdot \frac{d^2 r_4 \cdot \cancel{c^2}}{d^2\, d^2} \cdot$$

$$\cdot \sim \frac{d^2 r_3}{c^0}$$

$$\sim \frac{d^2 k_\perp}{k_\perp^4}$$

as expected for a hard scatt.

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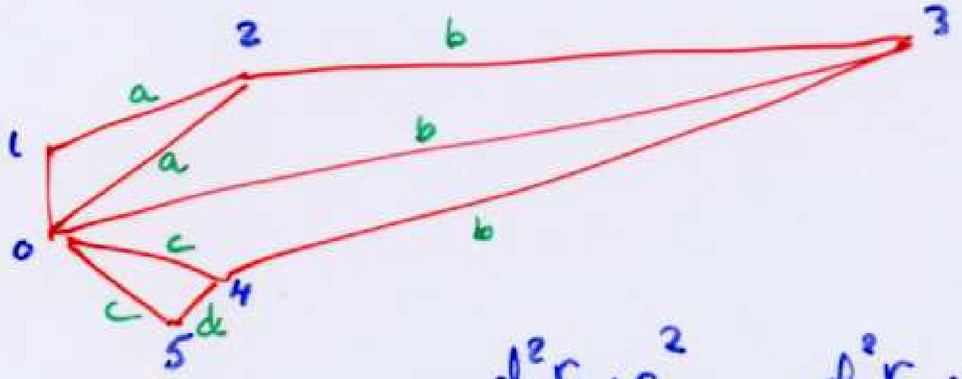
Maximum r, minimum $k_\perp$

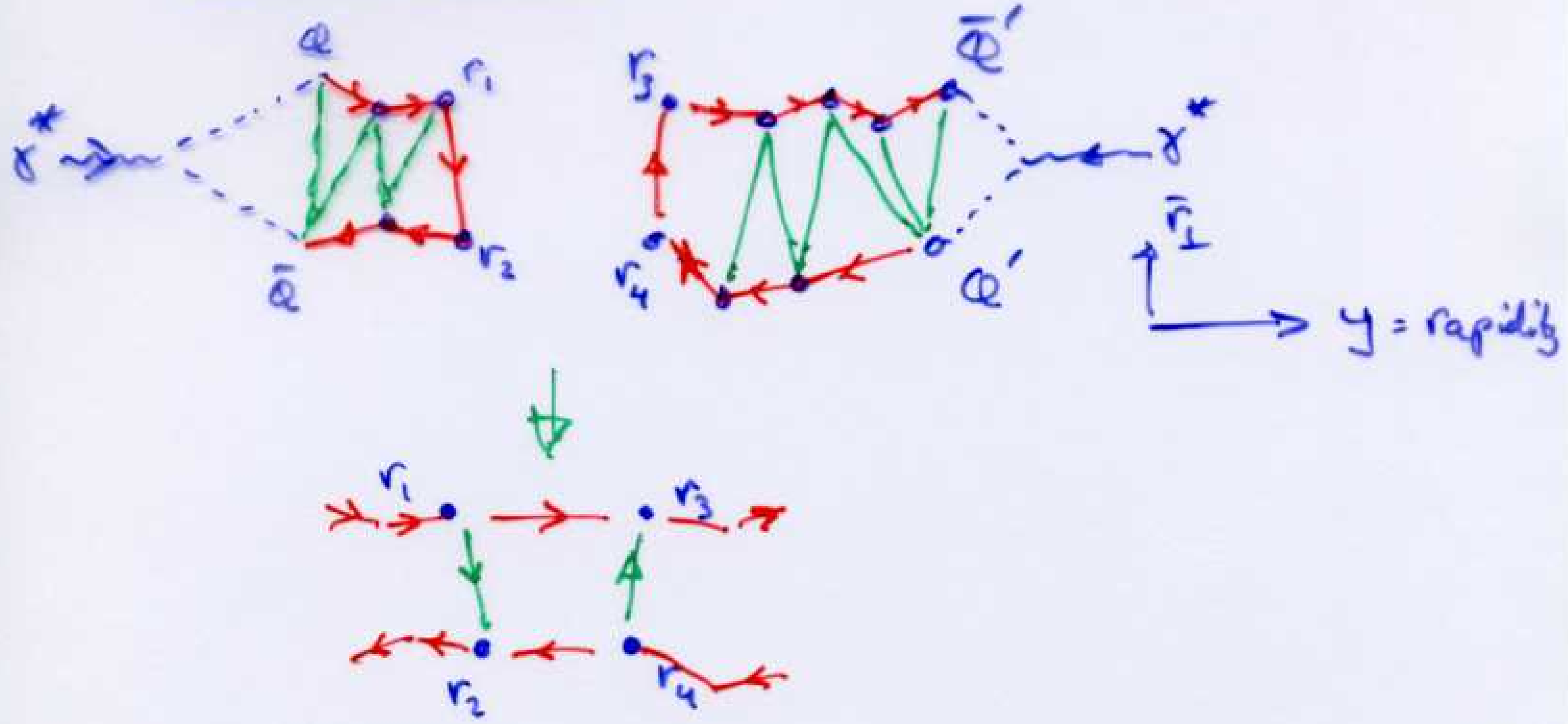


$$\ldots \frac{d^2 r_3 \cdot a^2}{b^4} \cdot \frac{d^2 r_4 \cdot b^2}{b^2 \cdot c^2} \ldots$$

$$\sim \frac{d^2 r_3}{b^4} \sim d^2 k_\perp$$

as in LDC, the Linked Dipole Chain model

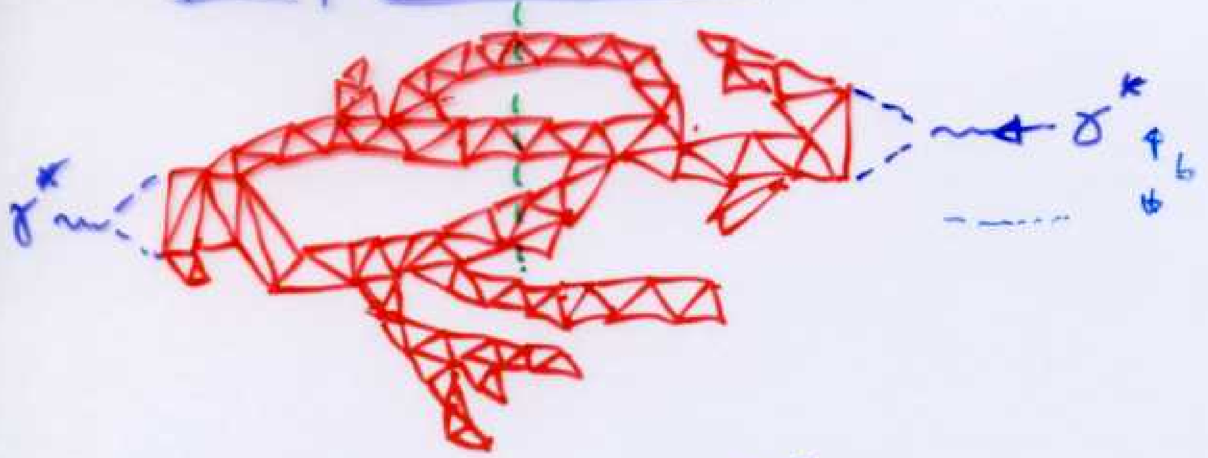
$\gamma^* \gamma^*$ scattering



Dipole-dipole scattering:

$$f = \frac{\alpha_s^2}{2} \left\{ \ln \left[\frac{|\bar{r}_1 - \bar{r}_3| \cdot |\bar{r}_2 - \bar{r}_4|}{|\bar{r}_1 - \bar{r}_4| \cdot |\bar{r}_2 - \bar{r}_3|} \right] \right\}^2$$

Multiple interactions



$$\sigma \sim \int d^2b \left(1 - e^{-\sum f_{ij}}\right)$$

i = left dipole
j = right "

Small cutoff ρ $(r > \rho)$ $\Rightarrow$

Very many small dipoles

Interpreted as real emissions $\Rightarrow$

$\Rightarrow$ Energy-momentum not conserved


must be regarded as a single "effective" dipole → energy conserv.

$\Rightarrow$ Dynamic cutoff $\rho(\Delta y)$

Left-right symmetric formalism:

p_- conservation ⇒ also max. value for r.

Suitable for MC simulation

Results: The # dipoles grows much more slowly with energy.

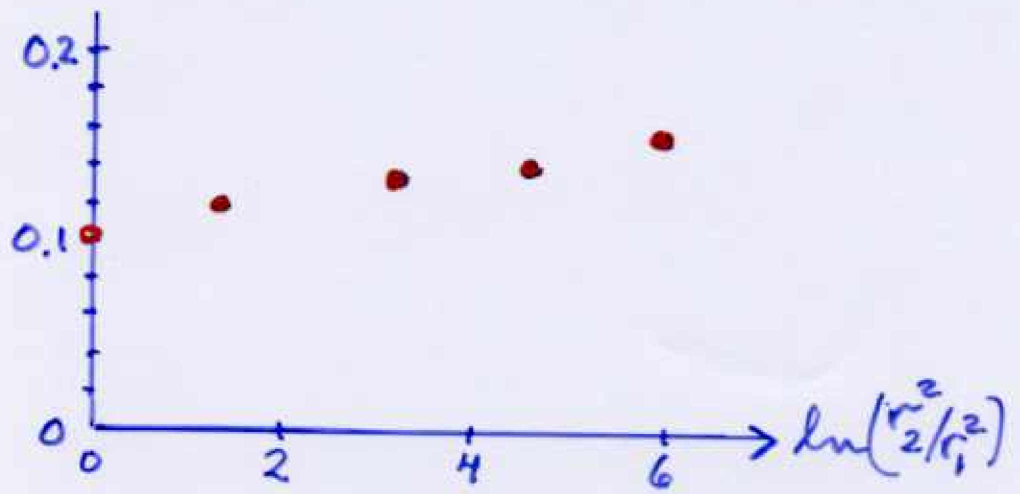
Straight forward to calculate cross sections and study saturation effects by comparing

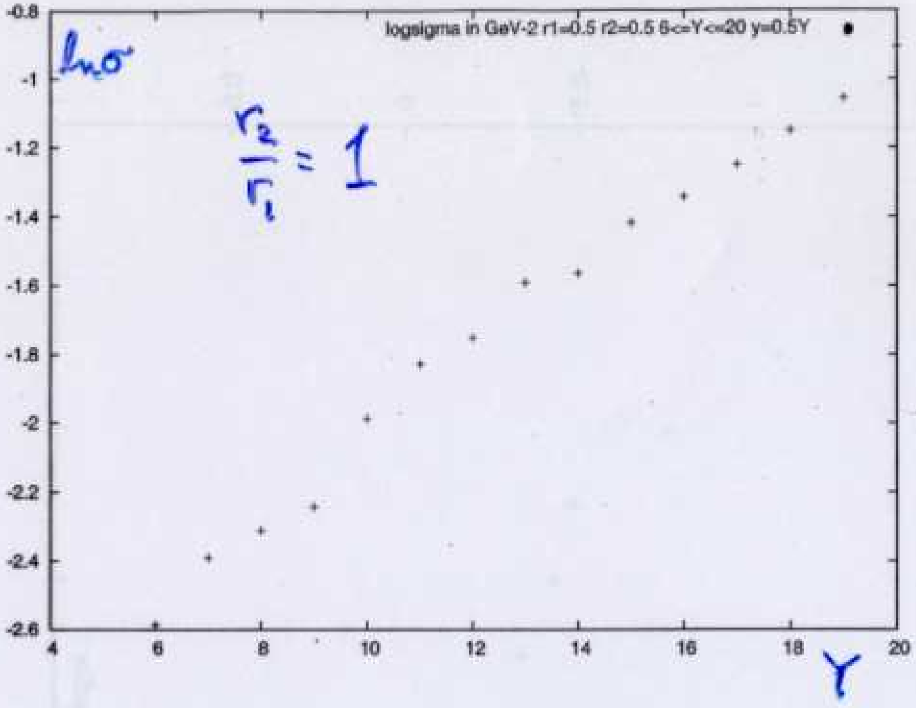
$$\int d^2b\,(1-e^{-\Sigma f_{ii}}) \text{ with } \int d^2b\,\Sigma f_{ij}$$

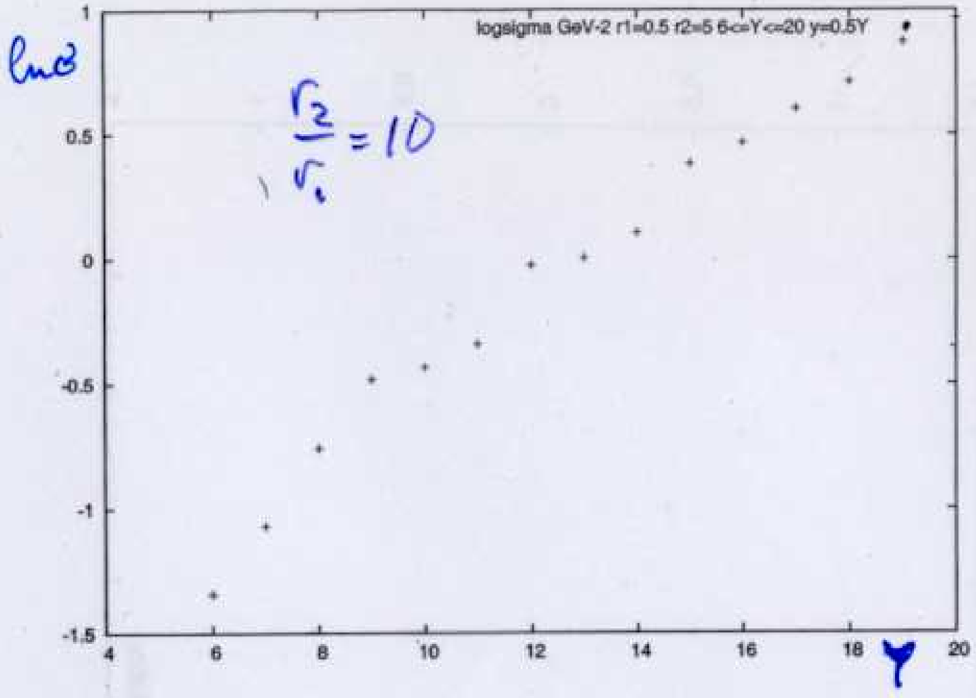
* * *

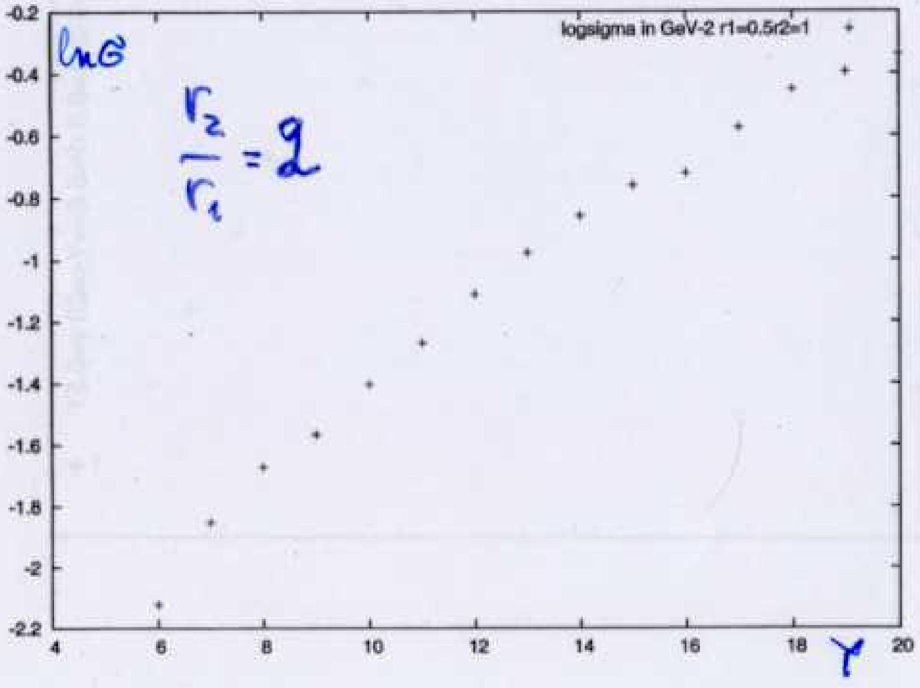
fig.

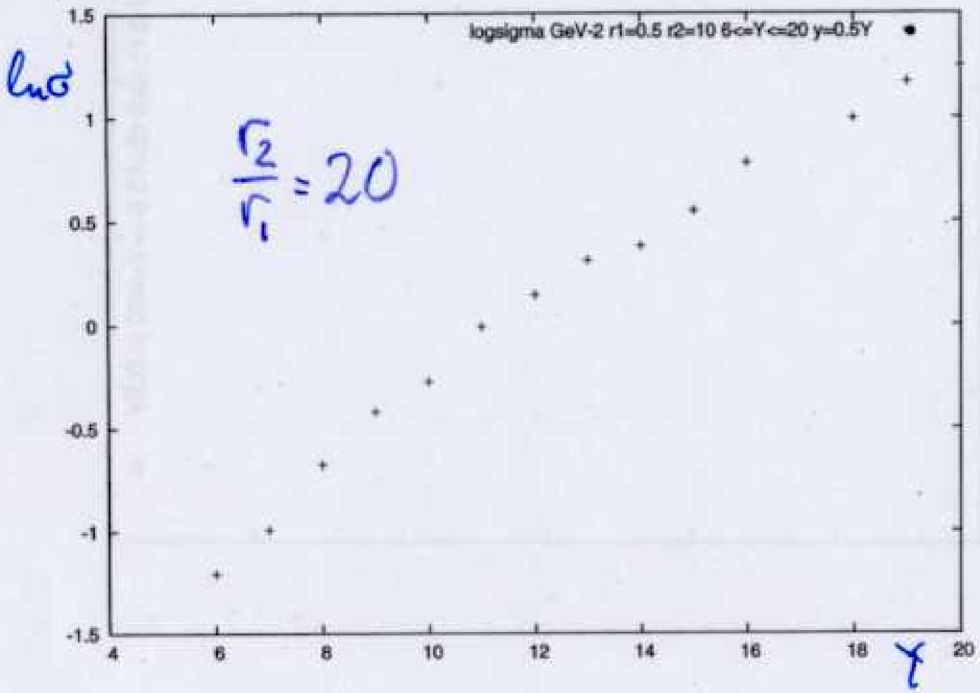
$\sigma \sim e^{\lambda y} \sim \frac{1}{x^\lambda}$; λ depends on r_2/r_1 = ratio between initial dipoles $\sim Q_1/Q_2$

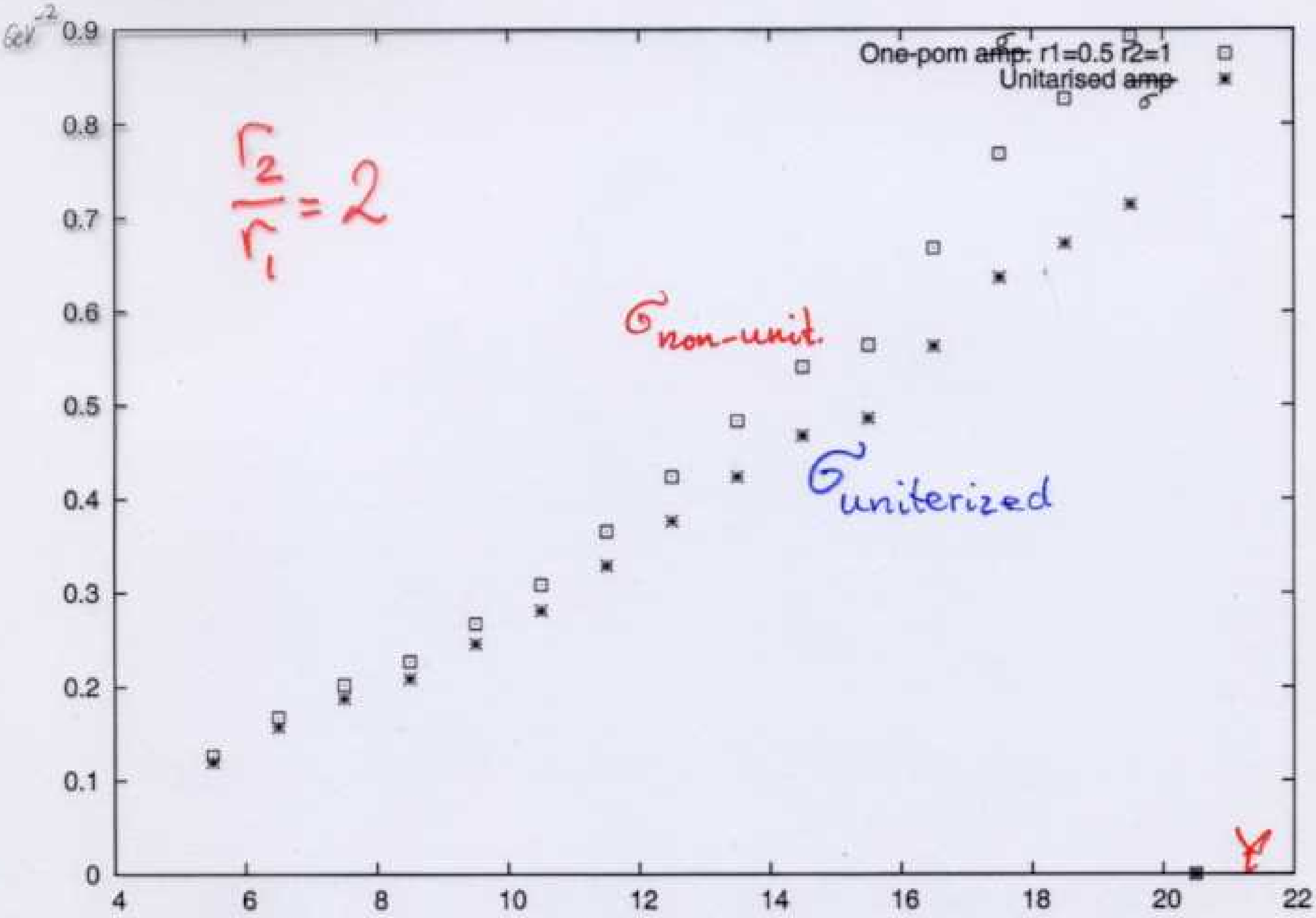


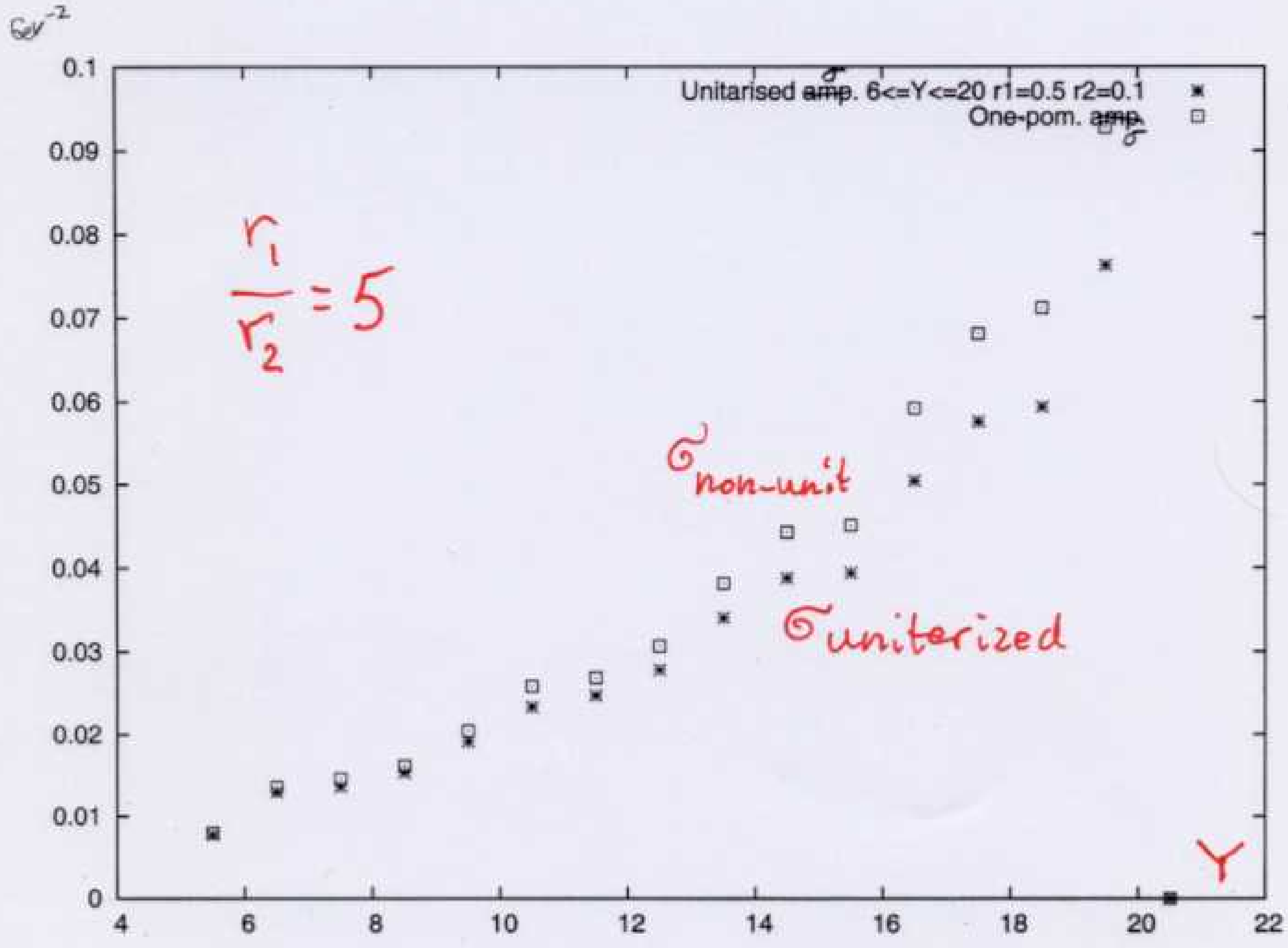

logsigma in GeV-2 r1=0.5 r2=0.5 6<=Y<=20 y=0.5Y
lnσ
$\frac{r_2}{r_1} = 1$
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
-2.4
-2.6
4
6
8
10
12
14
16
18
20
Y


logsigma GeV-2 r1=0.5 r2=5 6<=Y<=20 y=0.5Y
lnσ
$\frac{r_2}{r_1} = 10$
1
0.5
0
-0.5
-1
-1.5
4
6
8
10
12
14
16
18
20
Y


logsigma in GeV-2 r1=0.5r2=1
lnσ
$\frac{r_2}{r_1} = 2$
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
-2.2
4
6
8
10
12
14
16
18
20
Y


logsigma GeV-2 r1=0.5 r2=10 6<=Y<=20 y=0.5Y
lnσ
$\frac{r_2}{r_1} = 20$
1.5
1
0.5
0
-0.5
-1
-1.5
4
6
8
10
12
14
16
18
20
Y


GeV^{-2}
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
4
6
8
10
12
14
16
18
20
22
One-pom ~~amp.~~ σ r1=0.5 r2=1
Unitarised ~~amp~~ σ
$\frac{r_2}{r_1} = 2$
$\sigma_{\text{non-unit.}}$
$\sigma_{\text{uniterized}}$
Y


GeV^{-2}
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
4
6
8
10
12
14
16
18
20
22
Unitarised ~~amp.~~ σ 6<=Y<=20 r1=0.5 r2=0.1
One-pom. ~~amp.~~ σ
$\frac{r_1}{r_2} = 5$
$\sigma_{\text{non-unit}}$
$\sigma_{\text{uniterized}}$
Y

Note: Also non-colliding chains are treated as real final state sub-systems

⇒ The effects of energy conservation is somewhat overestimated in the results presented here.

Conclusions

Energy-momentum conservation ⇒

The # "effective gluons" grows much more slowly

$k_\perp$-ordered chains $\rightarrow \prod \frac{dk_\perp^2}{k_\perp^2}$ as in DGLAP

$$\left.\begin{array}{l} \text{Maximum } k_\perp \rightarrow \frac{dk_\perp^2}{k_\perp^4} \\ \text{Minimum } k_\perp \rightarrow dk_\perp^2 \end{array}\right\} \text{ as in LDC}$$

Preliminary results from a MC event generator:

- $\sigma \sim \exp(\lambda Y)$ where λ grows with $\frac{r_1^2}{r_2^2} \sim \frac{Q_2^2}{Q_1^2}$

- Saturation effects due to multiple collisions ~ 20% for $Y \sim 20$ (for $r_2/r_2 \sim 2-5$)