

Extended BK equation

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4. Talk based on:

- K.Kutak, J.Kwieciński
Eur.Phys.J.C29(2003) 521 [arXiv:hep-ph/0303209]
- K.Kutak, A.M.Staśto
to be published

1. Motivations

Reliable description of processes towards the region of **very small** values of x and **large** k^2



- **Constrains** on parton distributions implied by HERA data.
- **BFKL** dynamics with subleading corrections.
- Complete **DGLAP** evolution i.e. large k^2 .
- **Non-linear** screening effects built into the evolution equations.

2. Extended Balitzkij-Kovchegov(BK) equation

Nonlinear evolution equation for unintegrated gluon distribution(linked with the dipole cross-sections):

$$f(x, k^2) = \tilde{f}^{(0)}(x, k^2) + K^1 \otimes f - K^2 \otimes f^2 \quad (1)$$

where

$$\begin{aligned} \tilde{f}^{(0)}(x, k^2) &= \frac{\alpha_s(k^2)}{2\pi} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, k_0^2\right) \\ K^1 \otimes f &= 2N_c \frac{\alpha_s(k^2)}{2\pi} k^2 \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk'^2}{k'^2} \\ &\left\{ \frac{f\left(\frac{x}{z}, k'^2\right) \Theta\left(\frac{k^2}{z} - k'^2\right) - f\left(\frac{x}{z}, k^2\right)}{|k'^2 - k^2|} + \frac{f\left(\frac{x}{z}, k^2\right)}{[4k'^4 + k^4]^{\frac{1}{2}}} \right\} \\ &+ \frac{\alpha_s(k^2)}{2\pi} \int_x^1 \frac{dz}{z} \left[(zP_{gg}(z) - 2N_c) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f\left(\frac{x}{z}, k'^2\right) \right] \end{aligned}$$

$$\begin{aligned} K^2 \otimes f^2 &= \left(1 - k^2 \frac{d}{dk^2}\right)^2 \frac{k^2}{R^2} \\ &\int_x^1 \frac{dz}{z} \left[\int_{k^2}^{\infty} \frac{dk'^2}{k'^4} \alpha_s(k'^2) \ln\left(\frac{k'^2}{k^2}\right) f(z, k'^2) \right]^2 \end{aligned}$$

Extended because in linear part we include:

- kinematic constraint on gluon emission $k'^2 < \frac{k^2}{z}$ (dominant subleading BFKL effects)
- inclusion of DGLAP evolution



How to solve BK equation?

BK equation \rightarrow effective equation

$$\Theta(k^2/z - k'^2) \rightarrow \Theta(k^2 - k'^2) + \left(\frac{k^2}{k'^2}\right)^{\omega_{eff}} \Theta(k'^2 - k^2) \quad (2)$$

where $\omega_{eff} = 0.2$

$$\int_x^1 \frac{dz}{z} [zP_{gg}(z) - 2N_c] f\left(\frac{x}{z}, k'^2\right) \rightarrow \bar{P}_{gg}(\omega = 0) f(x, k'^2) \quad (3)$$

where $\bar{P}_{gg}(\omega)$ is a moment function of $zP_{gg}(z) - 2N_c$, i.e.

$$\bar{P}_{gg}(\omega) = \int_0^1 \frac{dz}{z} z^\omega [(zP_{gg}(z) - 2N_c)] \quad (4)$$

These approximations \rightarrow **nonlinear** BK becomes evolution equation in $\ln 1/x$.

Unintegrated gluon distribution

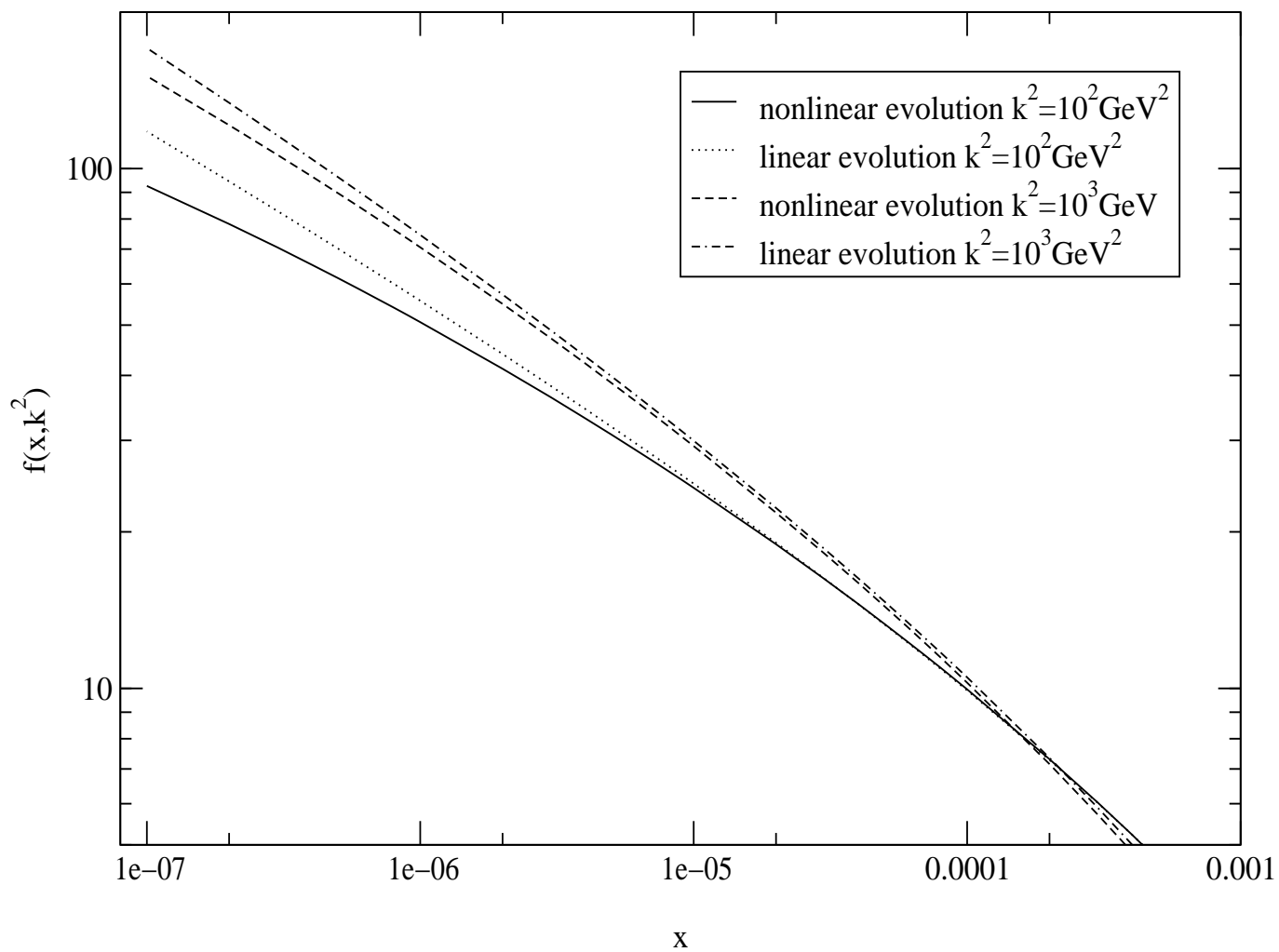


Figure 1: Linear equation vs nonlinear

Origin of suppression of $f(x, k^2)$

- At very small x the **linear** (DGLAP or BFKL) evolution generates strong increase of the gluon distributions for $x \rightarrow 0$ which eventually violates unitarity.
- Non-linear screening corrections tame the growth at small x .
- Emergence of the **saturation scale** $Q_s^2(x) \sim x^{-\lambda}$
- Non-linear effects in the unintegrated gluon distributions are small (eventually negligible) for $k^2 > Q_s^2(x)$.
- Non-linear effects are very strong \rightarrow **saturation** for $k^2 < Q_s^2(x)$
- For $k^2 \ll Q_s^2(x)$, $f \ll f^{linear}$.

Critical lines

$$cl = (f_{lin} - f_{EBK}) / f_{lin}$$

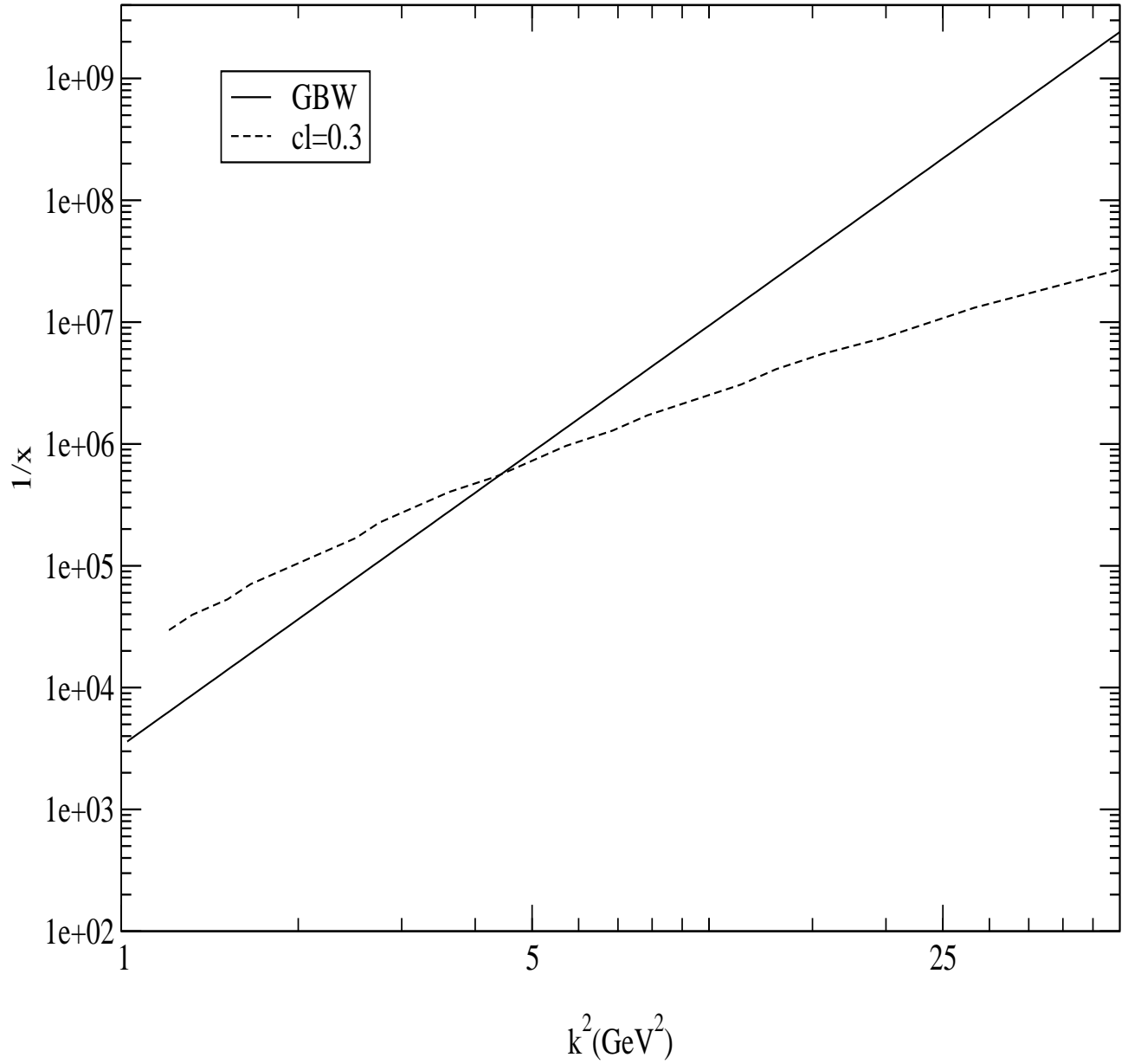


Figure 2: Critical line

4. Conclusions

Presented equation allows to study processes for very small x and large k^2 and takes into account nonlinear effects.

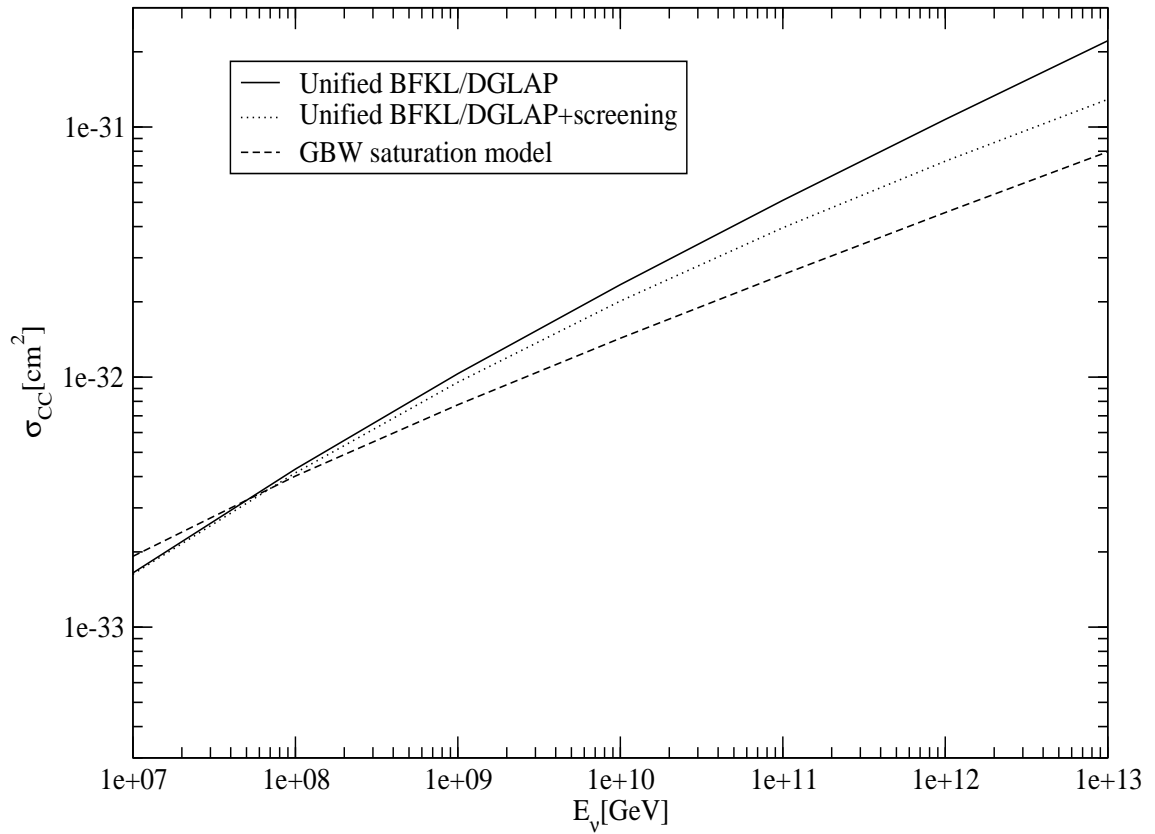


Figure 3: Ultrahigh energy neutrino-nucleon cross sections

LHC parton kinematics

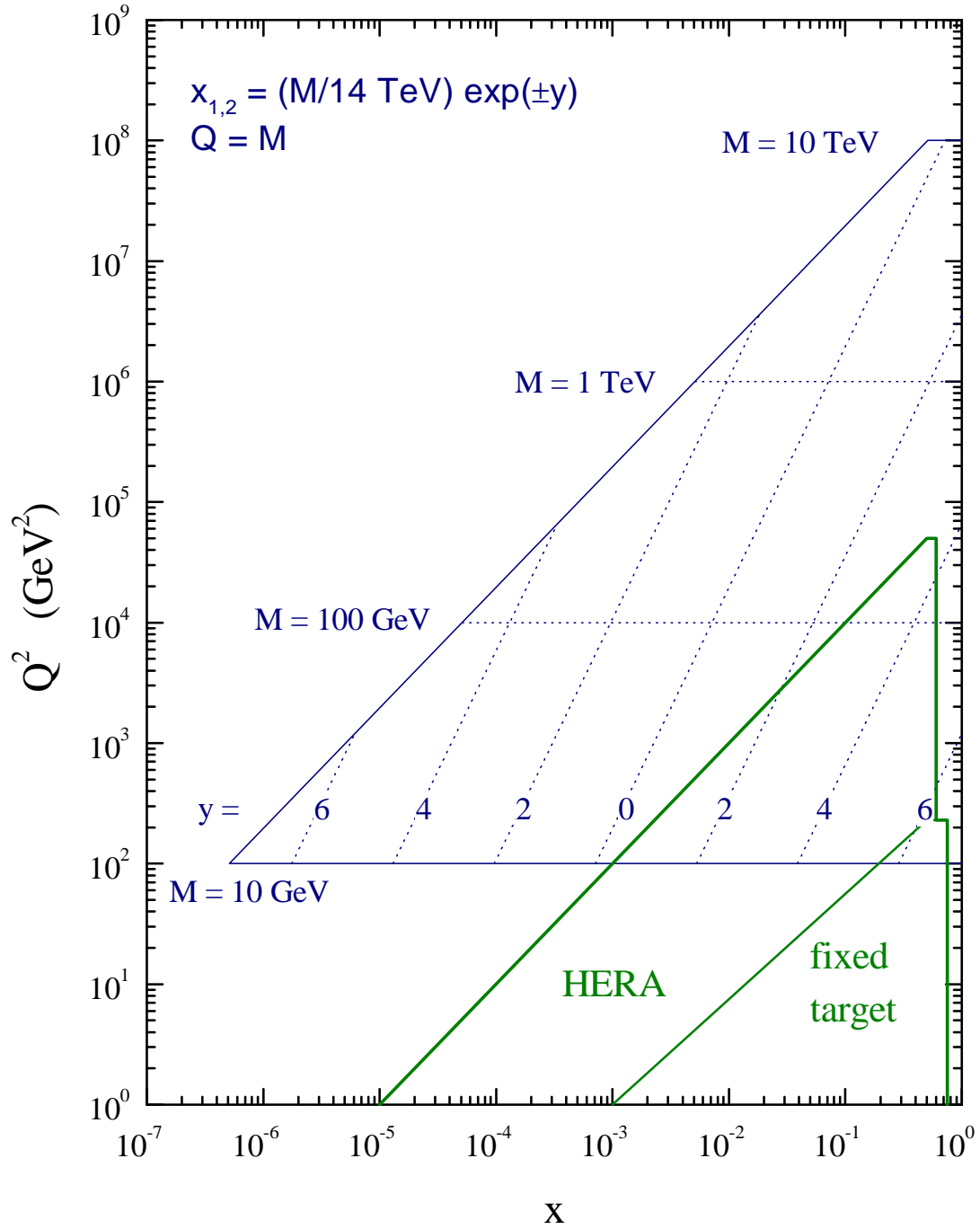


Figure 4: