Resummation at small x

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Collinear poles

Mellin variables:

$$\begin{array}{ccc} \gamma & \leftrightarrow & \ln k^2 \\ \omega & \leftrightarrow & \ln 1/x \end{array}$$

Kernel expansion:

$$\chi(\gamma) = \sum_k \bar{\alpha}_s^k \chi^{(k)}(\gamma)$$

LLx:

$$\chi^{(0)}(\gamma) \sim \overbrace{\frac{1}{\gamma}}^{collinear} + \overbrace{\frac{1}{1-\gamma}}^{anti-collinear} + \ldots$$

(where $\ldots$ denote other poles).
NLLx (symmetric scales):

$$\chi^{(1)}(\gamma) \sim \frac{A_1}{\gamma^2} + \frac{A_1}{(1-\gamma)^2} - \frac{1}{2\gamma^3} - \frac{1}{2(1-\gamma)^3}, \qquad A_1 = -\frac{11}{12}$$

Choice of scales

Green's function (ν energy, k, k' scales of two probes):

$$\mathcal{G}(\nu, k, k') = \int \frac{d\omega}{2\pi i} \left(\frac{\nu}{kk'}\right)^{\omega} \mathcal{G}_{\omega}(k, k')$$

Choice of scales:

$$\begin{array}{lll} k^2 \sim k'^2, & Q_0^2 = kk' & \underline{\mathrm{symmetric}} \\ k^2 > k'^2, & Q_0^2 = k^2 & \underline{\mathrm{DGLAP}} \\ k^2 < k'^2, & Q_0^2 = k'^2 & \underline{\mathrm{anti-DGLAP}} \end{array}$$

Look for kernel which satisfies similarity transformations:

$$\mathcal{K}_{\omega}(k, k') \quad \rightarrow \quad \mathcal{K}^{u}_{\omega}(k, k') = \mathcal{K}_{\omega}(k, k') \left(\frac{k}{k'}\right)^{\omega}, Q_0^2 = k^2$$

$$\mathcal{K}_{\omega}(k, k') \quad \rightarrow \quad \mathcal{K}^{l}_{\omega}(k, k') = \mathcal{K}_{\omega}(k, k') \left(\frac{k'}{k}\right)^{\omega}, Q_0^2 = k'^2$$

Shifted kernel

Multiplying $\mathcal{K}$ by $(k/k')^\omega$, leads to shifts in $\chi(\gamma)$:

$$\chi_\omega(\gamma) \simeq \frac{1}{\gamma + \frac{\omega}{2}} + \frac{1}{1-\gamma+\frac{\omega}{2}}$$

Expand in ω:

$$\chi_\omega(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma} - \frac{\omega}{2}\frac{1}{\gamma^2} - \frac{\omega}{2}\frac{1}{(1-\gamma)^2} + \dots$$

Use condition $\omega = \bar{\alpha}_s \chi(\gamma) \simeq \frac{\bar{\alpha}_s}{\gamma} + \frac{\bar{\alpha}_s}{1-\gamma}$:

$$\chi^{(0)}(\gamma) + \bar{\alpha}_s \chi^{(1)}(\gamma) \simeq \frac{1}{\gamma} + \frac{1}{1-\gamma} - \frac{\bar{\alpha}_s}{2}\frac{1}{\gamma^3} - \frac{\bar{\alpha}_s}{2}\frac{1}{(1-\gamma)^3} + \dots$$

Shift in ω generated cubic poles in γ. Recover partially NLLx result.

If the choice of scales was different $\rightarrow$ different form of NLLx kernel.

Non-singular DGLAP terms

Take DGLAP term with full splitting function $\omega P_{gg}(\omega) = 1 + \omega A_1(\omega)$ and expand in ω

$$\frac{\omega P_{gg}(\omega)}{\gamma} = \frac{1}{\gamma} + \omega \frac{A_1(\omega)}{\gamma} \simeq \frac{1}{\gamma} + \bar{\alpha}_s \frac{A_1(0)}{\gamma^2}$$

non-singular term (in ω) of the splitting function appears at NLLx.

Idea (roughly): (Ciafaloni, Colferai,Salam)
Take shifted kernel + nonsingular DGLAP terms.

Original CCS model not very practical since formulated in Mellin space.
New model written in $(\ln k^2, \ln 1/x)$ space.

Advantage: Can easily compute gluon Green's function.

Form of kernel in (x, k^2) *space*

ω shift $\leftrightarrow$ kinematical constraint
(cutoff onto real part of emissions in the kernel)

- symmetric: $kz < k' < k/z$
- DGLAP: $k'^2 < k^2/z$
- anti-DGLAP: $k'^2 > k^2 z$

$$K^{\rm resum}(z;k,k') = \bar{\alpha}_s(\mathbf{q}^2)K_0^{kc}(z,k,k') + \bar{\alpha}_s(k_>^2)K_c^{kc}(z,k,k') + \bar{\alpha}_s^2(k_>^2)\tilde{K}_1(z,k,k')$$

where:
$K_0^{kc}(z,k,k')$ LLx BFKL with kinematical constraint
$K_c^{kc}(z,k,k')$ LO non-singular(in x) DGLAP with kinematical constraint
$\tilde{K}_1(z,k,k')$ NLLx term with subtractions of $1/\gamma^2$,$1/\gamma^3$ poles because they are resummed in K_0^{kc} and in K_c^{kc}
Important: choice of scale in $\bar{\alpha}_s$

Resummed splitting function

Deconvolution of the integral equation $\rightarrow$ get the splitting function
Define integrated gluon density:

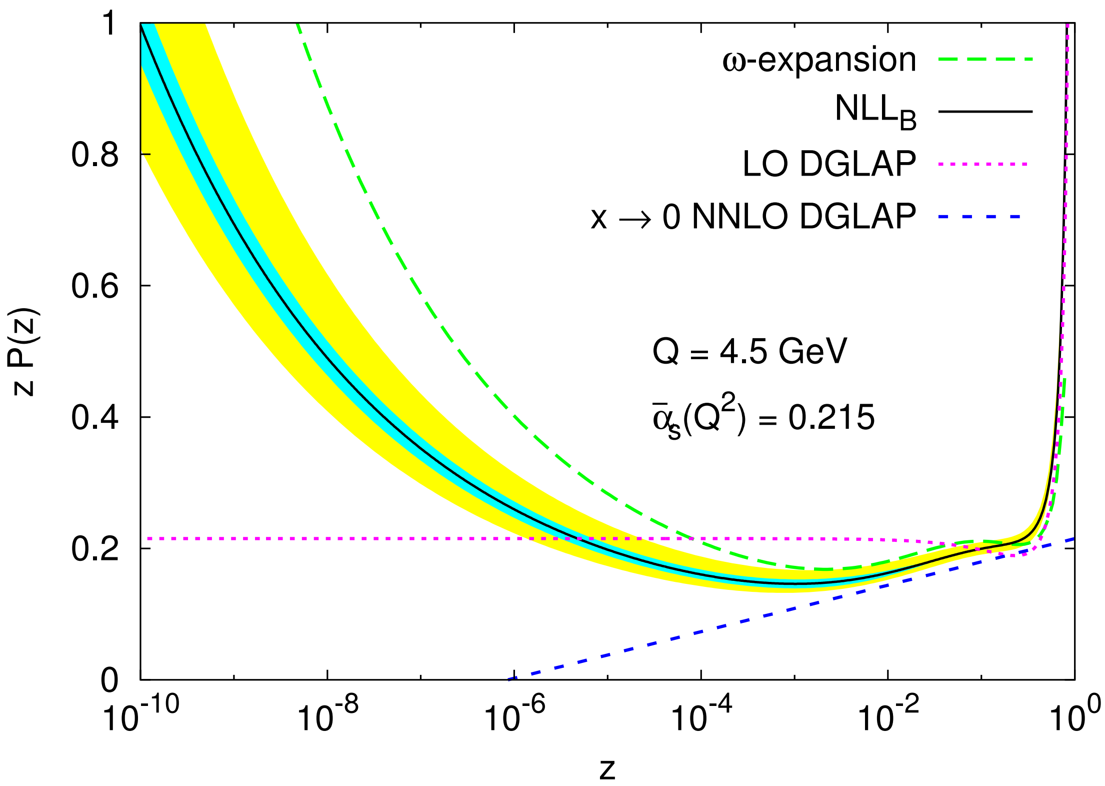
$$xg(x,Q^2) = \int^{Q^2} dk^2 G^{(Q_0^2=k^2)}(x;k,k_0)$$

Solve numerically for the effective splitting function

$$\frac{dg(x,Q^2)}{d\log Q^2} = \int \frac{dz}{z} P_{\mathrm{eff}}(z,Q^2) g(\frac{x}{z},Q^2)$$

P_{eff} in the limit $Q^2 \gg k_0^2$ (should be) independent of the regularisation of the coupling and of the choice of k_0

Resummed splitting function



- mild dependence on the infrared cutoff
- renormalisation scale dependence
- characterstic dip around $x = 10^{-3}$
- rise in x postponed to smaller values
- initial decrease co-incides with NNLO small x part

Can HERA data discriminate between NNLO and resummed answer?
Will resummed calculation make gluon positive?
How important is P_{qg}?