The real corrections to the photon impact factor

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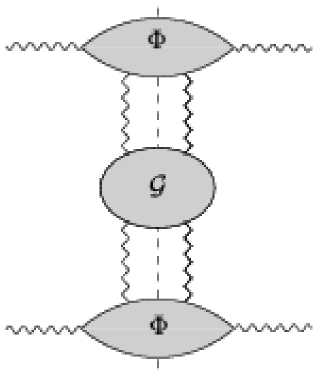
Motivation



Calculation of the
photon impact factor Φ_{γ^*} at NLO:

→ $\sigma^{tot}_{\gamma^*\gamma^*}$ in framework of NLO BFKL

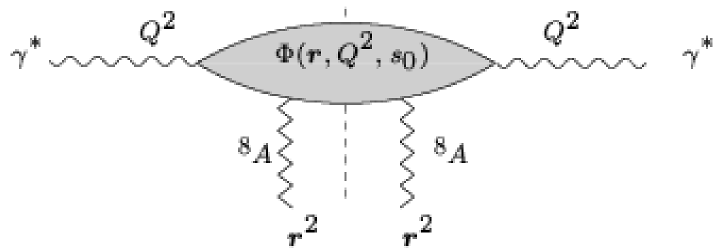
→ clean test of NLO BFKL

→ resummation of NLL(1/x) in splitting functions

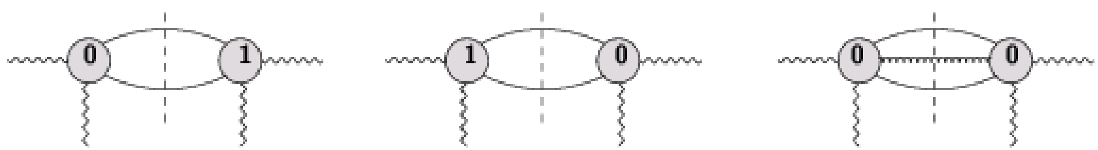
→ Color dipole picture at NLO

→ 2-jet (NLO), 3-jet (LO) in DIS (k_T-factorization scheme)

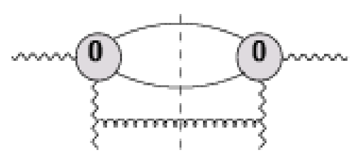
The NLO photon impact factor



The NLO corrections:



to do:

- calculate $\Gamma^{(1)}_{\gamma^* \to q\bar{q}}$
- calculate $|\Gamma^{(0)}_{\gamma^* \to q\bar{q}g}|^2$
- cancel divergencies
- exclude central region (LLA) $\longrightarrow$ 

* perform phase space integration

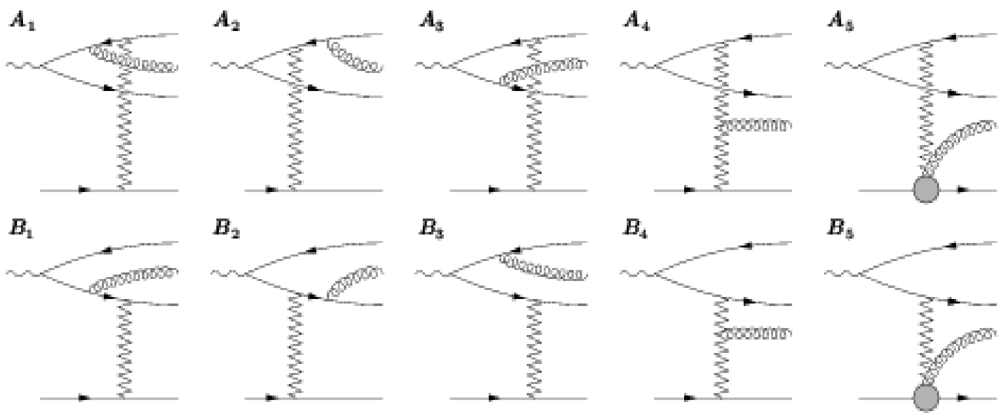
Virtual corrections

$\Gamma^{(1)}_{\gamma^* \to q\bar{q}}$ calculated in terms of standard integrals

[J.Bartels, S.Gieseke, C.F.Qiao, PRD **63** (2001) 056014]

Real corrections

Consider (in the Regge limit):



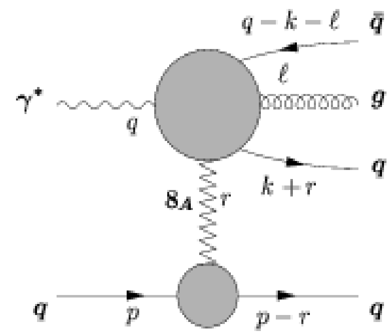
$|\Gamma^{(0)}_{\gamma^* \to q\bar{q}g}|^2$ is then given by

$$|\bar{\mathcal{M}}|^2 = |\Gamma^{(0)}_{\gamma^* \to q\bar{q}g}|^2 \, \frac{4s^2}{t^2} \, |\Gamma^{(0)}_{q \to q}|^2$$

[J. Bartels, S. Gieseke, A.K., PRD **65** (2002) 014006]

[J. Bartels, D. Colferei, S. Gieseke, A.K.,PRD **66** (2002) 094017]

Real Corrections



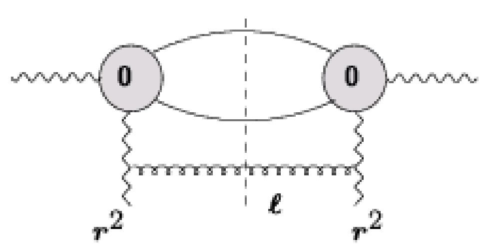
$$s = (p+q)^2, \quad t = r^2$$

Using p and $q' = q + xp$ with $p^2, q'^2 = 0$

→ Sudakov variables:

$$\begin{aligned} k =& \alpha q' + \beta p + k_\perp \\ \ell =& \alpha_\ell\, q' + \beta_\ell\, p + \ell_\perp \\ \boldsymbol{\ell}^2 =& - \ell_\perp^2 \end{aligned}$$

The **central limit :**

$|\Gamma^{(0)}_{\gamma^* \to q\bar{q}g}|^2 \quad \xrightarrow{\alpha_\ell \to 0}$ 

Finite Combinations

IR divergences from soft and collinear $(g||q, g||\bar{q})$ gluon:

soft: $\alpha_\ell \sim |\boldsymbol{\ell}| \to 0$

coll.: $|\boldsymbol{\ell}_c| \to 0 \quad (\bar{\mathrm{q}} : \boldsymbol{\ell}_c = \boldsymbol{\ell} + \alpha_\ell/(1-\alpha)\boldsymbol{k}$

$\mathrm{q} : \boldsymbol{\ell}_c = \boldsymbol{\ell} - \alpha_\ell/\alpha(\boldsymbol{k} + \boldsymbol{r}))$

The divergent parts are subtracted and re-added

For brevity: $\Gamma^2_{q\bar{q}g} := \frac{1}{(1-\alpha-\alpha_\ell)} \left| \Gamma^{(0)}_{\gamma^* \to q\bar{q}g} \right|^2$

$$\Phi^{(1,\mathrm{real})}_{\gamma^*} = \int \frac{d\alpha}{2\alpha} \int \frac{d\alpha_\ell}{2\alpha_\ell} \int \frac{d^{2-2\epsilon}\boldsymbol{k}}{(2\pi)^{3-2\epsilon}} \frac{d^{2-2\epsilon}\boldsymbol{\ell}}{(2\pi)^{3-2\epsilon}} \Bigg\{$$

$$\Gamma^2_{q\bar{q}g} - \Gamma^2_{q\bar{q}g}\Big|_{\mathrm{soft}} - \left(\Gamma^2_{q\bar{q}g}\Big|_{\mathrm{coll}} - \Gamma^2_{q\bar{q}g}\Big|_{\mathrm{coll,soft}} \right) \Theta(\alpha_\ell \Lambda - |\boldsymbol{\ell}_c|)$$

$$- \left(\Gamma^2_{q\bar{q}g}\Big|_{\mathrm{cen}} - \Gamma^2_{q\bar{q}g}\Big|_{\mathrm{cen,soft}} \right) \Theta(\frac{|\ell|}{\sqrt{s_0}} - \alpha_\ell) \Bigg\} + \Phi^{real}_{\gamma^*}\Big|^{\mathrm{divergent}}$$

$\Phi^{real}_{\gamma^*}\Big|^{\mathrm{divergent}}$ = all orange terms

Analytical integration of $\Phi_{\gamma^*}^{real}\Big|^{\text{divergent}}$

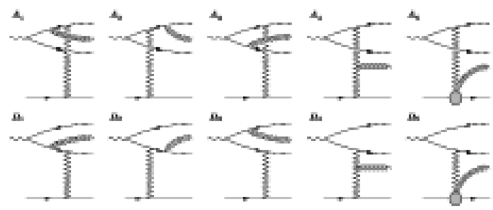
$$\longrightarrow \quad \epsilon - \text{poles and finite terms}$$

Cancellation of ϵ-poles from virtual and real corrections.

$\Rightarrow$ **Finite expressions for real and virtual parts**

[J. Bartels, D. Colferai, S. Gieseke, A.K.,PRD **66** (2002) 094017]

Approaching the integration



$$\Gamma^2_{q\bar{q}g} = c \sum_{\text{diagrams}} \mathcal{P}\,, \qquad \boldsymbol{\mathcal{P}} = \textbf{product of 2 amplitudes}$$

$$\Phi^{(1,\text{real})}_{\gamma^*}\Big|^{finite} = \text{const} \cdot \int_0^1 d\alpha \int_0^{1-\alpha} \frac{d\alpha_\ell}{\alpha_\ell} \int d^{2-2\epsilon}\boldsymbol{k}\, d^{2-2\epsilon}\boldsymbol{\ell}$$

$$\sum_{\text{diags.}} \Big\{ \mathcal{P} - \mathcal{P}_{\text{soft}} - (\mathcal{P}_{\text{coll}} - \mathcal{P}_{\text{coll,soft}})\, \Theta(\alpha_\ell \Lambda - |\boldsymbol{\ell}_c|)$$

$$- (\mathcal{P}_{\text{cen}} - \mathcal{P}_{\text{cen, soft}})\, \Theta(|\boldsymbol{\ell}| - \alpha_\ell \sqrt{s_0}) \Big\}$$

Analytical **Integration** over $\boldsymbol{k}, \boldsymbol{\ell}$

$\Downarrow$

Feynman paramerization of each $\mathcal{P}$ (or small groups)

$\Downarrow$

Addititional divergences due to separation of diagrams
(cancel in the sum)

The divergences

- $\boldsymbol{\ell} \to \infty \quad (\boldsymbol{\ell}-\mathrm{uv})$

$$\int d^d\boldsymbol{k}\, d^d\boldsymbol{\ell}\; \mathcal{P} = \int \prod \beta_i\; \Omega = \int \prod \beta_i\; (\Omega - \Omega|_{\boldsymbol{\ell}\text{–uv}}) + R_{\boldsymbol{\ell}\text{–uv}}$$

$$R_{\boldsymbol{\ell}\text{–uv}} = \int \prod \beta_i\; \Omega|_{\boldsymbol{\ell}\text{–uv}} = \frac{1}{\epsilon}\, c_1 + c_2$$

$$\sum_{\text{diagrams}} R_{\boldsymbol{\ell}\text{–uv}} = \text{finite}$$

- $\boldsymbol{\ell} \to \boldsymbol{r}$

Sum in central limit is finite :

$$\Gamma^2_{q\bar{q}g}\Big|_{\text{cen}} = \text{const} \cdot \left(\frac{1}{D(\boldsymbol{k}+\boldsymbol{r})} - \frac{1}{D(\boldsymbol{k}+\boldsymbol{\ell})}\right)^2 \cdot \frac{1}{(\boldsymbol{\ell}-\boldsymbol{r})^4}\, \frac{4C_A \boldsymbol{r}^2\, (\boldsymbol{\ell}-\boldsymbol{r})^2}{\boldsymbol{\ell}^2}$$

with $D(\boldsymbol{k}) = \alpha(1-\alpha)Q^2 + \boldsymbol{k}^2$

But single diagrams diverge as $\boldsymbol{\ell} \to \boldsymbol{r}$, therefore:
$*$ consider finite **pairs of diagrams**
$*$ common set $\{\beta_i\}$ for each pair
$*$ common integration over β_i

- $\boldsymbol{\ell} \sim \sqrt{\alpha_\ell} \to 0 \quad (\mathrm{is})$

$\to$ Subtraction

The result

A sum of convergent integrals

$$\Phi_{\gamma^*}^{(1,\mathrm{real})}\Big|^{finite} = \mathrm{const}\cdot \sum_{\mathrm{diagrams(pairs)}} \int_0^1 d\alpha \int_0^{1-\alpha} \frac{d\alpha_\ell}{\alpha_\ell} \int \prod d\beta_i$$

$$\Big\{ \Omega - \Omega_{\mathrm{soft}} - (\Omega_{\mathrm{coll}} - \Omega_{\mathrm{coll,soft}}) - (\Omega_{\mathrm{cen}} - \Omega_{\mathrm{cen,soft}})$$

$$- (\Omega - \Omega_{\mathrm{cen}})_{\boldsymbol{\ell}\text{–uv}} - (\Omega - \Omega_{\mathrm{soft}})_{\mathrm{is}} \Big\} + \mathrm{const}$$

the remaining integrations: numerically

The completey NLO corrections read:

$$\Phi_{\gamma^*}^{(1)} = \Phi_{\gamma^*}^{(1,virtual)}\Big|^{finite} + \Delta_\mu(\boldsymbol{r}^2, \mu) +$$

$$+ \underbrace{\Phi_{\gamma^*}^{(1,\mathrm{real})}\Big|^{finite}_{C_F} + \Phi_{\gamma^*}^{(1,\mathrm{real})}\Big|^{finite}_{C_A} + \Delta_{s_0}(\boldsymbol{r}^2, s_0) + \Delta_\Lambda(\boldsymbol{r}^2, \Lambda)}_{\Phi_{\gamma^*}^{real}}$$

def.: $\quad \Phi_{\gamma^*}^{real} = e^2 e_f^2 \left(\Phi_F + \Phi_A + \Delta_{s_0} + \Delta_\Lambda\right)$

The entire dependence of $\Phi_{\gamma^*}^{(1)}$ on Λ, s_0 is now gathered in $\Phi_{\gamma^*}^{real}$

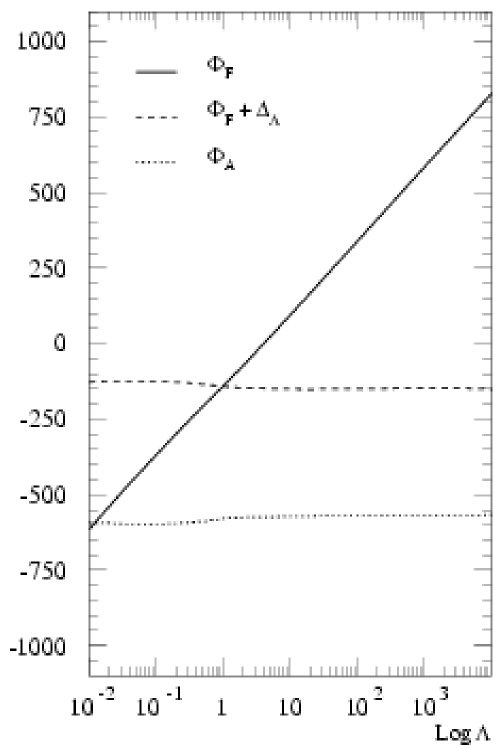
Numerical Results

use dimensionless variables: $\frac{1}{Q}(\boldsymbol{r}, \Lambda, s_0) \rightarrow (\boldsymbol{r}, \Lambda, s_0)$

check of the calculation: the Λ dependence

$$\Phi_{\gamma^*}^{real} = e^2 e_f^2 \left(\Phi_F + \Phi_A + \Delta_{s_0} + \Delta_\Lambda\right)$$

with : $$\Delta_\Lambda = -\frac{3\, C_F}{(2\pi)^2}\, \Phi_{\gamma^*}^{(0)}\; \ln\, \Lambda$$



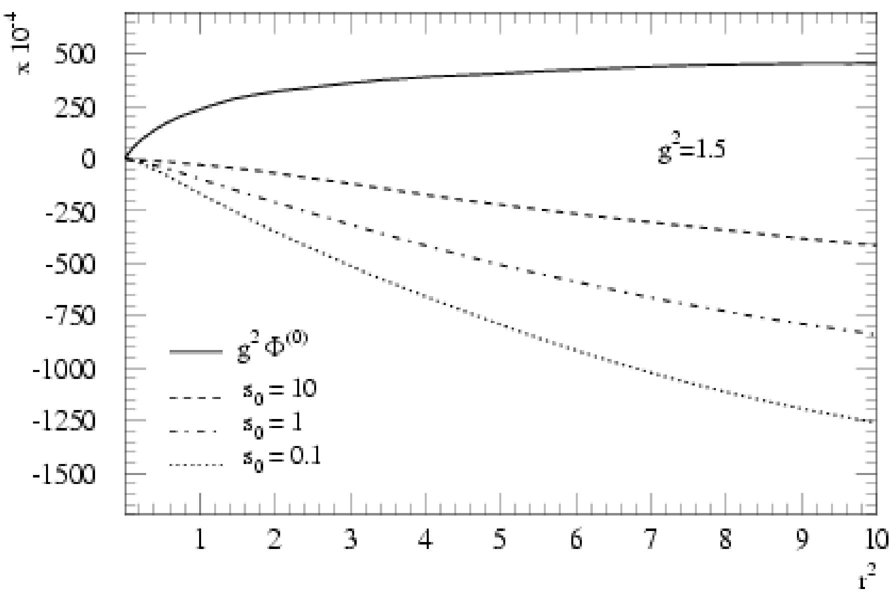
$\rightarrow$ real corrections independent from Λ

The dependence on the energy scale s_0

$$\Phi'_{\gamma*} = g^2 \Phi^{(0)}_{\gamma*} + g^4 \Phi^{\rm real}_{\gamma*}$$

at $Q^2 = 15$ GeV^2: $\alpha_s = 0.18 \Leftrightarrow g^2 = 1.5$

$\Phi'_{\gamma*}$ compared to the LO impact factor :



→ **photon impact factor ↘ as s_0 ↘**

$\sigma^{tot} = \Phi \otimes \mathcal{G} \otimes \Phi$ is independent of s_0.

$\mathcal{G} \to (s/s_0)^\omega \quad \Rightarrow \quad \Phi$ has to decrease towards smaller s_0

Summary

for longitudinal photon ploarization:

$$\Phi_{\gamma^*}^{(1,real)}\Big|^{finite} = \int d\alpha\, d\alpha_\ell\, d^d\boldsymbol{k}\; d^d\boldsymbol{\ell} \sum_{\text{diagrams}} \left(\mathcal{P} - \mathcal{P}_{\text{coll}}\Theta(..) - \ldots\right)$$

$$\downarrow$$

$$\downarrow$$

$$\Phi_{\gamma^*}^{(1,\text{real})}\Big|^{finite} = \sum_{\text{diagrams}} \int \underbrace{d\alpha\, d\alpha_\ell\, d\beta_i\; \left(\Omega - \Omega_{\text{coll}} - \ldots\right)}_{\text{convergent}}$$

Remaining integrations numerically

- real corrections independent from Λ
- photon impact factor $\searrow$ as $s_0 \searrow$