Overview

Basic Accelerator Principles :

- units and equations
- acceleration concepts
- storage rings
- trajectory stability
- collider concept
- vacuum requirements
- synchrotron radiation

design parameters for the LHC

http://bruening.home.cern.ch/bruening/summer−school/HST−Lecture.pdf

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- M.S. Livingston and J.P. Blewett, ’Partticle Accelerators’, McGraw-Hill, 1962
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- E.D. Courant and H.S. Snyder, ’Theory of the Alternating-Gradient Synchrotron’, Annals of Physics **3**, 1-48 (1958).
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- Lichtenberg and Lieberman, 'Regular and Stochastic Motion', Applied Mathematical Sciences 38, Springer Verlag.
- A.W. Chao, 'Physics of Collective Beam Instabilities in High Energy Accelerators', Wiley, New York 1993.
- M. Diens, M. Month and S. Turner, 'Frontiers of Particle Beams: Intensity Limitations', Springer-Verlag 1992, (ISBN 3-540-55250-2 or 0-387-55250-2) (Hilton Head Island 1990) 'Physics of Collective Beam Instabilities in High Energy Accelerators', Wiley, New York 1993.
- R.A. Carrigan, F.R. Huson and M. Month, 'The State of Particle Accelerators and High Energy Physics', American Institute of Physics New Yorkm 1982, (ISBN 0-88318-191-6) (AIP 92 1981) 'Physics of Collective Beam Instabilities in High Energy Accelerators', Wiley, New York 1993.

Choices for the LHC

- super conducting RF
- R = 2784 meter

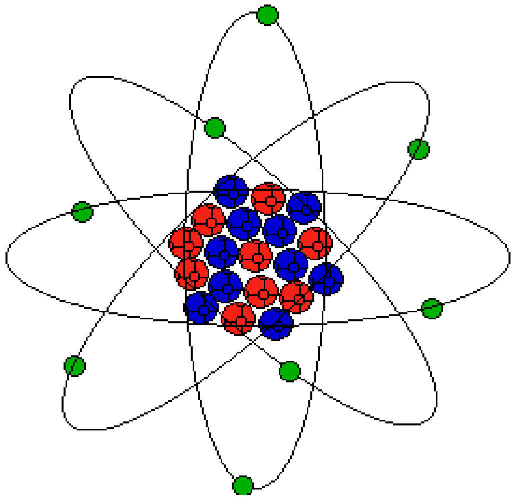
→ $B_{max} = 8.38$ T → *iron sarturation: 2 Tesla*
*earth: 0.3 * 10^{-4} Tesla*

- super conducting magnet technology
- FODO lattice
- proton collider
- 2 in 1 magnet design
- 2835 bunche with 10^{11} particles per bunch
- high luminosity insertions
- beam screen
- cryo pump at 2K

Search for Elementary Particles

Stage I:

Nuclear Physics



- **Chronology:**

- 1803: Dalton → Atom
- 1896: M & P Currie → Atoms can decay
- 1896: Thomson → Electron
- 1906: Rutherford → Nucleus + Electron
- 1911: Rutherford → $\alpha + N \rightarrow O + H^{+}$

→ Disintegration of Nuclei!

→ Particle Accelerators

Stage II:

Particle Physics

Chronology (Theory):

- 1905: **Einstein** → $E = mc^2$
- 1930: **Dirac** → **Antimatter**
- 1935: **Yukawa** → π - **Meson**

Chronology (Experiments):

(Cosmic Rays)

- 1932: **Anderson** → e^+
- 1937: **Anderson** → μ

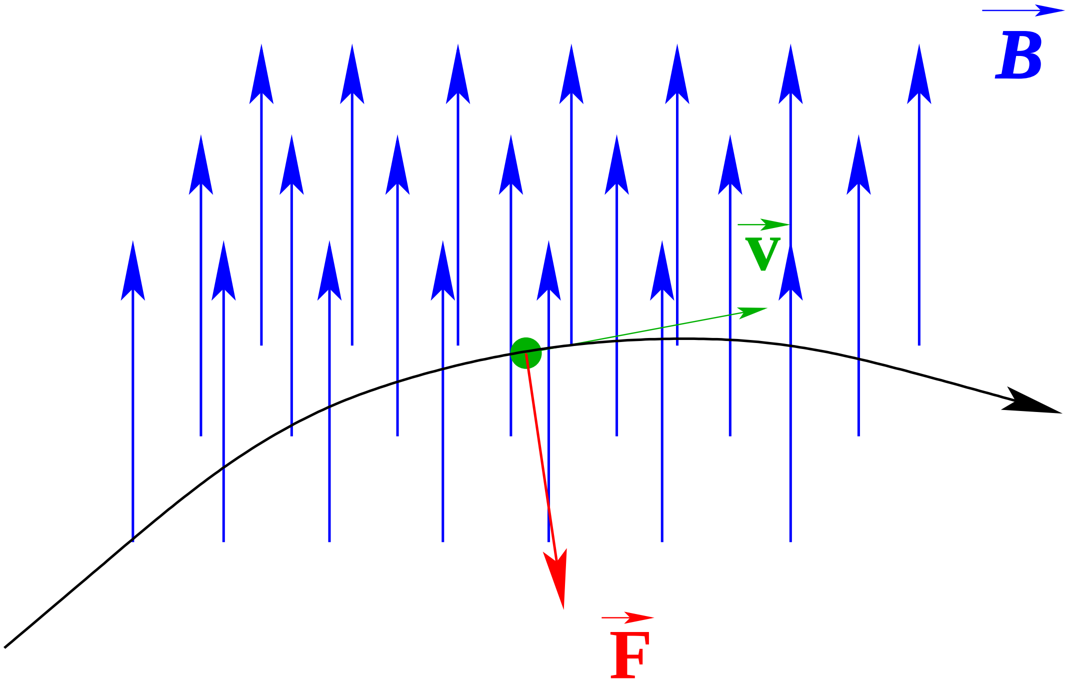
$$\left.\begin{array}{l} p^- \\ \pi \end{array}\right\} ? \longrightarrow \text{Accelerators}$$

Acceleration Concepts

● **Lorentz Force:**

$$\frac{d\vec{p}}{dt} = Q * \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

🟩 ***magnetic fields:***



→ ***Trajectory curvature due to B field!***

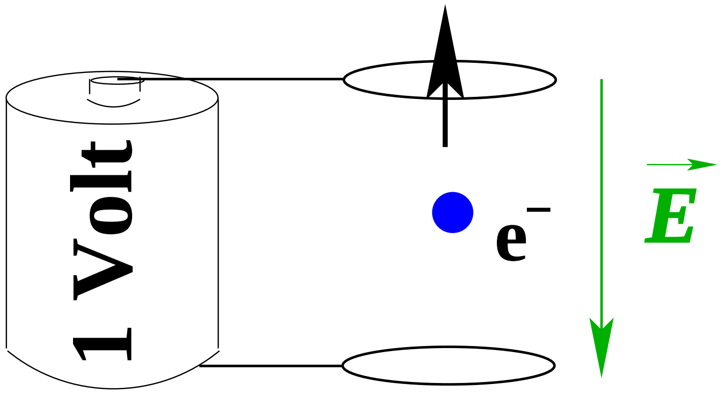
→ ***Energy gain only due to E field!***

Units

Energy Gain:

1 eV

→ $(1.6 * 10^{-19}\ J)$



Common Units:

keV, MeV, GeV, TeV

$(10^3, \quad 10^6, \quad 10^9, \quad 10^{12})$

Total Particle Energy:

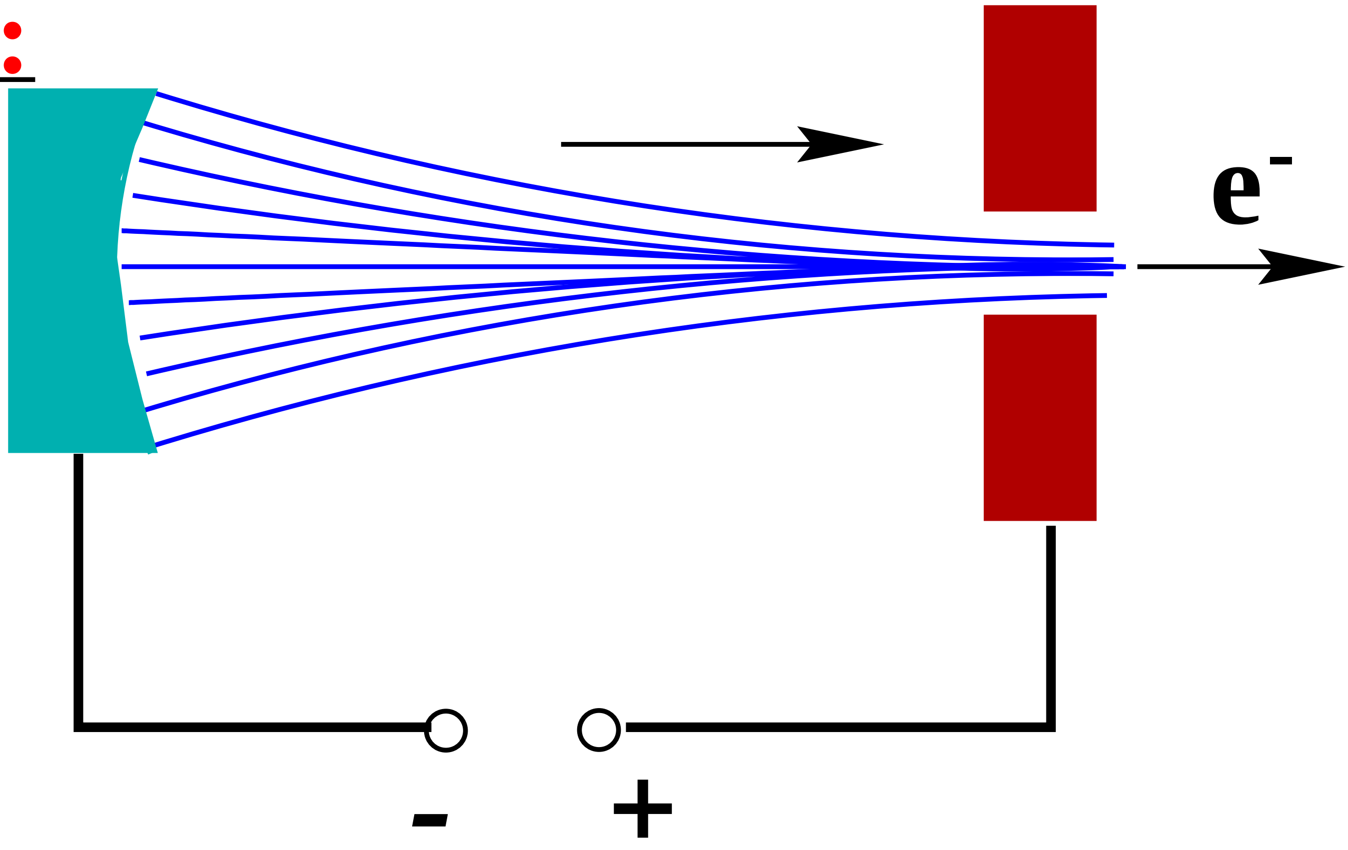
- **Relativity:** $E = mc^2 ; m = \gamma * m_0$

$$\gamma = 1/\sqrt{1 - \beta^2}; \quad \beta = v/c$$

electron: 0.51 MeV **proton:** 0.94 GeV

Particle Sources:

e^-:



→ ***Cathode Rays***

p^+: ***Cathode Tube with*** **H**

$$H_2 + e^- \rightarrow H_2^+ + 2\,e^-$$

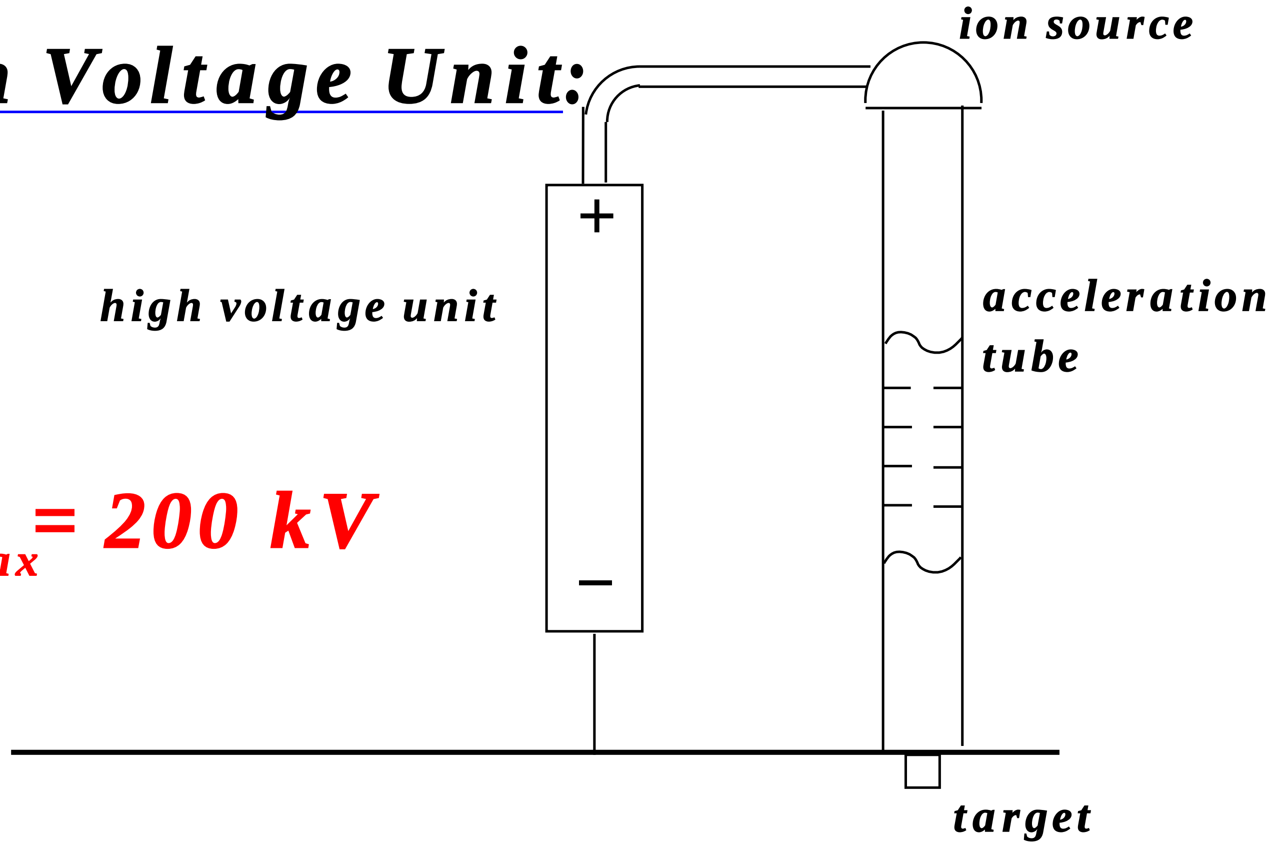
$$H_2^+ + e^- \rightarrow H^+ + H + e^-$$

$$H + e^- \rightarrow H^+ + 2\,e^-$$

Antimatter: ***Pair Production***

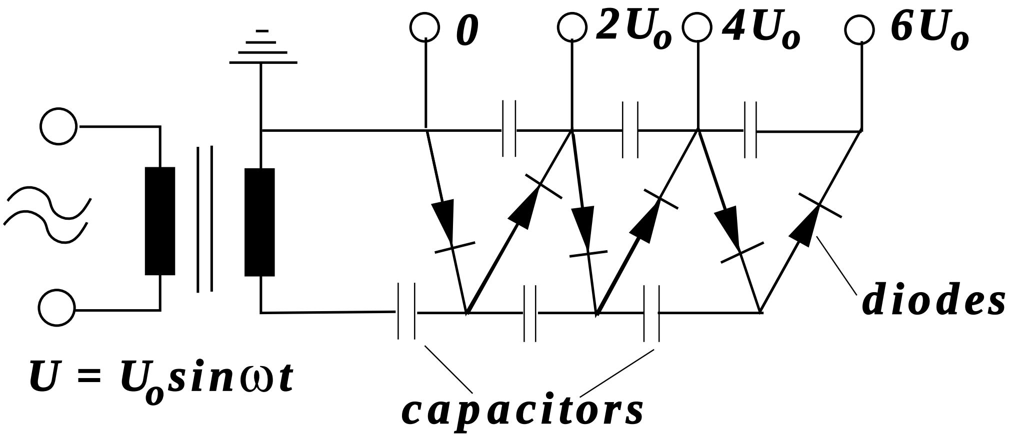
Electrostatic Fields

High Voltage Unit:



$V_{max} = 200\ kV$

Cascade Generator:



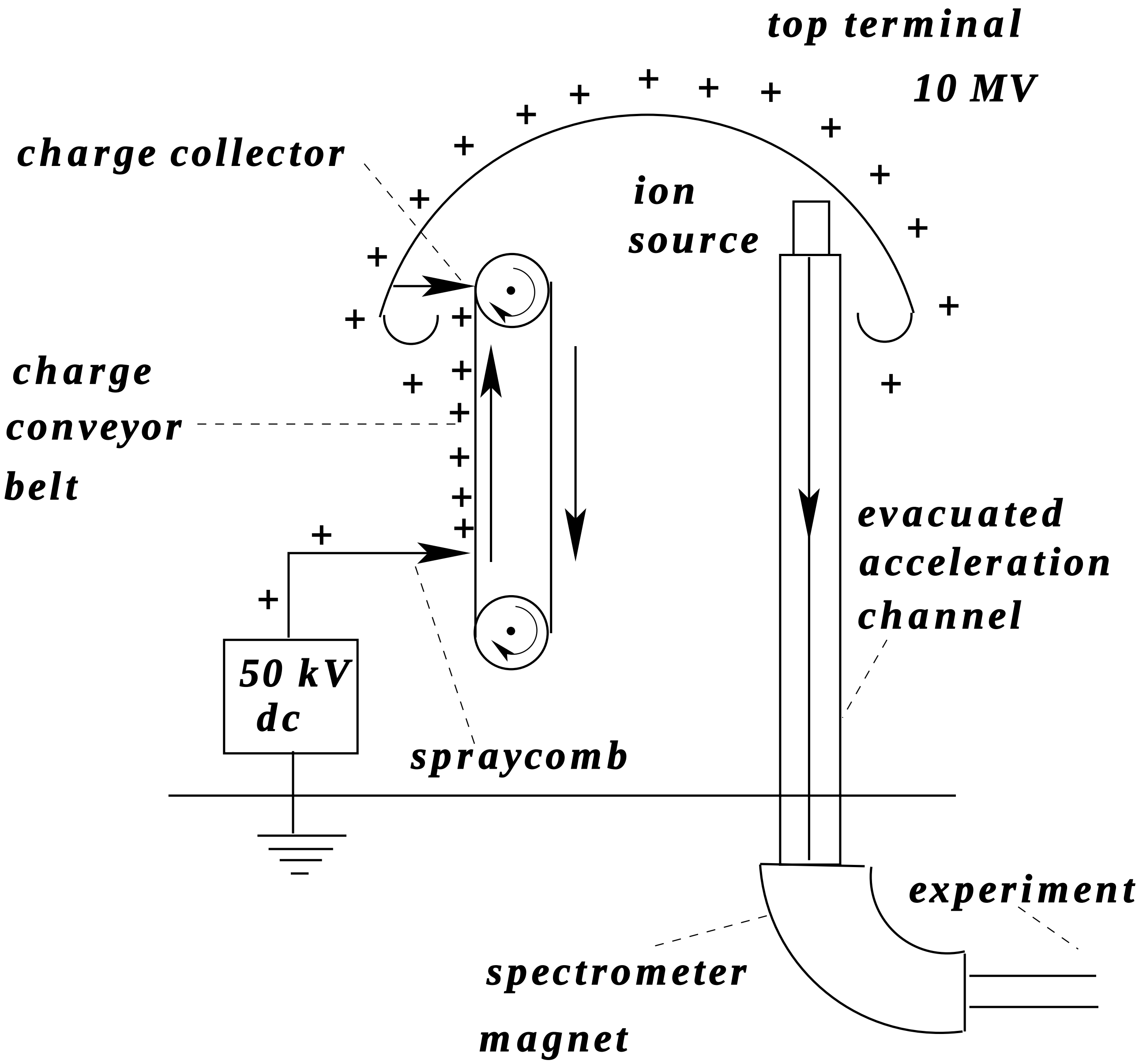
🟩 **1928: Cockroft + Walton** 800kV

🟩 **1932:** $p + Li \rightarrow 2\ He$ 700kV (p)

(Nobel Prize 1951)

Van de Graaf Generator

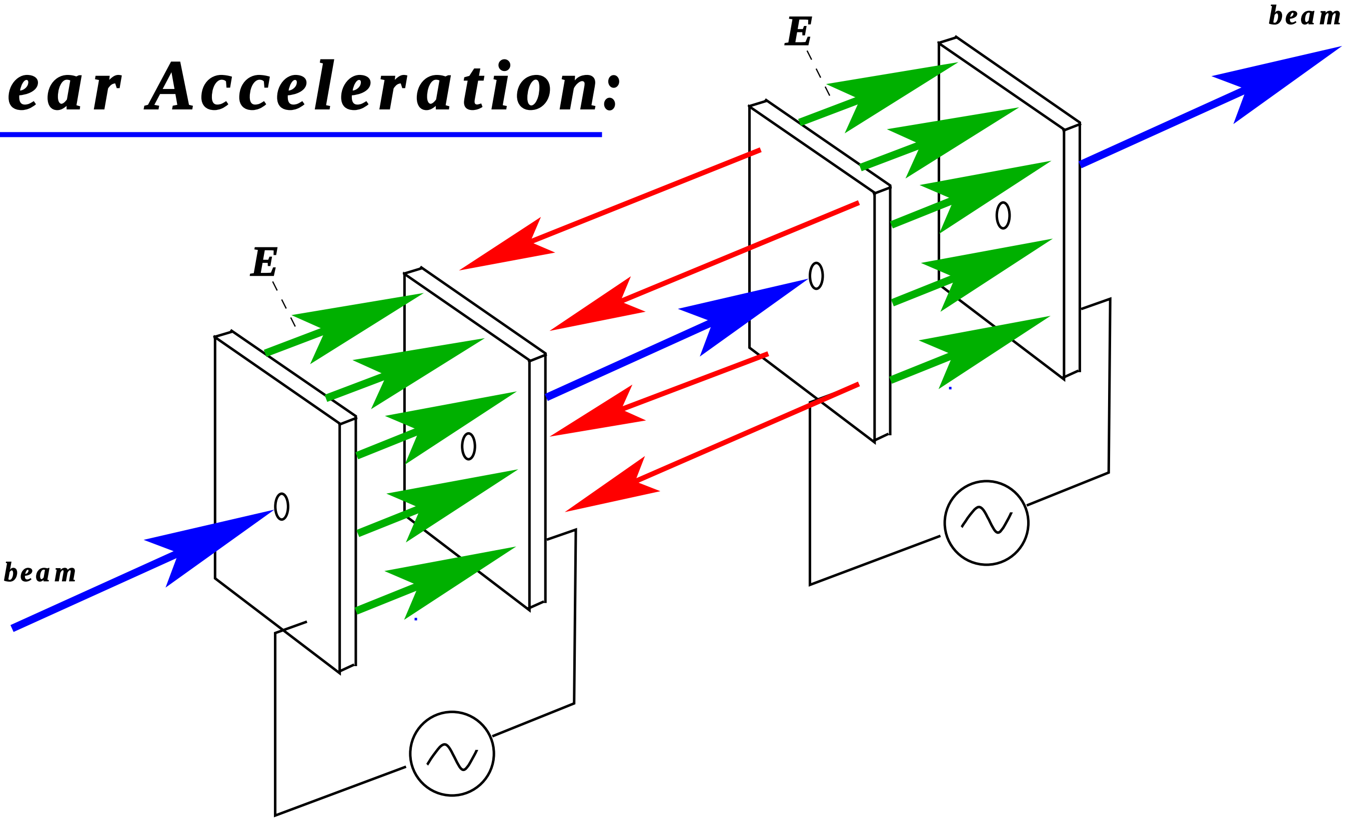
Single Unit:



→ $V_{max} = 10$ MVolt

Time Varying Fields

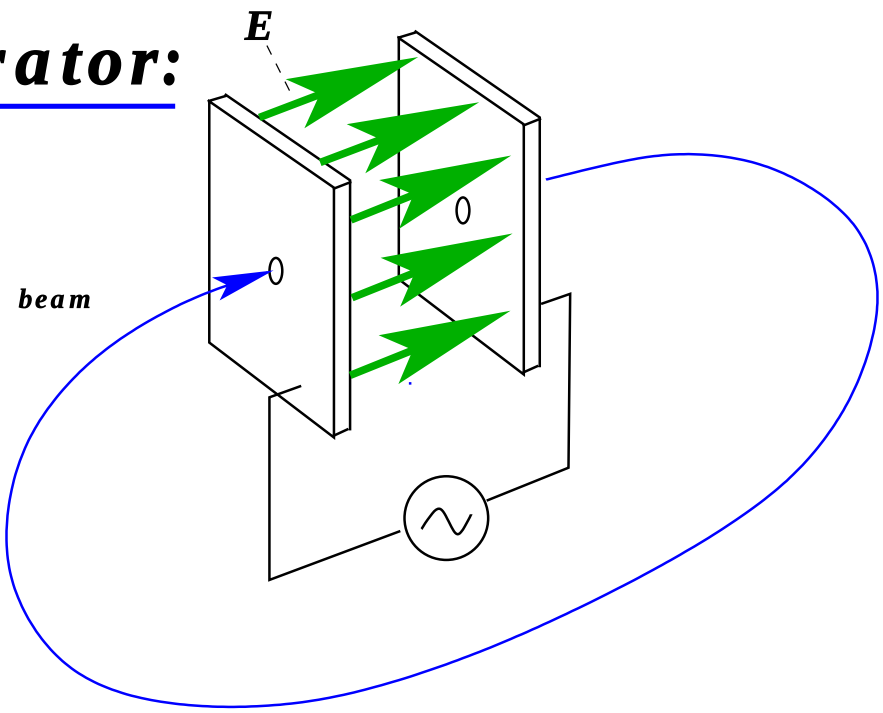
● Linear Acceleration:



→ bunched beam

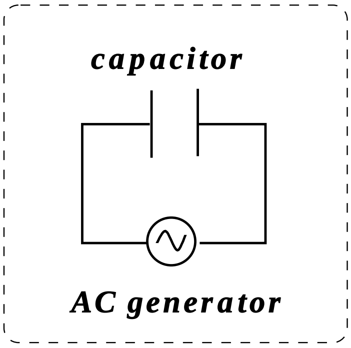
→ long accelerator!

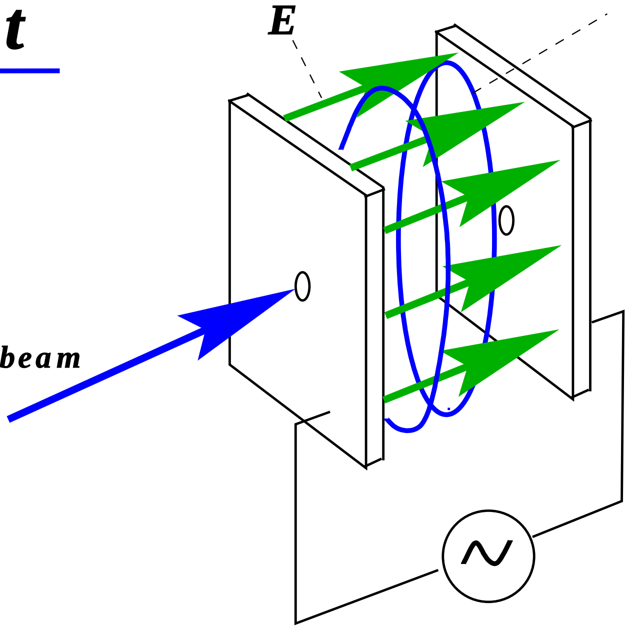
● Circular Accelerator:



Time Varying Fields

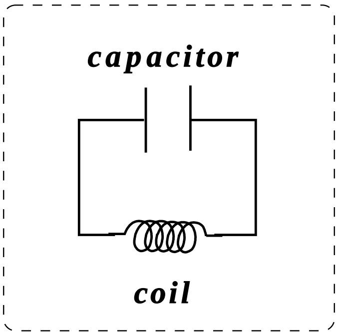
- $$E = -\frac{1}{c}\frac{\partial A}{\partial t}$$





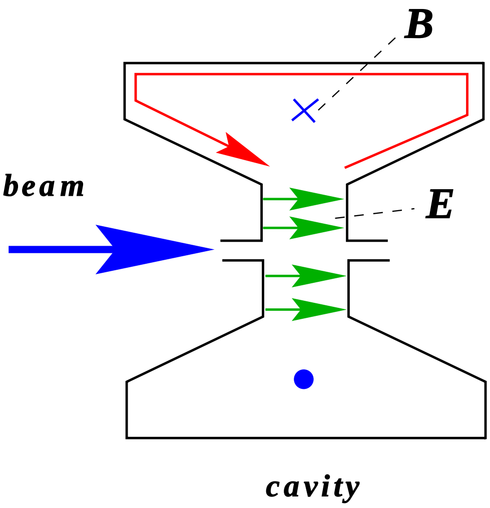
$$\text{rot } B = \frac{\mu\varepsilon}{c}\frac{\partial E}{\partial t}$$

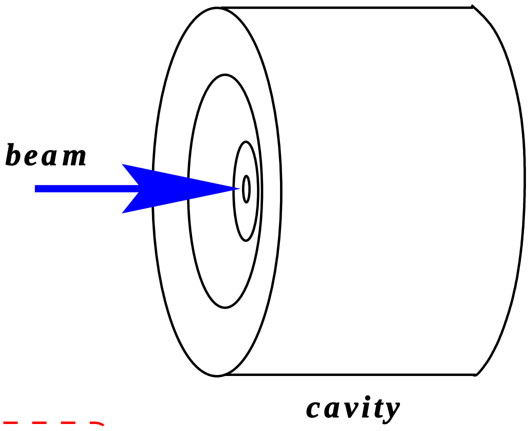
- ## Resonator:



$$L = \frac{\mu_0 \cdot N^2 \cdot A}{l}$$

$$C = \frac{\varepsilon_0 \cdot A}{d}$$





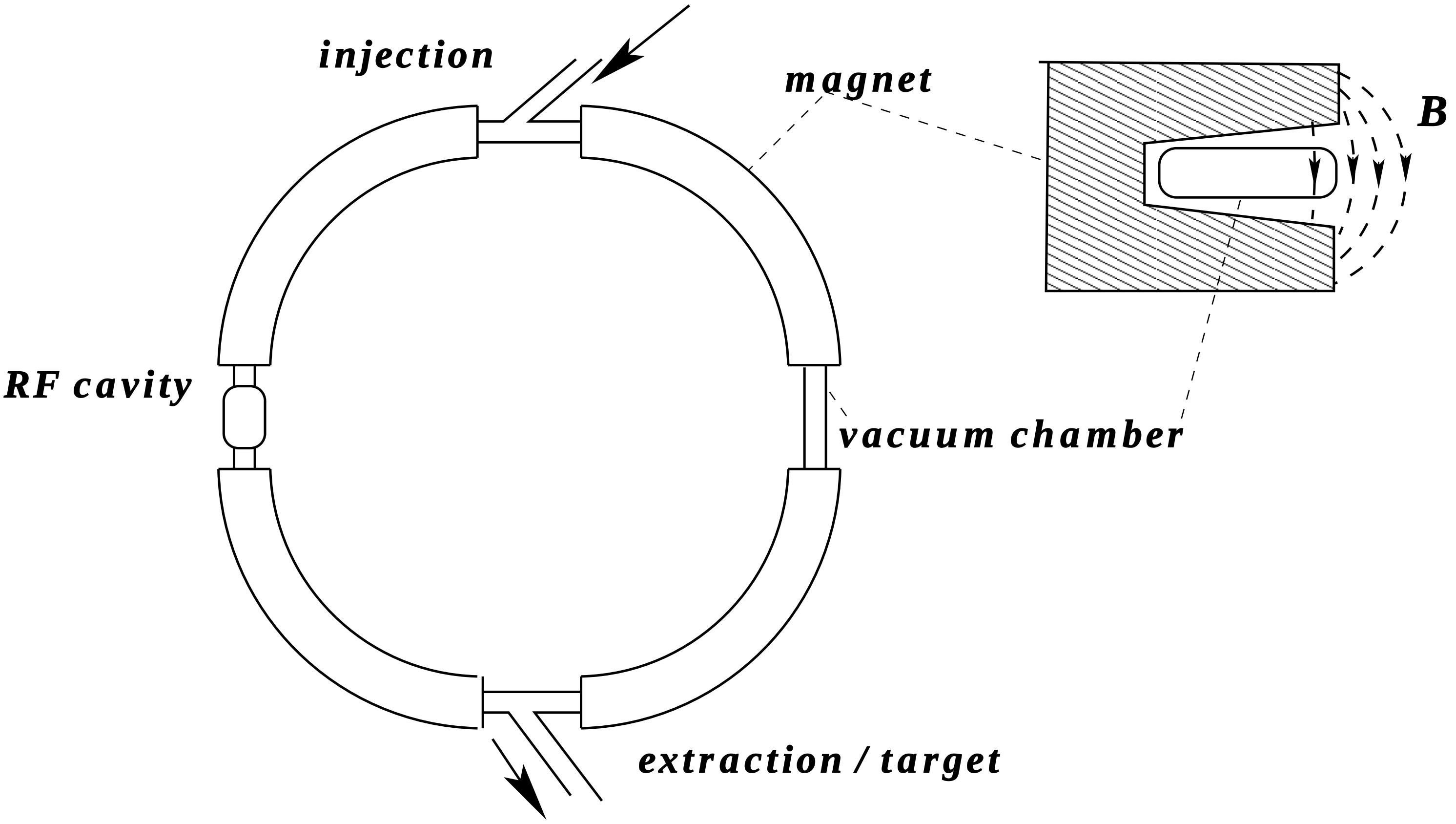
→ **f; Q; R**

Circular Accelerators

- **Synchrotron:** $R = const.$

$$\omega_0 = \frac{Q}{m_0} \cdot \frac{B}{\gamma} \qquad (\text{LHC/LEP: } \omega_0 = 11.3\text{kHz})$$

$$r = \frac{m_0}{Q} \cdot \frac{\gamma}{B} \cdot v \longrightarrow B \neq const.$$



Why 8.4 Tesla?

- ***Synchrotron:*** *R = const.*

$$r = \frac{m_0}{Q} \cdot \frac{\gamma}{B} \cdot v \longrightarrow B \propto \gamma$$

→

$$\boxed{B[T] = \frac{1}{0.3} \cdot \frac{p[GeV/c]}{R[meter]}}$$

- ***Physics:*** → p = 7000 GeV/c
- ***LEP tunel:*** L = 27000 meter

→ arcs: L = 22200 meter

→ R = 3500 meter

- **Bending and Focusing:** → R = 2784 meter

→ $\boxed{B_{max} = 8.38\ T}$ →

iron sarturation: 2 Tesla

earth: $0.3 * 10^{-4}$ Tesla

Power Consumption

- ***LEP:***

B = 0.135 Tesla $\boxed{P = R \cdot I^2}$

I = 4500A; R = 1mΩ → P = 20 kW / magnet

ca. 500 magnets → P = 10 MW

- ***LHC:*** $\boxed{B \propto I}$

→ $\mathbf{B_{max} = 8.38\ T}$ → I = 280000 A

→ P = 78 MW / magnet

ca. 500 magnets → $\boxed{P > 39\ \text{GW}}$

→ ***superconducting technology!***

8.4 T is at the limit of available technology!

Trajectory Stability

● Vertical Plane:

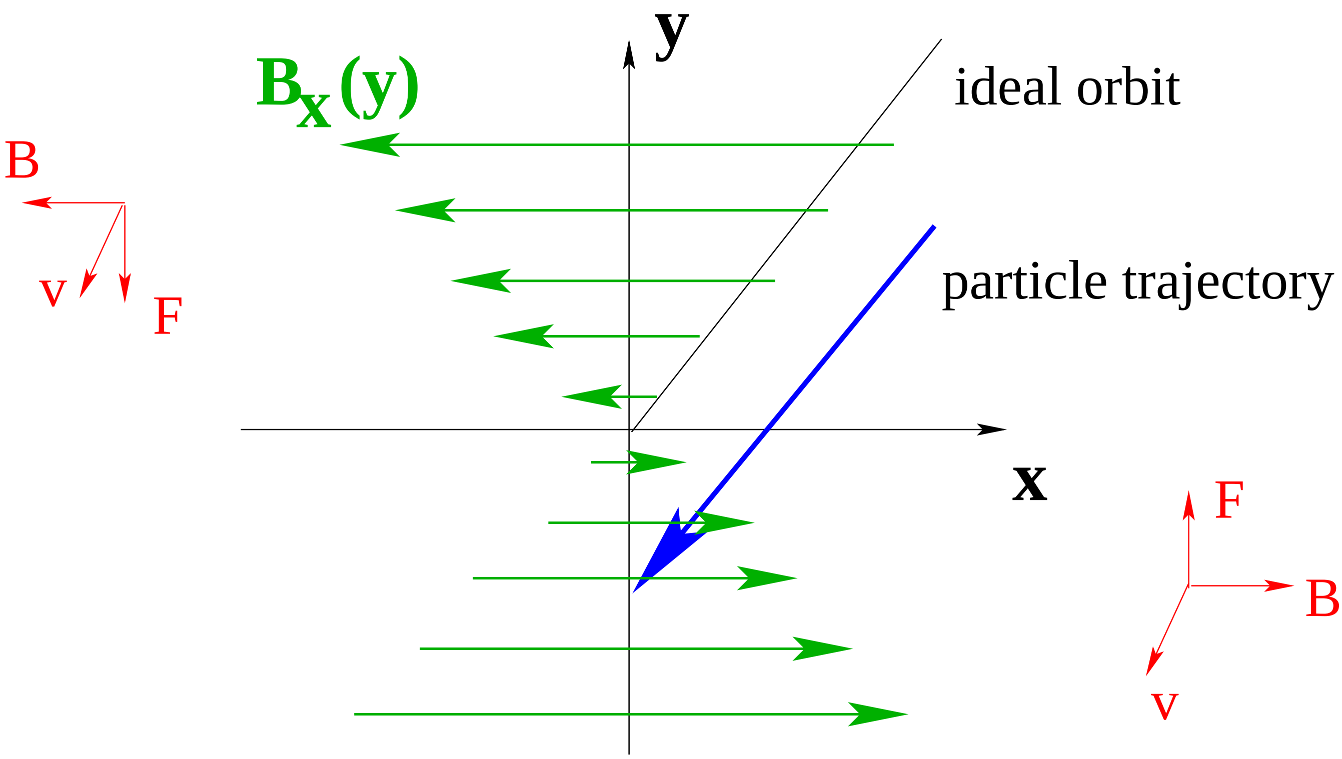
- **gravitation:** $\Delta s = \frac{1}{2} \cdot g \cdot \Delta t^2$

$g = 10 \cdot m \cdot s^{-2}$

$\Delta s = 18\ mm$ $\Delta t = 60\ msec$

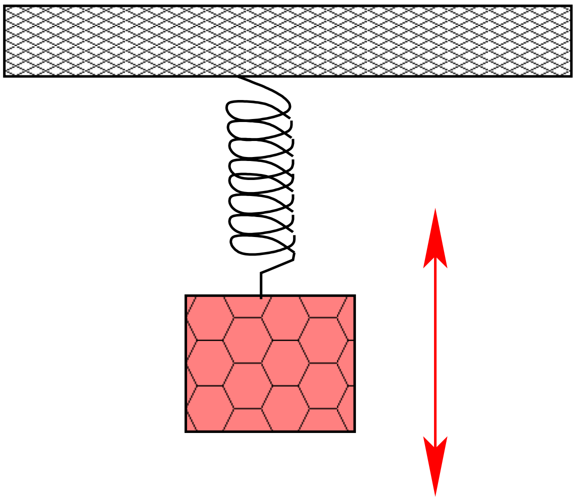
→ **660 Turns!**

→ **requires focusing!**



Strong Focusing

- *oscillator (spring):*



$$F = -g \cdot y$$

→ $\Omega^2 \propto g$

$A \propto \frac{1}{g}$ for a fixed energy

- *strong focusing:*

→ **small amplitudes**

→ **small vacuum chamber**

→ **efficient magnets**

→ **high oscillation frequency**

Quadrupole Focusing

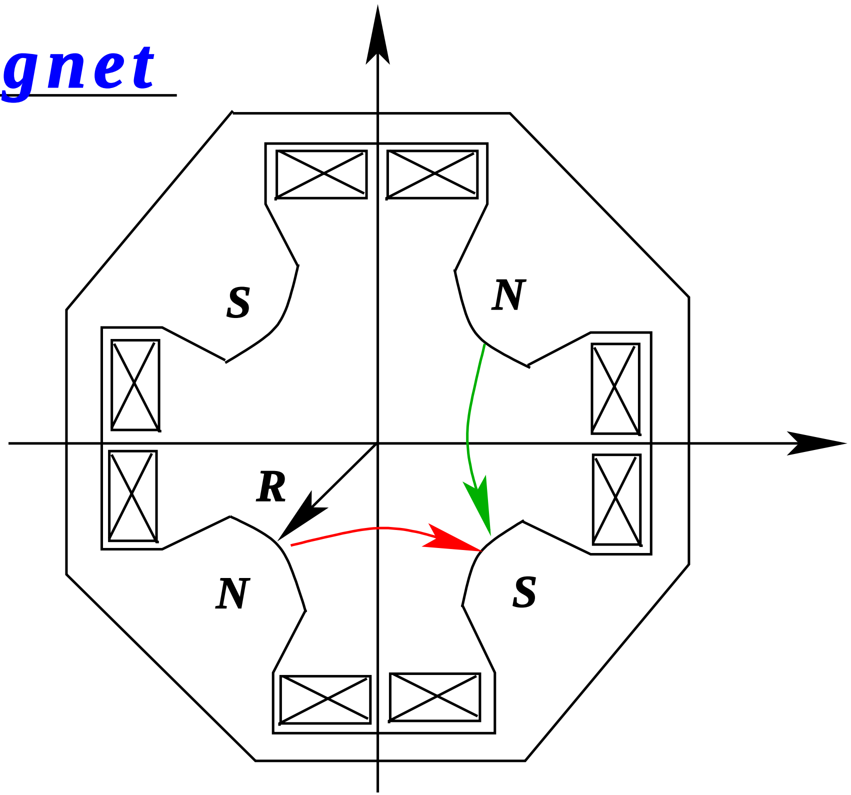
Quadrupole Magnet

$B_x = -g \cdot y$

$B_y = -g \cdot x$

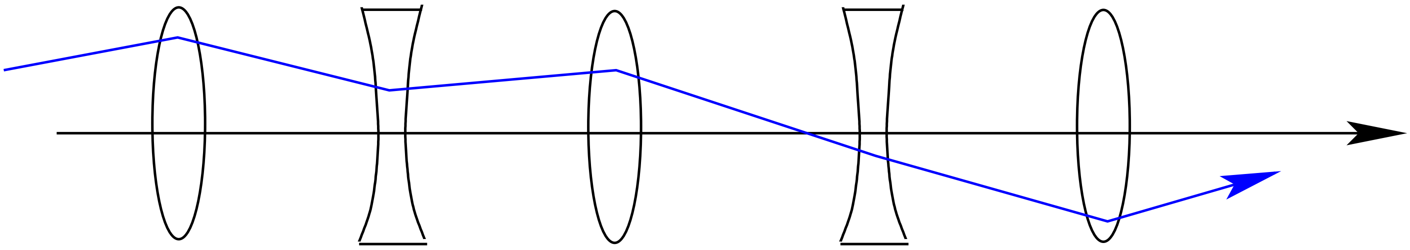
$F_x = g \cdot x$

$F_y = -g \cdot y$

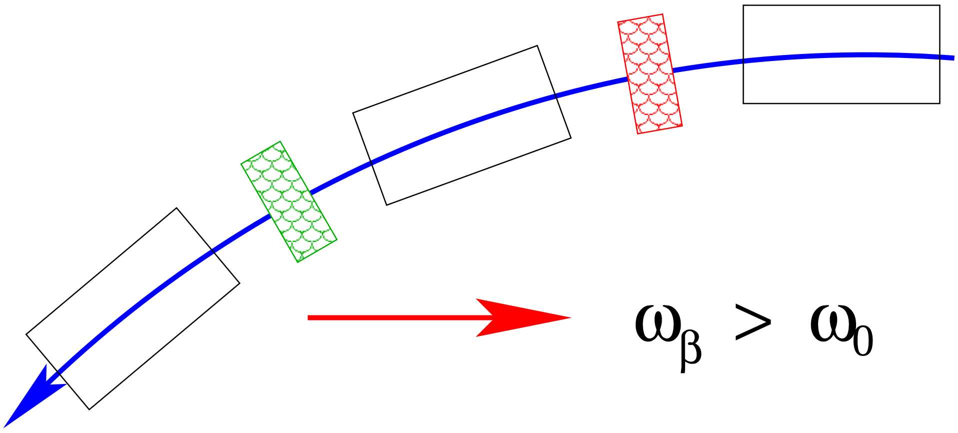


→ defocusing in horizontal plane!

Alternate Gradient Focusing



Idea: cut the arc sections in focusing and defocusing elements



→ $\omega_\beta > \omega_0$

Storage Ring

Tune:

$$Q = \frac{\text{number of oscillations}}{\text{turn}}$$

→ Q_x ; Q_y ; Q_s

Envelope Function:

- $$y(s) = \sqrt{A \cdot \beta} \cdot \sin\left(\frac{2\pi}{L} \cdot Q \cdot s + \phi_0\right)$$

  amplitude term due to injector

  amplitude term due to focusing

  sorage ring circumference

- $$\beta(s + L) = \beta(s)$$

- $$Q = \frac{1}{2\pi} \cdot \oint \frac{1}{\beta(s)} \, ds$$

Circular Accelerators

- **uniform B−field:** $R = \text{const.}$

$$r = \frac{m_0}{Q} \bullet \frac{\gamma}{B} \bullet v$$

$$p = Q \bullet \frac{B \bullet L}{2\pi} \approx E / c \quad \text{for } E >> E_0$$

- **realistic synchrotron:**

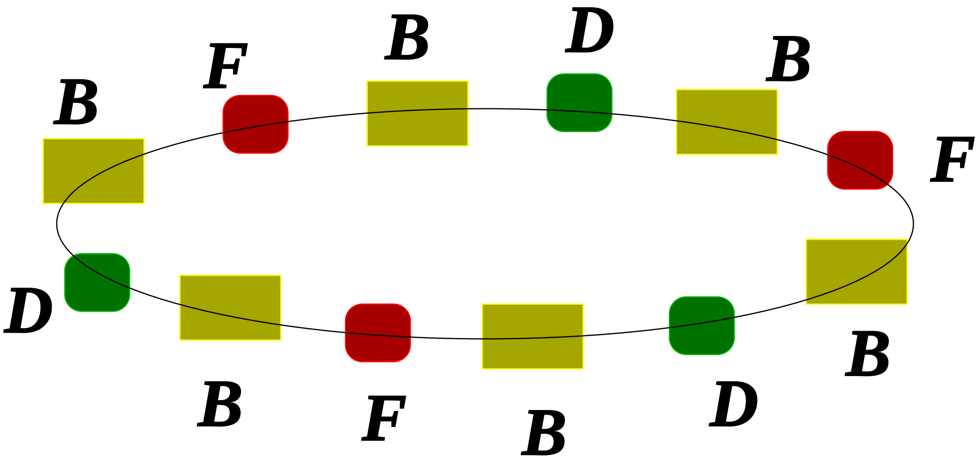
B−field is not uniform:
- −drift space for installation
- −different types of magnets
- −space for experiments etc

$$E = \frac{Q \bullet c}{2\pi} \bullet \oint \vec{B} \bullet d\,\vec{l}$$

→ high beam energy requires:

- −high magnetic field
- −large packing factor ’F’

Closed Orbit



$$B_x = -g \cdot y$$

$$B_y = -g \cdot x$$

- **Orbit Offset in Quadrupole:**

$$x = x_0 + \tilde{x}$$

quadrupole

$$B_x = -g \cdot \tilde{y}$$

$$B_y = -g \cdot x_0 - g \cdot \tilde{x}$$

dipole component

→ orbit error

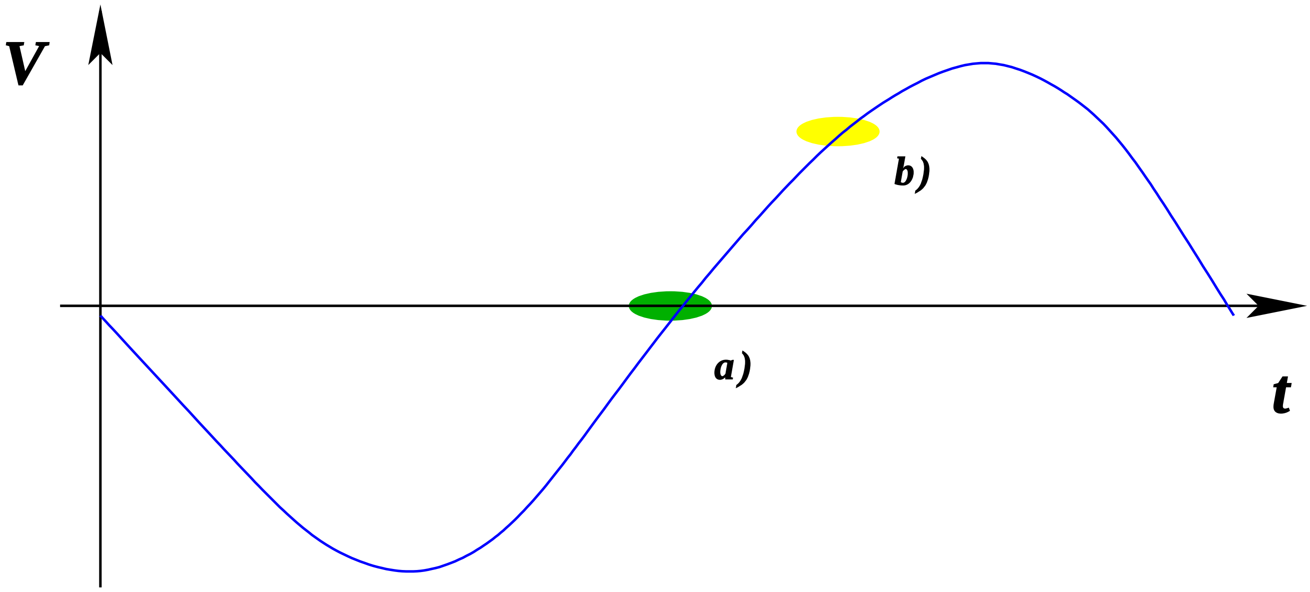
Sources for Orbit Errors

- **Alignment:** +/− 0.1 mm
- **Ground motion**
  - slow drift
  - civilisation
  - moon
  - seasons
  - civil engineering
- **Error in dipole strength**
  - power supplies
  - calibration
- **Energy error of particles**

Synchrotron:

→ the orbit determines the particle energy!

- assume: L > design orbit



→ energy increase

Equilibrium:

$$f_{RF} = h \cdot f_{rev}$$

$$f_{rev} = \frac{1}{2 \cdot \pi} \cdot \frac{q}{m \cdot \gamma} \cdot B$$

→ E depends on orbit and magnetic field!

momentum compaction factor:

- **increase particle energy**

  → velocity increase

  shorter revolution time

  → momentum increase

  longer revolution time

- **transition energy**

$$\frac{\Delta R}{R} = \alpha \cdot \frac{\Delta p}{p}$$

$$\alpha = \frac{1}{\gamma_t^2} \qquad \alpha \approx \frac{1}{Q^2}$$

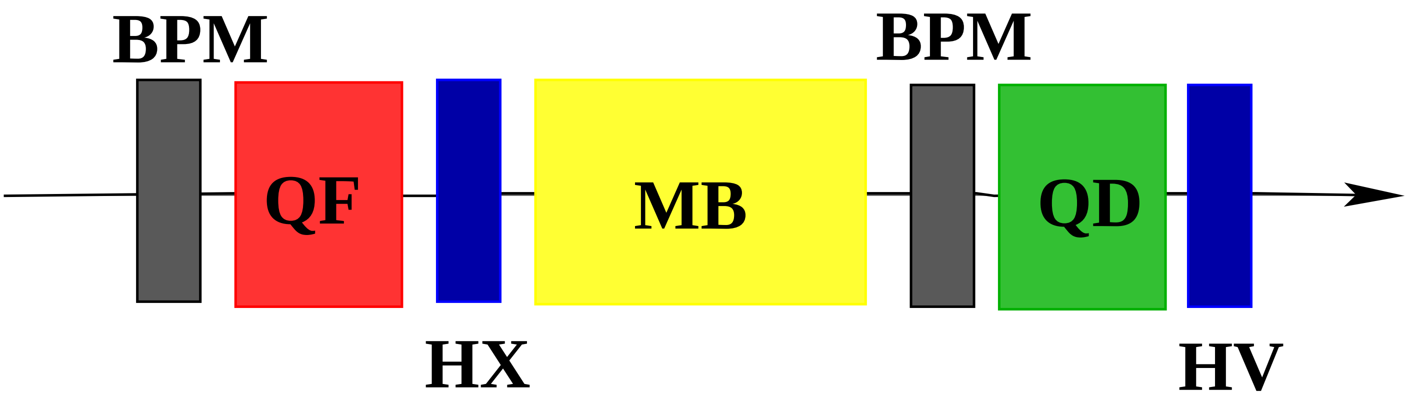
→ E error depends on transition energy!

Orbit Correction

- aim at a local correction of the dipole error due to the quadrupole alignment errors

→ place orbit corrector and BPM next to the main quadrupoles

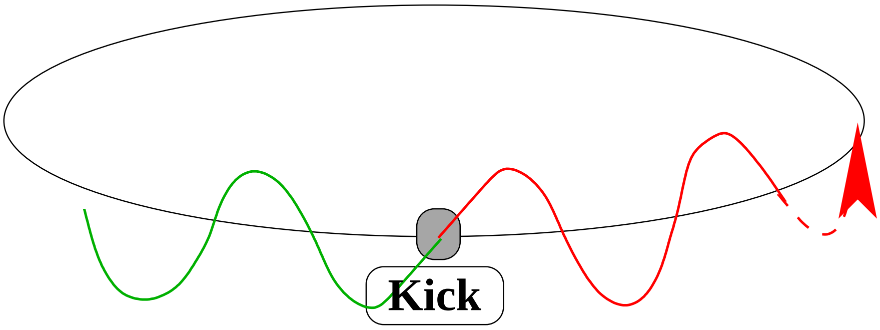
→ horizontal BPM and corrector next to QF
vertical BPM and corrector next to QD



→ **relative alignment of BPM and quadrupole?**

Orbit Stability

🟩 dipole error and Q = N:



→ the perturbation adds up

watch out for integer tunes!

🟩 field errors:

the perturbations add up for $Q = 1/n$

watch out for fractional tunes!

→ minimise field errors

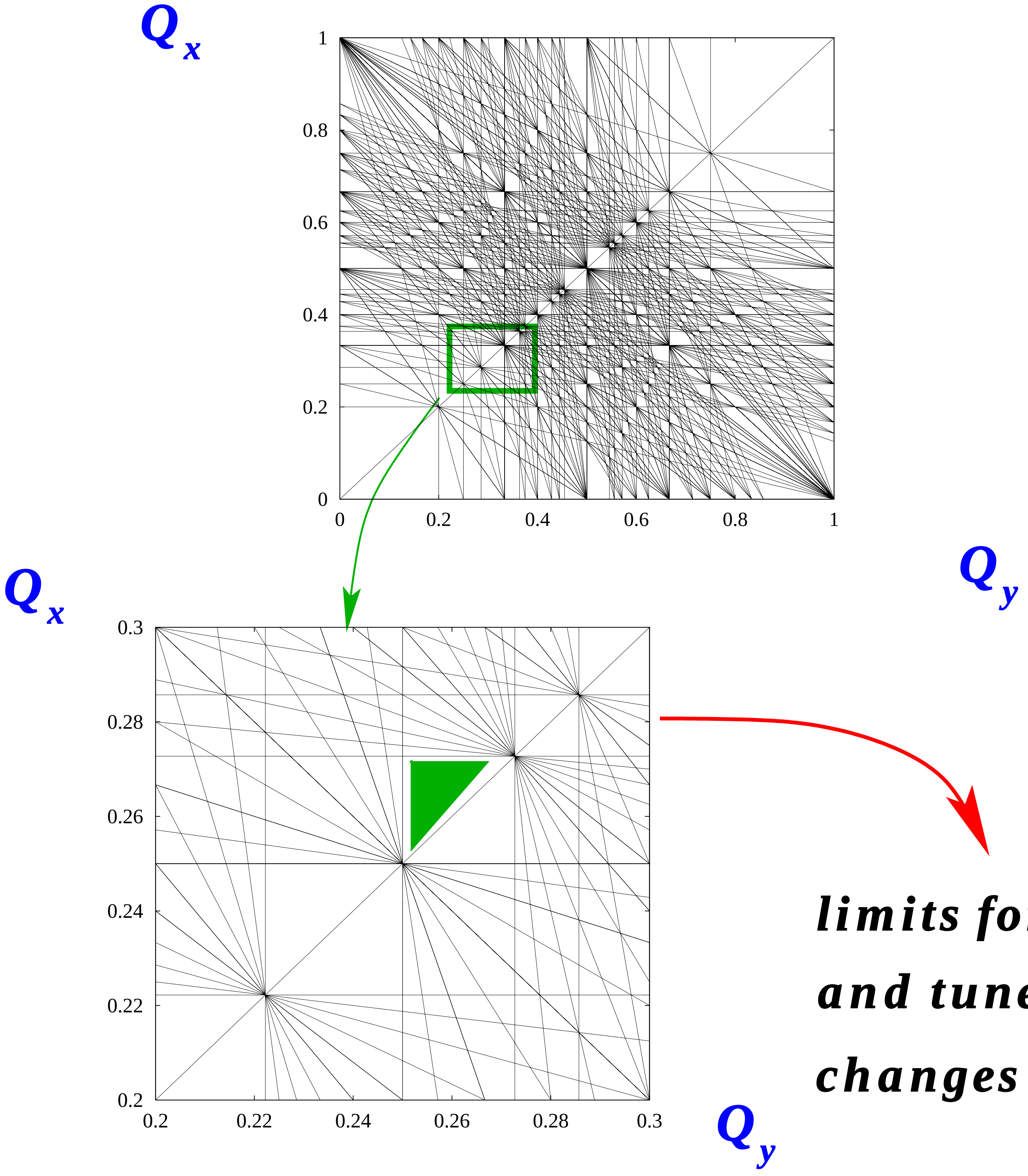
and avoid strong resonances!

Tune Diagram

🟩 **resonances:** $n \cdot Q_x + m \cdot Q_y + r \cdot Q_s = p$

strength: $h \propto A^{n+m+s}$

→ *avoid low order resonances!*



limits for b_n and tune changes

Collider Rings

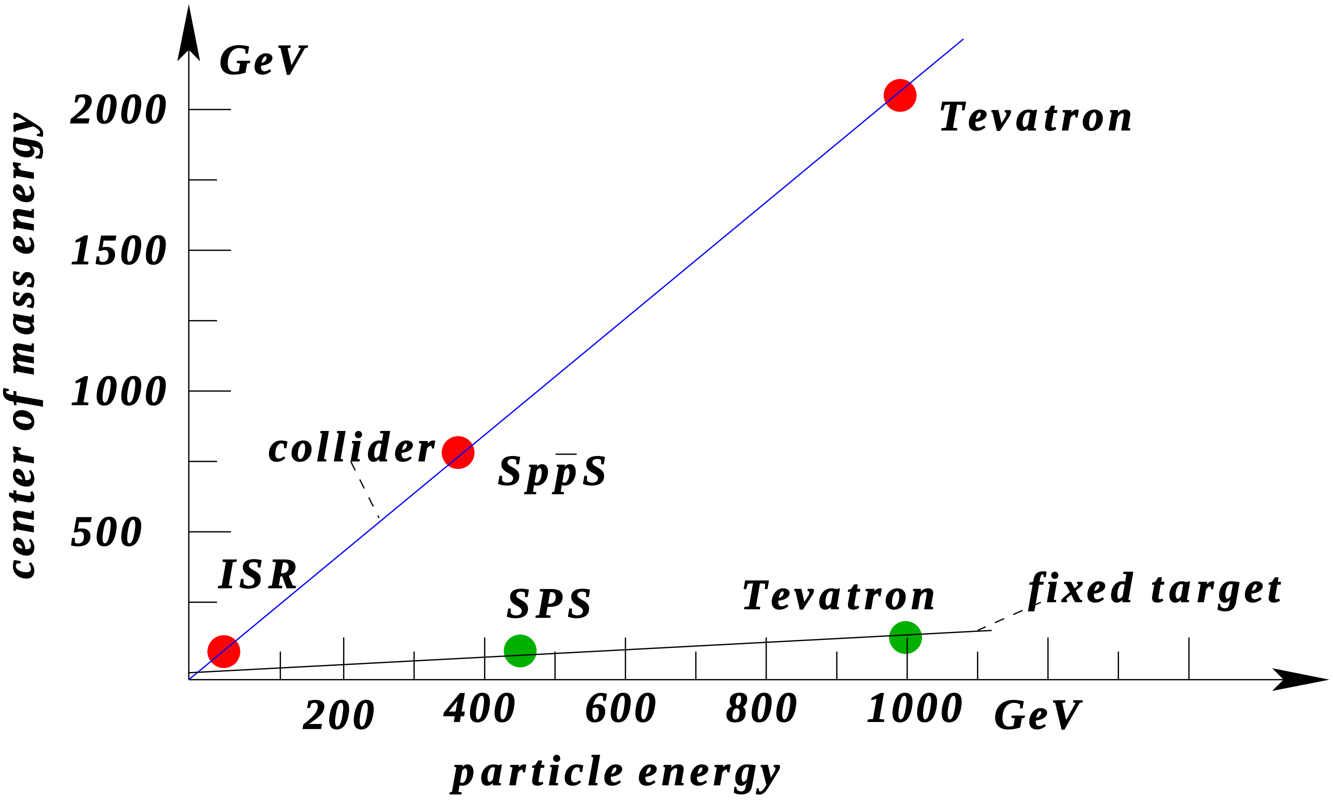
→ **1960:** fixed target physics

(bubble chamber)

- **But:**

$$E_{cm} = 2 \cdot m_0 \cdot c^2 \cdot \left(\sqrt{1 + \frac{E}{2 \cdot m_0 \cdot c^2}} - 1 \right)$$

- **Collider:** $E_{CM} = 2 \cdot E_p$



1960 ↗: e^+/e^- collider

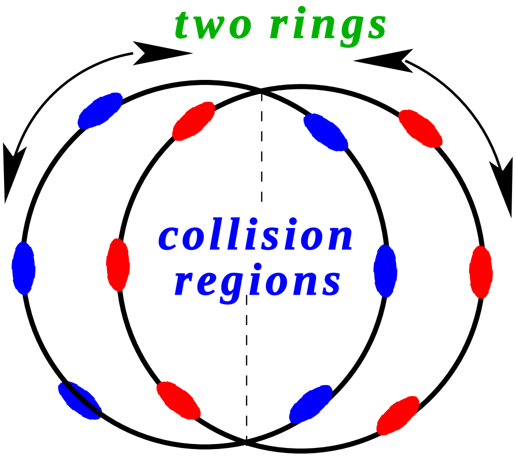
1970 ↗: p^+/p^- collider

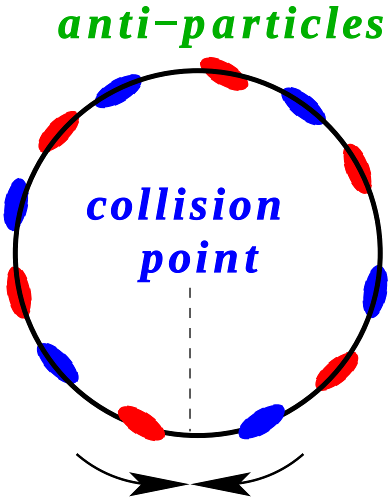
Features (+/−)

- **not all particles collide in one crossing**

→ long storage times

- **requires 2 beams:**





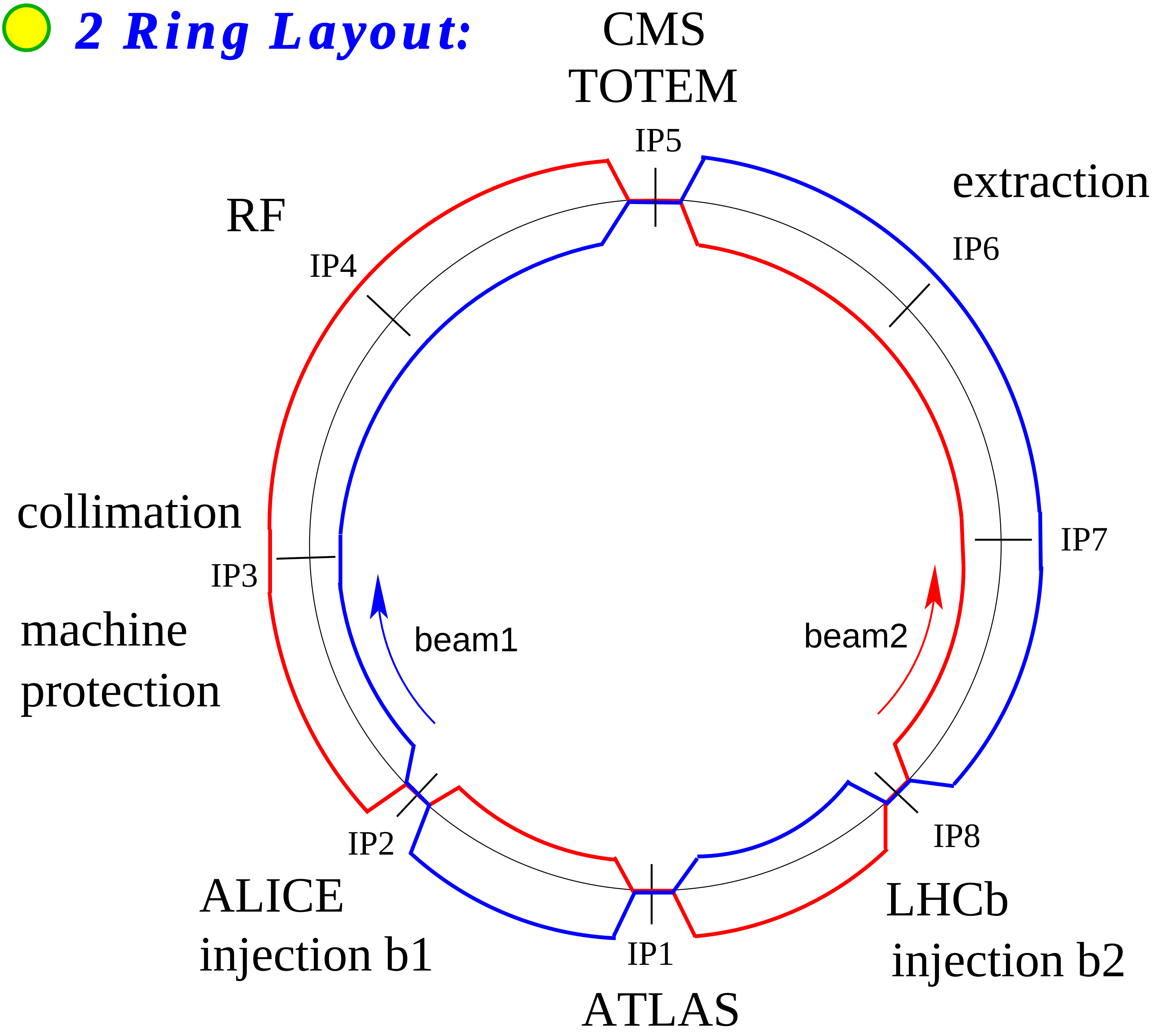
→ anti−particles hard to produc

- **beam−beam interaction**

→ requires beam separation

LHC Layout

● 2 Ring Layout:



🟩 2−in−1 magnet design

🟩 4 proton experiments + 1 ion experiment

→ beam cross−over in 4 IR’s

Lepton versus Hadron Collider

- **Leptons:** (e^+ / $\bar{e}$)
  - *elementary particles*
    - → *well defined energy*
    - → *precision experiments*

- **Hadrons:** (p^+ / $\bar{p}$)
  - *multi particle collisions*
    - → *energy spread*
    - → *discovery potential*

- **Example:**

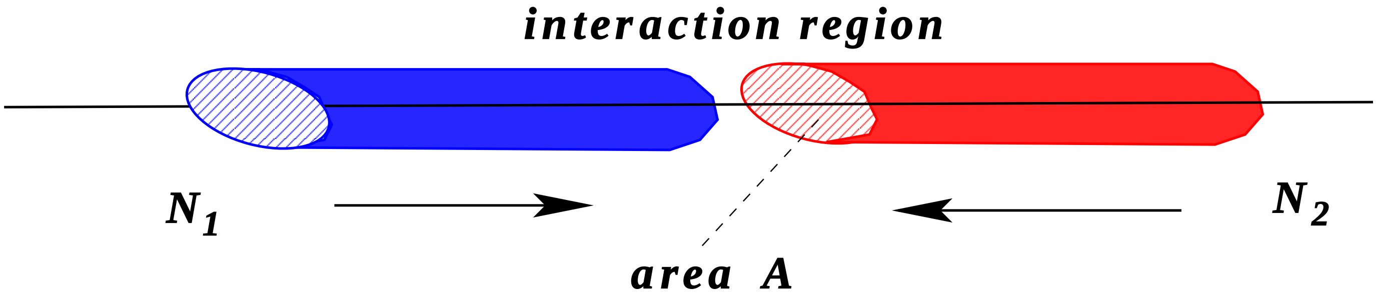
Z_0

1985 S$p\bar{p}$S p^+p^-

1990 LEP e^+e^-

Luminosity

- $N_{ev}/sec = \sigma \cdot L$ $\quad [\,L\,] = cm^{-2} \cdot s^{-1}$



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

- **high bunch current**

  beam−beam; collective effects

- **many bunches**

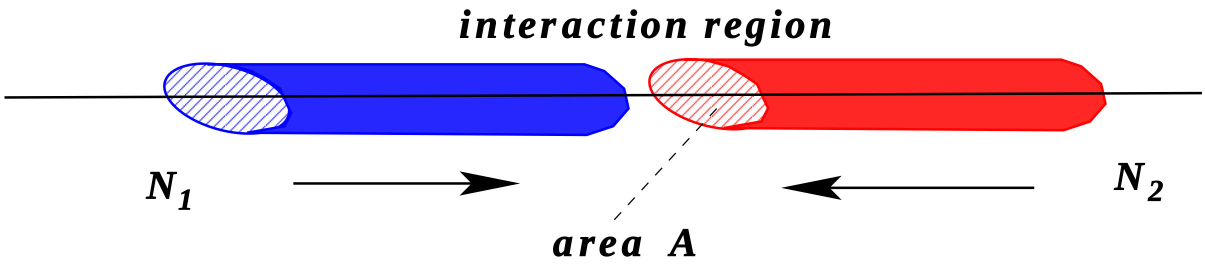
  total current (RF); collective effects

- **small beam size**

  coupling; dispersion; hardware

Beam Size

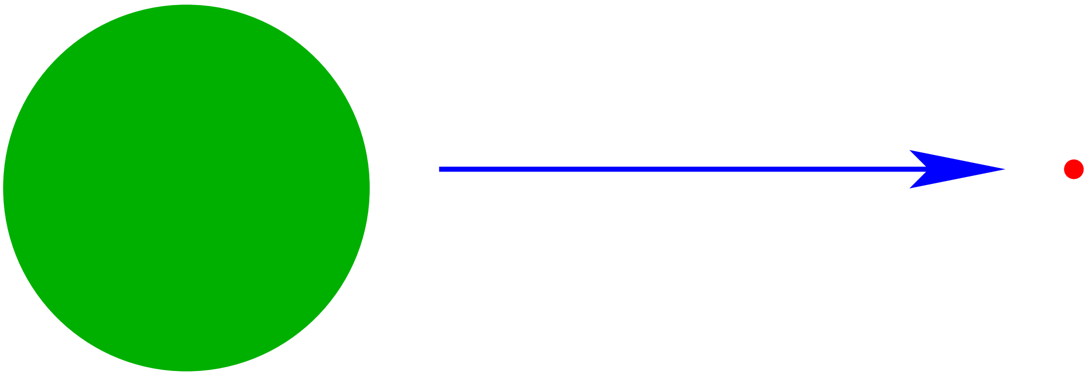
Luminosity:



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

$$A = \pi \cdot \beta \cdot \varepsilon$$

LHC:



$\langle\beta\rangle_{arc}$ = 80 meter

β_{IP} = 0.5 meter

Limit:

- 🟩 magnet strength
- 🟩 aperture

$$x = \sqrt{A \cdot \beta} \cdot \sin(\phi)$$

$$x' = \sqrt{\frac{A}{\beta}} \cdot \sin(\phi)$$

Synchrotron Radiation

Electro−Magnetic Waves :

- accelerated charge emits electro−magnetic waves

→ radio signal

→ X−rays

- radiation fan in bending plane bending plane

opening angle $\propto \frac{1}{\gamma}$

$$\left(\begin{array}{l} \text{LEP: } \gamma = 200000 \\ \text{LHC: } \gamma = 7000 \end{array} \right)$$

- $P \propto \frac{\gamma^4}{\rho^2}$

- $<E_\gamma> \propto \frac{\gamma^3}{\rho}$

Examples

	E [GeV]	ρ [km]	N [10^{12}]	U [MeV]	P [MW]	u_c [keV]
LEP 1	45	3.1	4.7	260	1.2	90
LEP 2	100	3.1	4.7	2900	30	715
LEP2+	110	3.1	312	3900	44	952
LHC	7000	3.1	312	0.007	0.005	0.04

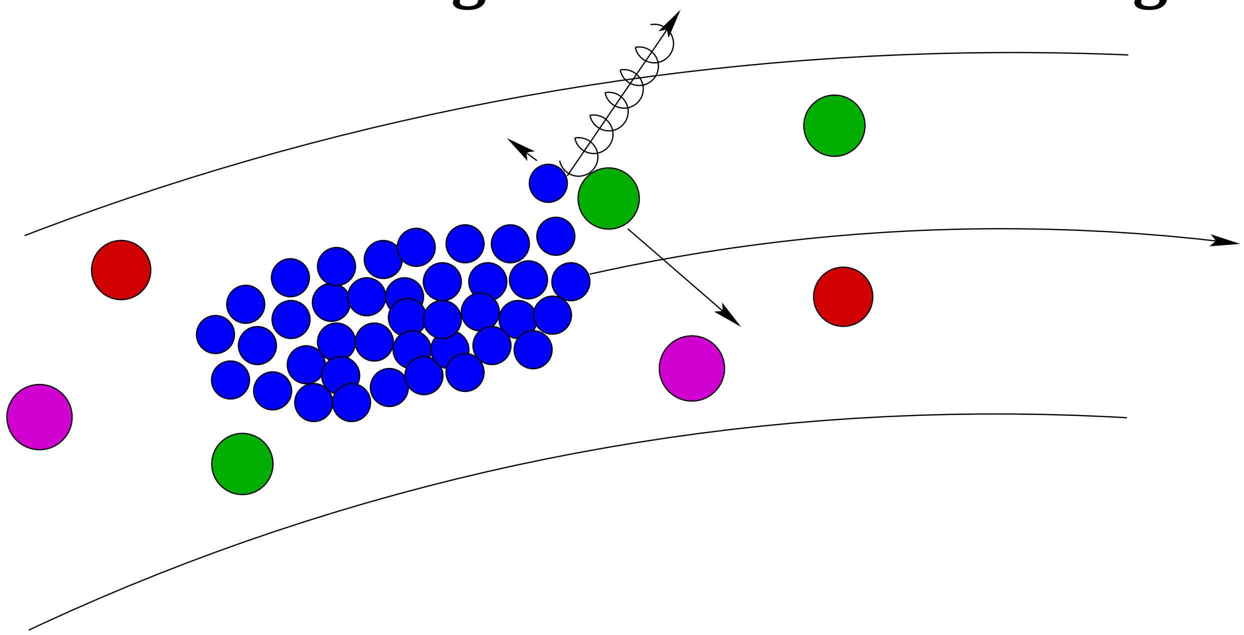
LEP 1 → X−rays

LEP 2 → γ −rays

LHC → UV light

Vacuum

Bremsstrahlung + Coulomb Scattering



→ ***beam blow-up***

→ ***particle loss***

→ ***background in experiments***

→ ***loss in luminosity!***

equipment damage!

LHC – Beam Parameter

$$L = \frac{N_p^2 \cdot n_b}{\varepsilon \cdot \beta} \cdot \frac{f_{rev}}{2\cdot\pi}$$

$$\boxed{L = 10^{34}\, cm^{-2}\, s^{-1}}$$

- **Beam–Beam Interaction:**

$$\Delta Q \propto \frac{N_b}{\varepsilon} < 5\cdot 10^{-3} \rightarrow \boxed{N_p = 10^{11}}$$

- **Beam Size:**

magnet quality + aperture $\rightarrow \varepsilon \rightarrow N_p$

- **β:** quadrupole strength + aperture

$\rightarrow \beta = 0.5$ meter

$\rightarrow n_b = 2835$

$\rightarrow I_{beam} = 0.5$ A

Beam Power

$E = 300$ MJ

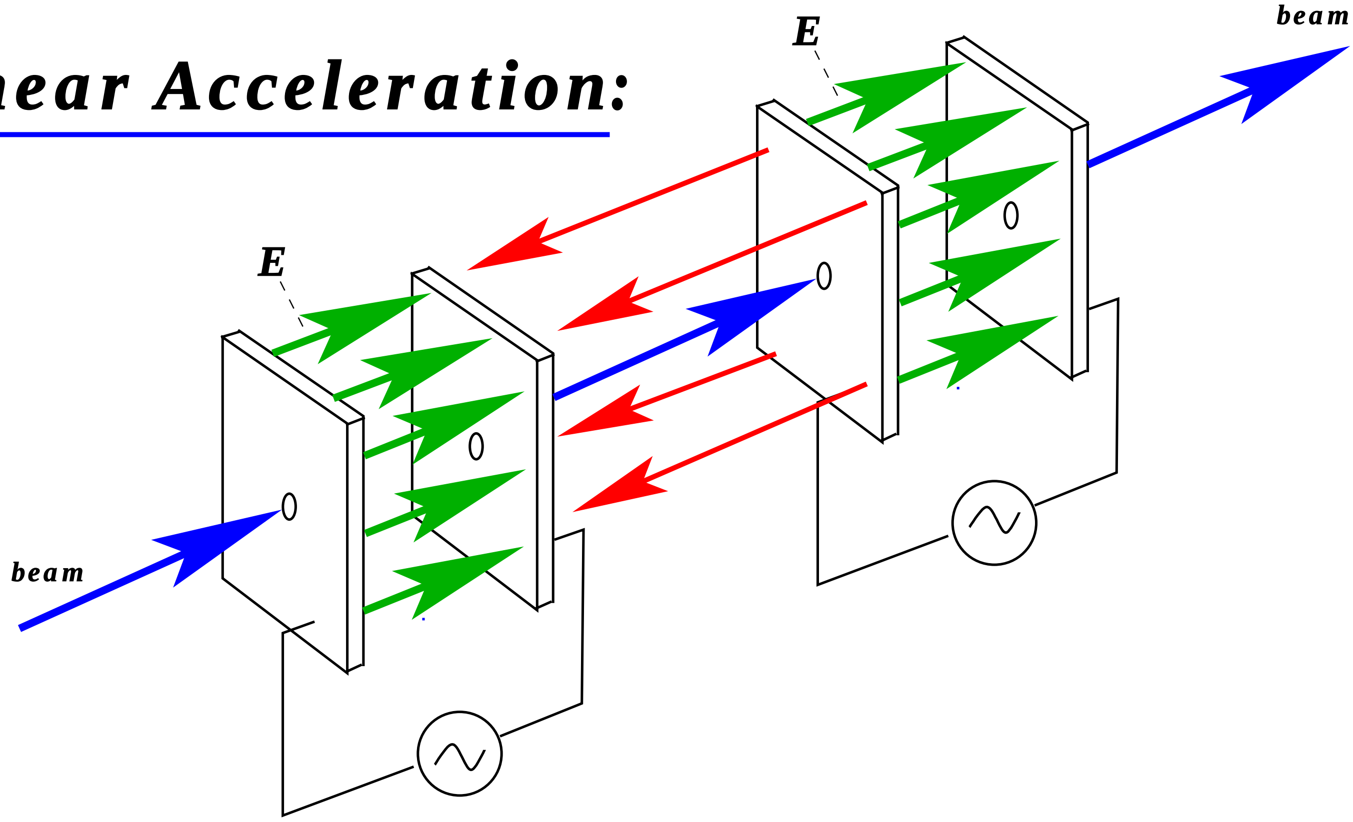
$\triangleq 120$ kg TnT

Synchrotron Radiation

$P = 0.5$ W/m

Time Varying Fields

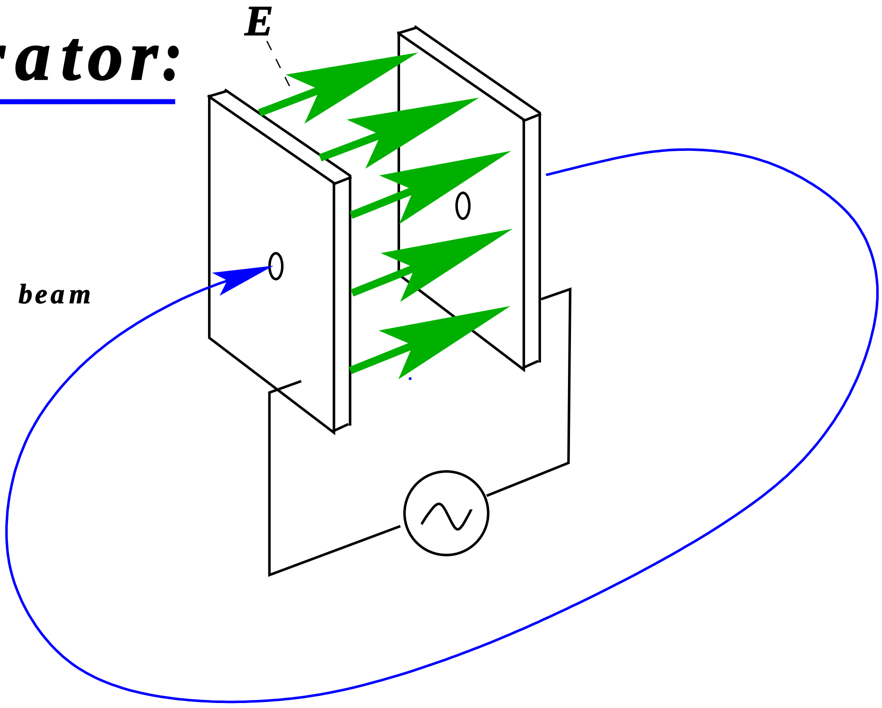
- **Linear Acceleration:**



→ bunched beam

→ long accelerator!

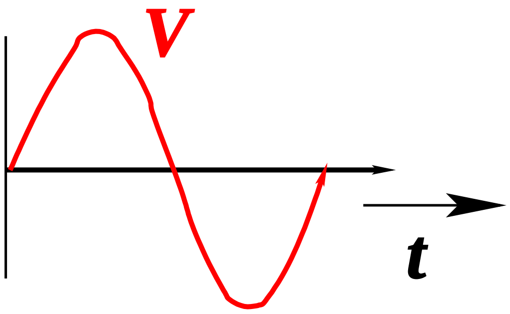
- **Circular Accelerator:**



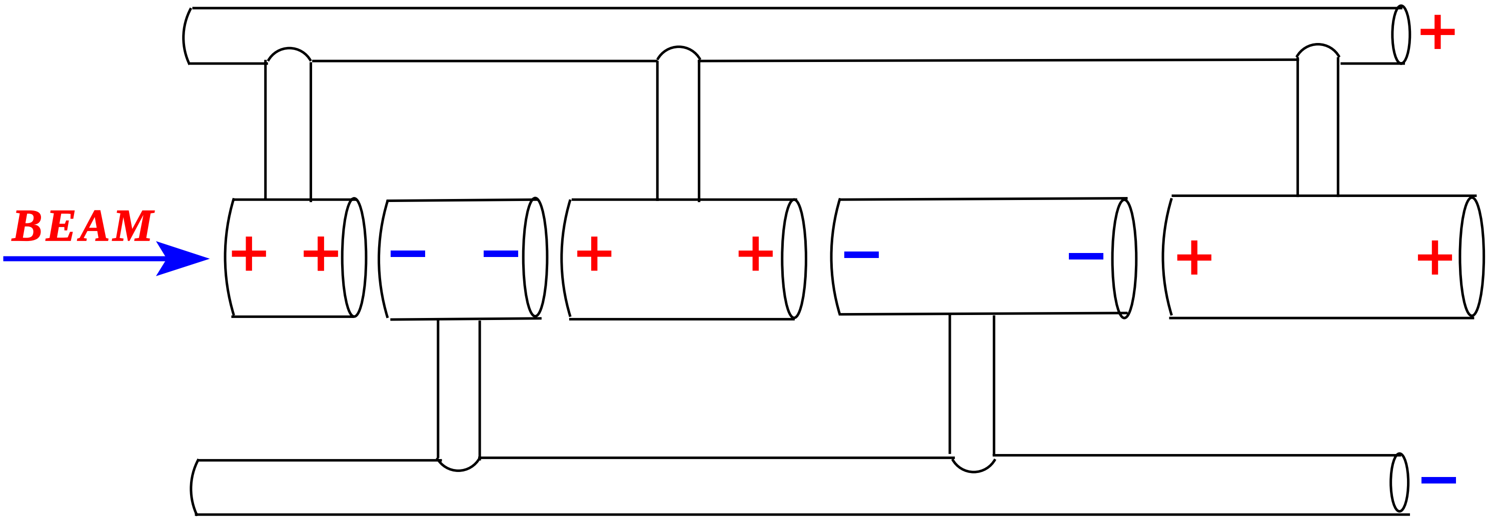
Drift Tubes

- **1924:** Ising

AC Voltage:



Symmetric line:



→ $l = v_{part} \cdot T/2$

- **1928:** demonstrated by Wideroe

1MHz, 25kV oscillator

→ 50kV potassium ions

Lawrance:

1.3MV mercury ions with 48kV

- **But:** $f < 7MHz$ ($l = 21$ meter)!

Time Varying Fields

○ *Maxwell Equations without Sources*

a) $\vec{\nabla} * \vec{E} = 0$

b) $\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$

c) $\vec{\nabla} * \vec{B} = 0$

d) $\vec{\nabla} \times \vec{B} - \frac{\mu\varepsilon}{c}\frac{\partial \vec{E}}{\partial t} = 0$

○ *Rotation on* b) *and* d)

plus: $\vec{\nabla} \times (\vec{\nabla} \times \vec{V}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{V}) - \vec{\nabla} \cdot \vec{V}$

→ *Wave equation:*

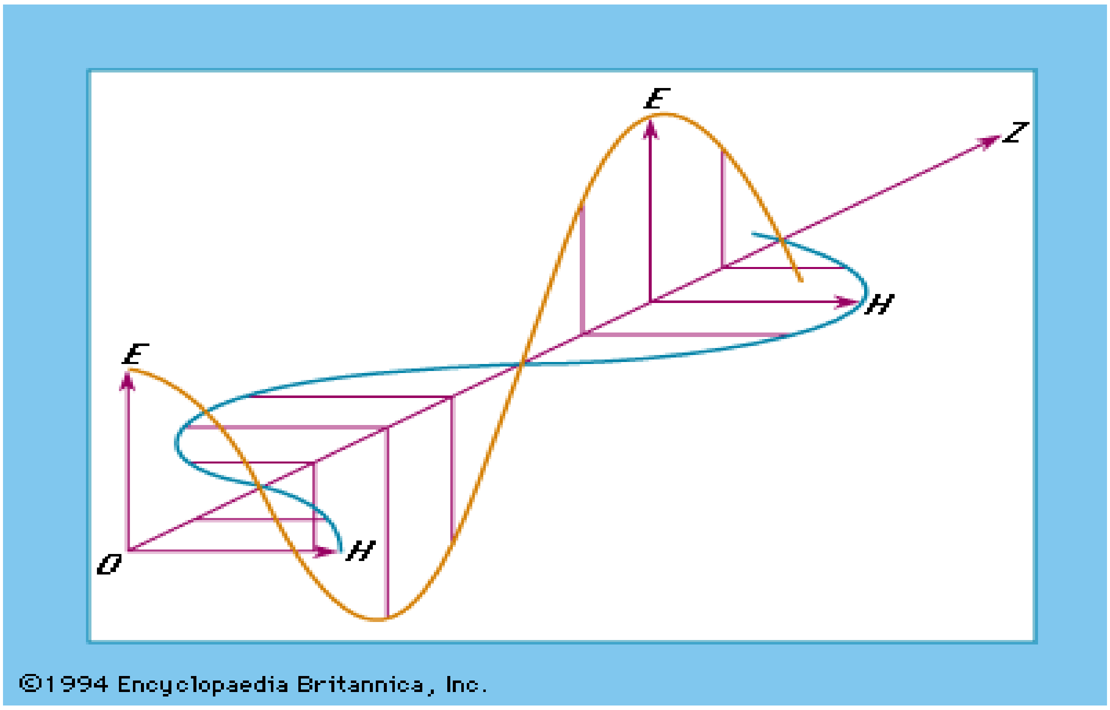
$$\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{E} \qquad \frac{\partial^2 \vec{B}}{\partial t^2} = \frac{c^2}{\mu\varepsilon} \nabla^2 \vec{B}$$

Time Varying Fields

Plane Electro Magnetic Wave:

$$\vec{E} = \vec{E}_0 \cdot e^{ik\,\vec{n}\cdot\vec{x} - \omega t} \qquad \vec{B} = \vec{B}_0 \cdot e^{ik\,\vec{n}\cdot\,\vec{x} - \omega t}$$

$$\vec{B}_0 = \sqrt{\mu\varepsilon} \cdot \vec{n} \times \vec{E}_0 \qquad k = \frac{2\,\pi}{\lambda}$$



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No acceleration in the direction of propagation!

Boundary Conditions I

Transverse Electric Waves (TE):

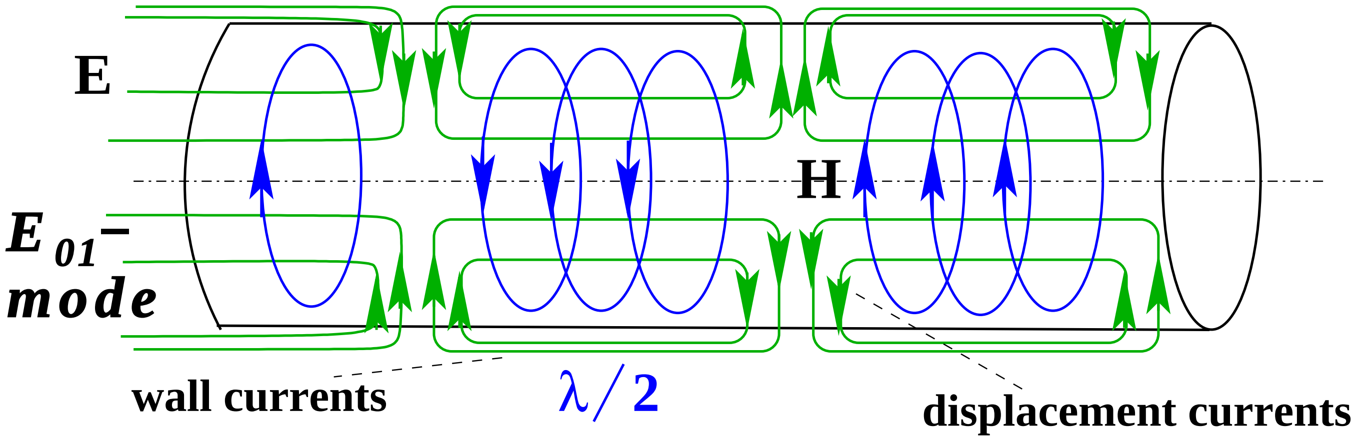
$E_z = 0$ everywhere;

Boundary condition: $\left.\frac{\partial B}{\partial n}\right|_s = 0$

Transverse Magnetic Waves (TM):

$B_z = 0$ everywhere;

Boundary condition: $\left.E_n\right|_s = 0$



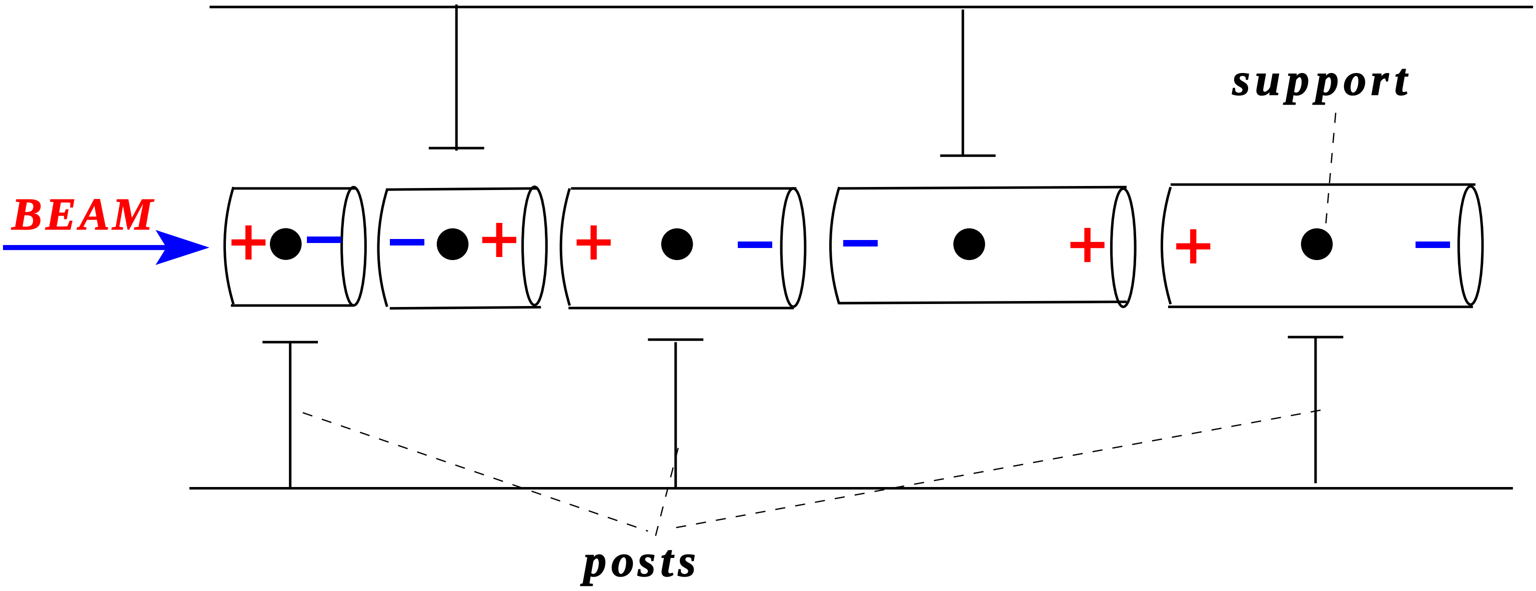
🟩 **Problem:** $v_{ph} > c$

→ Shielding or change v_{ph}

Resonance Tank

- Alvarez:

$l = v_{part} \cdot T$



- Tubes are passive

$\longrightarrow$ higher frequencies!

(f = 200 MHz gives good tube size)

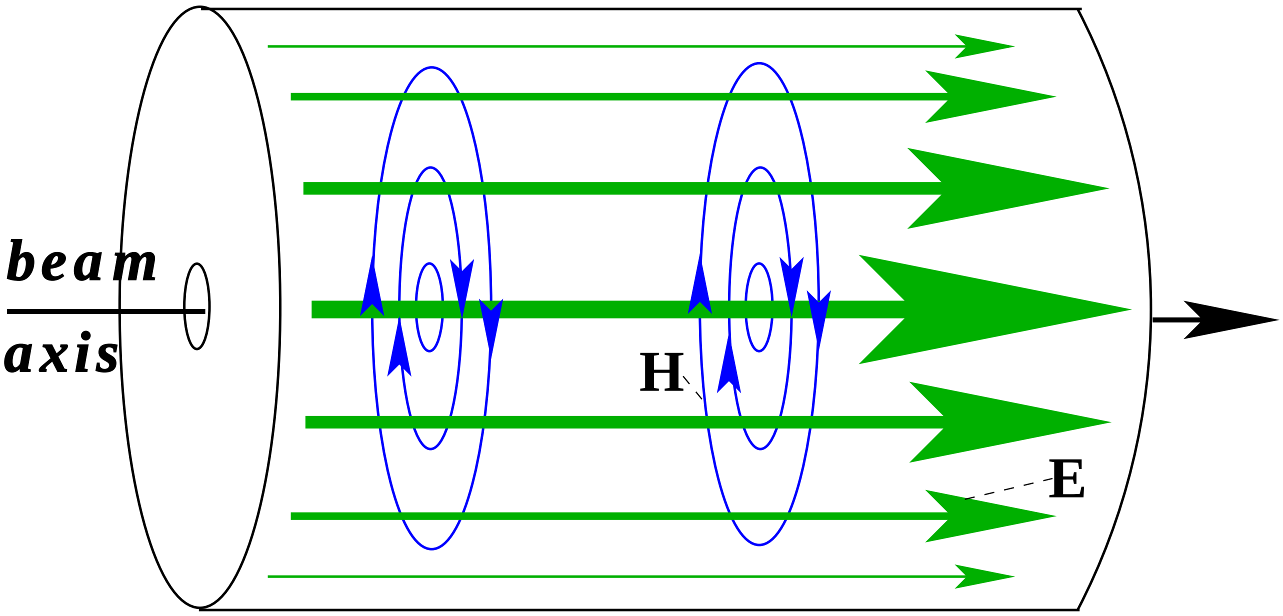
- Posts $\longrightarrow$ $v_{gr} \neq 0$

- Pre−accelerator for most proton acclelerators

Boundary Conditions II

● *Cavity Resonator:*

TM mode with longitudinal boundary;



🟩 *Short Section:*

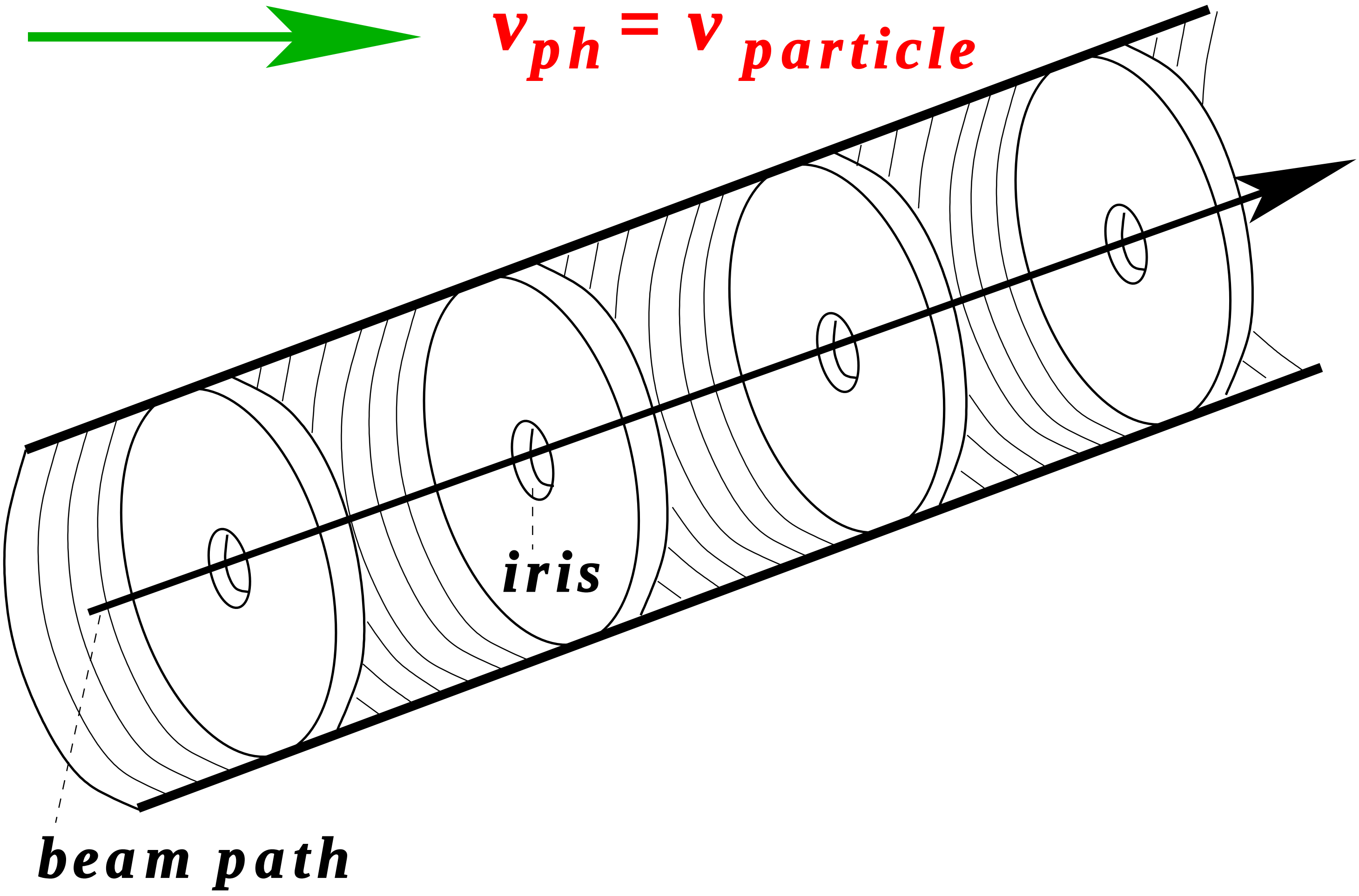
→ ***multi-cell***

→ ***multi-passage***

Boundary Conditions III

- **Loaded Wave Guide:**

→ $v_{ph} = v_{particle}$



- **But:**

Concept of linear acceleration is limited by power of RF generator!

→ Not feasible before World War II