Color superconducting quark phases in compact stars

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in collaboration with

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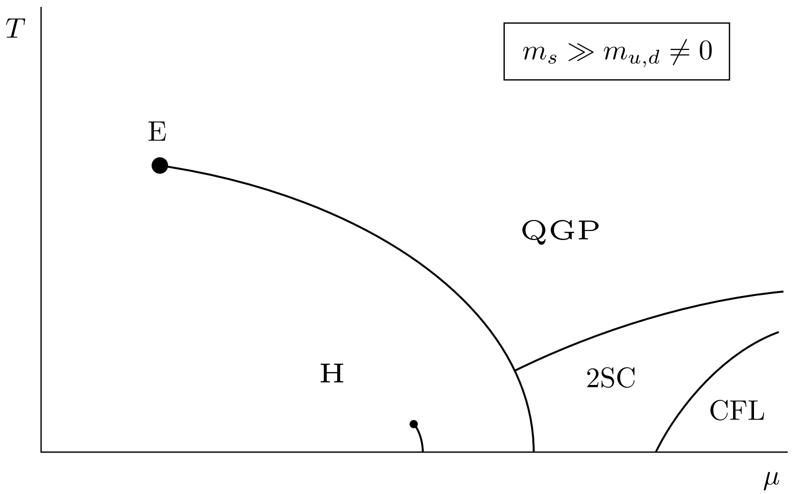
Skopelos, Greece, June 2004

Introduction: The QCD Phase Diagram ...

- schematic QCD phase diagram (2+1 flavors)

(Rajagopal, Wilczek, hep-ph/0011333)



- hadronic phase (H):

  $\langle\bar{\psi}\psi\rangle \neq 0, \ \langle\psi\psi\rangle = 0$

- quark-gluon plasma (QGP):

  $\langle\bar{\psi}\psi\rangle \approx 0, \langle\psi\psi\rangle = 0$

- two-flavor color superconductor (2SC):

  $\langle\bar{\psi}\psi\rangle \approx 0 \ \langle ud\rangle \neq 0$

- color-flavor locking (CFL):

  $\langle ud\rangle \approx \langle us\rangle = \langle ds\rangle \neq 0$

- further possible phases:

  color superconding crystals, CFL + kaon condensate, spin-1 condensates, gapless color superconductors ...

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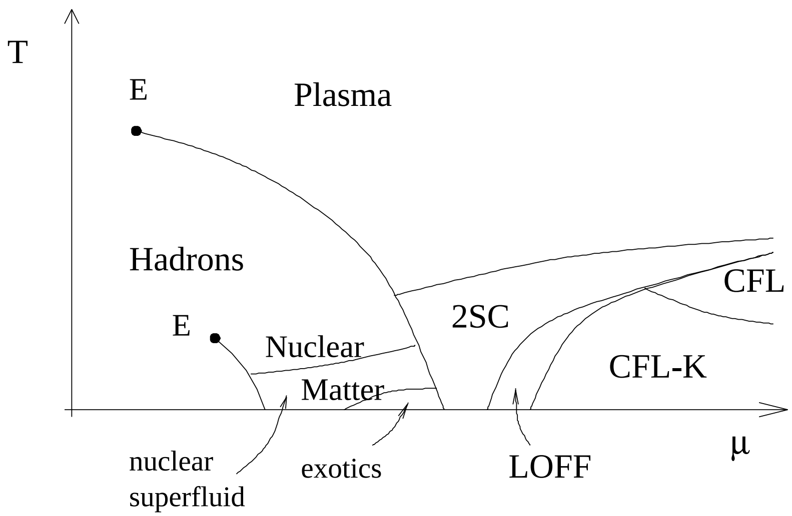
(T. Schäfer, hep-ph/0304281)



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From Theory to 'Experiment'

- Quark phases in compact stars:
  - Existence ?
  - Signatures: cooling, magnetic fields, ... ? (not discussed here)
- Prerequisite: electric and color neutrality
  - nonequal chemical potentials, e.g., $\mu_u \neq \mu_d$
  - stability of the 2SC phase ?
- This talk:

  study these issues within NJL-type models

Diquark condensates

Diquark channels

- diquark condensates: $\langle \psi^T \hat{\mathcal{O}} \psi \rangle$
  - ψ : quark field operator
  - $\hat{\mathcal{O}}$: operator in color, flavor, and Dirac space
- Pauli principle: $\hat{\mathcal{O}}$ must be **totally antisymmetric**.

	symmetric	antisymmetric
Dirac	$C\gamma^\mu, \; C\sigma^{\mu\nu}$ A T	$C, \; C\gamma_5, \; C\gamma_5\gamma^\mu$ P S V
$U(2)$	$\underbrace{\mathbb{1}, \tau_1, \tau_3}_{3}$	$\underbrace{\tau_2}_{1}$
$U(3)$	$\underbrace{\mathbb{1}, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8}_{6}$	$\underbrace{\lambda_2, \lambda_5, \lambda_7}_{\bar{3}}$

(C : charge conjugation)

- many allowed combinations ! $\rightarrow$ The interaction must decide ...

scalar color-$\bar{3}$ condensates

- most attractive diquark channel for many interactions (e.g., instantons, one-gluon exchange):

$$s_{AA'} = \langle \psi^T \, C\gamma_5 \, \tau_A \, \lambda_{A'} \, \psi \rangle$$

  - τ_A : antisymmetric flavor $SU(N_f)$-generator
  - $\lambda_{A'}$: antisymmetric color $SU(N_c)$-generator

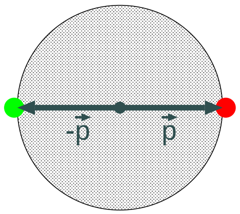
- 2 flavor color-superconductor (2SC): $\tau_A = \tau_2$
  - We can always choose $\lambda_{A'} = \lambda_2$

    $\rightarrow \quad s_{22} \neq 0, \quad s_{ij} = 0 \quad \text{for} \quad (i,j) \neq (2,2)$

- 3 degenerate flavors: $\tau_A = \tau_2, \tau_5, \tau_7$
  - various non-equivalent color-flavor combinations
  - most favored at large μ:

    $s_{22} = s_{55} = s_{77} \neq 0 \,, \quad s_{ij} = 0 \quad \text{for} \quad i \neq j$ "color-flavor locking"

more realistic case: $M_u \simeq M_d < M_s < \infty$



- precondition for standard BCS pairing: $|p_F^a - p_F^b| \lesssim \sqrt{2}\Delta_{ab}$
  (but see Mei Huang's talk for exceptions)
  - $\underline{\mu \gg M_s}$ $\Rightarrow$ $p_F^u = \sqrt{\mu^2 - M_u^2} \approx \sqrt{\mu^2 - M_s^2} = p_F^s$ $\longrightarrow$ CFL
  - $\underline{\mu \sim M_s}$ $\Rightarrow$ $p_F^u \gg p_F^s$ $\longrightarrow$ 2SC phase
    (with or without unpaired s-quarks)
- favored state at intermediate densities?
- M_u, M_d, M_s: effective ("constituent") quark masses
  - related to $\langle \bar{u}u \rangle$, $\langle \bar{u}d \rangle$, $\langle \bar{s}s \rangle$
  - T and μ dependent
  - interdependence: masses $\leftrightarrow$ diquark condensates

Interaction

- microscopic treatment within QCD:
  - asymptotic densities $\rightarrow$ α_s = small $\rightarrow$ gluon exchange
  - optimistic estimate: $\mu > 1.5$ GeV $\rightarrow$ $\rho_B > 175\ \rho_0$
  - Rajagopal and Shuster, PRD (2000): $\mu \gg 10^5$ GeV !!!
- "model independent" studies:
  - expansions in Δ/μ, M_s/μ
  - expansion parameters not necessarily small
  - misses μ-dependence of Δ or M_s
- model calculations:
  - based on vacuum phenomenology
    $\rightarrow$ extrapolation of parameters into an unknown regime
  - relatively simple $\rightarrow$ allows for tackling more complex problems
  - NJL model: naturally suited for studying the competion of
    $\langle qq \rangle$ and $\langle \bar{q}q \rangle$ condensates

Model calculation

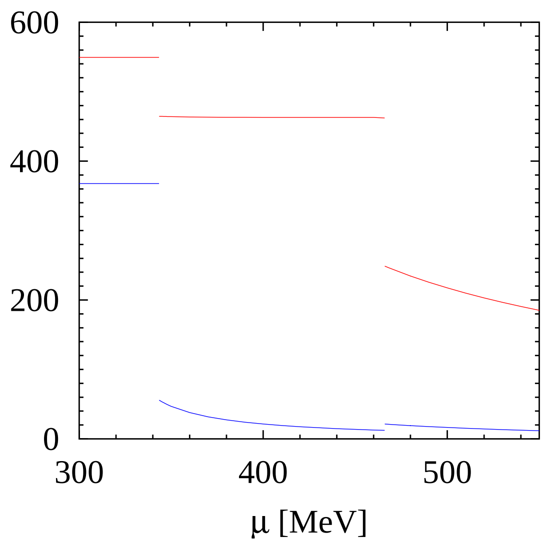
- NJL-type Lagrangian: $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$
  - free part:
$$\mathcal{L}_0 = \bar{\psi}(i\partial\!\!\!/ - \hat{m})\psi \ , \qquad \hat{m} = diag_f(m_u, m_d, m_s)$$
  - quark-antiquark interaction:
$$\mathcal{L}_{\bar{q}q} = G\left\{(\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2\right\} - K\left\{\det{}_f\Big(\bar{\psi}(1+\gamma_5)\psi\Big) + \det{}_f\Big(\bar{\psi}(1-\gamma_5)\psi\Big)\right\}$$
  - quark-quark interaction:
$$\mathcal{L}_{qq} = H\,(\bar{\psi}\,i\gamma_5\tau_A\lambda_{A'}\,C\bar{\psi}^T)(\psi^T C\,i\gamma_5\tau_A\lambda_{A'}\,\psi)$$
- mean-field approximation:
  - six condensates: $\langle\bar{u}u\rangle, \quad \langle\bar{d}d\rangle, \quad \langle\bar{s}s\rangle; \quad \langle ud\rangle, \quad \langle us\rangle, \quad \langle ds\rangle$

  $\rightarrow$ six coupled gap equations for $M_u, \quad M_d, \quad M_s; \quad \Delta_{ud}, \quad \Delta_{us}, \quad \Delta_{ds}$

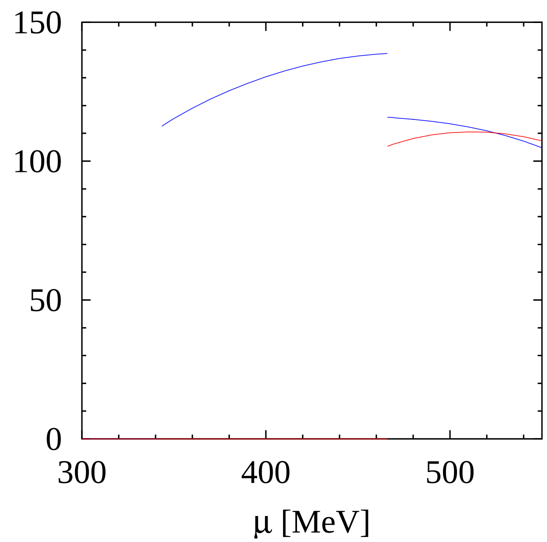
Numerical results

- Parameters fixed to reproduce reasonable vacuum properties
- $T = 0$, equal chemical potentials:

$M_u = M_d$, M_s



Δ_{ud}, $\Delta_{us} = \Delta_{ds}$



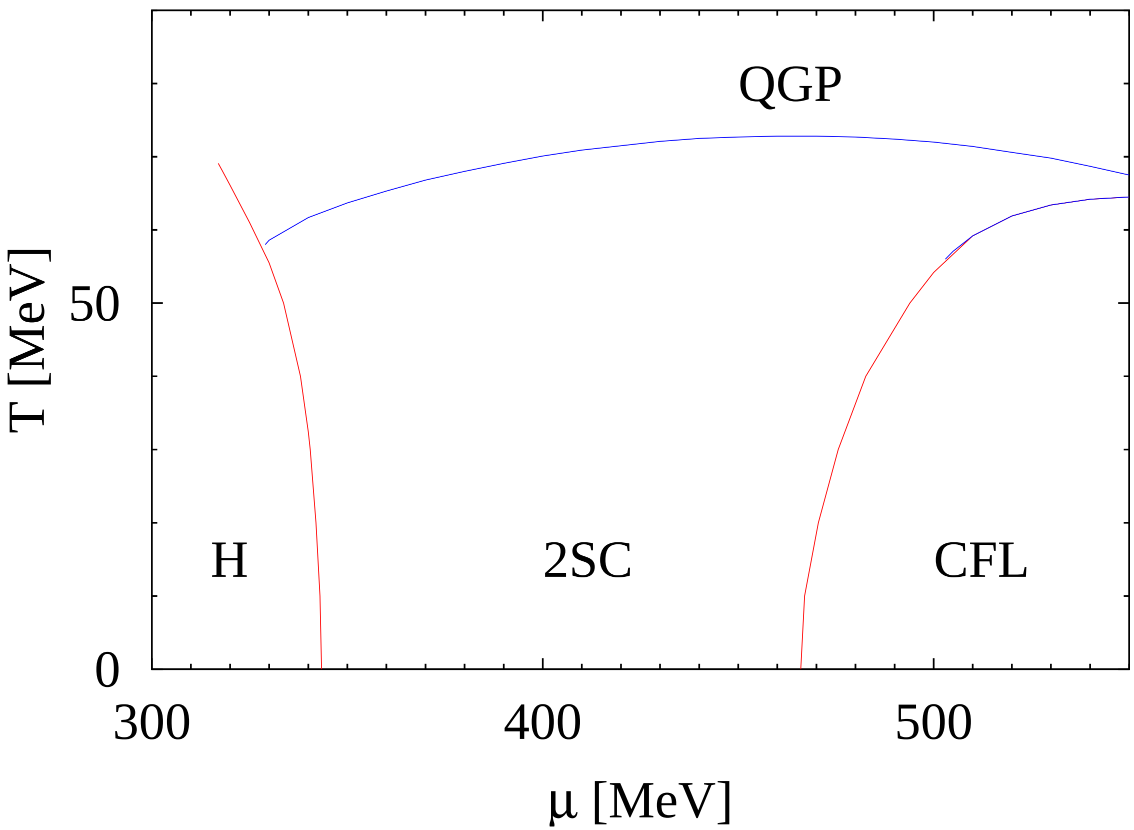
- Two distinct first-order phase transitions:

  normal $\longrightarrow$ 2SC $\longrightarrow$ CFL

- strong interdependence masses $\leftrightarrow$ diquark condensates

Phase diagram

first and second order phase transitions:



Quark matter in compact stars

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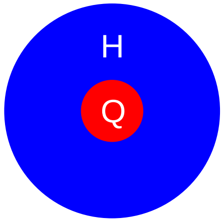


- quark core of a neutron star:
  - quarks (u, d, s) + leptons
  - after a few minutes: neutrinos untrapped
- additional constraints:
  - β equilibrium: $d, s \leftrightarrow u + e^- + \bar{\nu}_e \quad \Rightarrow \quad \mu_d = \mu_s = \mu_u + \mu_e$
  - electric charge neutrality: $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$
  - color singletness $\Rightarrow$ color neutrality: $n_r = n_g = n_b$
- consequences:
  - unequal Fermi momenta for u and d
  - instability of the ud condensate (no 2SC phase) ??

    (M. Alford K. Rajagopal, JHEP 0206 (2002) 031)

Limiting cases:

- case 1: M_s small (Alford, Rajagopal, '02)
  - $M_s = 0$: $n_u = n_d = n_s$
  - Taylor expansion in M_s:
    $p_F^d = p_F^u + \frac{M_s^2}{4\bar{\mu}}$, $p_F^s = p_F^u - \frac{M_s^2}{4\bar{\mu}}$ equidistant Fermi momenta!

  $\Rightarrow$ us pairing as likely as ud pairing

  $\Rightarrow$ whenever ud pairing is more favored than no pairing, CFL is even more favored

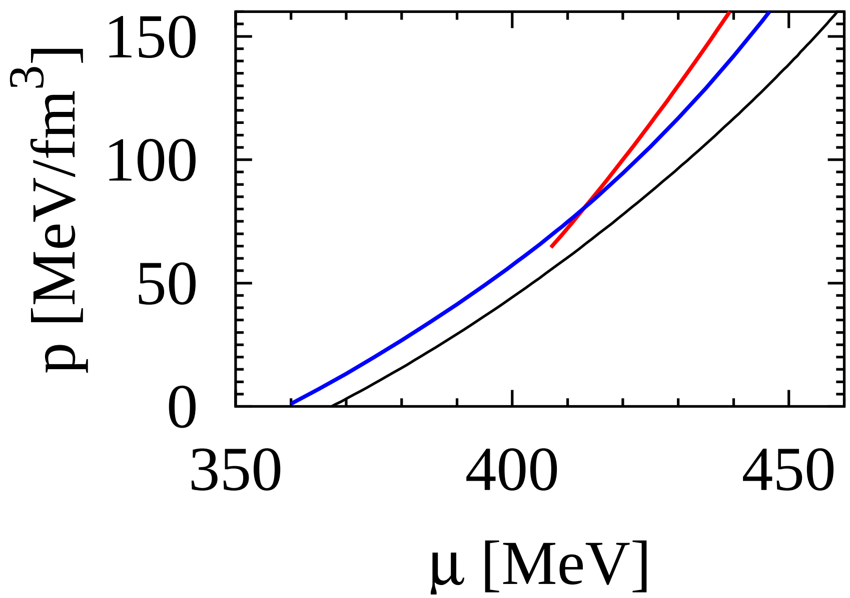
  $\Rightarrow$ no 2SC phase

- case 2: M_s large $\Rightarrow$ no strange quarks
  - $n_d \simeq 2\, n_u$ $\Rightarrow$ $p_F^d \simeq 2^{1/3}\, p_F^u \simeq \frac{5}{4}\, p_F^u$,
  - stability criterion for standard BCS pairing: $\Delta > \delta\mu/\sqrt{2}$
  - example: $p_F^u = 400$ MeV $\Rightarrow$ $p_F^d = 500$ MeV $\Rightarrow$ $\Delta > 70$ MeV

  $\Rightarrow$ 2SC phase possible if interaction strong enough

Homogeneous neutral matter: numerical results

- pressure: (N, 2SC, CFL)

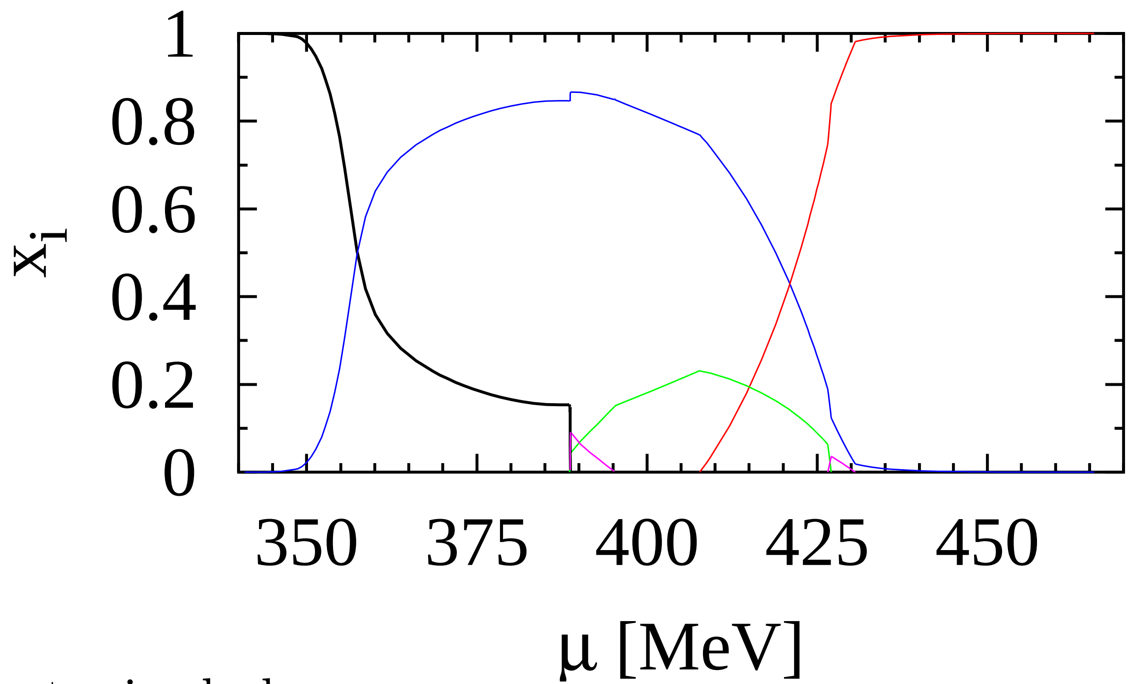


- CFL favored for large μ
- 2SC favored for small μ
- normal quark matter never favored

- masses and gaps in the 2SC phase:
  - $\Delta_{ud} \sim 100$ MeV
  - $M_s \sim \mu \gg M_u, M_d$ $\Rightarrow$ Taylor expansion in M_s fails

Mixed quark phases

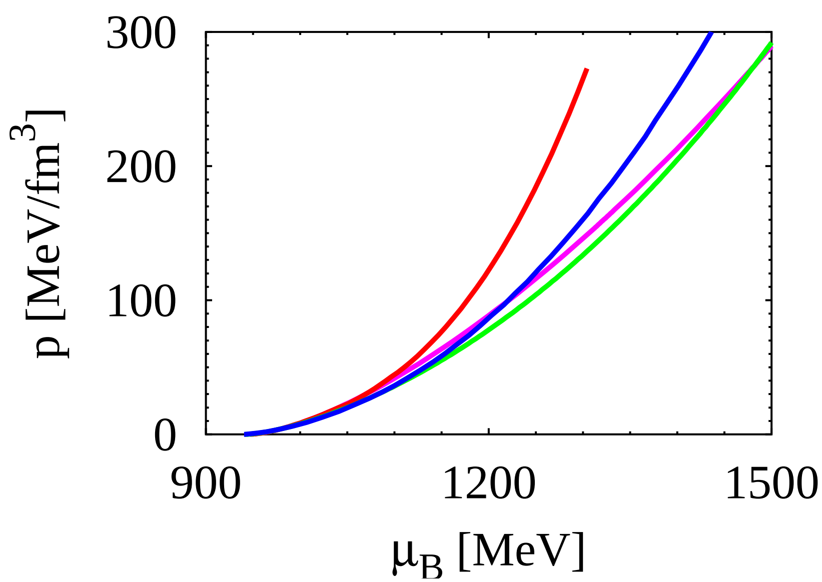
- composition: N 2SC $2SC_{us}$ SC_{us+ds} CFL



  - 9 different mixed phases
  - 2-, 3-, and 4-component systems
  - "exotic" components: SC_{us+ds}, $2SC_{us}$
- BUT: likely to be unstable if surface and Coulomb effects are included

Application to compact stars

- homogeneous neutral NJL quark matter
- various hadronic EOS:

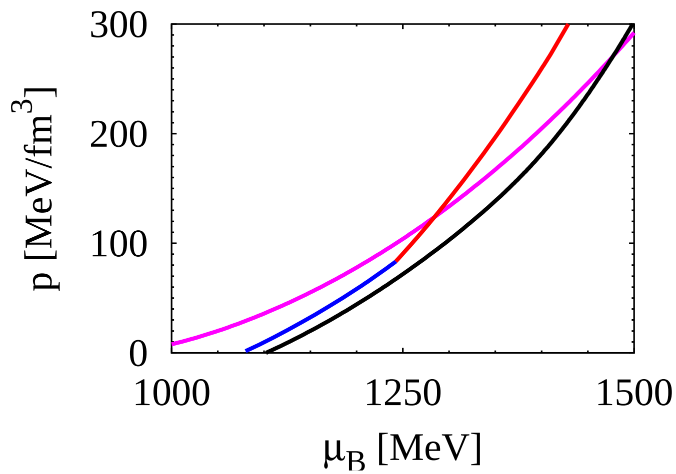


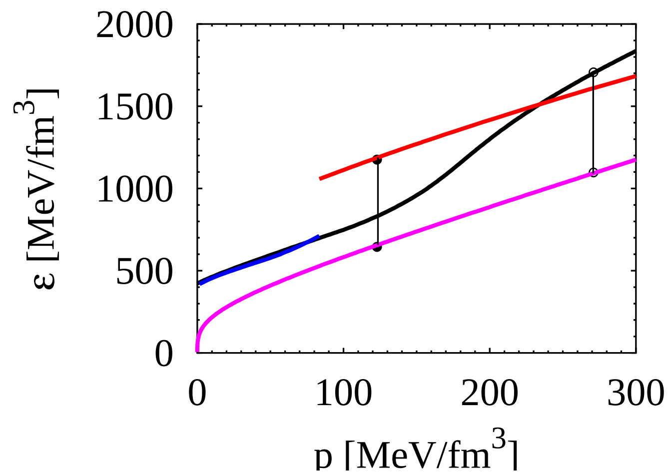
- BHF (nucleons and leptons only) (Baldo et al.)
- BHF (nucleons, hyperons, and leptons) (Baldo et al.)
- relativistic mean field w/ hyperons (Glendenning)
- chiral SU(3) model (Hanauske et al.)

- construct sharp phase transition
- solve Toman-Oppenheimer-Volkoff equation

Example: chiral SU(3) hadronic EOS

- hadron-quark phase transition (H, N, 2SC, CFL)



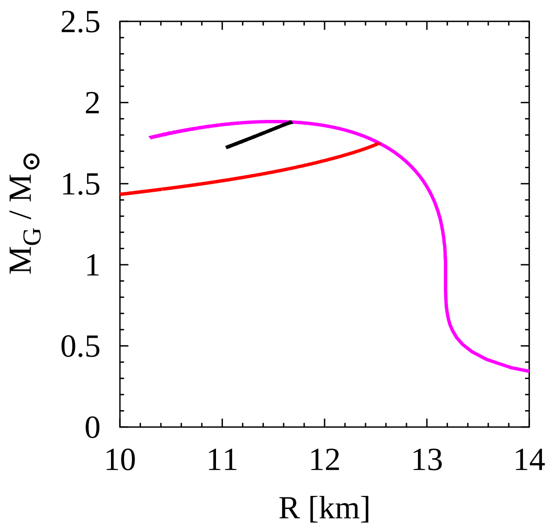


- μ_{crit}(H $\rightarrow$ CFL) $<$ μ_{crit}(H $\rightarrow$ N)
- 2SC solution irrelevant

- strong discontinuity of ϵ at μ_{crit}

- solutions of the TOV equation:

no stable configuration
with pure quark matter core!



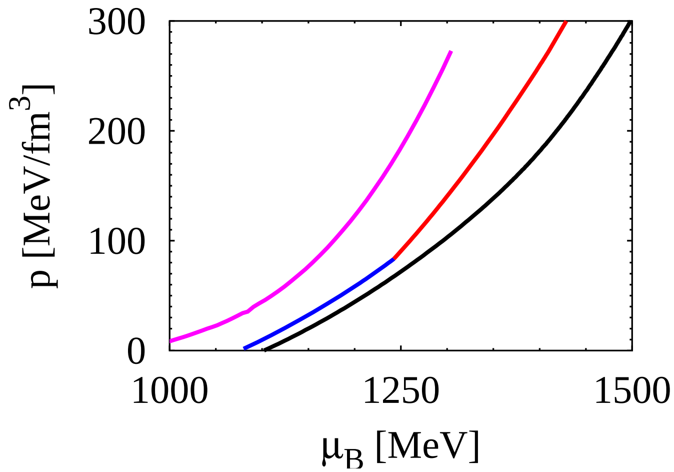
Other hadronic EOS

- BHF without hyperons: practically the same result

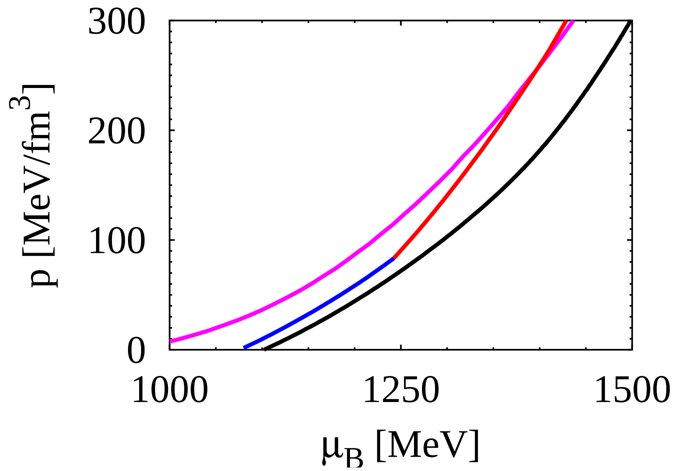
- BHF with hyperons:

  (H, N, 2SC, CFL)

  no phase transition at all!



- relativistic mean field:
  - H $\not\to$ N
  - H $\to$ CFL
  - phase transition renders star unstable



Discussion

- Summarizing the results up to this point:

  NJL quark matter can compete with hadronic matter only
  if there is a non-negligible fraction of strange quarks.

  $\longrightarrow$ strong increase of the energy density at the phase transition

  $\longrightarrow$ star gets unstable

  $\longrightarrow$ no pure quark matter core in compact stars

- stable hybrid stars in the bag model:

  strange quark masses and bag constant typically smaller than in NJL

- BUT: recent example for stable hybrid stars in **two-flavor** NJL

  (Shovkovy et al., PRD (2003))

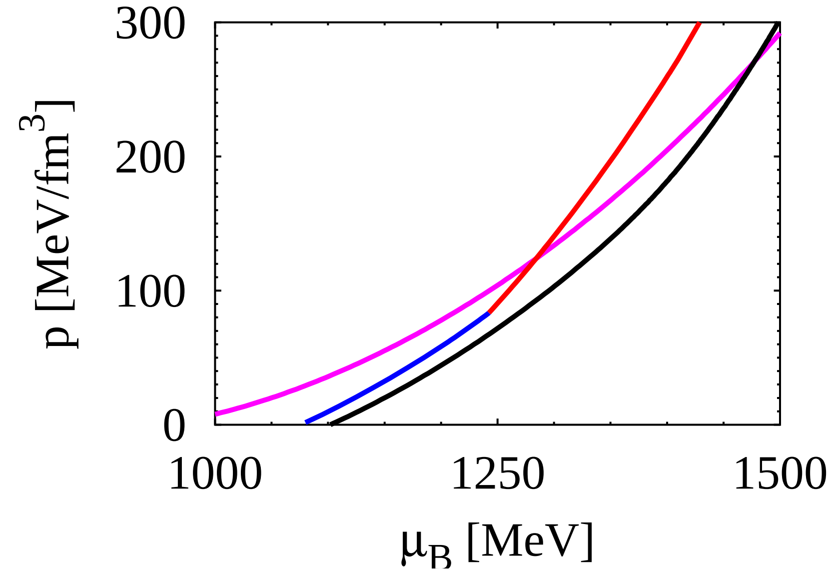
  - still possible if strange quarks are included ?
  - parameter dependence ?

Different NJL-model parameters

- literature fits to pseudoscalar spectrum in vacuum
- so far: $M_u^{vac} = 368$ MeV, $M_s^{vac} = 550$ MeV
  (Rehberg, Klevansky, Hüfner, PRC '96)
- alternative: $M_u^{vac} = 335$ MeV, $M_s^{vac} = 527$ MeV
  (Hatsuda & Kunihiro, Phys. Rep. '94)
- impact on the pressure:

Rehberg, Klevansky, Hüfner:

(N, 2SC, CFL, $H = \chi SU(3)$)



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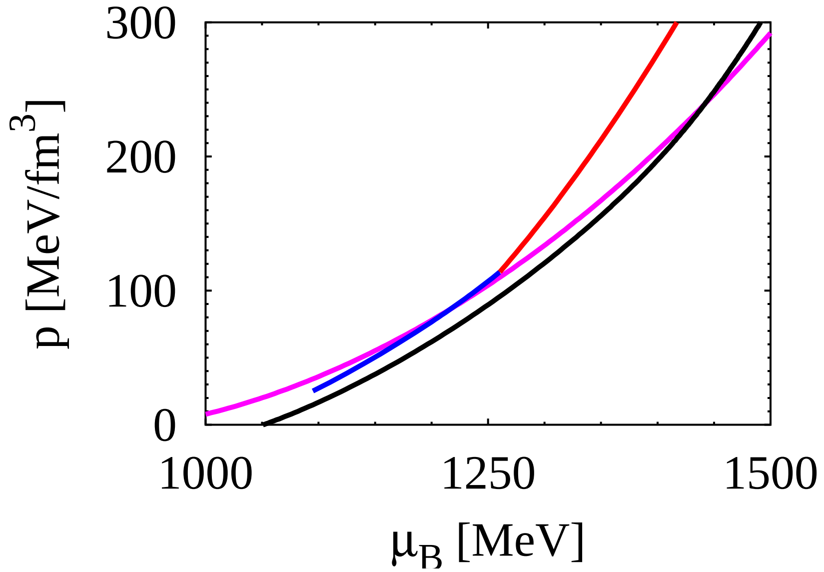
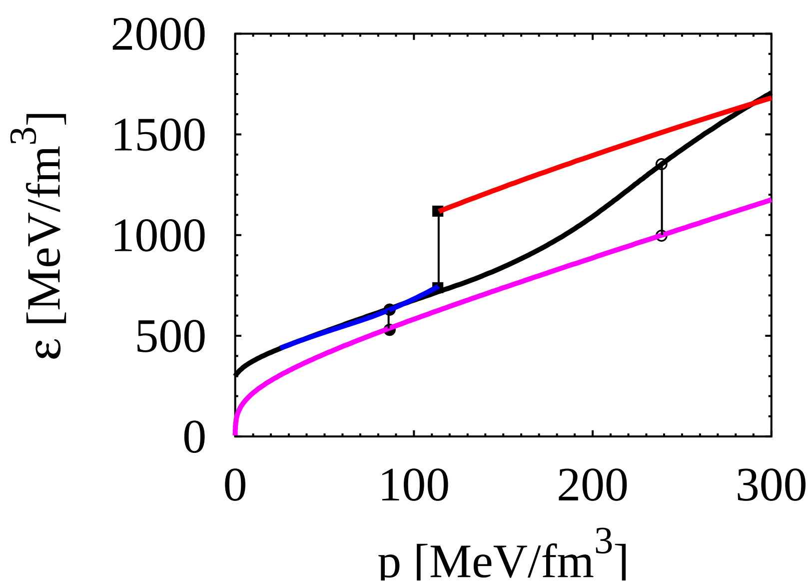
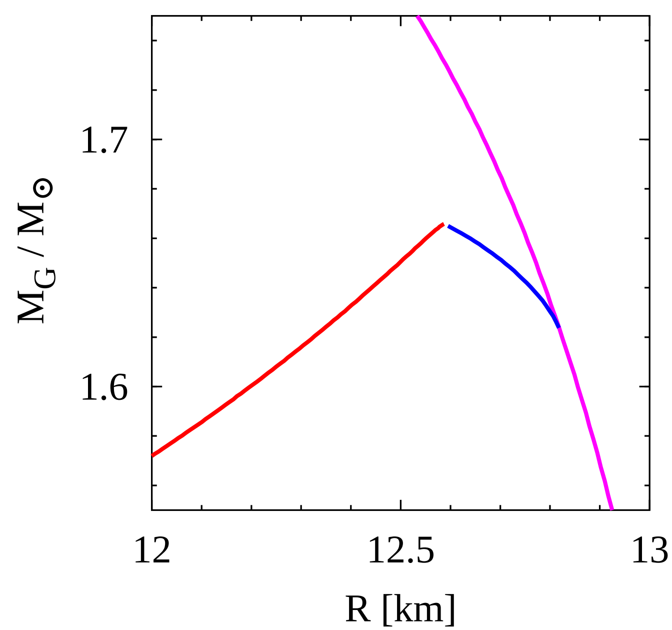


Results

- most results qualitatively unchanged
- only exception: χSU(3) $\to$ 2SC $\to$ CFL
- in this case:
  - modest increase of the energy density at H $\to$ 2SC, strong increase at 2SC $\to$ CFL
  - TOV: stable 2SC core, unstable CFL core





Conclusions

- NJL-model study of quark-matter cores in compact stars:
  - three $\langle qq \rangle$ and three $\langle \bar{q}q \rangle$ condensates under the constraints imposed by electric and color neutrality
  - 2 quark $\times$ 4 hadronic EOS w/ and w/o diquark condensation: only one case with stable pure quark matter core
  - stable case: 2SC phase with very few strange quarks
  - no stable CFL-matter core
- These results can at best be strong hints because the model parameters fixed in vacuum may be completely off at high densities.
- However, they
  - provide a counter example to the "model independent" prediction of absence of the 2SC phase in compact stars.
  - demonstrate the possible importance of μ-dependent constituent masses and gaps and their interplay.