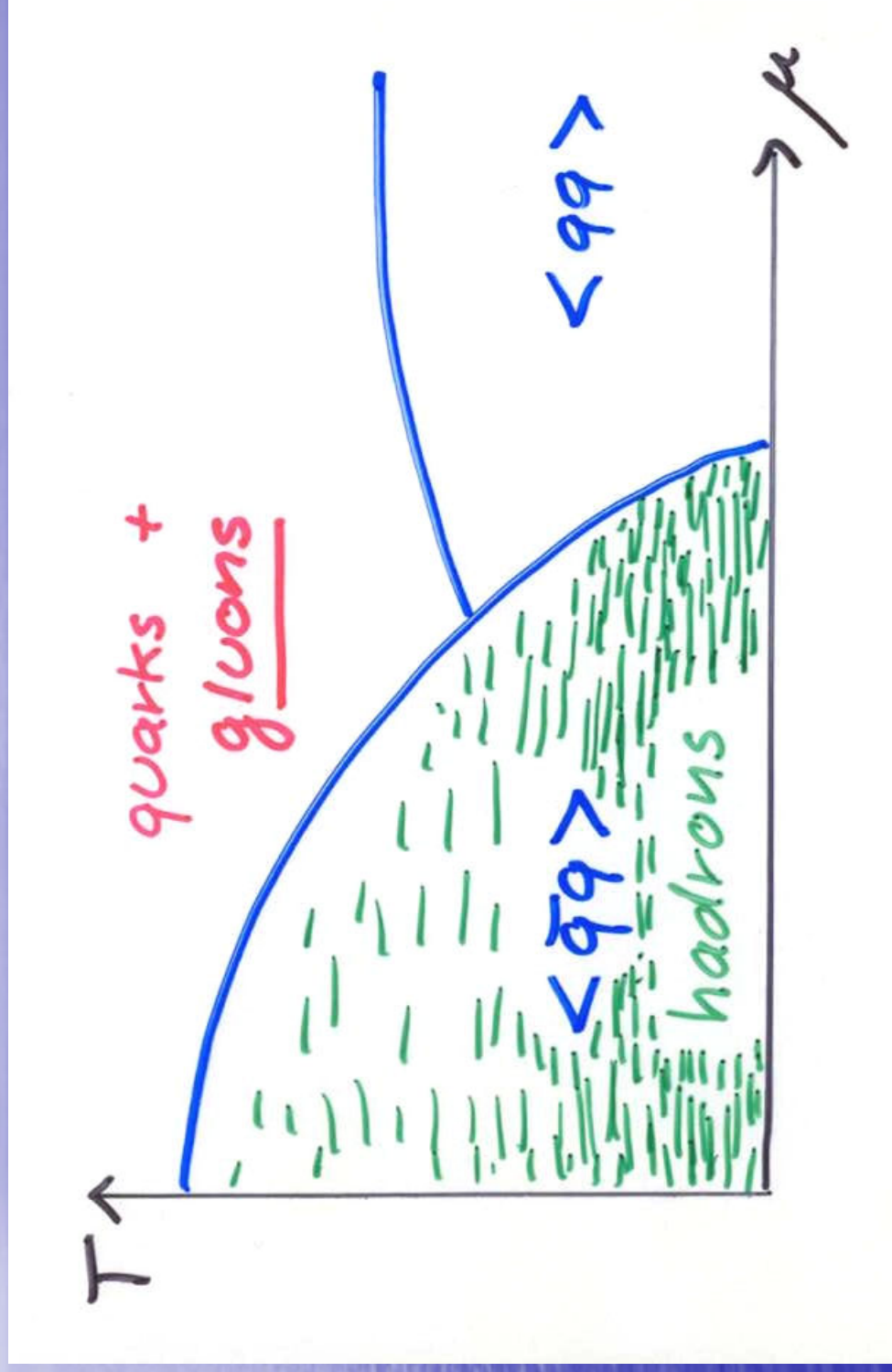


Critical lines and points

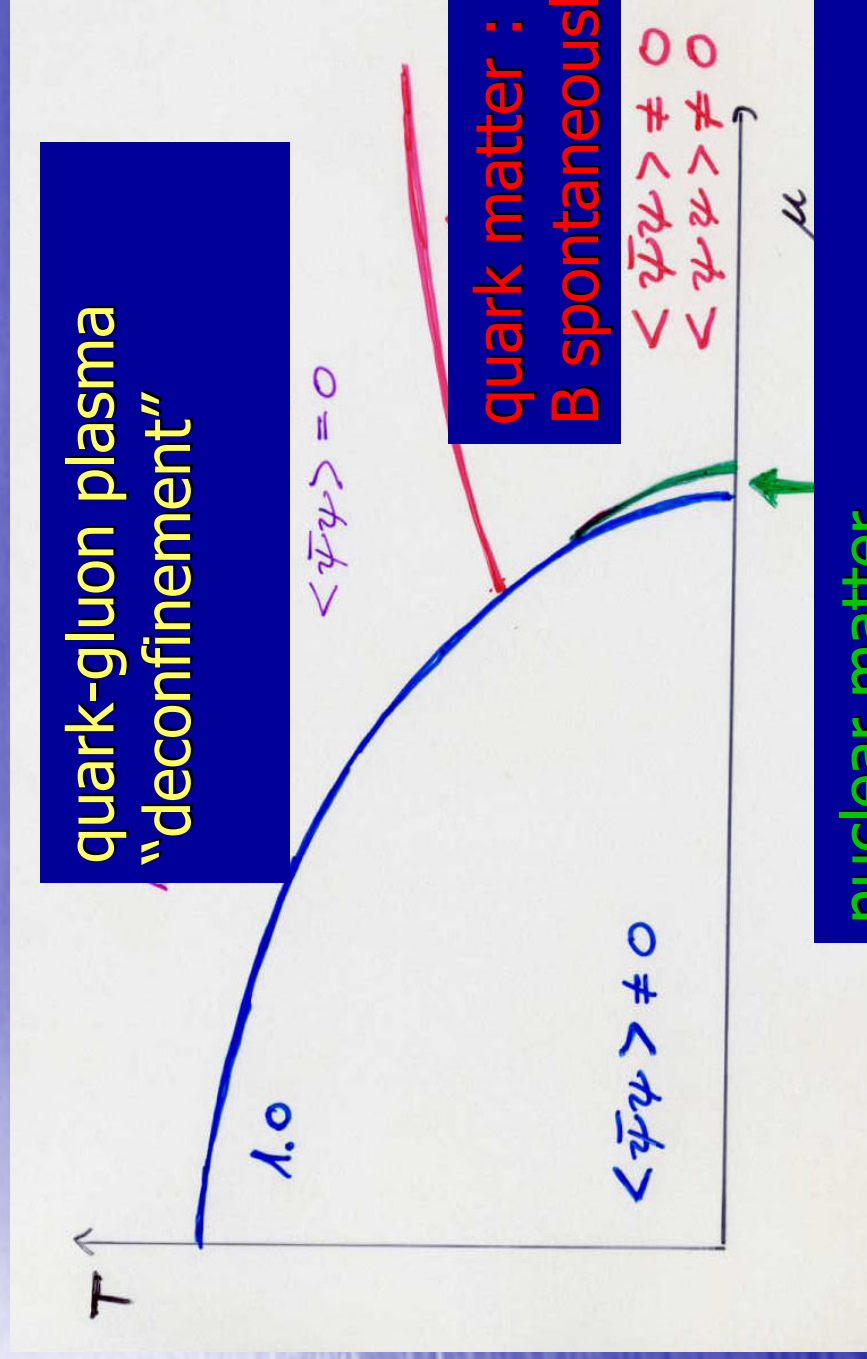
in the

QCD phase diagram

Understanding the phase diagram



Phase diagram for $m_s > m_{u,d}$



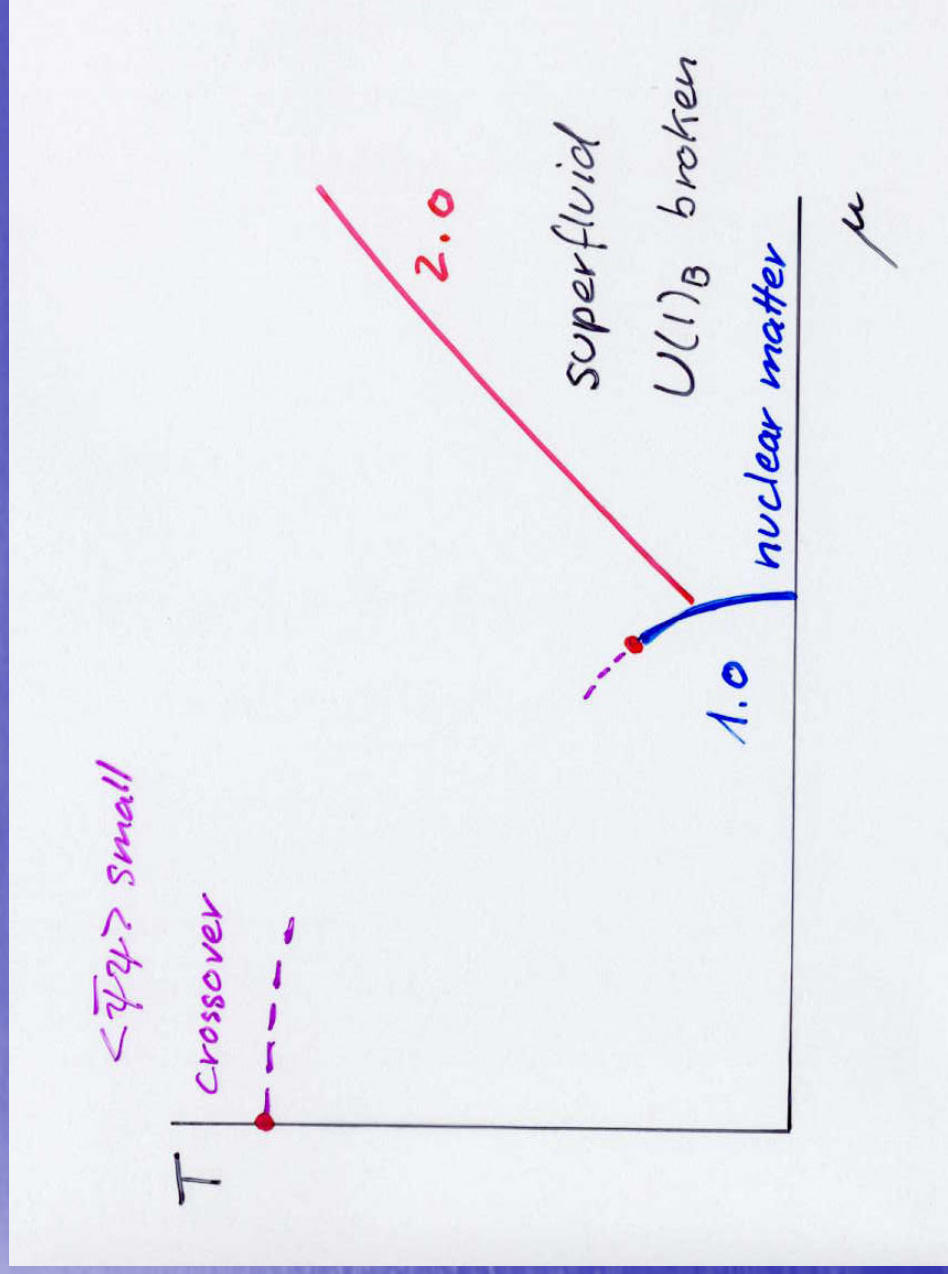
nuclear matter
 B, isospin (I_3) spontaneously broken, S conserved

Order parameters

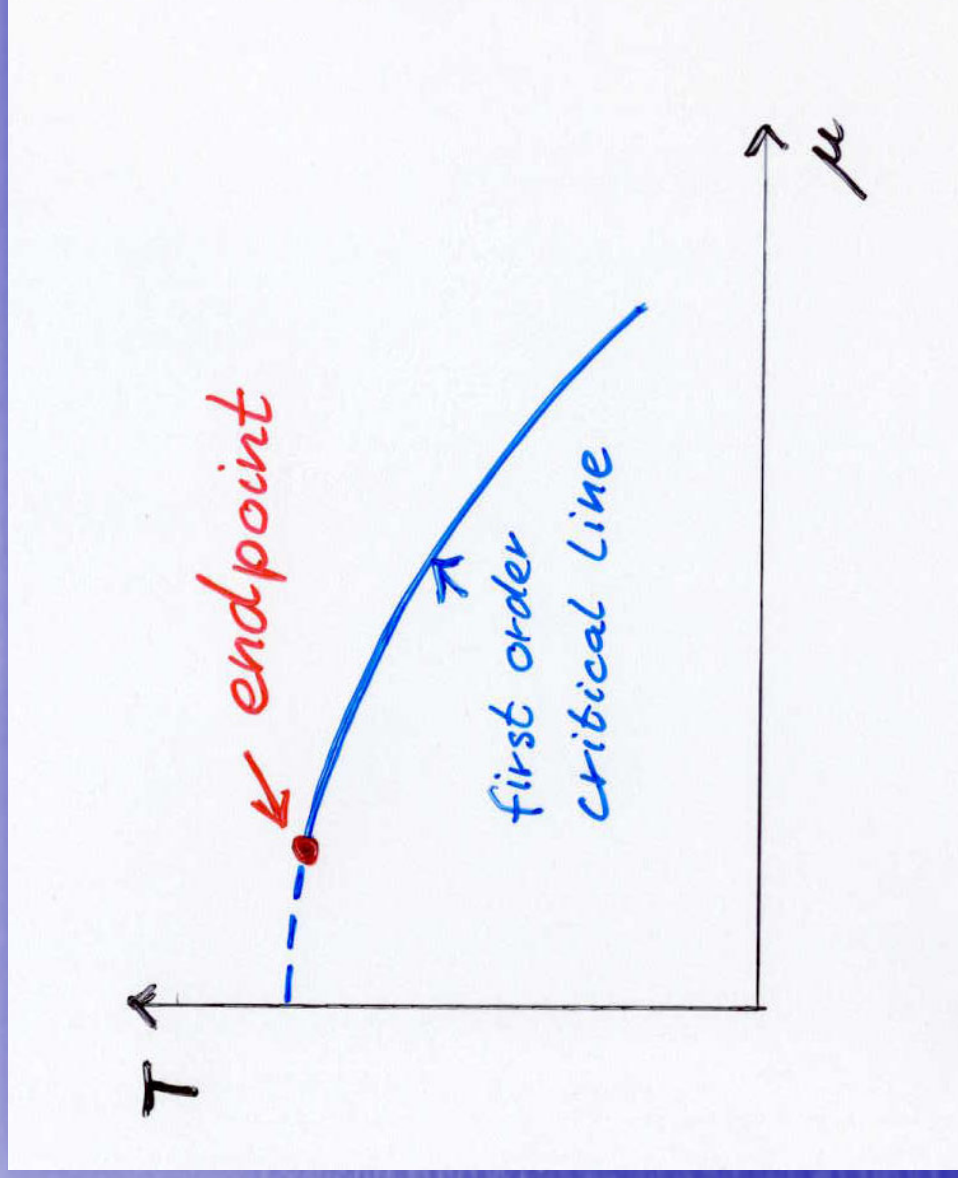
- Nuclear matter and quark matter are separated from other phases by true **critical lines**
- Different realizations of **global** symmetries
- Quark matter: SSB of baryon number B
- Nuclear matter: SSB of combination of B and isospin I_3
neutron-neutron condensate

"minimal" phase diagram

for equal nonzero quark masses



Endpoint of critical line ?





How to find out ?

Methods

- **Lattice :** You have to wait until chiral limit is properly implemented !
- **Models :** Quark meson models cannot work
Higgs picture of QCD ?
- **Experiment :** Has T_c been measured ?
Indications for first order transition !

Lattice



Lattice results

e.g. Karsch, Laermann, Peikert

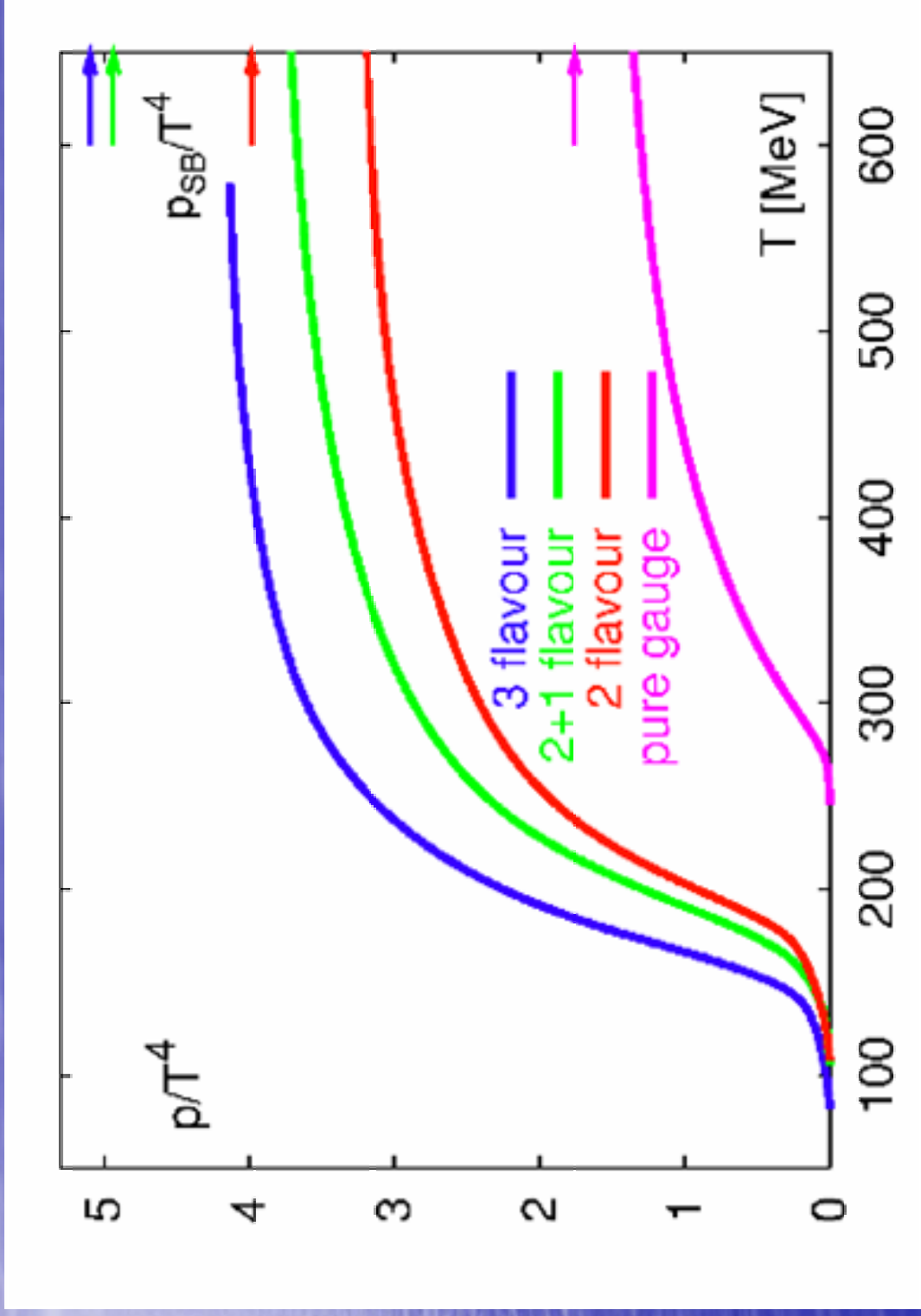
Critical temperature in chiral limit :

$$N_f = 3 : T_c = (154 \pm 8) \text{ MeV}$$

$$N_f = 2 : T_c = (173 \pm 8) \text{ MeV}$$

Chiral symmetry restoration and deconfinement at same T_c

pressure



realistic QCD

- precise lattice results not yet available for first order transition vs. crossover
- also uncertainties in determination of critical temperature (chiral limit ...)
- extension to nonvanishing baryon number only for QCD with relatively heavy quarks

Models



Analytical description of phase transition

- Needs model that can account simultaneously for the correct degrees of freedom below and above the transition temperature.
- Partial aspects can be described by more limited models, e.g. chiral properties at small momenta.

Chiral quark meson model

- Limitation to chiral behavior
- Small up and down quark mass
 - large strange quark mass
- Particularly useful for critical behavior of second order phase transition or near endpoints of critical lines

(see N. Tetradis for possible QCD-endpoint)

Quark descriptions (NJL-model) fail to describe the high temperature and high density phase transitions correctly

High T : chiral aspects could be ok , but glue ...
(pion gas to quark gas)

High density transition : different Fermi surface for quarks and baryons ($T=0$)

– in mean field theory factor 27 for density at given chemical potential –

Confinement is important : baryon enhancement

Berges,Junghnickel,...

Chiral perturbation theory even less complete

Universe cools below 170 MeV...

Both gluons and quarks disappear from thermal equilibrium : mass generation

Chiral symmetry breaking

→ mass for fermions

Gluons ?

Analogous situation in electroweak phase transition understood by Higgs mechanism

Higgs description of QCD vacuum ?

Higgs picture of QCD

"spontaneous breaking of color"
in the QCD – vacuum

octet condensate

for $N_f = 3$ (u,d,s)

Higgs phase and confinement

can be equivalent –
then simply two different descriptions
(pictures) of the same physical situation

Is this realized for QCD ?

Necessary condition : spectrum of
excitations with the same quantum
numbers in both pictures

- known for QCD : mesons + baryons -

Quark - antiquark condensate

$$\text{quarks : } \psi_{L,R} a_i$$

↑ color
↑ flavor

condensate in vacuum :

$$\langle \bar{\psi}_L \psi_R \rangle = \frac{1}{16} \bar{\xi}_0 (\delta_{ia} \delta_{jb} - \frac{1}{3} \delta_{ij} \delta_{ab})$$

color octet

$$+ \frac{1}{13} \bar{\xi}_0 \delta_{ij} \delta_{ab}$$

color singlet

Octet condensate

$\langle \text{octet} \rangle \neq 0$:

- "Spontaneous breaking of color"
- Higgs mechanism
- Massive Gluons – all masses equal
- Eight octets have vev
- Infrared regulator for QCD

Flavor symmetry

for equal quark masses :

octet preserves global $SU(3)$ -symmetry
"diagonal in color and flavor"
"color-flavor-locking"

(cf. Alford, Rajagopal, Wilczek ; Schaefer, Wilczek)

All particles fall into representations of
the "eightfold way"

quarks : $8 + 1$, gluons : 8

Quarks and gluons carry the
observed quantum numbers of
isospin and strangeness
of the baryon and
vector meson octets !

They are integer charged!

Low energy effective action

$$\begin{aligned}
 \mathcal{L} = & Z_T \{ i \bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i + \bar{\Psi}_i \gamma^\mu A_{ij\mu} \Psi_j \} \\
 & + \frac{1}{2} G_{ij}^{\mu\nu} G_{\mu\nu ij} \quad \text{--- } \mathcal{L}_{\text{GCD}} \\
 & + \text{Tr} \{ (D^\mu \gamma_{ij})^\dagger (D_\mu \gamma_{ij}) \} + \underline{U}(\gamma) \\
 & + Z_T \bar{\Psi}_i \left[\underline{h} \varphi \delta_{ij} + \tilde{h} \chi_{ij} \right] \frac{1+\gamma_5}{2} \\
 & - (\underline{h} \varphi^\dagger \delta_{ij} + \tilde{h} \chi_{ij}^\dagger) \frac{1-\gamma_5}{2} \Psi_j
 \end{aligned}$$

$$\gamma = \varphi + \chi$$

$$A_{ij\mu} = \frac{1}{2} A_\mu^2(\lambda_2)_{ij}$$

...accounts for masses and
couplings of light pseudoscalars,
vector-mesons and baryons !

Phenomenological parameters

- 5 undetermined parameters
- predictions

$$\chi_0, \bar{g}, h, \bar{h}$$

fixed by 5 observable quantities

(for $m_q = 0$, averages over $SU(3)$ multiplets)

$$\bar{M}_p = 850 \text{ MeV}$$

$$\bar{M}_N = 1150 \text{ MeV}$$

$$M_1 = 1400 \text{ MeV}$$

$$\bar{f} = 110 \text{ MeV} \quad (\bar{f} = \frac{2}{3} f_K + \frac{1}{3} f_\pi)$$

$$\Gamma(p \rightarrow \mu^+ \nu), \quad \Gamma(p \rightarrow e^+ e^-) = 7 \text{ keV}$$

$$* \Gamma(p \rightarrow 2\pi) \approx 150 \text{ MeV}$$

$$* \beta\text{-decay of neutrons: } g_A = 1 \quad (\text{Exp: } g_A = 1.26)$$

* vector dominance in electromagnetic interactions of pions, $g_{\gamma\pi\pi}/e = 0.04$

Chiral perturbation theory

- + all predictions of chiral perturbation theory
- + determination of parameters

		"Expt"
L_1	0.87	0.7 ± 0.3
L_2	1.74	1.7 ± 0.7
L_3	-5.2	$-(4.4 \pm 2.5)$

Chiral phase transition at high temperature

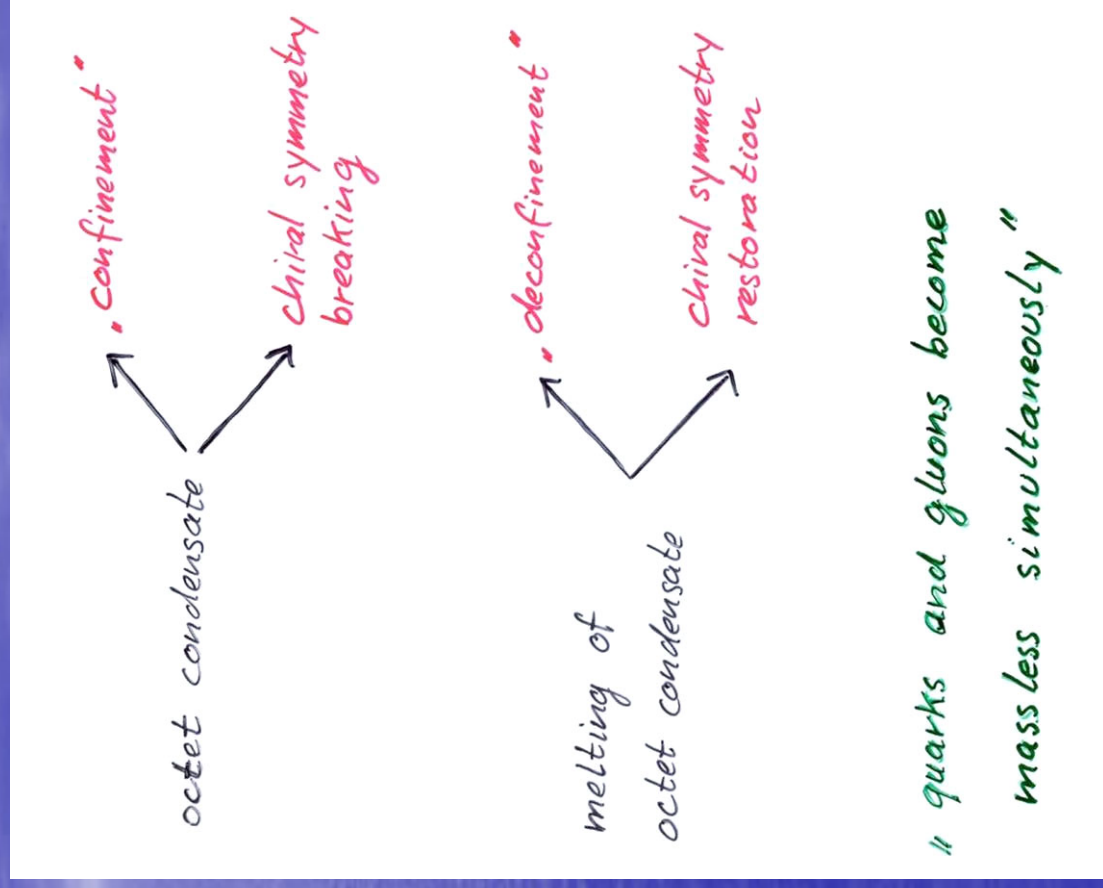
High temperature phase transition in QCD :
Melting of octet condensate

Lattice simulations :

Deconfinement temperature = critical
temperature for restoration of chiral
symmetry

Why ?

Simple explanation :



Higgs picture of the QCD-phase transition

A simple mean field calculation gives roughly reasonable description that should be improved.

$$T_c = 170 \text{ MeV}$$

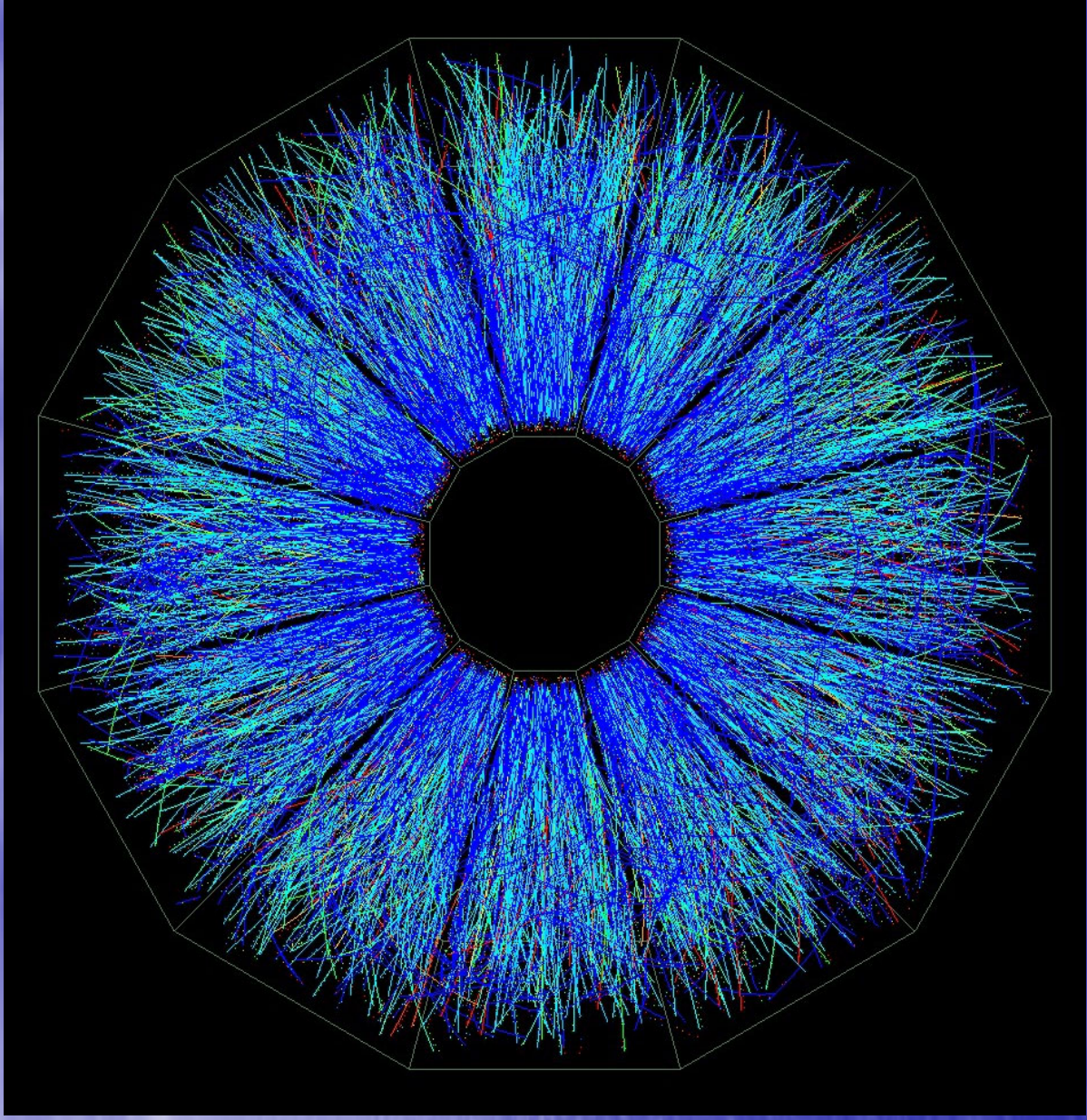
First order transition

Experiment



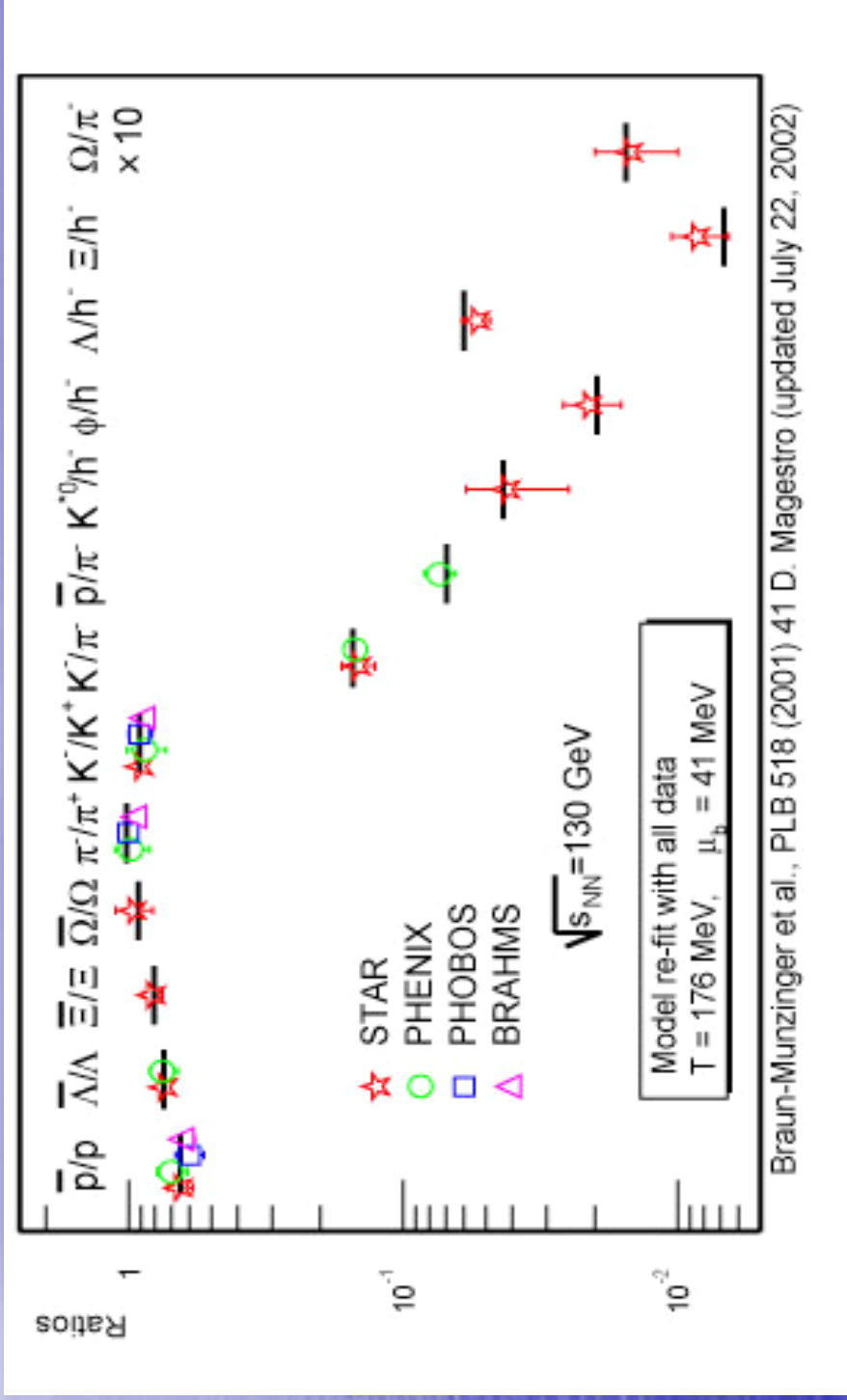
Has the
critical temperature of the
QCD phase transition
been measured ?

Heavy ion collision



Chemical freeze-out temperature

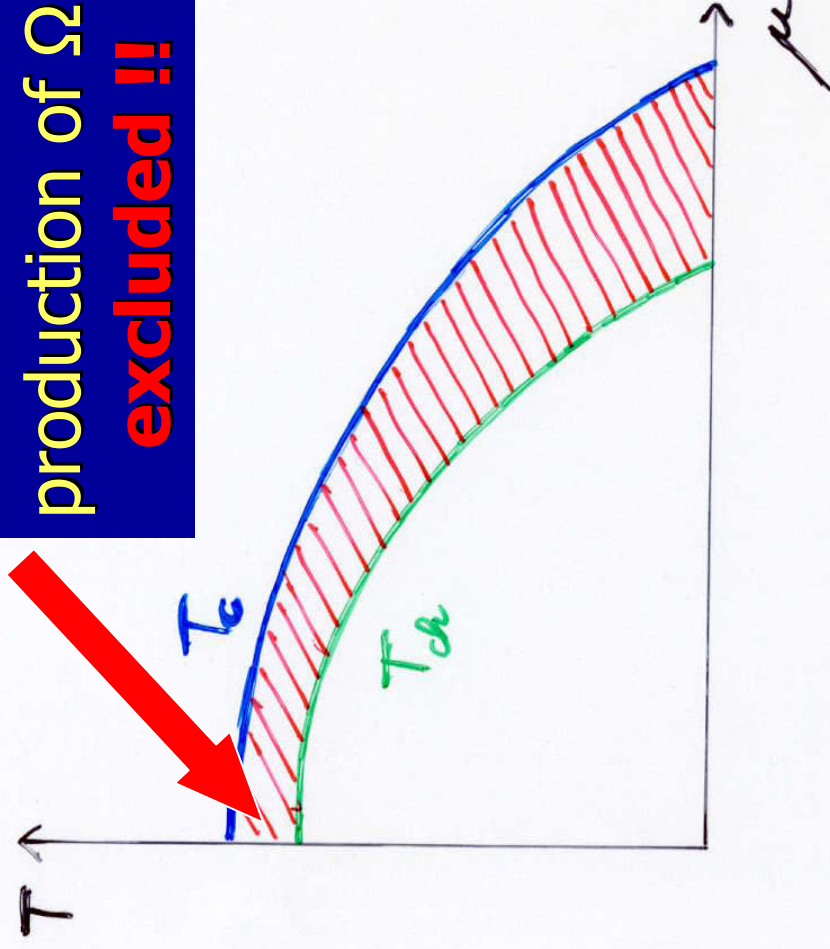
$T_{\text{ch}} = 176 \text{ MeV}$



hadron abundancies

Exclusion argument

hadronic phase
with sufficient
production of Ω :
excluded !!



Exclusion argument

Assume T is a meaningful concept -
complex issue, to be discussed later

$T_{\text{ch}} < T_c$: hadrochemical equilibrium

Exclude T_{ch} much smaller than T_c :

say $T_{\text{ch}} > 0.95 T_c$

$$0.95 < T_{\text{ch}} / T_c < 1$$

Has T_c been measured ?

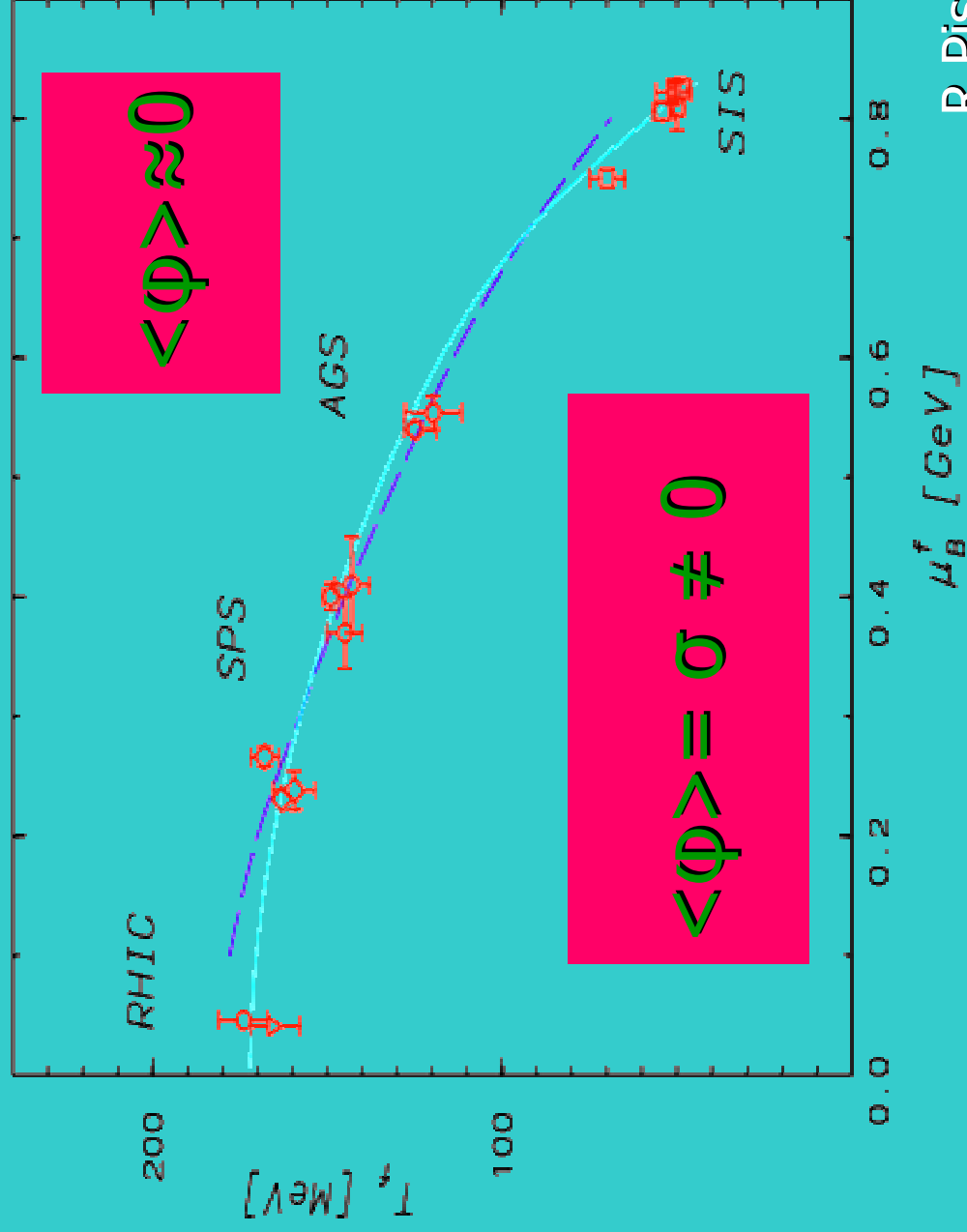
- Observation : statistical distribution of hadron species with "chemical freeze out temperature" $T_{ch} = 176$ MeV
- T_{ch} cannot be much smaller than T_c : hadronic rates for $T < T_c$ are too small to produce multistrange hadrons (Ω, \dots)
- Only near T_c multiparticle scattering becomes important (collective excitations ...) – proportional to high power of density



$$T_{ch} \approx T_c$$

$$T_{ch} \approx T_c$$

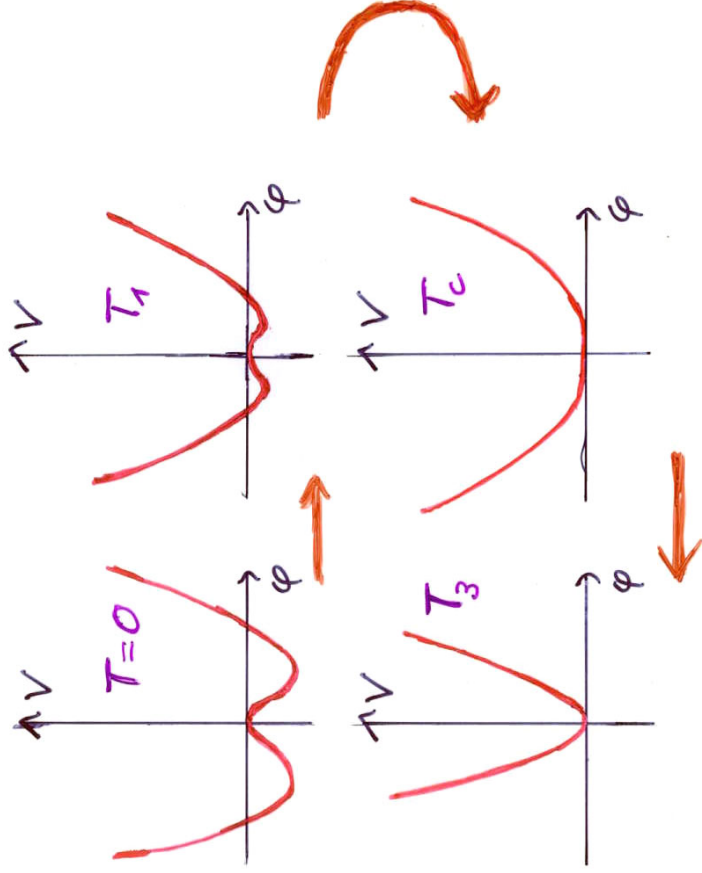
Phase diagram



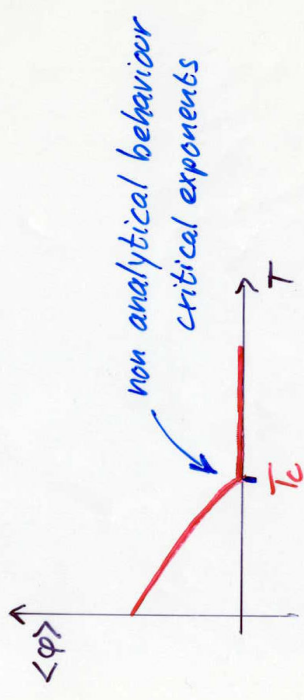
Temperature dependence of chiral order parameter

Does experiment indicate a
first order phase transition
for $\mu = 0$?

Second order phase transition



$$V = -\mu_0^2 \varphi^4 + cT^2 \varphi^2 + \frac{1}{2}(\varphi^6)$$



Second order phase transition

for T only somewhat below T_c :
the order parameter σ is expected to
be close to zero and
deviate substantially from its vacuum
value

This seems to be disfavored by observation
of chemical freeze out !

Temperature dependent masses

- Chiral order parameter σ depends on T
- Particle masses depend on σ
- Chemical freeze out measures m/T for many species
- Mass ratios at T just below T_c are close to vacuum ratios

Ratios of particle masses and chemical freeze out

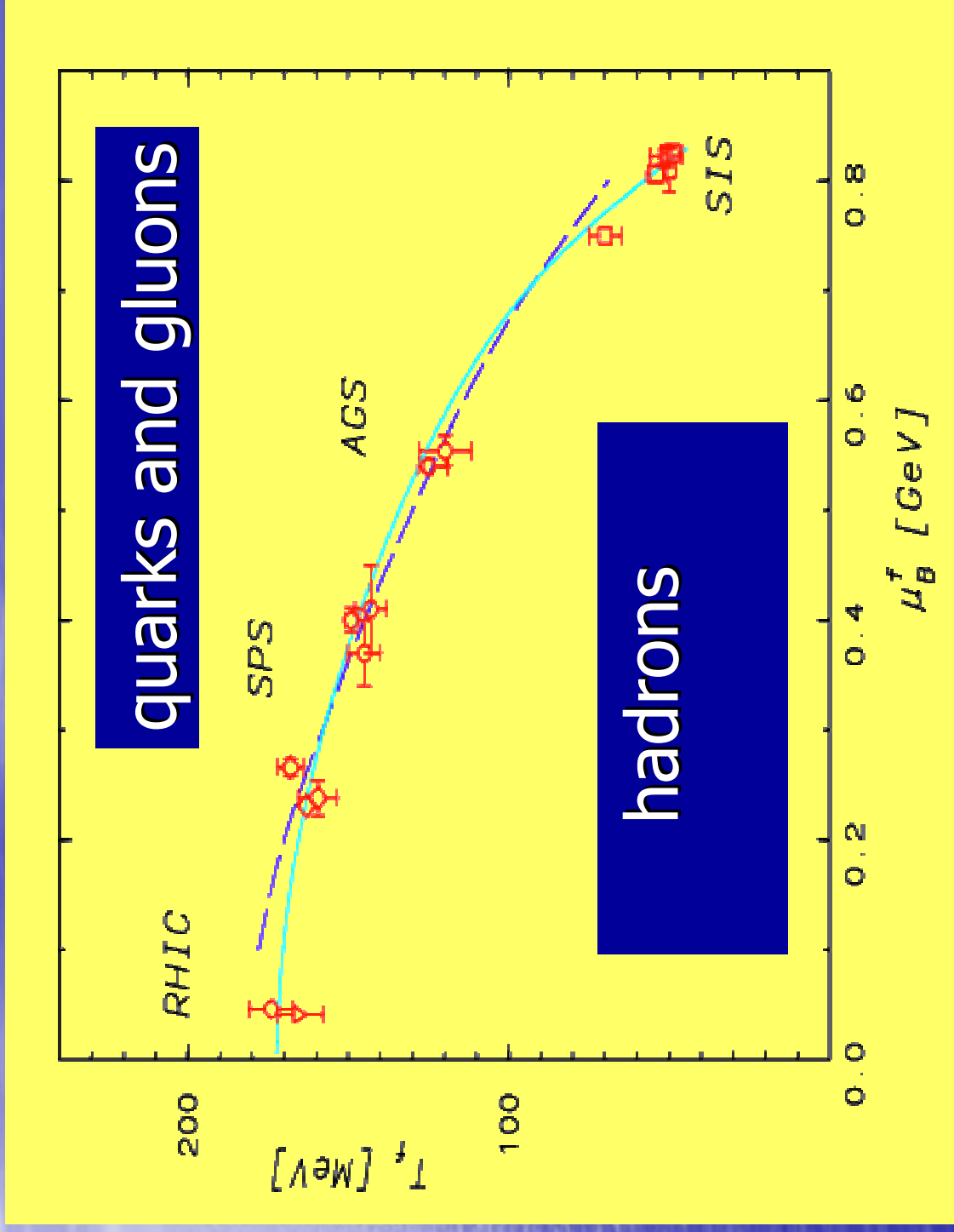
at chemical freeze out :

- ratios of hadron masses seem to be close to vacuum values
- nucleon and meson masses have different characteristic dependence on σ
- $m_{\text{nucleon}} \sim \sigma$, $m_{\pi} \sim \sigma^{-1/2}$
- $\Delta\sigma/\sigma < 0.1$ (conservative)

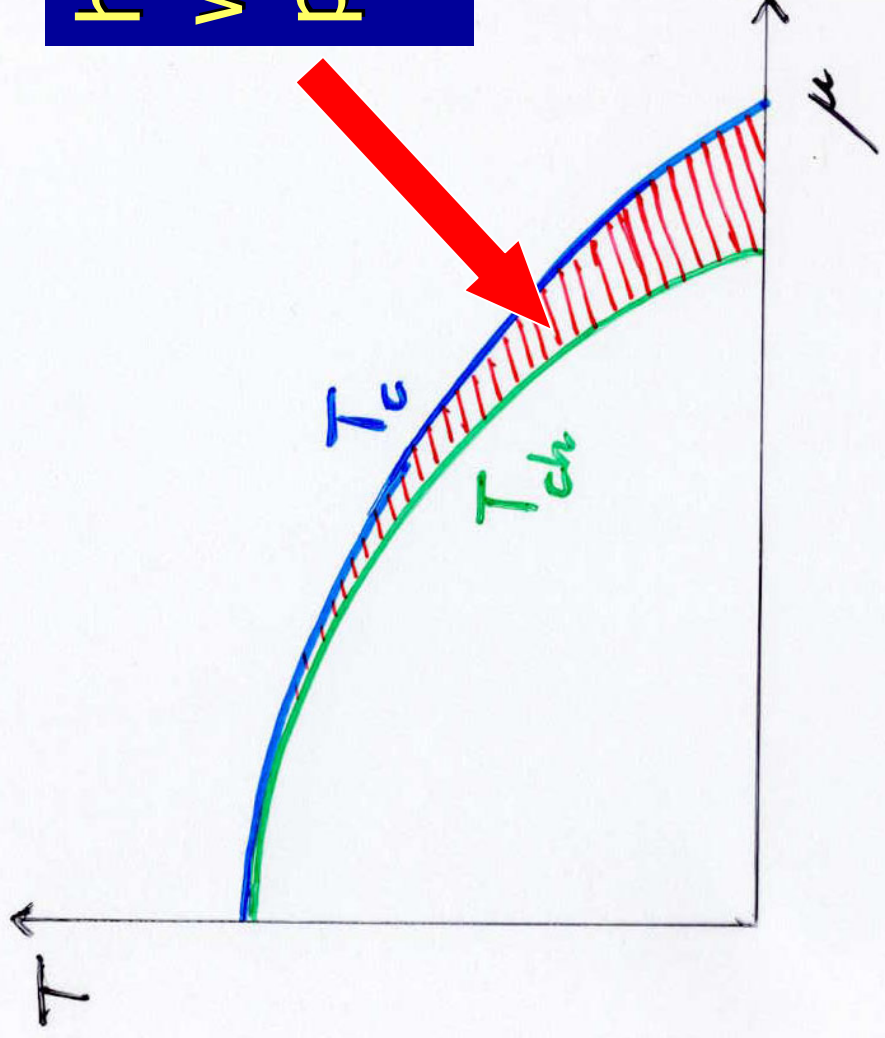
**first order phase transition
seems to be favored by
chemical freeze out**

...or extremely rapid crossover

How far has first order line been measured?

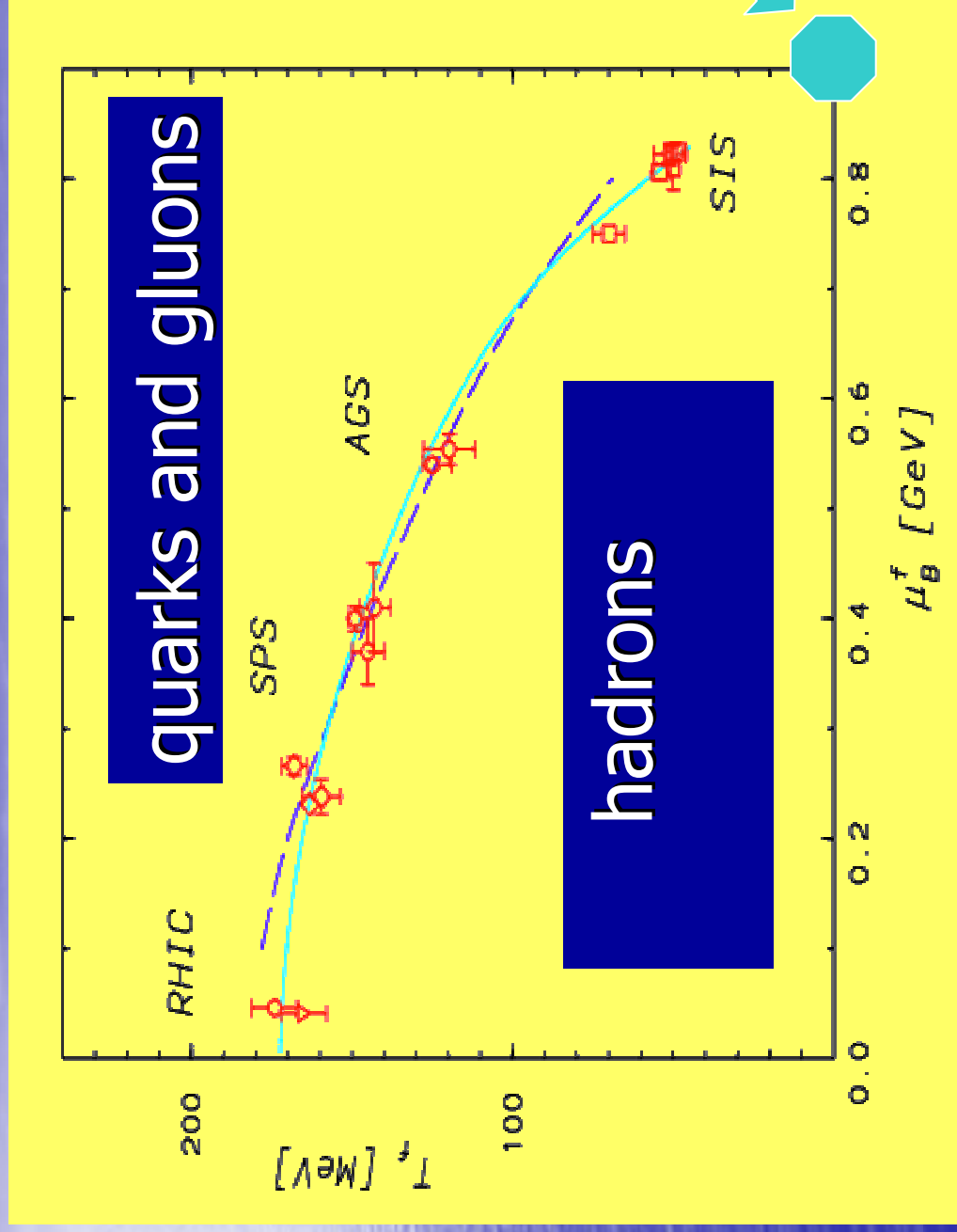


Exclusion argument for large density



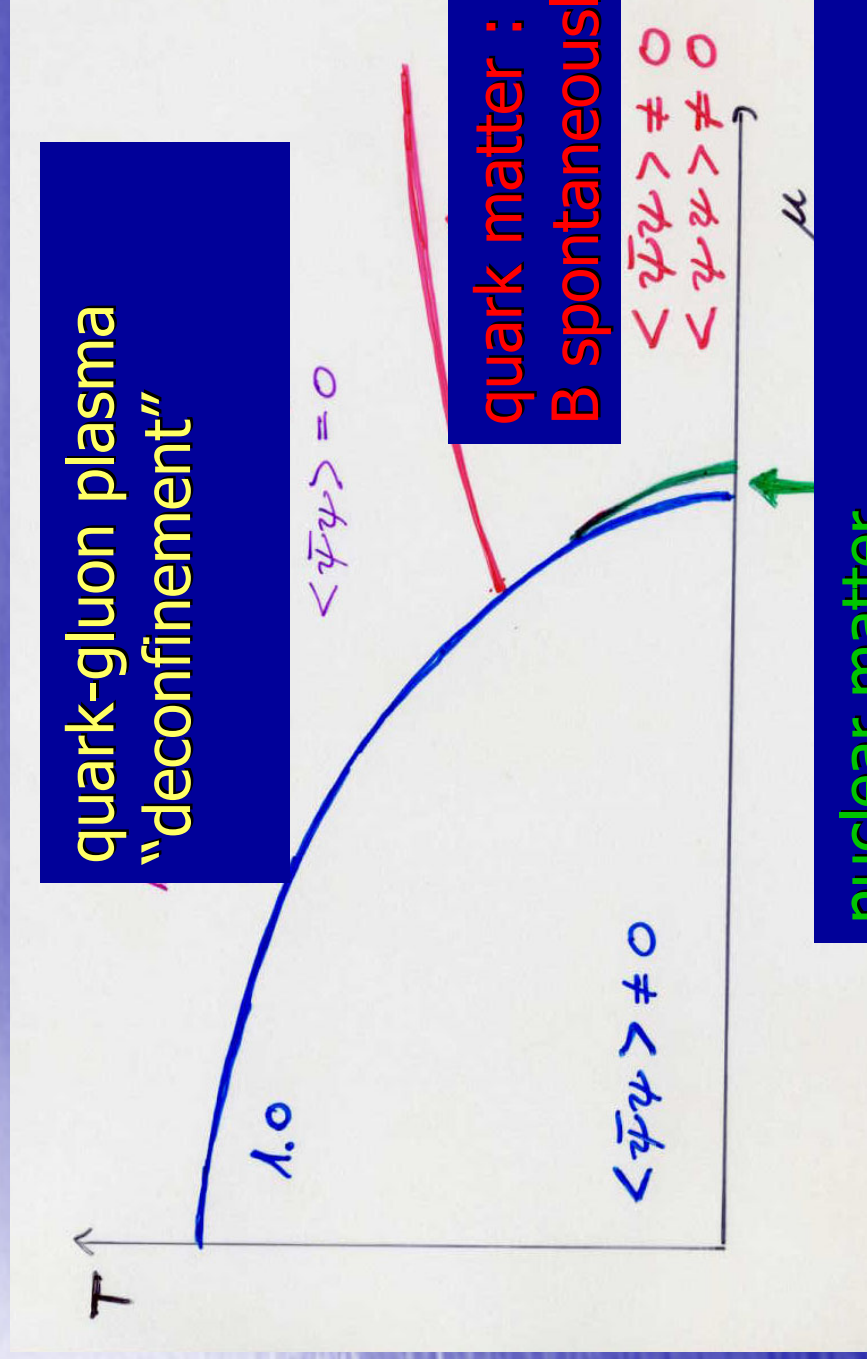
hadronic phase
with sufficient
production of Ω :
excluded !!

First order phase transition line



$\mu = 923 \text{ MeV}$
transition to
nuclear
matter

Phase diagram for $m_s > m_{u,d}$



quark-gluon plasma
"deconfinement"

quark matter : superfluid
B spontaneously broken

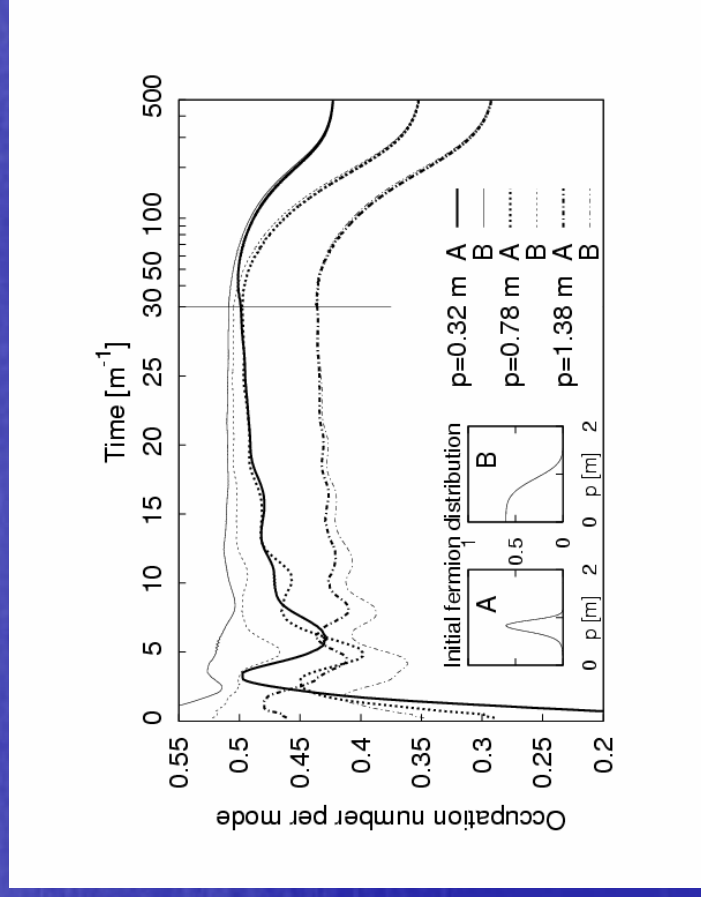
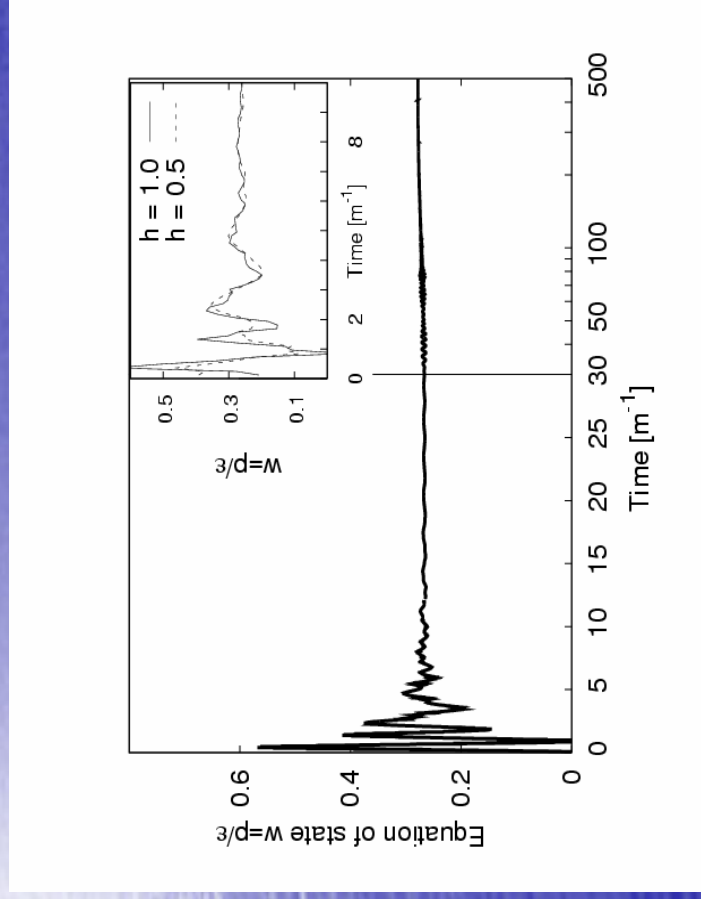
nuclear matter
B, isospin (I_3) spontaneously broken, S conserved

Is temperature defined ?

**Does comparison with
equilibrium critical temperature
make sense ?**

Prethermalization

J. Berges, Sz. Borsanyi, CW



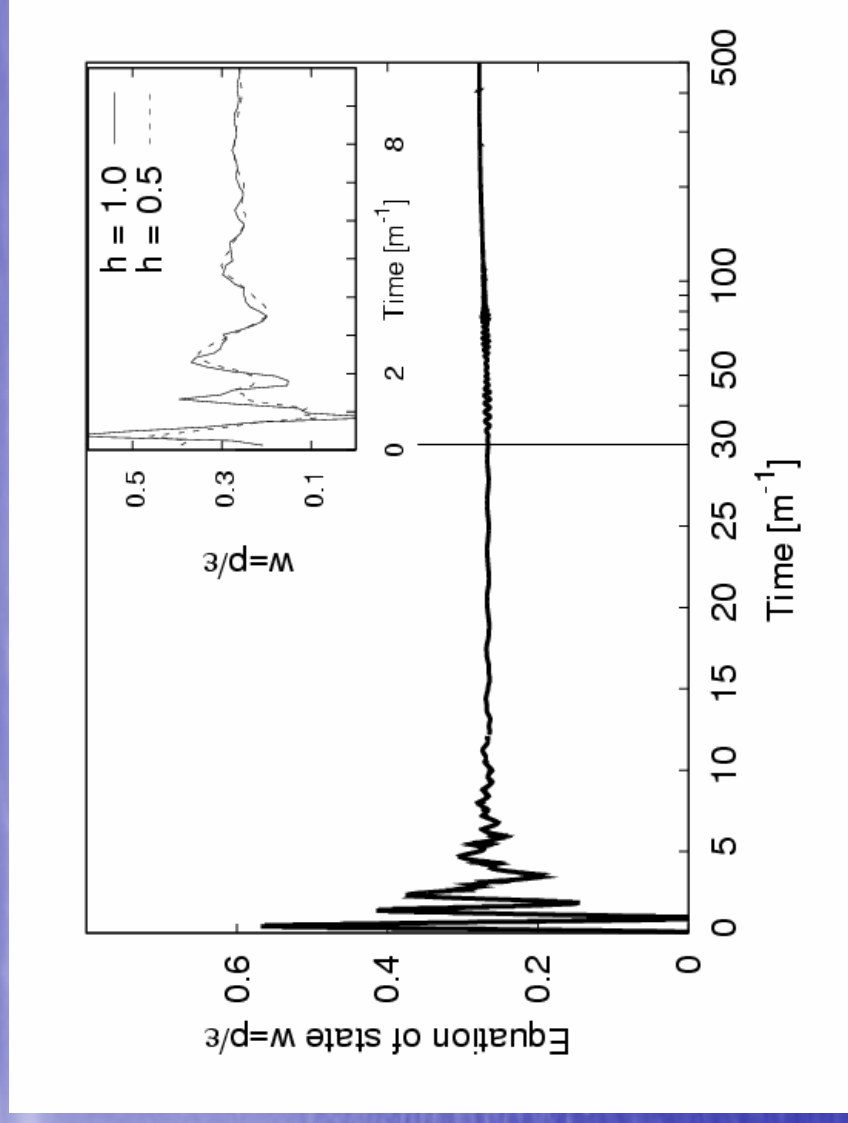
Vastly different time scales

for “thermalization” of different quantities

here : scalar with mass m coupled to fermions
(linear quark-meson-model)

method : two particle irreducible non-
equilibrium effective action (J.Berges et al)

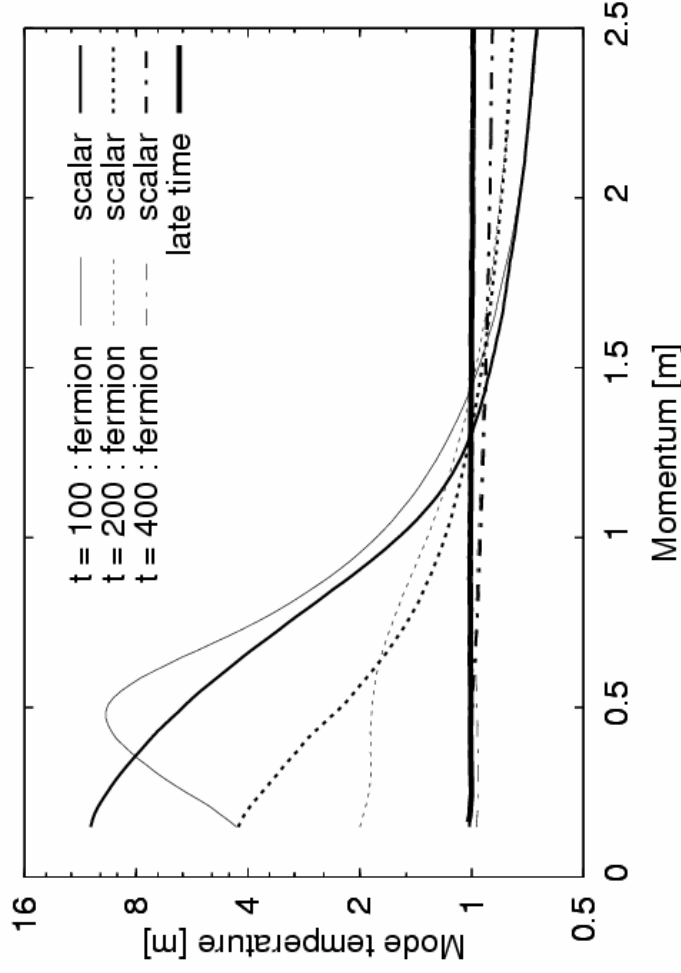
Prethermalization equation of state $w=p/\epsilon$



similar for kinetic temperature

different "temperatures"

Mode temperature



$$n_p(t) \stackrel{!}{=} \frac{1}{\exp[\omega_p(t)/T_p(t)] \pm 1}$$

$\omega_p^{(f,s)}(t)$ determined by peak of spectral function

n_p : occupation number
for momentum p

late time:
**Bose-Einstein or
Fermi-Dirac distribution**

Global kinetic temperature T_{kin}

Practical definition:

- association of temperature with average kinetic energy per d.o.f.
- $T_{\text{kin}}(t) = E_{\text{kin}}(t)/c_{\text{eq}}$
- $c_{\text{eq}} = E_{\text{kin,eq}}/T_{\text{eq}}$ is given solely in terms of equilibrium quantities
(E.g. relativistic plasma: $E_{\text{kin}}/N = \epsilon/n = \alpha T$)

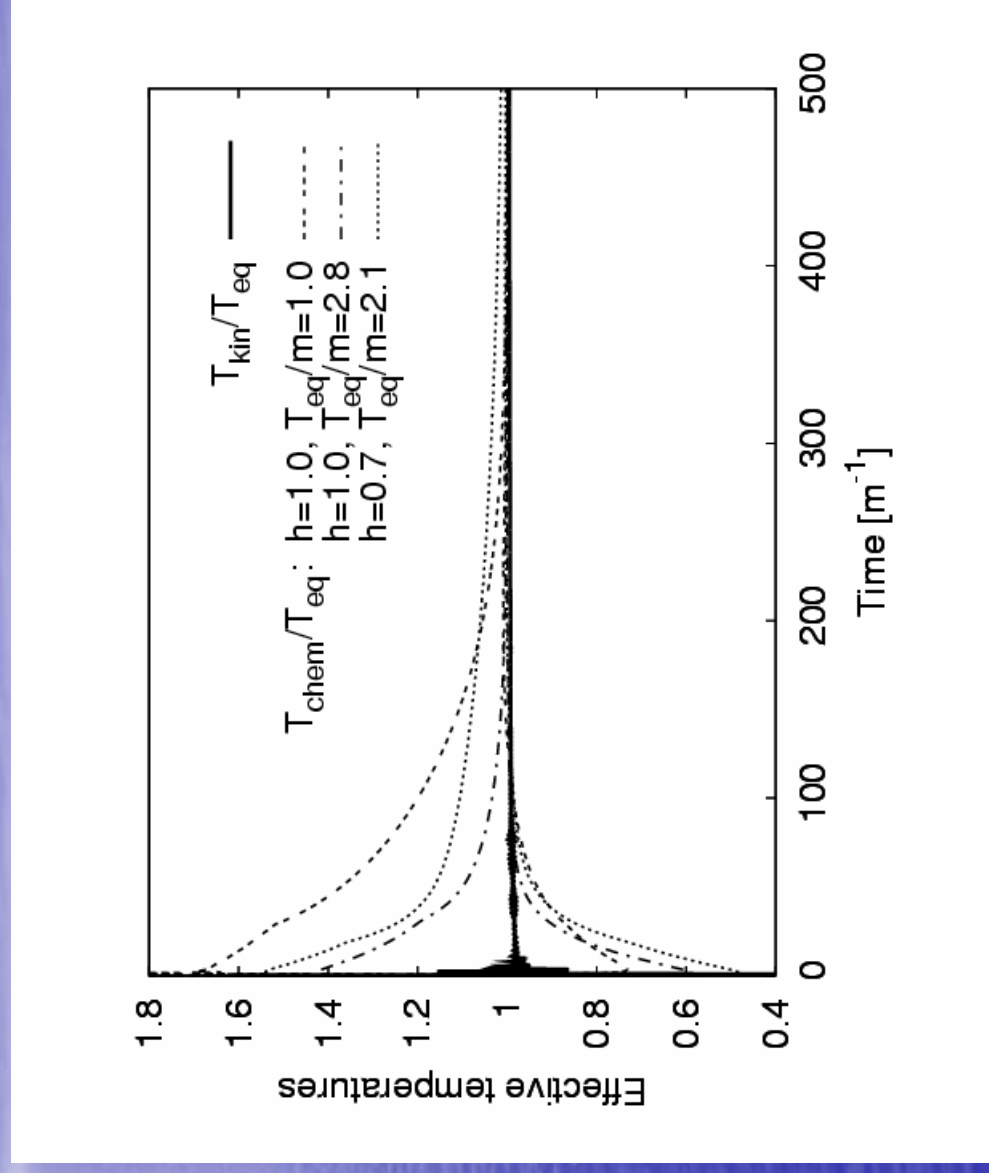
Kinetic equilibration: $T_{\text{kin}}(t) = T_{\text{eq}}$

Consider also *chemical temperatures* $T_{\text{ch}}^{(f,s)}$ from integrated number density of each species, $n^{(f,s)}(t) = g^{(f,s)} \int d^3p / (2\pi)^3 n_p^{(f,s)}(t)$:

$$n(t) \doteq \frac{g}{2\pi^2} \int_0^\infty dp p^2 [\exp(\omega_p(t)/T_{\text{ch}}(t)) \pm 1]^{-1}$$

Chemical equilibration: $T_{\text{ch}}^{(f)}(t) = T_{\text{ch}}^{(s)}(t)$

Kinetic equilibration before chemical equilibration



Once a temperature becomes stationary it takes the value of the equilibrium temperature.

Once chemical equilibration has been reached the chemical temperature equals the kinetic temperature and can be associated with the overall equilibrium temperature.

Comparison of chemical freeze out temperature with critical temperature of phase transition makes sense

Short and long distance
degrees of freedom
are different !

Short distances : quarks and gluons
Long distances : baryons and mesons

How to make the transition?

How to come from quarks and gluons to baryons and mesons ?

Find effective description where relevant degrees of freedom depend on momentum scale or resolution in space.

Microscope with variable resolution:

- High resolution , small piece of volume: quarks and gluons
- Low resolution, large volume : hadrons

Functional Renormalization Group

from small to large scales

Exact renormalization group equation

Exact flow equation

for scale dependence of average action

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2} \text{Tr} \left\{ \left(\Gamma_k^{(2)}[\varphi] + R_k \right)^{-1} \partial_k R_k \right\}$$

'92

$$\left(\Gamma_k^{(2)} \right)_{ab}(q, q') = \frac{\delta^2 \Gamma_k}{\delta \varphi_a(-q) \delta \varphi_b(q')}$$

$$\text{Tr} : \sum_a \int \frac{d^d q}{(2\pi)^d}$$

(fermions : STR)

$$\partial_k U_k = \frac{1}{2}$$

$$\left(Z_k q^2 + M_k^2 + R_k(q^2) \right)^{-1}$$

Infrared cutoff

R_k : IR-cutoff

$$\text{e.g. } R_k = \frac{Z_k q^2}{e^{q^2/k^2} - 1}$$

$$\text{or } R_k = Z_k(k^2 - q^2)\Theta(k^2 - q^2) \quad (\text{Litim})$$

$$\lim_{k \rightarrow 0} R_k = 0$$

$$\lim_{k \rightarrow \infty} R_k \rightarrow \infty$$

Nambu Jona-Lasinio model

$$S = \int d^4x \left\{ i \bar{\psi}_a^i \gamma^\mu \partial_\mu \psi_a^i \right. \\ \left. + 2\lambda_G \left(\bar{\psi}_{Lb}^i \psi_{Ra}^i \right) \left(\bar{\psi}_{Ra}^j \psi_{Lb}^j \right) \right\}$$
$$\psi_{L,R} = \frac{1 \pm \gamma^5}{2} \psi$$

$$i, j = 1 \dots N_C \quad \text{color} \quad (N_C = 3)$$
$$a, b = 1 \dots N_F \quad \text{flavor} \quad (N_F = 3, 2)$$

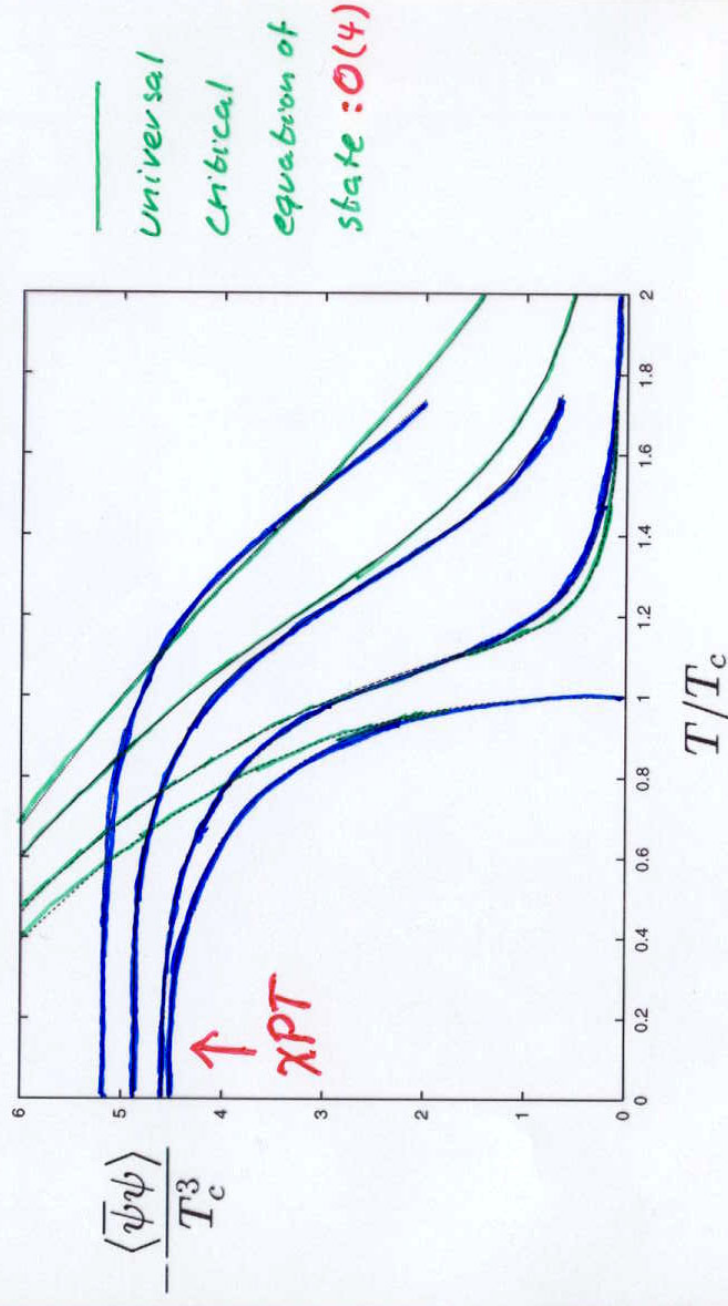
chiral flavor symmetry :

$$SU_L(N_F) \times SU_R(N_F)$$

...and more general quark meson models

Chiral condensate

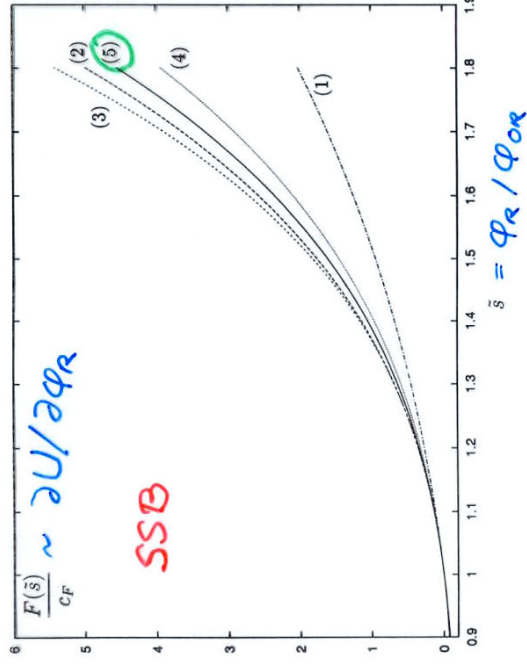
2nd order PT (expected for $O(4)$ Heisenberg model)



\Rightarrow Explicit link between χ PT domain of validity (4d) and critical (universal) domain near T_c (3d)

Scaling form of equation of state

Berges,
Tetradis,...



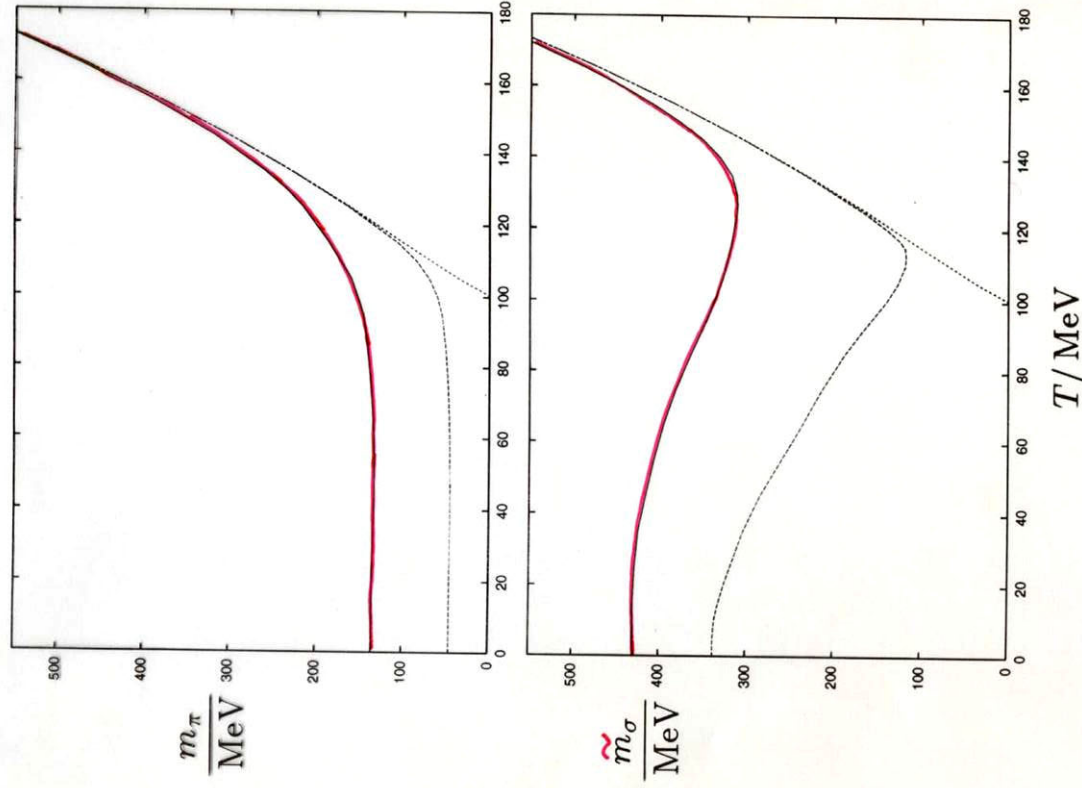
critical equation of state

- (2) ERGE, lowest order derivative exp.; Berges, Tetradis, ...
- (5) ERGE, first order derivative exp.; Seide, ...
- (1) mean field.
- (4) high-T-series, loop expansion, ϵ -expansion
- (3) Monte Carlo

temperature dependent masses

pion mass

sigma mass



? $m_\sigma < 2m_\pi$ for $T \gtrsim 100$ MeV ?

No long pion correlation length in thermal equilibrium!

conclusion

- Experimental determination of critical temperature may be more precise than lattice results
- Rather simple phase structure is suggested
- Analytical understanding is only at beginning



end

Cosmological phase transition...

...when the universe cools below 175 MeV

10^{-5} seconds after the big bang

QCD at high density

Nuclear matter

Heavy nuclei

Neutron stars

Quark stars ...

QCD at high temperature

- Quark – gluon plasma
- Chiral symmetry restored
- Deconfinement (no linear heavy quark potential at large distances)
- Lattice simulations : both effects happen at the same temperature

"Solution" of QCD

Effective action (for suitable fields) contains all the relevant information of the solution of QCD

Gauge singlet fields, low momenta:

Order parameters, meson-(baryon-) propagators

Gluon and quark fields, high momenta:

Perturbative QCD

Aim: Computation of effective action

QCD – phase transition

Quark –gluon plasma

Hadron gas

- Gluons : $8 \times 2 = 16$
- Quarks : $9 \times 7/2 = 12.5$
- Dof : 28.5
- Light mesons : 8
- (pions : 3)
- Dof : 8

Chiral symmetry

Chiral sym. broken

Large difference in number of degrees of freedom !

Strong increase of density and energy density at T_c !

Spontaneous breaking of color

- Condensate of colored scalar field
- Equivalence of Higgs and confinement description in real ($N_f=3$) QCD vacuum
- Gauge symmetries not spontaneously broken in formal sense (only for fixed gauge)
 - Similar situation as in electroweak theory
- No “fundamental” scalars
- Symmetry breaking by quark-antiquark-condensate

A simple mean field calculation

vanishing quark masses		equal $m_U = m_D = m_S \neq 0$ $2M_K^2 + M_\pi^2 = (390 \text{ MeV})^2$
\bar{M}_p	700 MeV	770 MeV
f	68 MeV	116 MeV
T_c	154 MeV	<u>170 MeV</u>
$\bar{M}_p(T_c)$	290 MeV	290 MeV
$\bar{M}_g(T_c)$	580 MeV	600 MeV

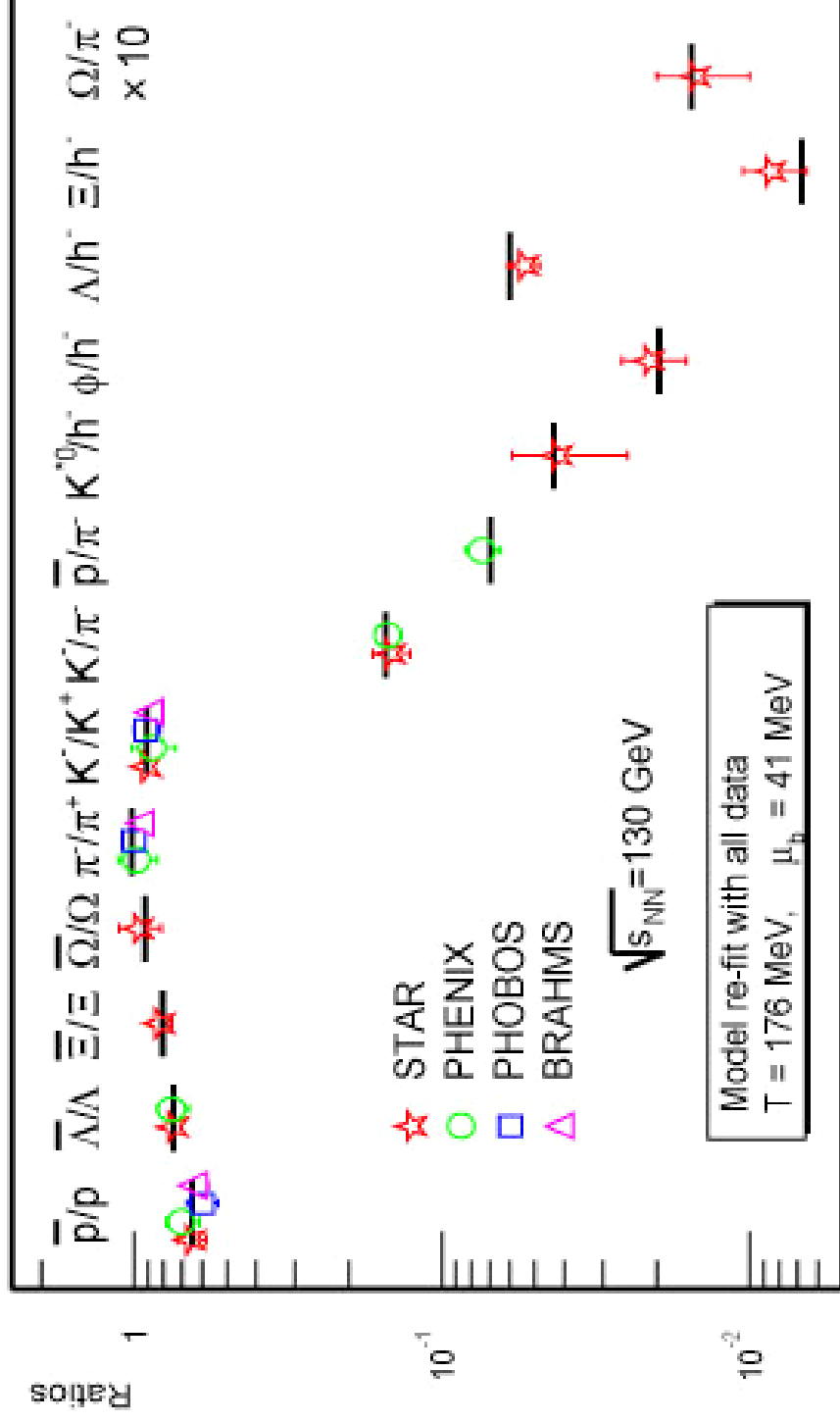
} screening masses

equation of state : pion gas \rightarrow QGP

$$\frac{\epsilon - 3P}{\epsilon + P} \approx \tau(T_c) \frac{T_c^4}{T^4} \quad (T \gg T_c)$$

$\tau(T_c)$	0.37	0.53
-------------	------	------

Hadron abundancies



Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

Bound for critical temperature

$$0.95 T_c < T_{ch} < T_c$$

- not : " I have a model where $T_c \approx T_{ch}$ "
- not : " I use T_c as a free parameter and find that in a model simulation it is close to the lattice value (or T_{ch}) "

$$T_{ch} \approx 176 \text{ MeV} \quad (?)$$

Estimate of critical temperature

For $T_{\text{ch}} \approx 176 \text{ MeV}$:

$$0.95 < T_{\text{ch}} / T_c$$

- $176 \text{ MeV} < T_c < 185 \text{ MeV}$

$$0.75 < T_{\text{ch}} / T_c$$

- $176 \text{ MeV} < T_c < 235 \text{ MeV}$

Quantitative issue matters!

Key argument

- Two particle scattering rates not sufficient to produce Ω
- “multiparticle scattering for Ω -production ” : dominant only in **immediate** vicinity of T_c

needed :

lower bound on

$$T_{ch} / T_c$$

Exclude the hypothesis of a hadronic phase
where multistrange particles are produced
at T substantially smaller than T_c

Mechanisms for production of multistrange hadrons

Many proposals

- Hadronization
- Quark-hadron equilibrium
- Decay of collective excitation (σ – field)
- Multi-hadron-scattering

Different pictures !

Hadronic picture of Ω - production

Should exist, at least semi-quantitatively, if $T_{ch} < T_c$

(for $T_{ch} = T_c$: $T_{ch} > 0.95 T_c$ is fulfilled anyhow)

e.g. collective excitations \approx multi-hadron-scattering

(not necessarily the best and simplest picture)

multihadron $\rightarrow \Omega + X$ should have sufficient rate

Check of consistency for many models

Necessary if $T_{ch} \neq T_c$ and temperature is defined

Way to give quantitative bound on T_{ch} / T_c

Rates for multiparticle scattering

2 pions + 3 kaons -> Ω + antiproton

$$r(n_{in}, n_{out}) = \bar{n}(\Gamma)^{n_{in}} |\mathcal{M}|^2 \phi$$

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

$$r_\Omega = n_\pi^5 (n_K/n_\pi)^3 |\mathcal{M}|^2 \phi.$$

Very rapid density increase

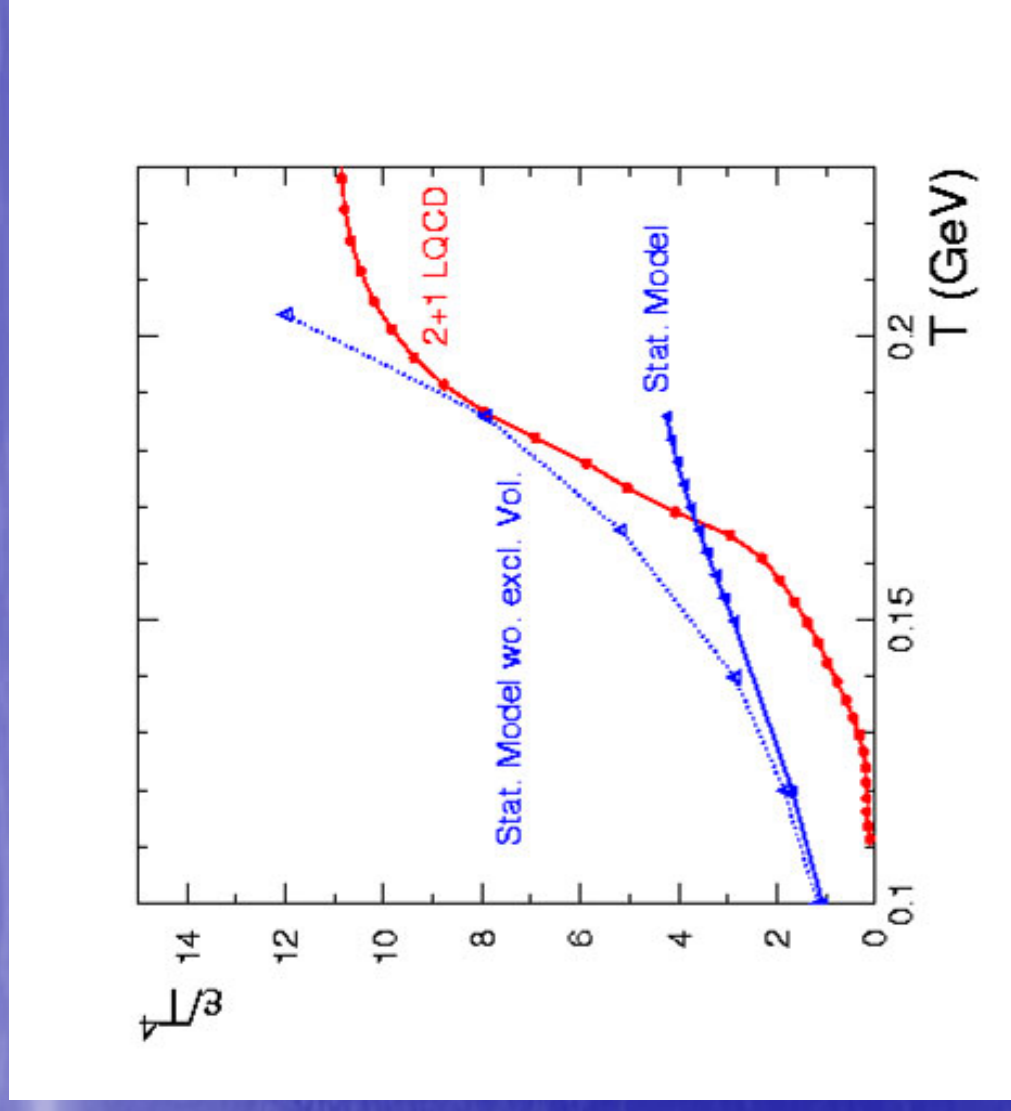
...in vicinity of critical temperature

**Extremely rapid increase of rate of
multiparticle scattering processes**

(proportional to very high power of density)

Energy density

Lattice simulations
Karsch et al
even more dramatic
for first order
transition



Phase space

- increases very rapidly with energy and therefore with temperature
- effective dependence of time needed to produce Ω

$$T_{\Omega} \sim T^{-60} !$$

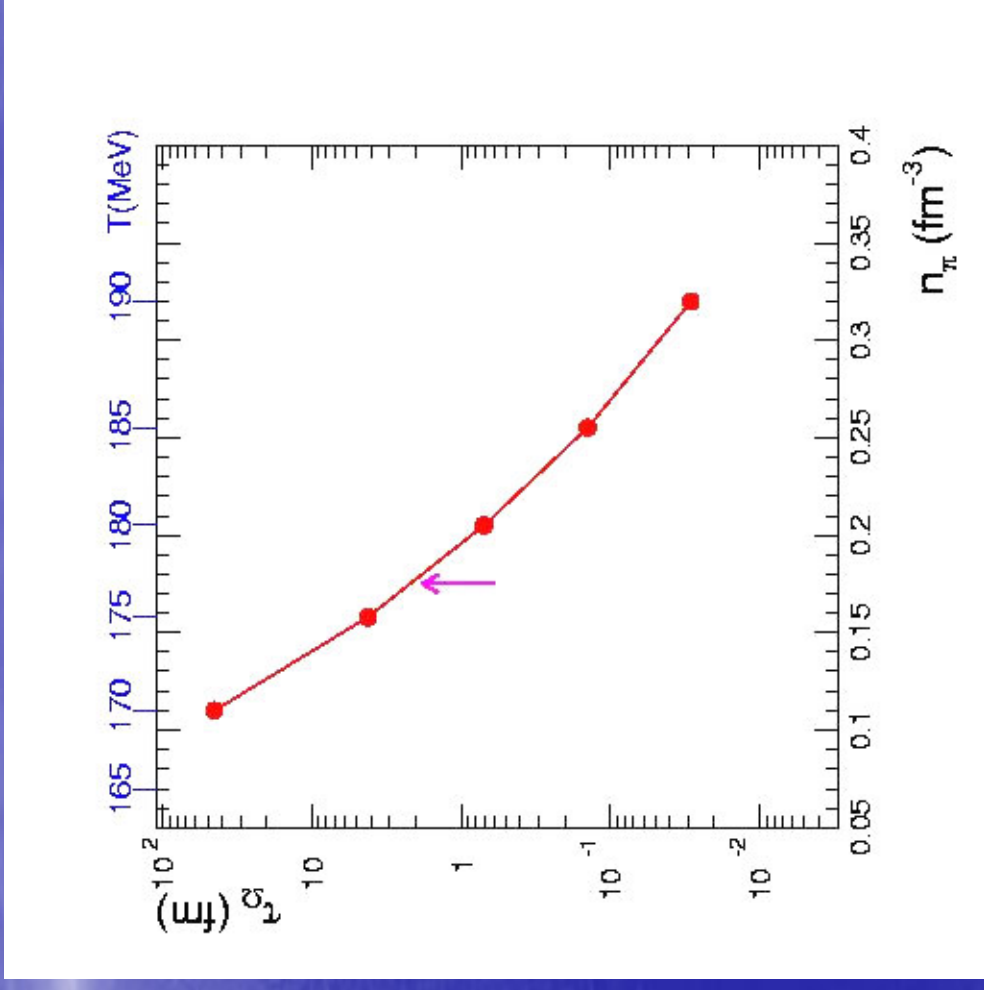
This will even be more dramatic if transition is closer to first order phase transition

Production time for Ω

multi-meson
scattering

$\pi+\pi+\pi+K+K \rightarrow$
 $\Omega+p$

strong
dependence on
"pion" density



P. Braun-Munzinger, J. Stachel, CW

extremely rapid change

lowering T by 5 MeV below critical temperature :

rate of Ω – production decreases by
factor 10

This restricts chemical freeze out to close vicinity
of critical temperature

$$0.95 < T_{\text{ch}} / T_c < 1$$

enough time for Ω - production

at $T=176$ MeV :

$$T_{\Omega} \sim 2.3 \text{ fm}$$

consistency !

Relevant time scale in hadronic phase

rates needed for equilibration of Ω and kaons:

$$\bar{r}_j = \frac{\dot{N}_j}{V} = \dot{n}_j + n_j \dot{V}/V.$$

$$\left| \frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K} \right| = \frac{\ln F_{\Omega K}}{\tau_T} \frac{T_{\text{ch}}}{\Delta T} = (1.10 - 0.55)/\text{fm}$$

$$\begin{aligned} \Delta T &= 5 \text{ MeV}, \\ F_{\Omega K} &= 1.13, \\ \tau_T &= 8 \text{ fm} \end{aligned}$$

two-particle – scattering :

$$\left| \frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K} \right| = (0.02 - 0.2)/\text{fm}$$

**A possible source of error :
temperature-dependent particle masses**

Chiral order parameter σ depends on T

**chemical
freeze out
measures
T/m !**

$$M_j(T) = h_j(T, \mu) \sigma(T, \mu)$$

$$\frac{\sigma(T_{\text{ch}}, \mu)}{T_{\text{ch}}} = \frac{\sigma(0, 0)}{T_{\text{obs}}}$$

$$T_c = 176_{-18}^{+5} \text{ MeV}.$$

uncertainty in $m(T)$



uncertainty in critical temperature

systematic uncertainty :

$$\Delta\sigma/\sigma = \Delta T_c / T_c$$

$\Delta\sigma$ is negative

$$M_j(\Gamma) = h_j(\Gamma, \mu)\sigma(\Gamma, \mu)$$

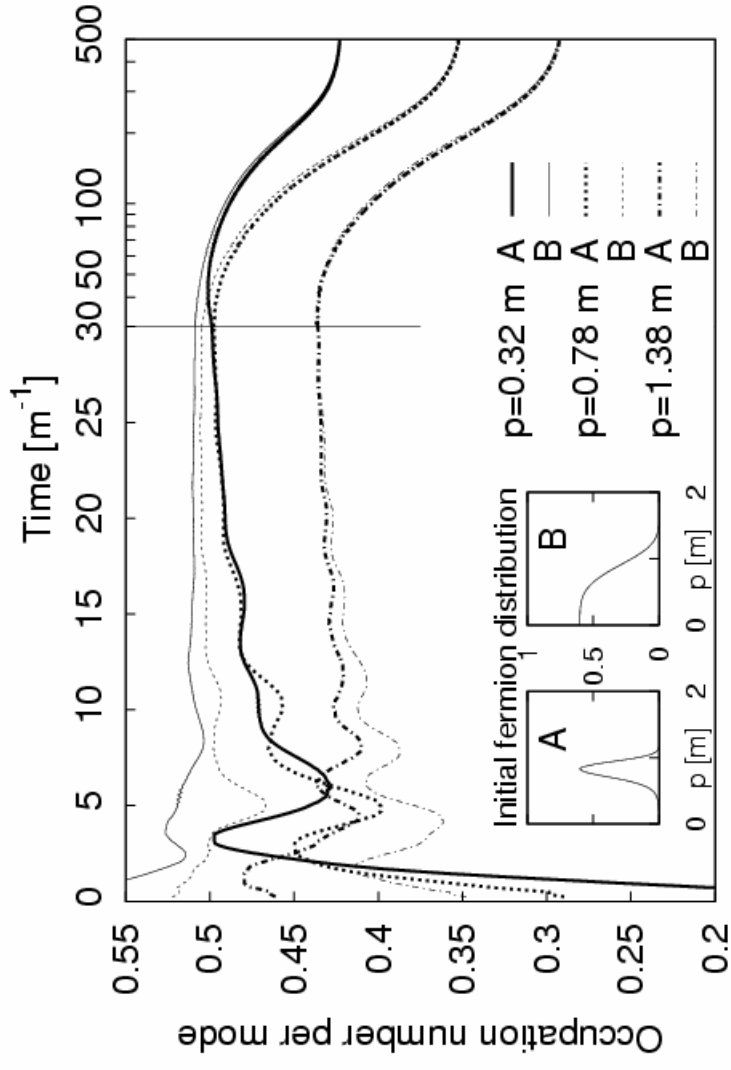
$$\frac{\sigma(T_{\text{ch}}, \mu)}{T_{\text{ch}}} = \frac{\sigma(0, 0)}{T_{\text{obs}}}$$

$$T_c = 176_{-18}^{+5} \text{ MeV}.$$

conclusion

- experimental determination of critical temperature may be more precise than lattice results
- error estimate becomes crucial

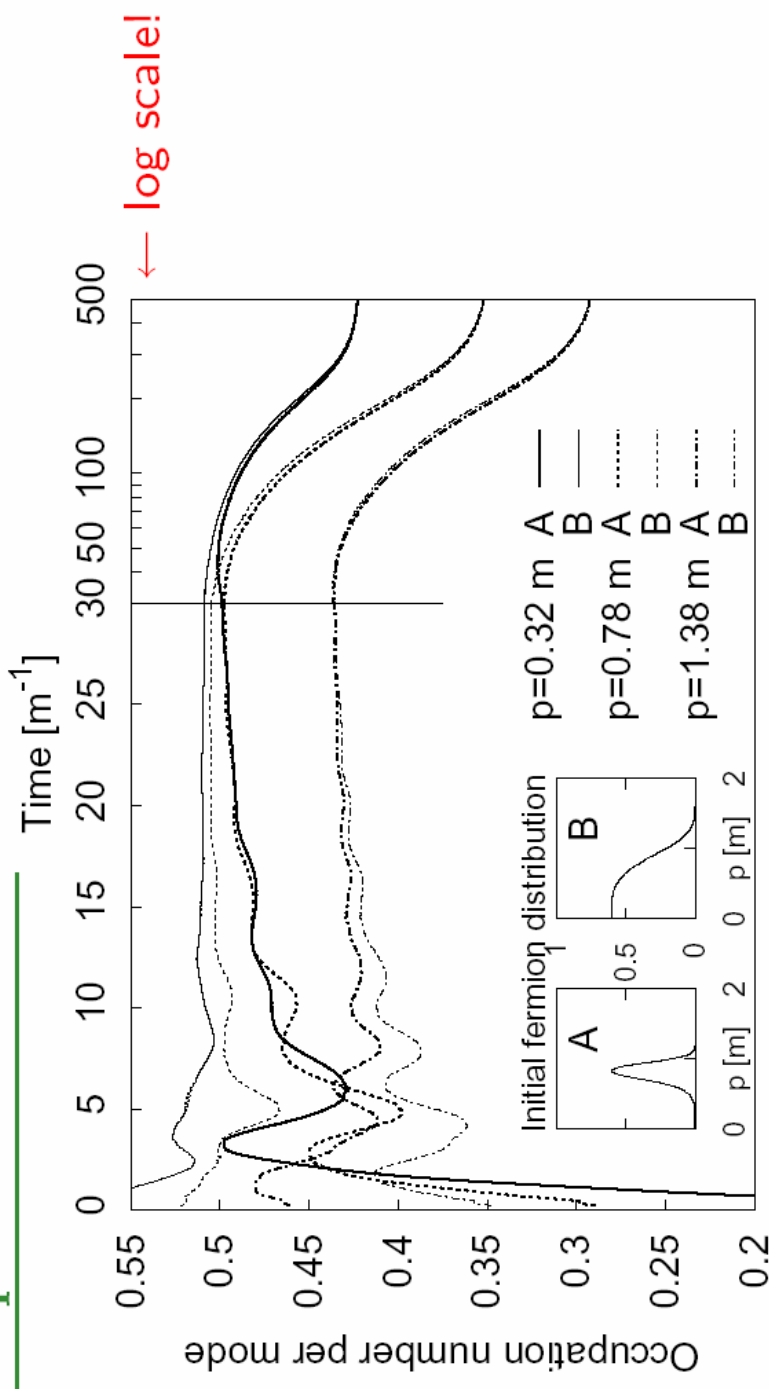
Thermal equilibration : occupation numbers



Effective loss of details of initial conditions (thermalization)

- Two different initial conditions (A), (B) with same energy density

Fermion occupation number:



Characteristic damping time: $t_{\text{damp}}(p/m \simeq 1) \simeq 25 m^{-1}$,

thermalization time: $t_{\text{eq}} \simeq 95 m^{-1}$

in units of scalar *thermal* mass m ($n(p) \sim \text{tr} \frac{p^i \chi^i}{p} \langle [\psi, \bar{\psi}] \rangle_p ; 0 \leq n(p) \leq 1$)

$\rightsquigarrow t_{\text{pt}}$ is of the order of the characteristic inverse mass scale m^{-1}

- consequence of rapid loss of phase information (“dephasing”)
- unrelated to the scattering-driven process of thermalization

$\rightsquigarrow t_{\text{pt}} \ll t_{\text{damp}} \ll t_{\text{eq}}$

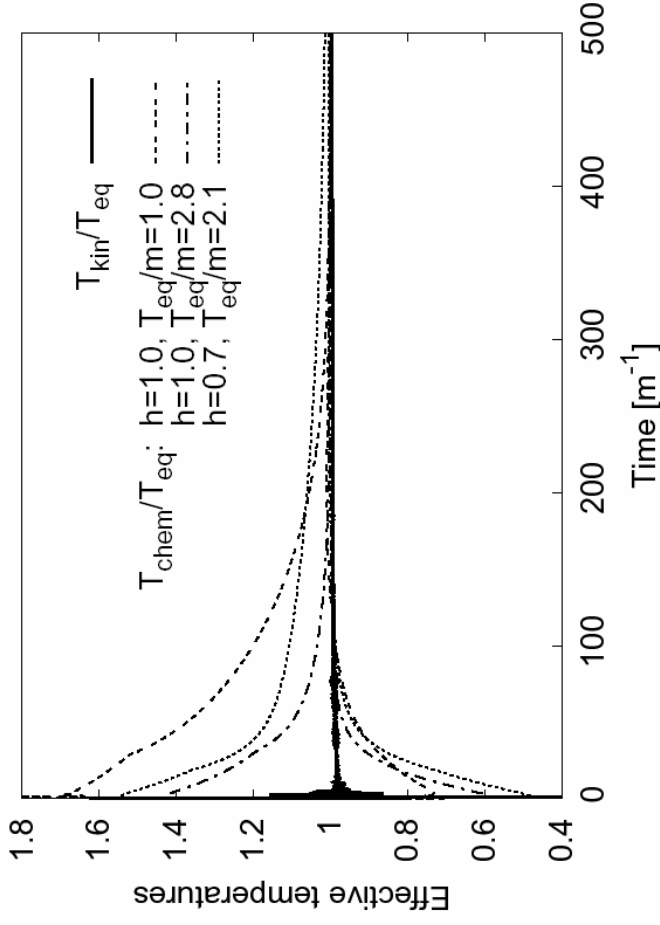
Prethermalization of the equation of state occurs on time scales dramatically shorter than the thermal equilibration time!

Given an EOS, the crucial question arises:

- *Does a suitable global kinetic temperature T_{kin} also exist at t_{pt} ?*

\rightsquigarrow “quasi-thermal” description in a far-from-equilibrium situation!

Kinetic vs chemical equilibration:



upper curves: scalars

lower curves: fermions

↪ $T_{\text{kin}}(t)$ prethermalizes on a very short time scale $\sim m^{-1}$
in contrast to chemical equilibration

↪ late-time chemical equilibration for $t_{\text{ch}} \simeq t_{\text{eq}}$
(t_{ch} depends on details of particle number changing interactions;
deviation from thermal result can become relatively small for $t \ll t_{\text{eq}}$)

Prethermalization: *far-from-equilibrium phenomenon* which describes

- very rapid establishment of an approximately constant ratio of pressure over energy density (equation of state)
- as well as a kinetic temperature based on average kinetic energy

↪ *Crucial for the use of efficient hydrodynamic descriptions!*

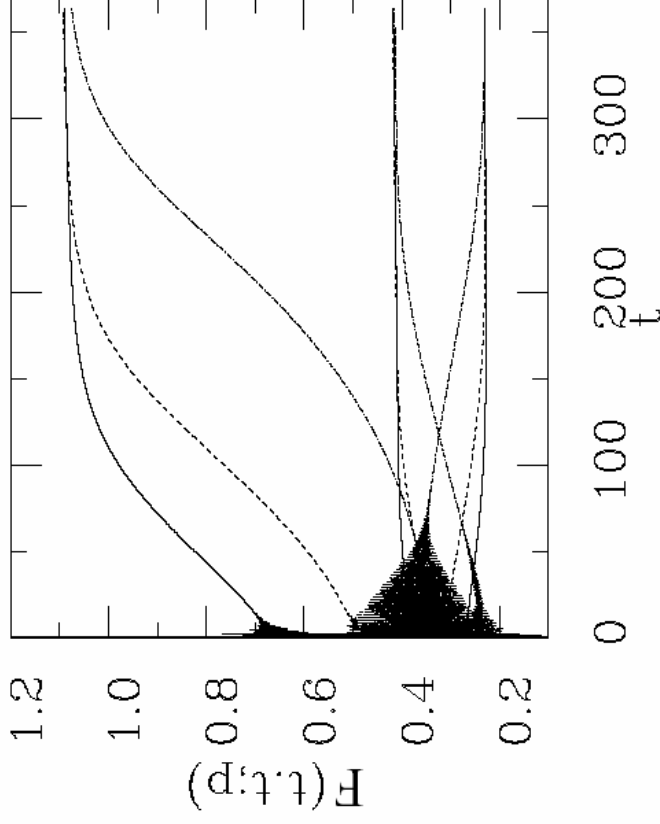
($p = p(e)$ important ingredient to close system of equations $\partial_\mu T^{\mu\nu} = 0$)

More generally:

Extremely different time scales for loss of initial conditions for

- certain “bulk quantities” which average over all momentum modes
- “mode quantities” characterizing the evolution of individual modes

Compare with 2PI:



Propagator for (very) different initial conditions with same $\langle E \rangle$

(momenta $p = 0, 3, 5$; all in initial mass units, $\phi = 0$)

here: scalar ϕ^4 to 3-loop

J.B., Cox, PLB517 (2001) 369

'Damping'

exponential suppression of correlations with initial time

rate: $\gamma^{(\text{damp})}$

$$F(t, 0) = \langle \phi(t) \phi(0) \rangle \rightarrow 0^+$$

'Drifting'

smooth, parametrically slow change of modes

weak dependence of $F(t, t')$ on $t + t'$

'Thermalization'

approach to quantum thermal equilibrium

rate: $\gamma^{(\text{therm})} \neq \gamma^{(\text{damp})}$

$$F(t, t') \rightarrow F^{(\text{eq})}(t-t')$$

Effective loss of initial conditions

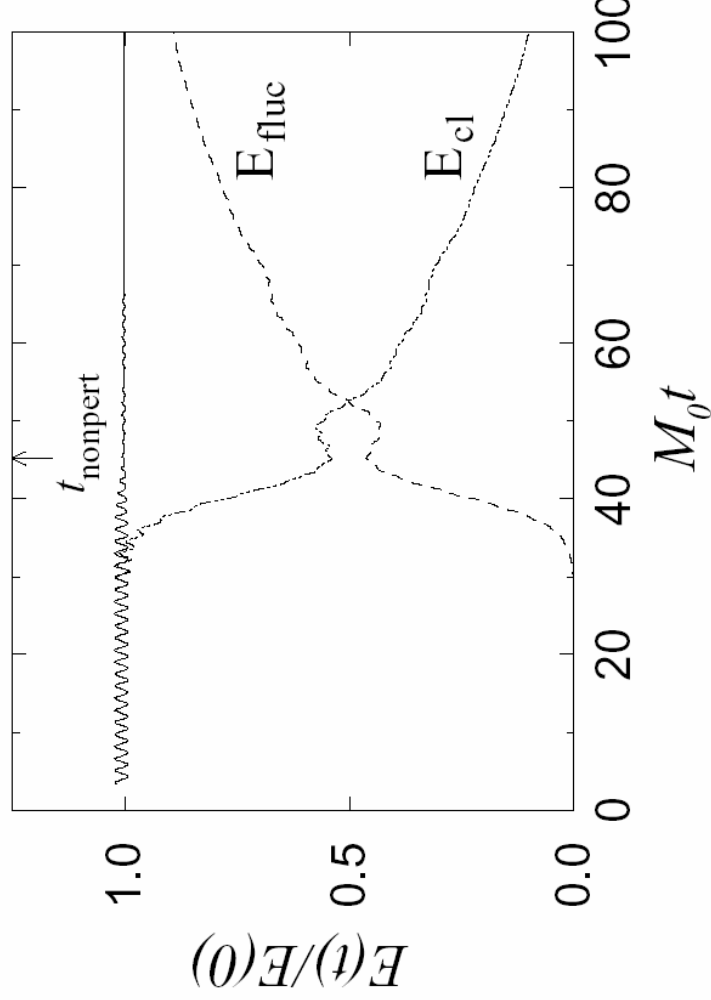
early

intermediate

late

time

Coherent field regime $\xrightarrow{\text{'particle production'}}$ fluctuation dominated regime:



$$t_{\text{nonpert}}: E_{\text{fluc}} \simeq E_{\text{cl}}$$

here: $N = 4, \lambda = 10^{-6}$

\rightsquigarrow dramatic phenomenon for 'arbitrarily' weakly coupled theory!

Chiral symmetry restoration at high temperature

Low T

SSB

$$\langle \varphi \rangle = \varphi_0 \neq 0$$

High T

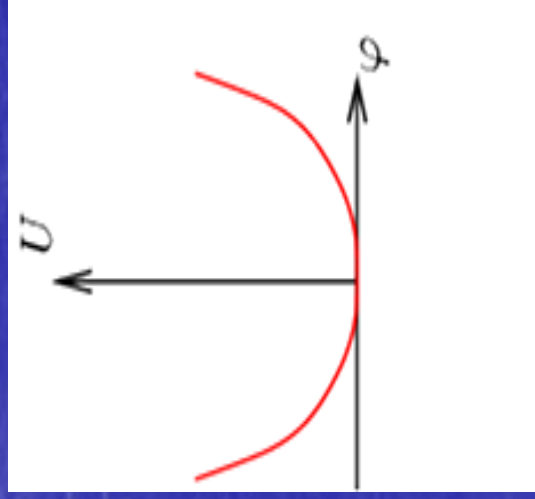
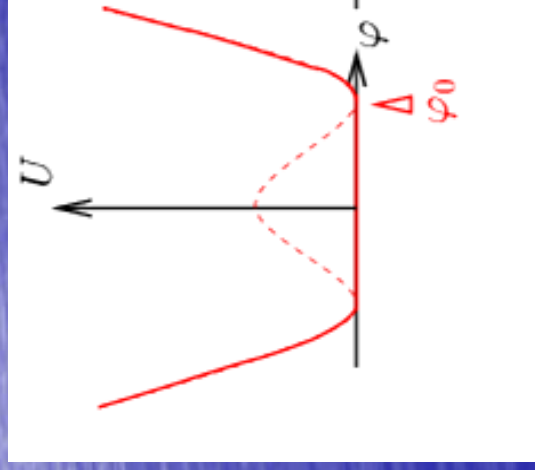
SYM

$$\langle \varphi \rangle = 0$$

at high T :
less order
more symmetry

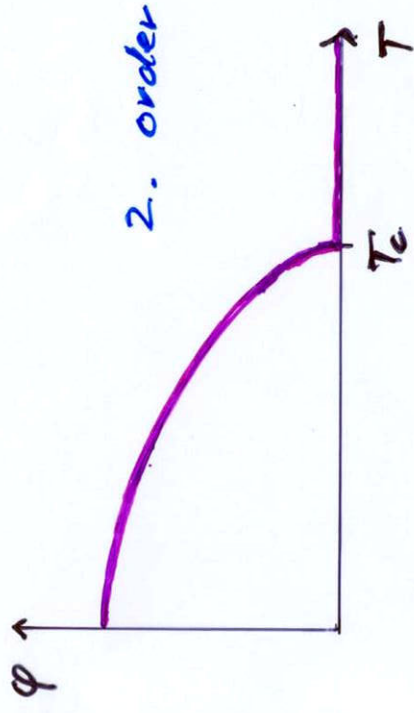
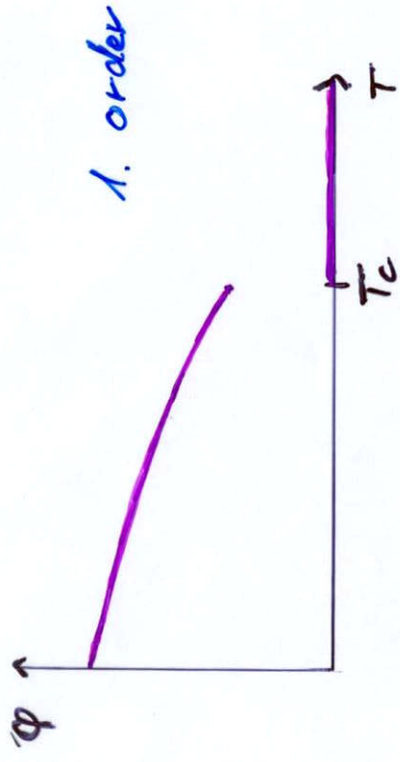
examples:

magnets, crystals

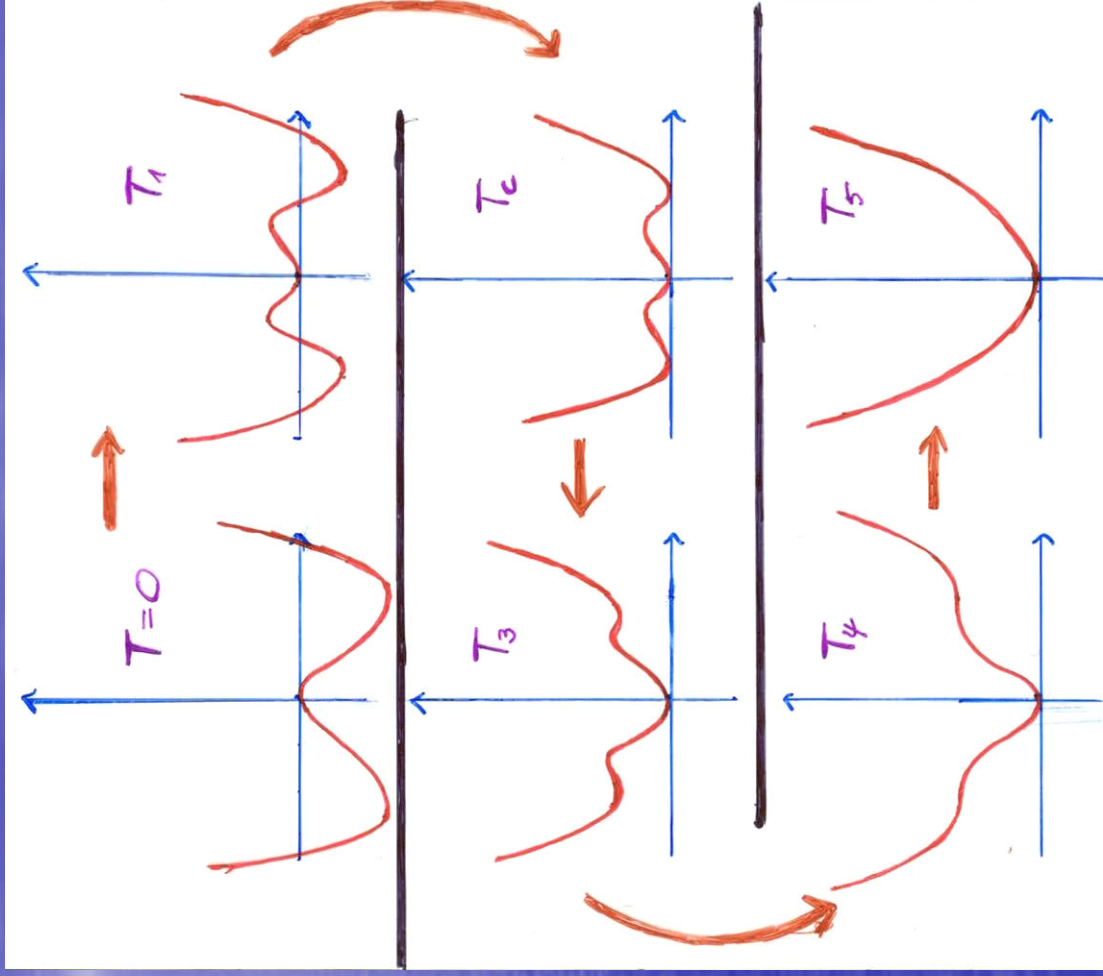


**Order of the phase transition is
crucial ingredient for experiments
(heavy ion collisions)
and cosmological phase transition**

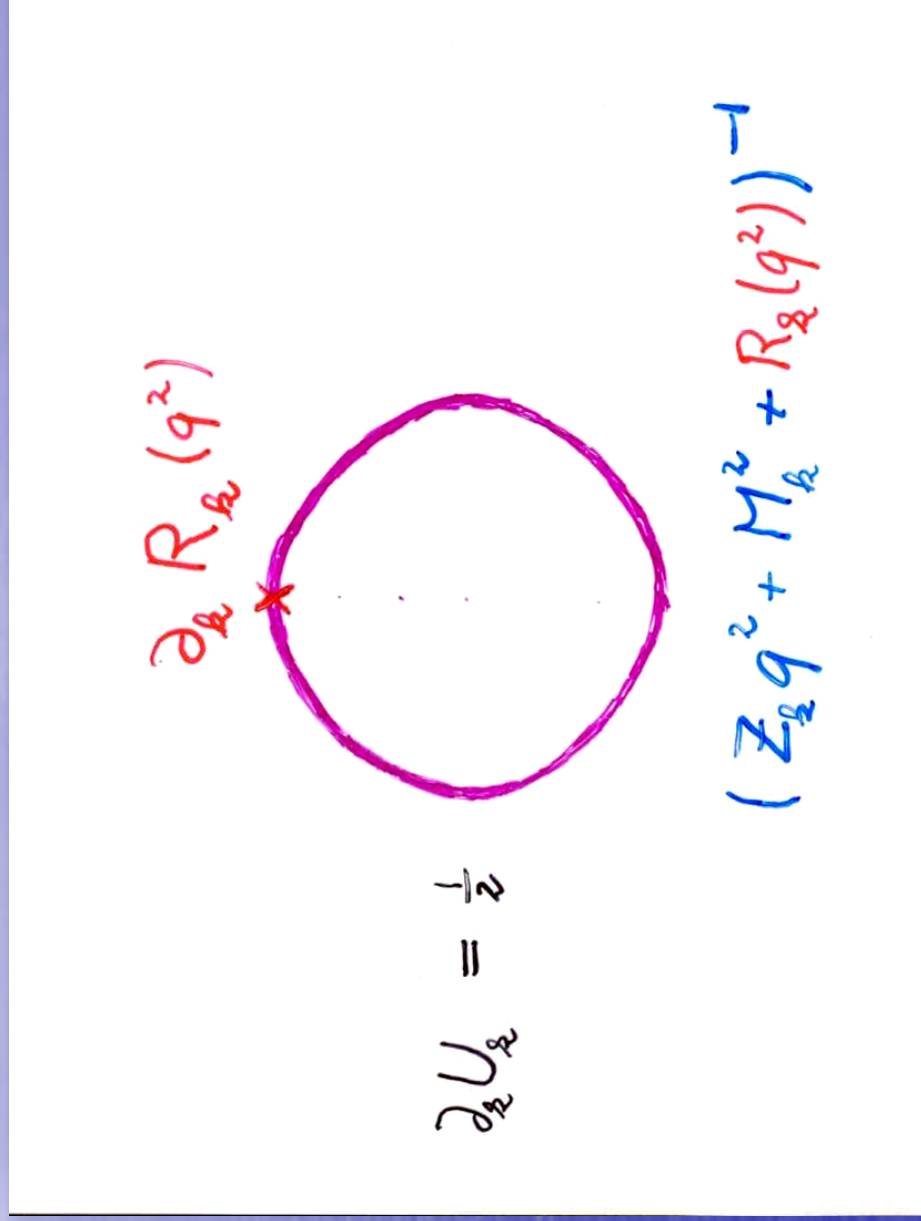
Order of the phase transition



First order phase transition



Simple one loop structure – nevertheless (almost) exact



Flow equation for average potential

$$\partial_k U_k(\varphi) = \frac{1}{2} \sum_i \int \frac{d^d q}{(2\pi)^d} \frac{\partial_k R_k(q^2)}{Z_k q^2 + R_k(q^2) + \bar{M}_{k,i}^2(\varphi)}$$

$$\bar{M}_{k,ab}^2 = \frac{\partial^2 U_k}{\partial \varphi_a \partial \varphi_b} \quad : \quad \text{Mass matrix}$$

$$\bar{M}_{k,i}^2 \quad : \quad \text{Eigenvalues of mass matrix}$$

Critical temperature, $N_f = 2$

$\frac{m_\pi}{\text{MeV}}$	0	45	135	230
$\frac{T_{pc}}{\text{MeV}}$	100.7	$\simeq 110$	$\simeq 130$	$\simeq 150$

for $f_\pi = 93 \text{ MeV}$



Lattice simulation

J.Berges, D.Jungnickel, ...