Paramagnetic Meissner Effect in the g2SC Phase

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Collaborate with I. Shovkovy

``The QCD-phase diagram", Skopelos, May 29 – June 2, 2004

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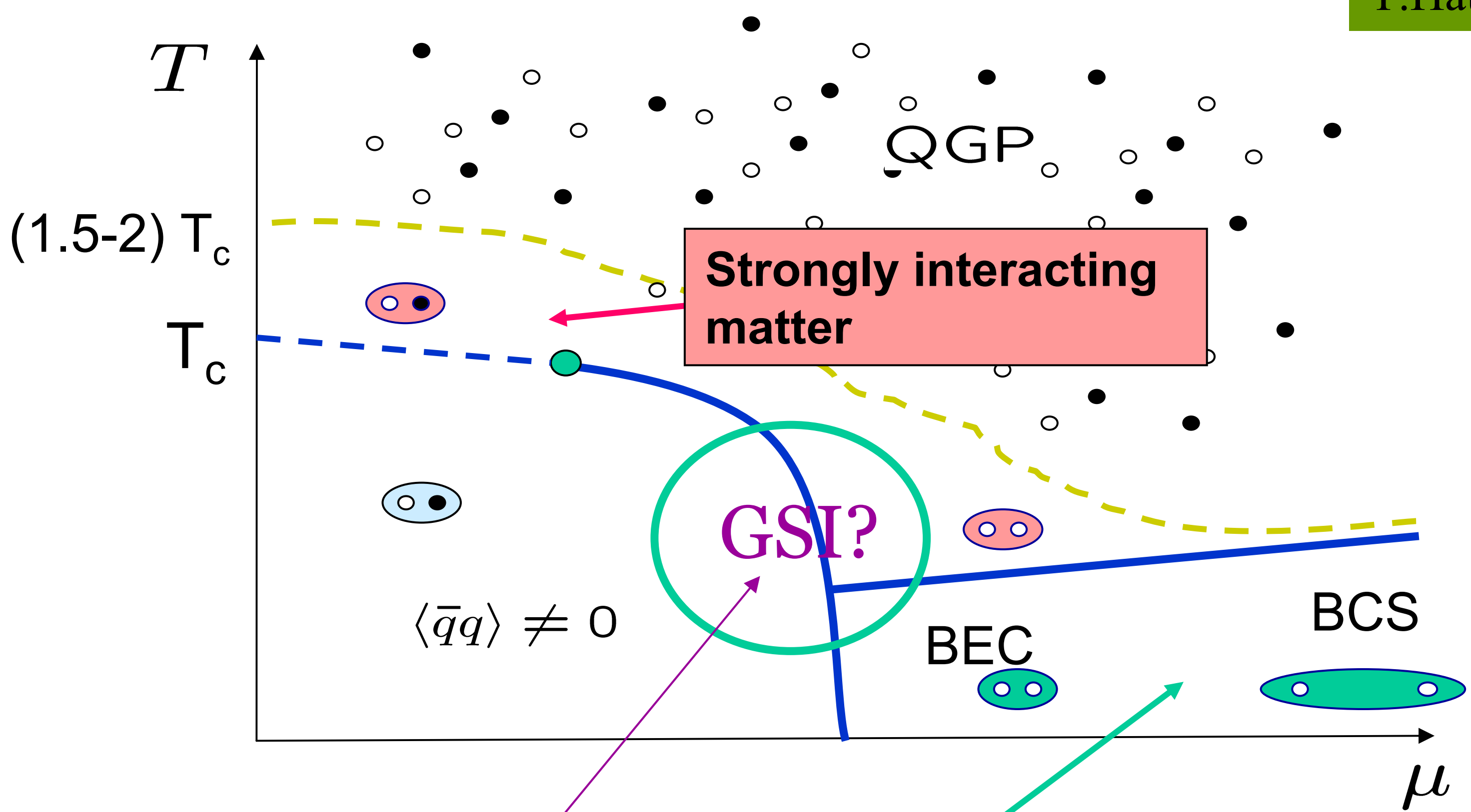
QCD Phase Diagram: from weak to strong coupling

Based on T.Hatsuda's talk



M.H, S. Scherer, H. Weber, M. Bleicher, H. Stoecker

Crossover from BCS to BEC ?

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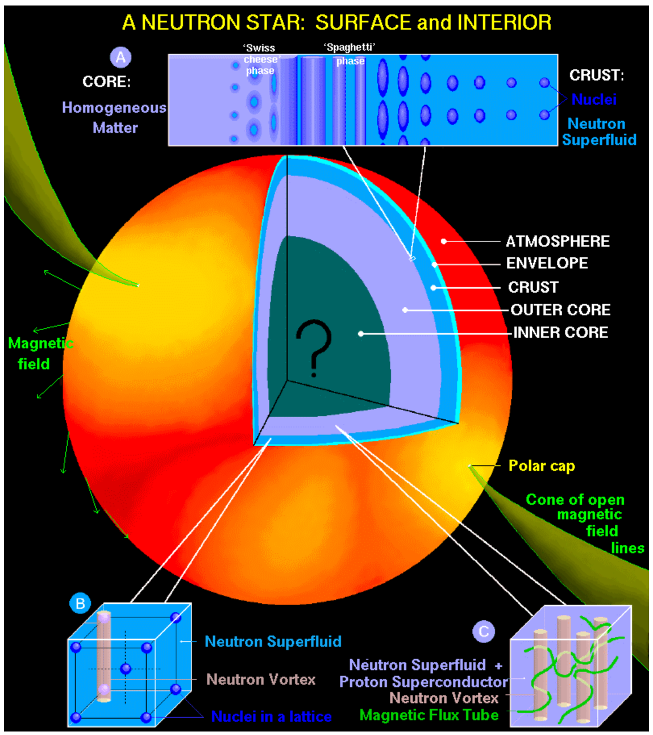
It is possible that CS exists in the core of compact stars



$$\rho / \rho_0 \approx 5 \sim 10$$

Charge neutrality !

1. **Global charge neutrality:**

   mixed phase

M. Buballa,
I. Shovkovy, M. Hanauske, M.H
Phys.Rev.D67:103004,2003

2. **Local charge neutrality:**

   homogeneous phase

g2SC:

M. H., P.F. Zhuang, W.Q. Chao
Phys.Rev.D67:0650152,2003
I. Shovkovy, M.H
Phys.Lett.B564:205,2003
M.H, I. Shovkovy
Nucl.Phys.A729:835,2003

Charge Neutral 2-flavor Dense Quark Matter

- Neutral 2-flavor quark matter:

$$n_d \approx 2n_u, n_e \approx \frac{1}{4^3}\frac{n_u}{3} \ll n_u$$

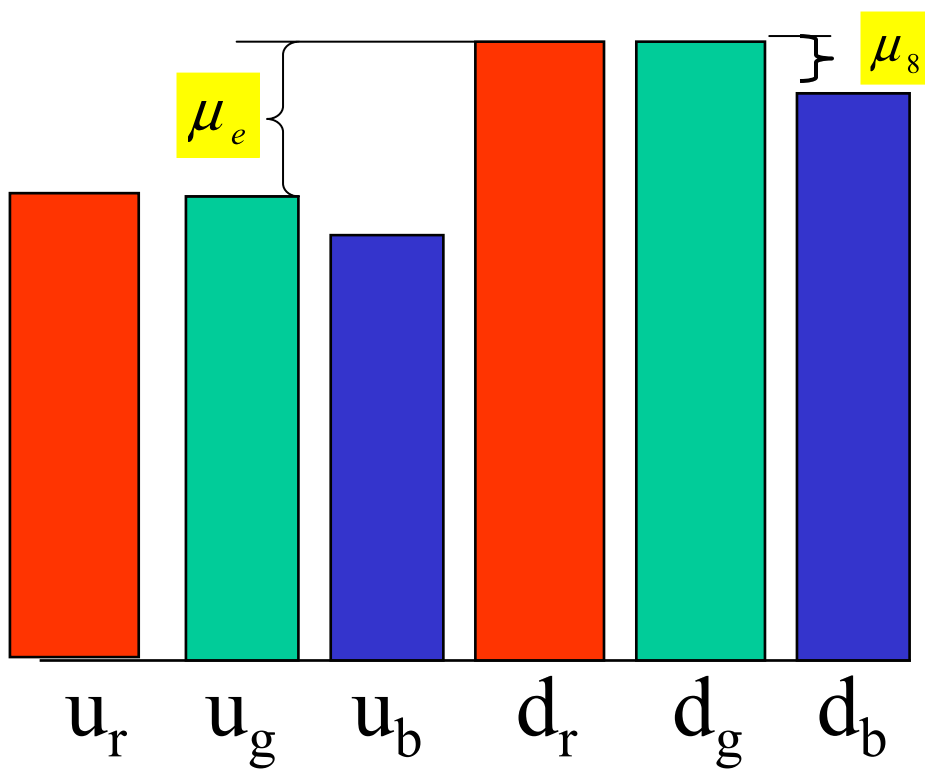
- Cooper pairing with a mismatch between Fermi surfaces of pairing quarks:

$$\mu_d - \mu_u = \mu_e = 2\delta\mu$$

Whether Cooper pairing will be destroyed by mismatch?

β-equilibrium :

$$\mu_{i\alpha} = \mu - \mu_e Q_{ii} + \mu_3 (Q_3)_{\alpha\alpha} + \mu_8 (Q_8)_{\alpha\alpha}$$



$\mu_3 = 0, SU(2)_C$

♣ SU(2) NJL Model

The Model

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] \\ &+G_D\left[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)\right]\end{aligned}$$

Model Parameters

$$m_0 = 0, G_S = 5.0163 GeV^{-2}, \Lambda = 653.3 MeV, \eta = G_D/G_S$$

Thermodynamical Potential:

$$\Omega = \Omega_0 - \frac{1}{12\pi^2}\left(\mu_e^4 + 2\pi^2 T^2 \mu_e^2 + \frac{7\pi^4}{15}T^4\right) + \frac{(m-m_0)^2}{4G_S} + \frac{\Delta^2}{4G_D} - \sum_a \int \frac{d^3p}{(2\pi)^3}\left[E_a + 2T\ln\left(1+e^{-E_a/T}\right)\right]$$

Dispersion Relations:

$$E_{ub}^{\pm} = E(p) \pm \mu_{ub}, \qquad [\times 1]$$

$$E_{db}^{\pm} = E(p) \pm \mu_{db}, \qquad [\times 1]$$

$$E_{\Delta^{\pm}}^{\pm} = E_{\Delta}^{\pm}(p) \pm \delta\mu. \qquad [\times 2]$$

Whether there is still Cooper pairing ?

Diquark Gap Equation and Charge Neutrality:

$$\frac{\partial \Omega}{\partial \Delta} = 0, \quad n_8 \equiv -\frac{\partial \Omega}{\partial \mu_8} = 0, \quad n_Q \equiv -\frac{\partial \Omega}{\partial \mu_e} = 0$$

- η Dependent Solutions at $T = 0$:

a) "strong":

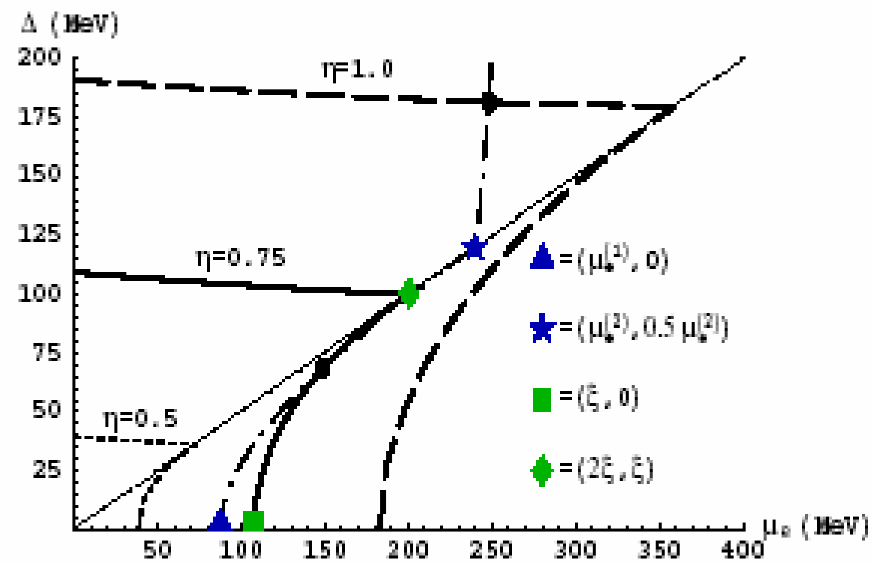
$\eta > \eta_2^{cr} \longrightarrow$ 2SC

b) "weak":

$\eta < \eta_1^{cr} \longrightarrow$ normal

c) "intermediate":

$\eta_1^{cr} < \eta < \eta_2^{cr} \rightarrow$ "g2SC"



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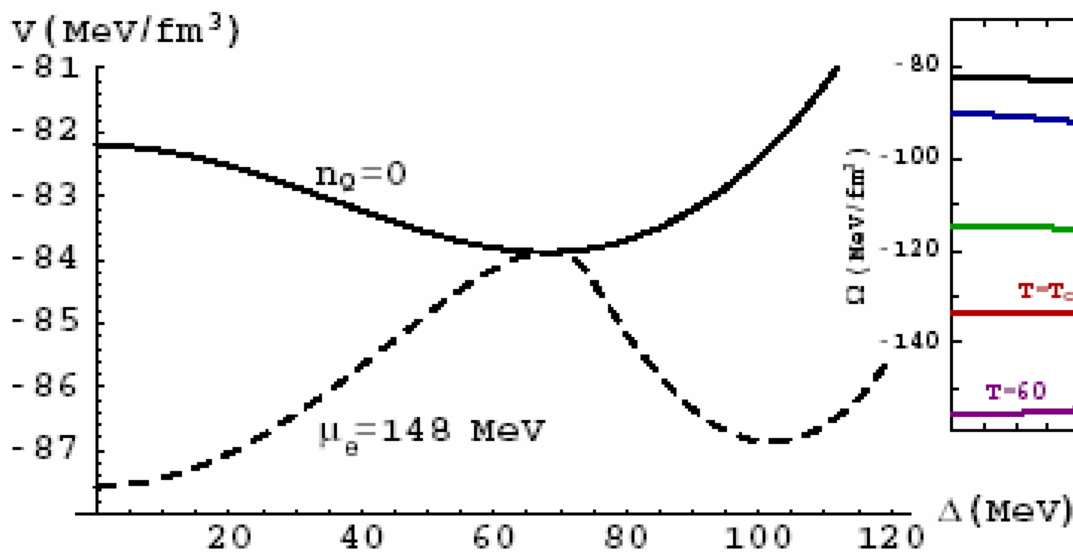
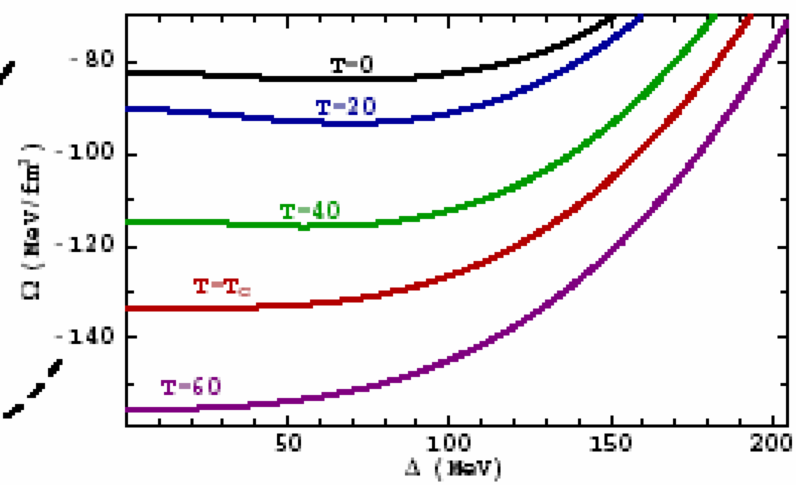
Is "g2SC" Stable?

Eff. potential at $T=0$ | **Eff. potential at $T \neq 0$**

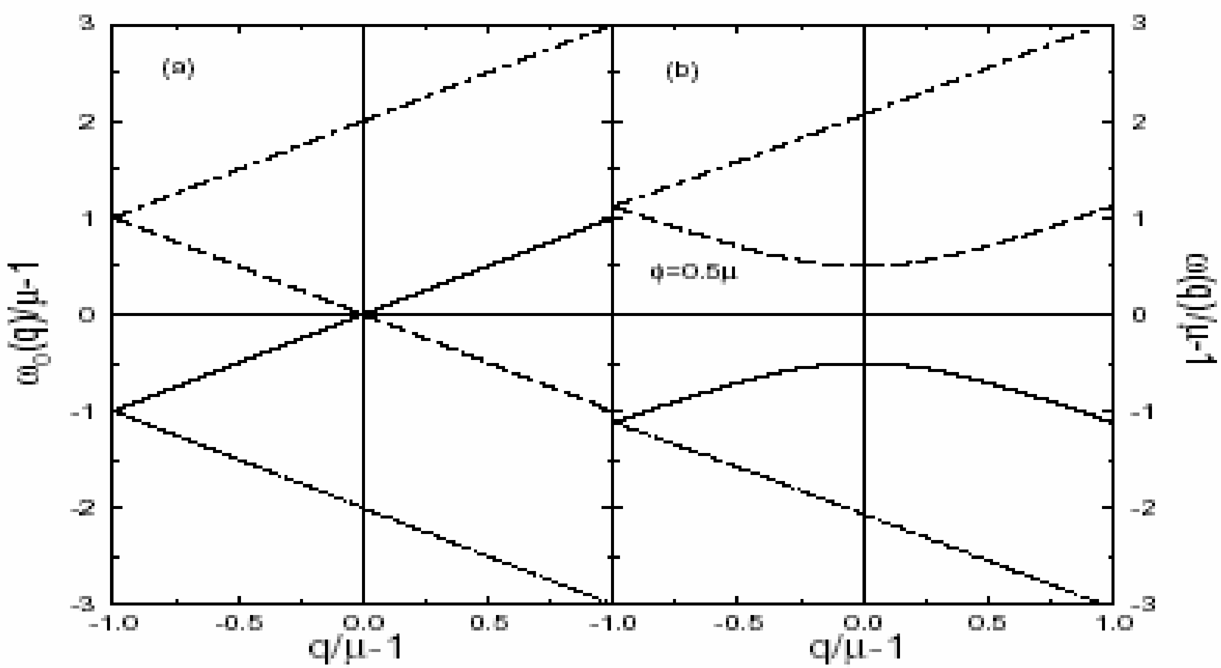




"g2SC" is stable provided $n_Q = 0$ is enforced *locally*!!

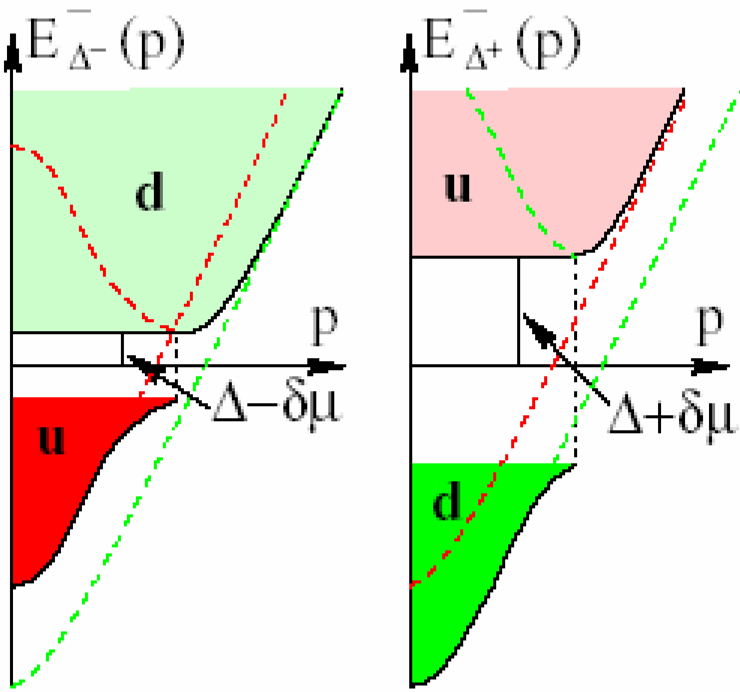
Why it is called "g2SC"?

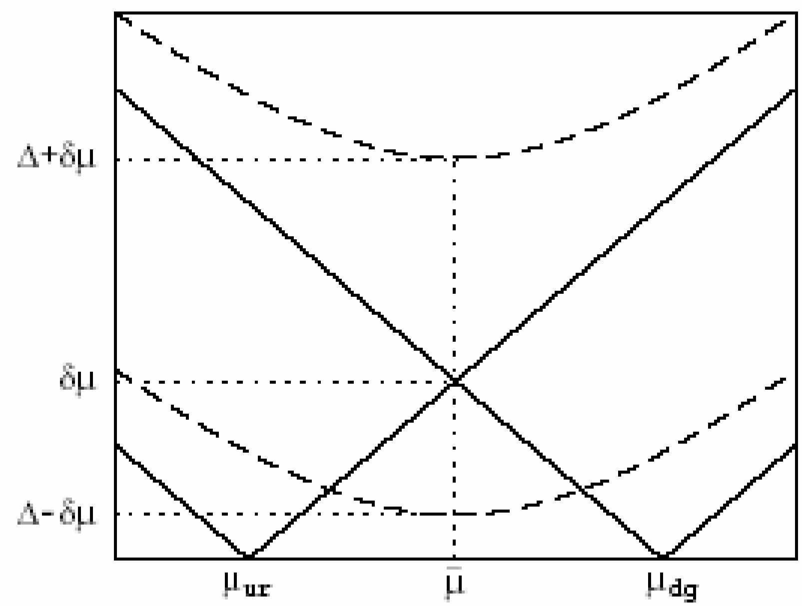
1. 2SC Quasiparticle Spectrum if $\delta\mu = 0$



R.D. Pisarski, D.H. Rischke, Phys.Rev.D60:094013,1999

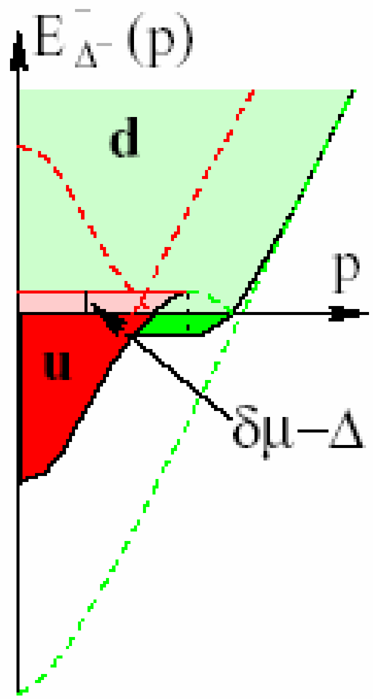
2. 2SC Quasiparticle Spectrum if $\delta\mu < \Delta$

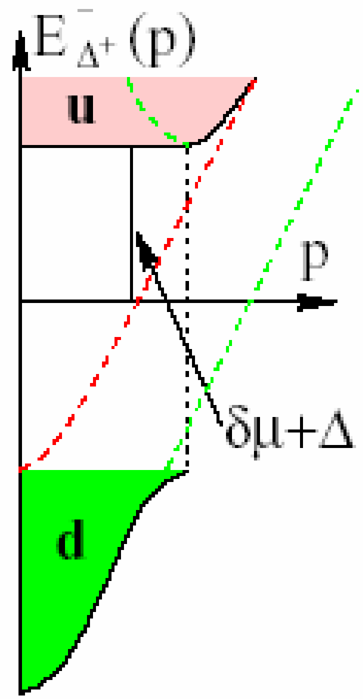


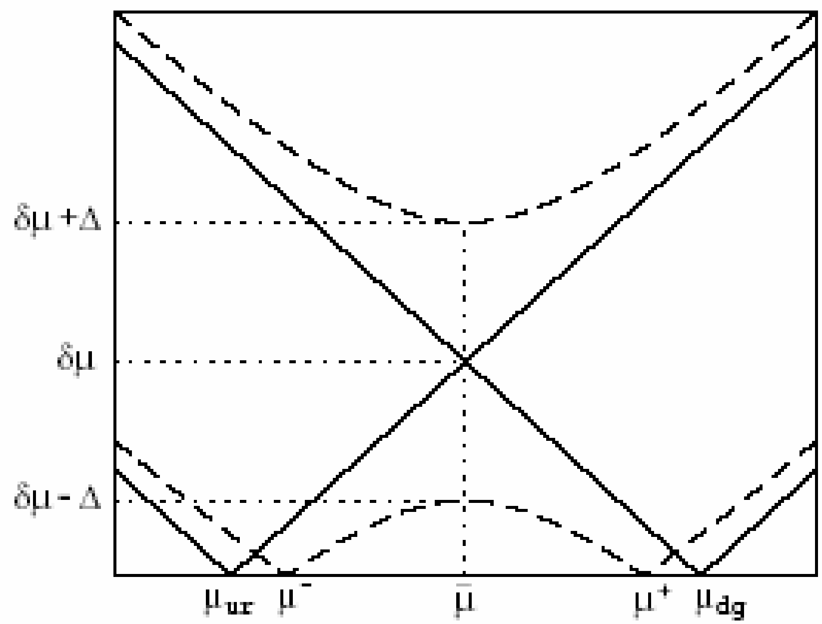


$\delta\mu$ **induces two branches of excitations!**

3. 2SC Quasiparticle Spectrum if $\delta\mu > \Delta$





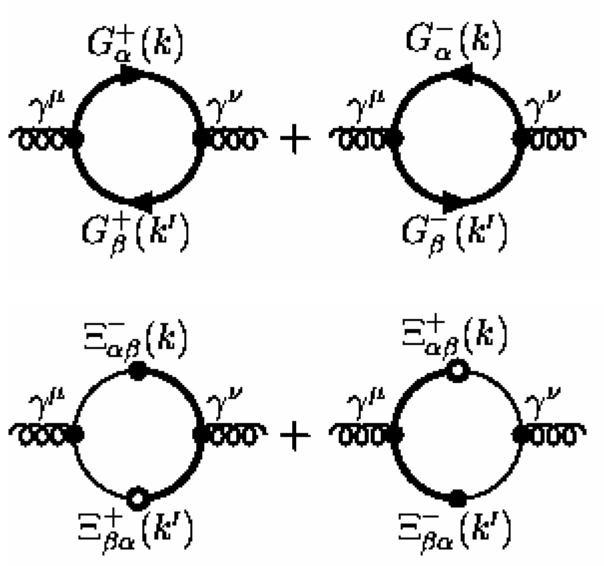


Appearance of Gapless Mode $\longrightarrow$ g2SC !!!

Gluon self-energy in g2SC

M.H. , I. Shovkovy, hep-ph/0406…, will appear in arXiv

$$\Pi^{\mu\nu}_{ab}(P) = \frac{1}{2} g_1 g_2 \frac{T}{V} \sum_K \mathrm{Tr}_{s,c,f,NG} \left[\hat{\Gamma}^{\mu}_{a} \mathcal{S}(K) \hat{\Gamma}^{\nu}_{b} \mathcal{S}(K-P) \right]$$



$$\mathcal{S}(K) = \begin{pmatrix} G^+(K) & \Xi^-(K) \\ \Xi^+(K) & G^-(K) \end{pmatrix}$$

$$\hat{\Gamma}^{\mu}_{a} \equiv \begin{pmatrix} \Gamma^{\mu}_{a} & 0 \\ 0 & \bar{\Gamma}^{\mu}_{a} \end{pmatrix}$$

Debye Mass:

$$M_D^2 = -\Pi^{00}(0,\vec{0})$$

Meissner Mass:

$$M_M^2 = \Pi^{ij}(0,\vec{0})\delta_{ij}$$

Special properties of g2SC

1. Pairing quarks have different number densities at T=0

$$n_{ur} = n_{ug} \neq n_{dr} = n_{dg}$$

2. Finite temperature properties:

nonmonotonic behavior

not a BCS ratio $r_{BCS} = T_c / \Delta = 0.567$

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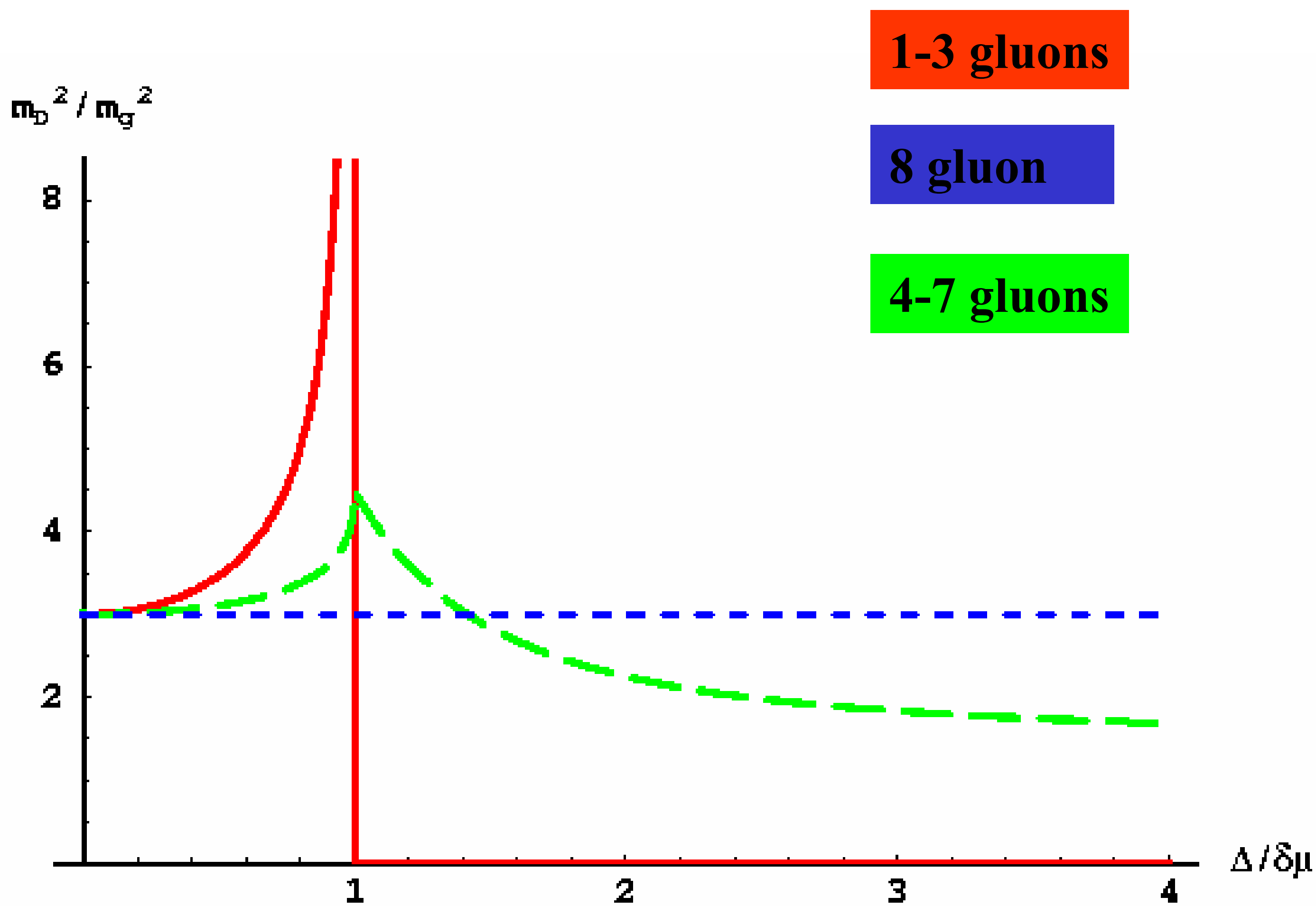

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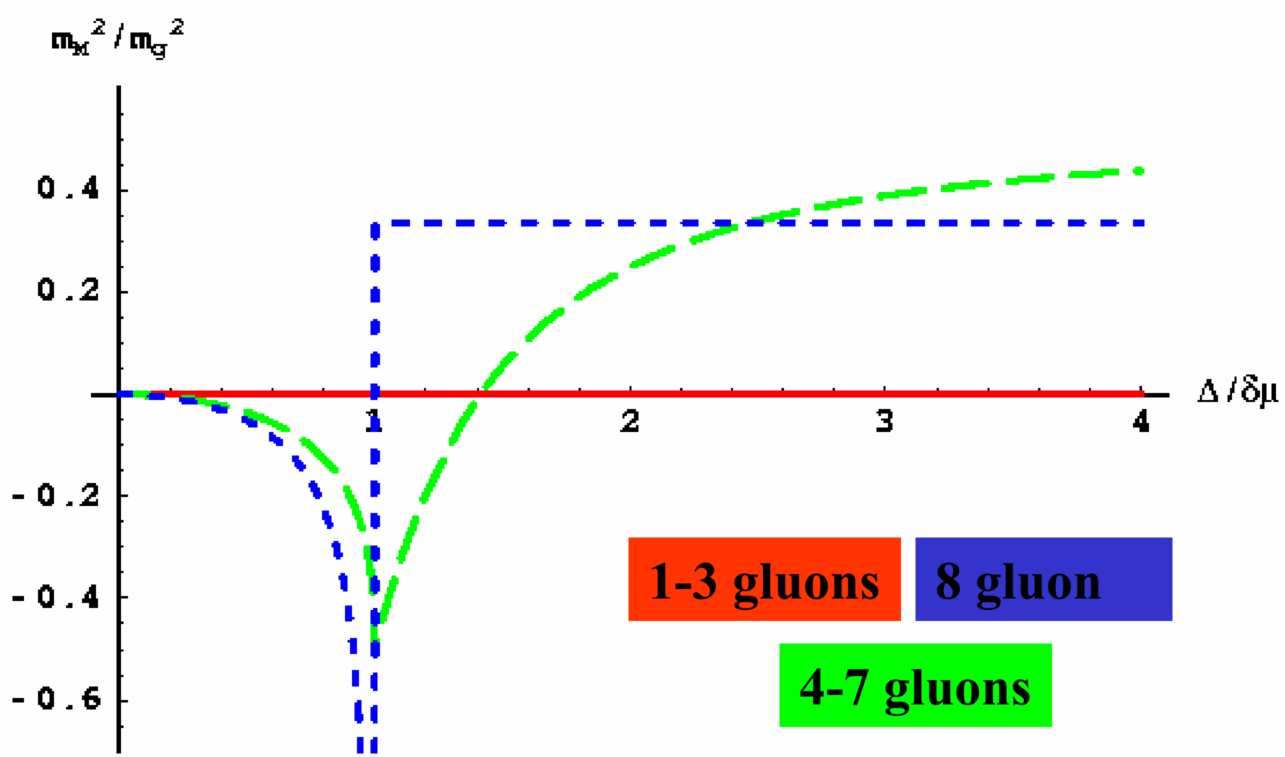

Meissner Masses for the 8th gluon and 4th-7th are negative!!!

How to understand the minus sign?

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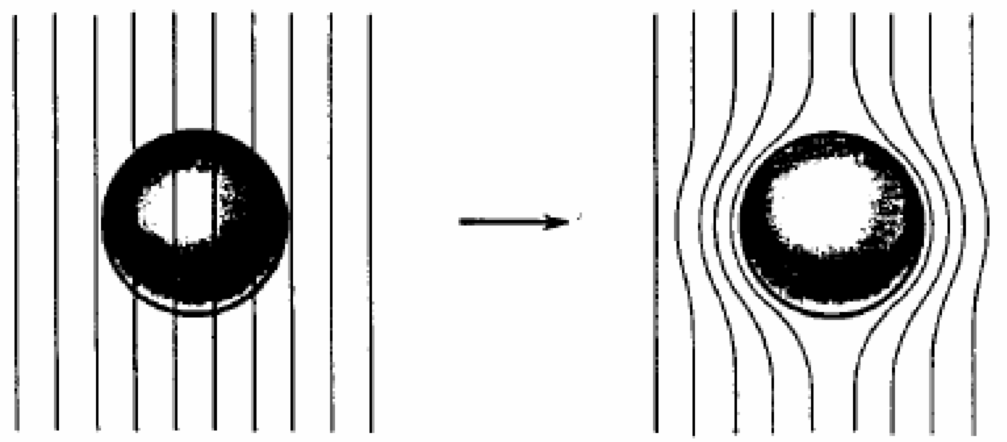
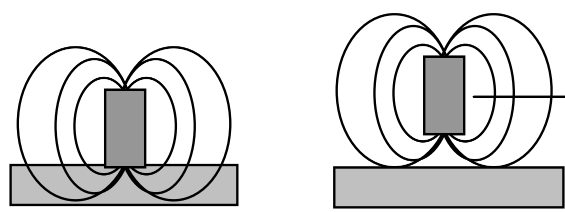
Meissner Effect

1933: Meissner & Ochsenfeld



Normal

S/C



$T > T_C$ $T < T_C$

Why Meissner Effect in regular superconductor ?

Response function: related to matter's properties

$$J_\mu(\vec{q},\omega) = -\frac{1}{4\pi} K_{\mu\nu}(\vec{q},\omega) A^\nu(\vec{q},\omega).$$

Induced current

External Field

Paramagnetic current and diamagnetic current

$$K_{\mu\nu} = K^{p}_{\mu\nu} + K^{d}_{\mu\nu}$$

1. Normal metal

$$K^{p}_{\mu\nu}(0,\vec{0}) + K^{d}_{\mu\nu}(0,\vec{0}) = 0$$

free eletrons

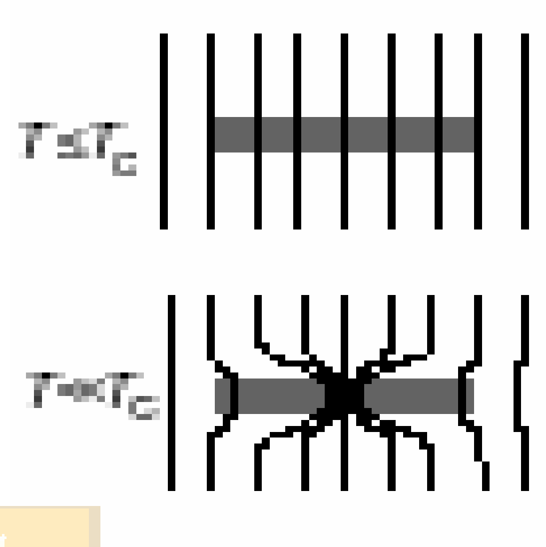
2. Superconductor

$$K^{p}_{\mu\nu}(0,\vec{0}) < K^{d}_{\mu\nu}(0,\vec{0})$$

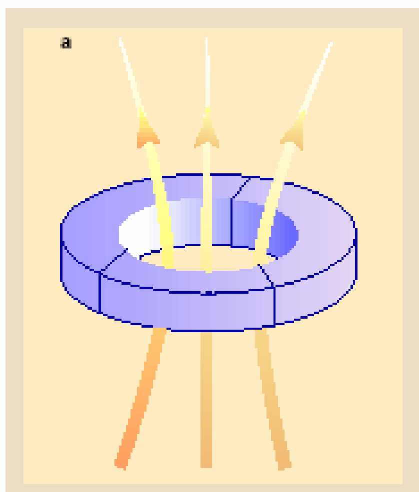
BCS pair, gap parameter

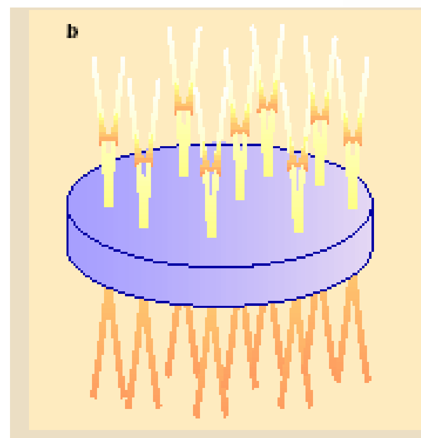
Perfect Diamagnet

3. Paramagnetic Meissner Effect



$$K^{p}_{\mu\nu}(0,\vec{0}) > K^{d}_{\mu\nu}(0,\vec{0})$$





Mesoscopic superconductors: a) A loop made of three crystals of HTSC; b) Aluminium disc

1. Braunisch, W. *et al.* Paramagnetic Meissner effect in Bi high-temperature superconductors. *Phys. Rev. Lett.* **68**, 1908–1911 (1992).
2. Schliepe, B., Stindtmann, M., Nikolic, I. & Baberschke, K. Positive field-cooled susceptibility in high-T_C superconductors. *Phys. Rev. B* **47**, 8331–8334 (1993).
3. Heizel, C., Theiling, T. & Zieman, P. Paramagnetic Meissner effect analyzed by 2nd harmonics of the magnetic susceptibiity. *Phys. Rev. B* **48**, 3445–3454 (1993).
4. Magnusson, J. *et al.* Time-dependence of the magnetization of BiSrCaCuO displaying the paramagnetic Meissner effect. *Phys. Rev. B* **52**, 7675–7681 (1995).
5. Riedling, S. *et al.* Observation of the Wohlleben effect in YBaCuO single crystals. *Phys. Rev. B* **49**, 13283–13286 (1994).
6. Thompson, D. J., Minhaj, M. S. M., Wenger, L. E. & Chen, J. T. Observation of paramagnetic Meissner effect in niobium disks. *Phys. Rev. Lett.* **75**, 529–532 (1995).
7. Kostic, P. *et al.* Paramagnetic Meissner effect in Nb. *Phys. Rev. B* **53**, 791–801 (1996).

in small superconductors *Nature* **396**, 144 - 146 (1998); doi:10.1038/24110

mixed state of Y–Ba–Cu–O ceramics Low Temperature Physics (1996) -- V22, 539-541

$\mathbf{Nd_{2-x}Ce_xCuO_y}$ J. Phys.: Condens. Matter **9** (1997) L525-L531

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Paramagnetic contributions

	$m^2_{M,8}$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	0	$+1$	0	-1	0
2SC	$+1$	$+1$	0	$-1/2$	$+1/2$
g2SC	$1-\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$+1$	0	$-\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$	$\frac{1}{2}-\frac{1}{2}\frac{\delta\mu}{\sqrt{\delta\mu^2-\Delta^2}}$

Paramagnetic contribution

	$m^2_{M,4}$	P-A, G	P-A, Ξ	P-H, G	P-H, Ξ
NQM	$+1$	$+2$	0	-1	0
2SC	$\frac{\Delta^2-2\delta\mu^2}{\Delta^2}$	$+2$	0	$-\frac{\Delta^2+2\delta\mu^2}{\Delta^2}$	0
g2SC	$\frac{\Delta^2-2\delta\mu^2}{\Delta^2}+\frac{\delta\mu\sqrt{\delta\mu^2-\Delta^2}}{\Delta^2}$	$+2$	0	$-\frac{\Delta^2+2\delta\mu^2}{\Delta^2}+2\frac{\delta\mu\sqrt{\delta\mu^2-\Delta^2}}{\Delta^2}$	0

Paramagnetic contribution

Summary

1. Cold-dense charge neutral 2-flavor quark matter at intermediate coupling region is in g2SC phase, it is a thermal stable state.
2. Unstable chromo-magnetic gluon modes appear in g2SC phase.

   Paramagnetic Meissner effect !

   Spontaneous magnetization ?

   Need gluon condensate to stabilize ?