Equilibration near the QCD Phase Boundary

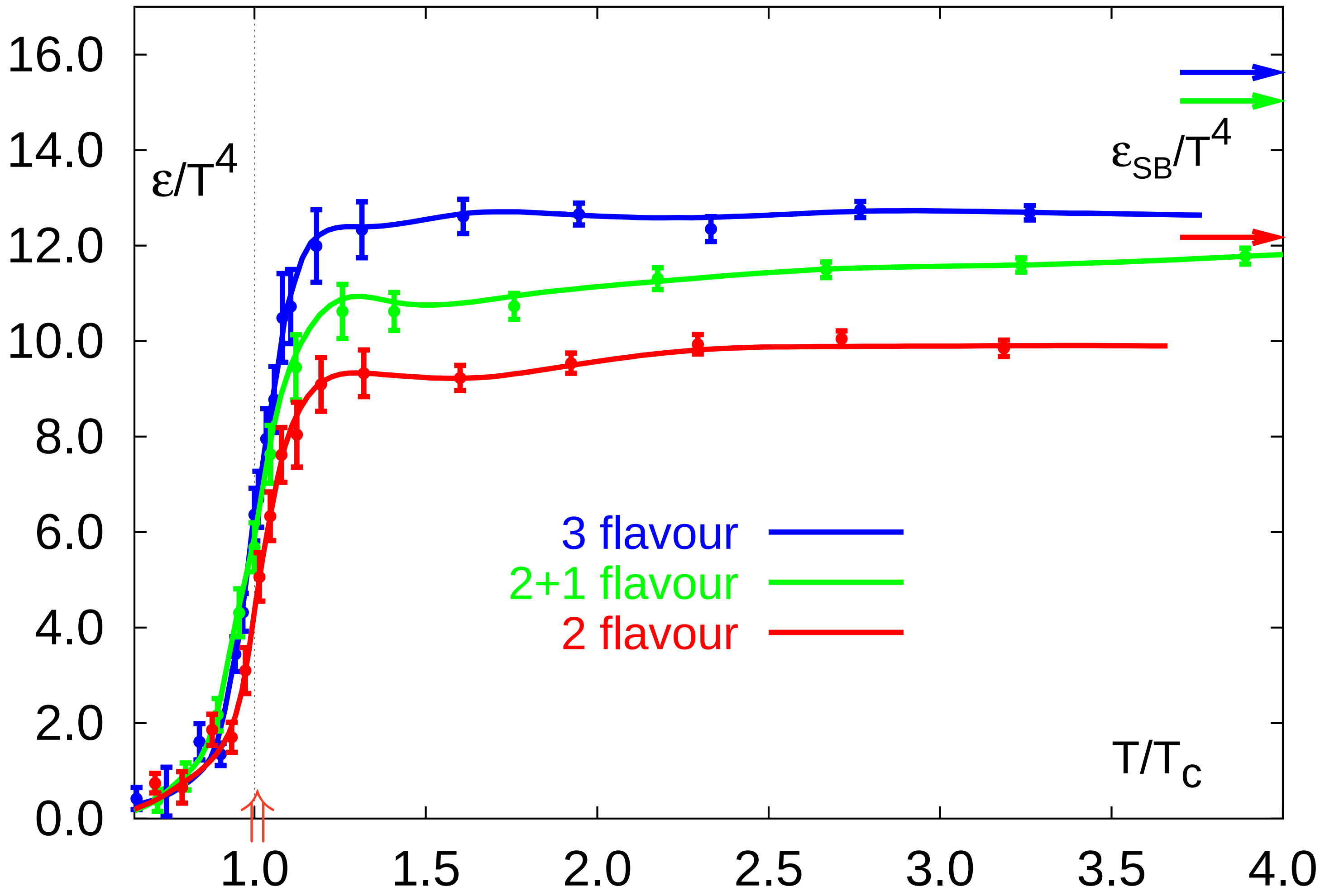
Johanna Stachel

Physikalisches Institut, Universität Heidelberg

- **Knowlegde about the QCD Phase Boundary and T_c**
- **Hadrochemical Equilibration and T_{ch}**
- **Space-Time Dynamics and T_f**
- **Model for Rapid Equilibration $T_{ch} \approx T_c$**
- **Summary**

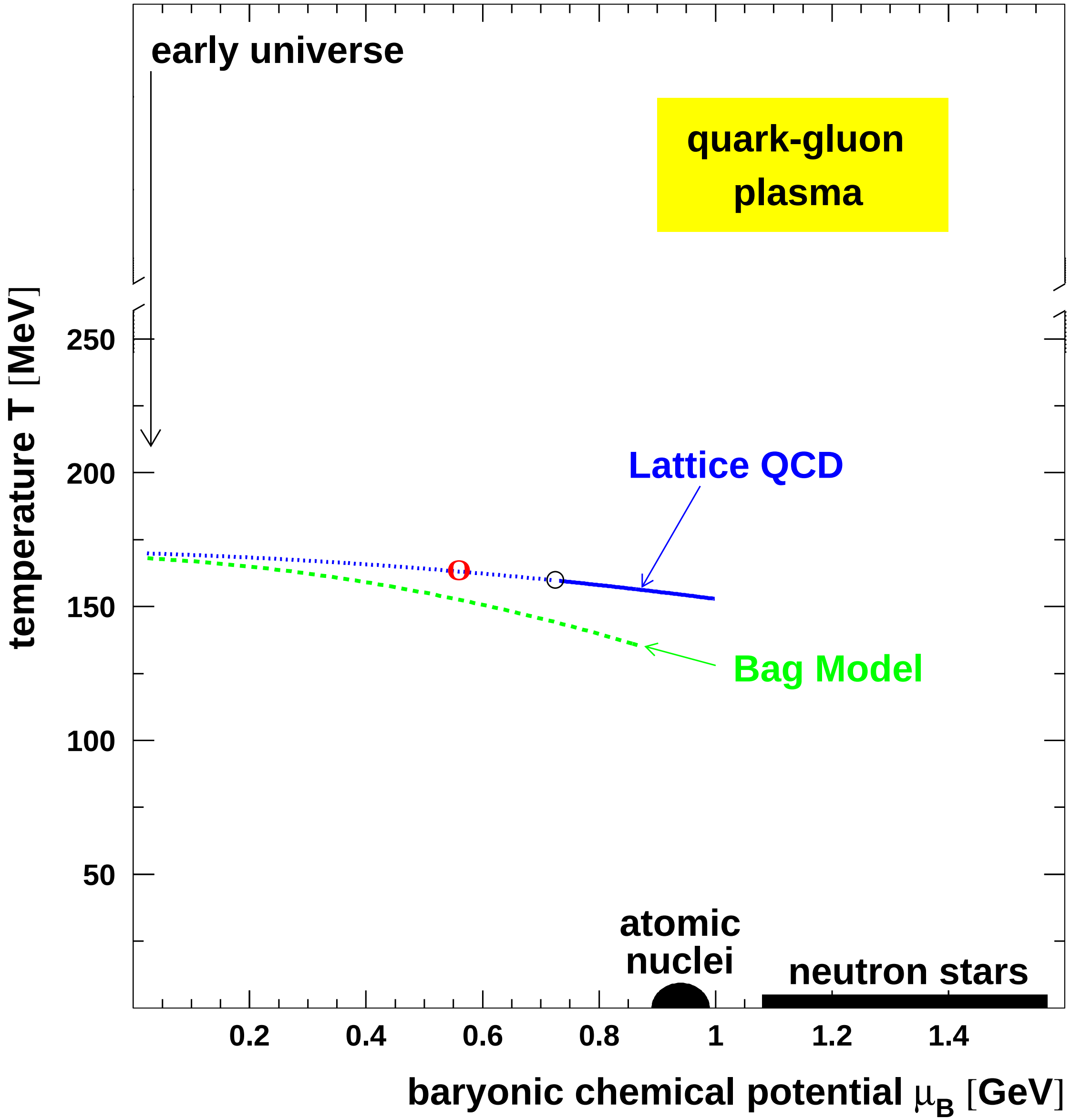
Energy Density from Finite Temperature Lattice QCD

F. Karsch et al. Bielefeld Group, Phys.Lett. **B478** (2000) 447; Nucl. Phys. **A698** (2002) 199c.
$16^3 \times$ 4 lattice, $m_{ql}/T=0.4$, $m_{qh}/T=1$



rapid rise at $T_c = 173 \pm 8$ MeV

The Phase Diagram of Nuclear Matter



Z.Fodor, S.D.Katz, hep-lat/0106002

C.R.Allton et al., hep-lat/0204010

pbm & js, Nucl.Phys.A606(1996)220

Statistical Description of Hadron Abundancies

Grand Canonical Ensemble

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp[-(E_i - \mu_i)/T]]$$

$$n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

for every conserved quantum number there is a chemical potential μ

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain:

- Baryon number: $V \sum_i n_i B_i = Z + N \quad \rightarrow V$
- Strangeness: $V \sum_i n_i S_i = 0 \quad \rightarrow \mu_S$
- Charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2} \quad \rightarrow \mu_{I_3}$

Only μ_b and T free parameter when 4π considered
for rapidity slice fix volume e.g. by dN_{ch}/dy

CERN SPS Data and Thermal Model

P. Braun-Munzinger, I. Heppe, J.Stachel, Phys.Lett.**B465** (1999) 15 + reanalysis in 2003 with more data

free parameters:
T = 0.170 ± 0.005 GeV
μ_b **= 0.255 ± 0.010 GeV**

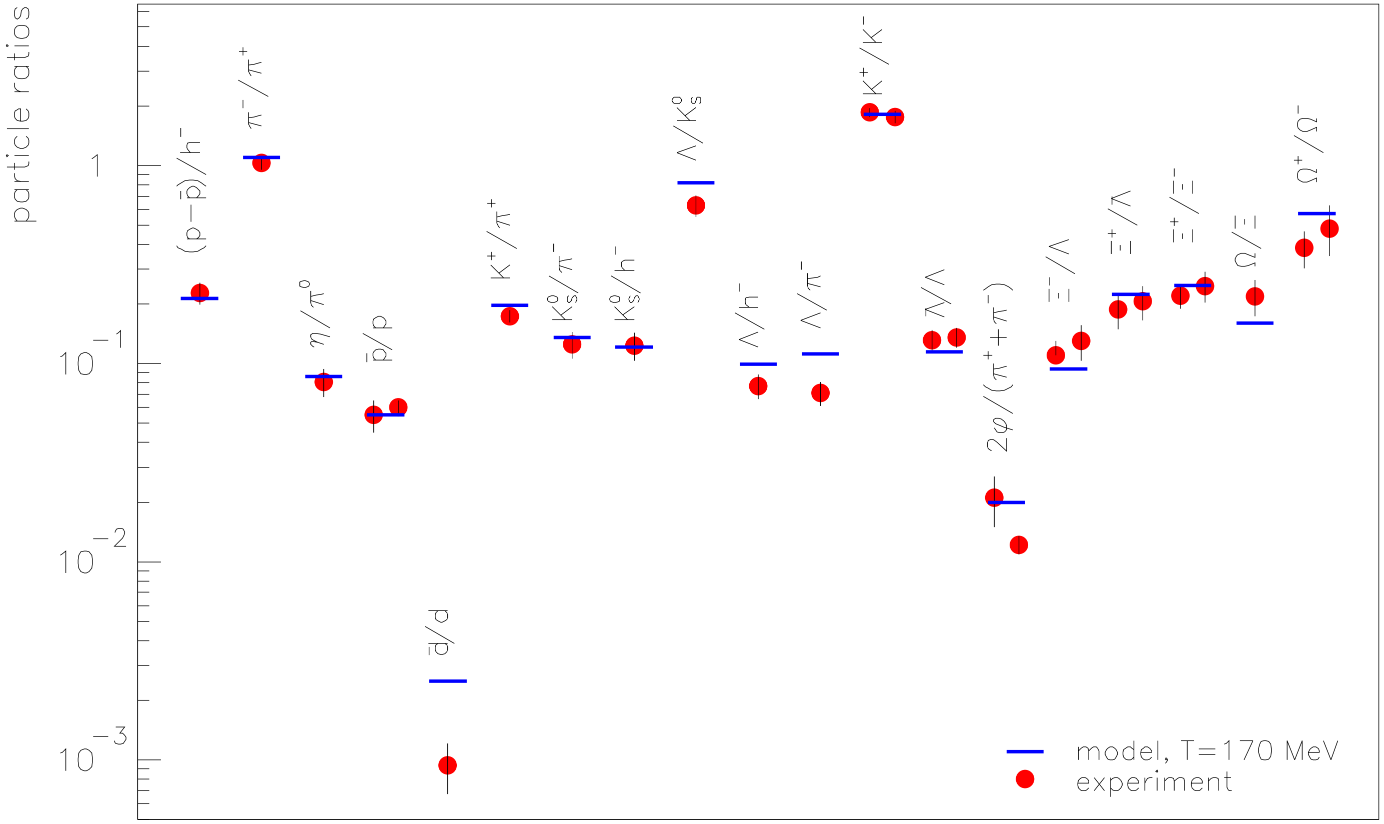
fixed by conservation laws:
μ_s = 0.074 GeV from ΔS=0
μ_{I_3}=0.005 GeV from ΔQ=0

reduced χ^2 (excl. ϕ and $\bar{\mathrm{d}}$) 2.0

largest contribution:

Λ/π, Λ/h^-, Λ/K^0_s

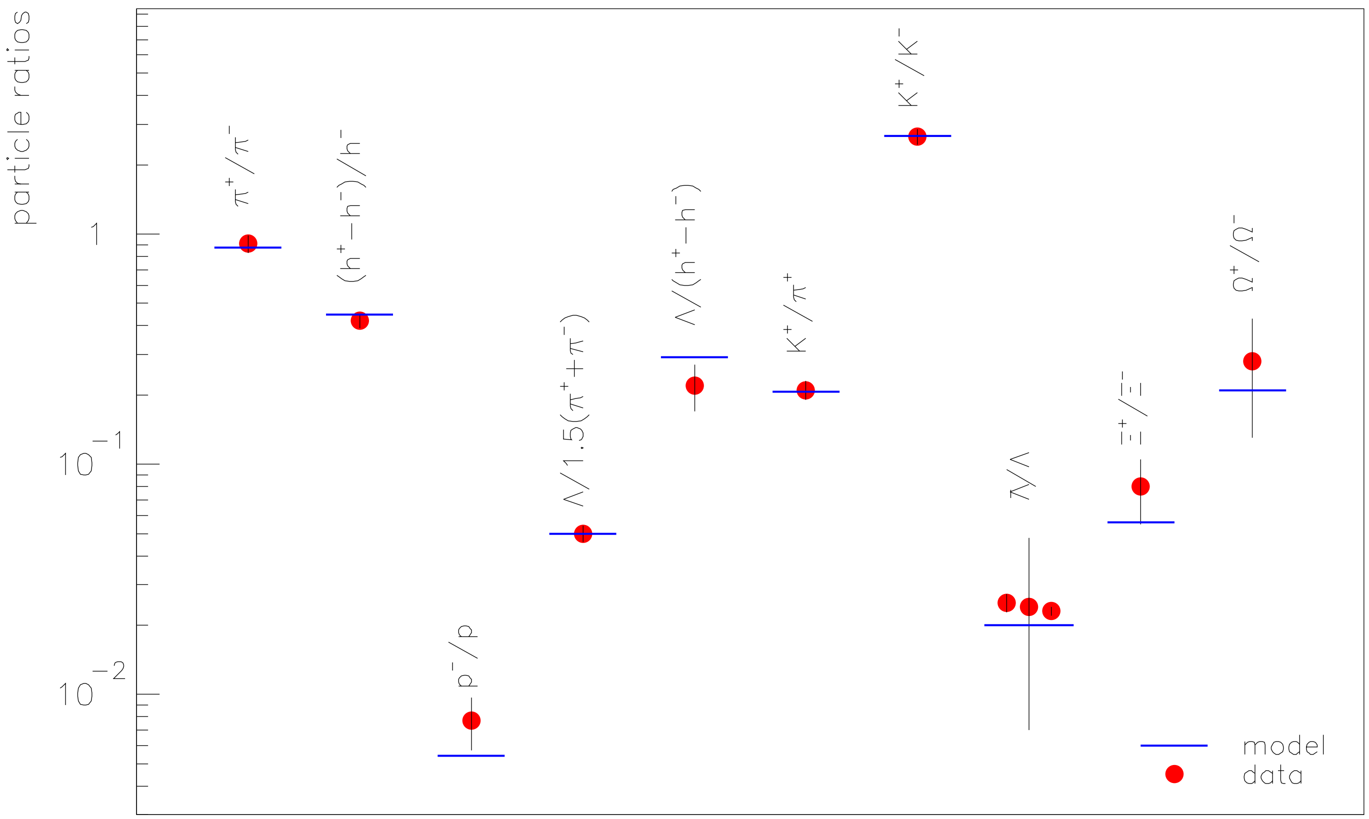
central 158 A GeV/c Pb + Pb collisions



Hadron Yields at SPS 40 A GeV/c and Thermal Model

P. Braun-Munzinger, D. Magestro, J. Stachel, Dec. 02

central 40 A GeV/c Pb + Pb collisions - thermal model parameters: T = 148 MeV, μ_b = 400 MeV

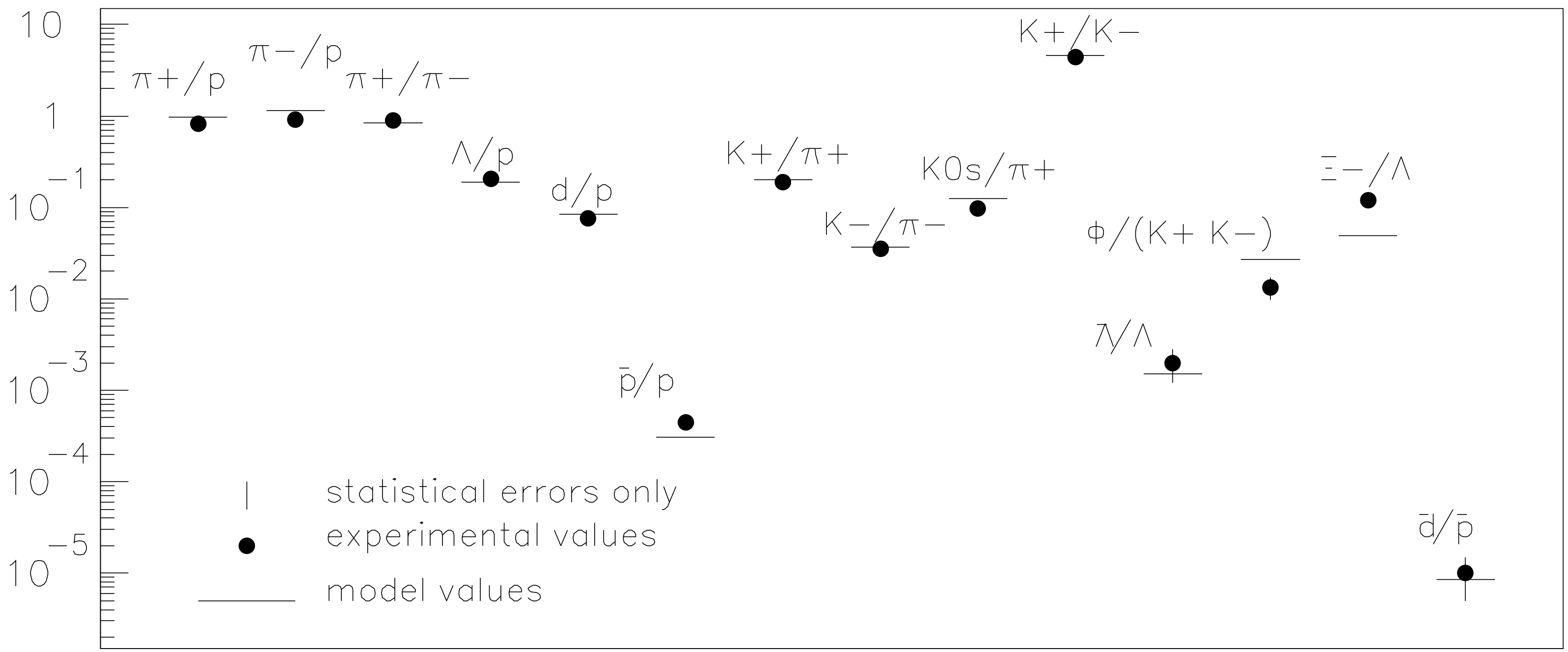


reduced $\chi^2 = 1.1$

Hadron Yields at AGS and Thermal Model

P. Braun-Munzinger, I. Heppe, J. Stachel, Phys. Lett. **B465** (1999) 5
and I. Heppe, Diploma thesis, U. Heidelberg 1998

central 14.6 A GeV/c Si + Au collisions
thermal model parameters: T = 125 MeV, μ_b = 540 MeV



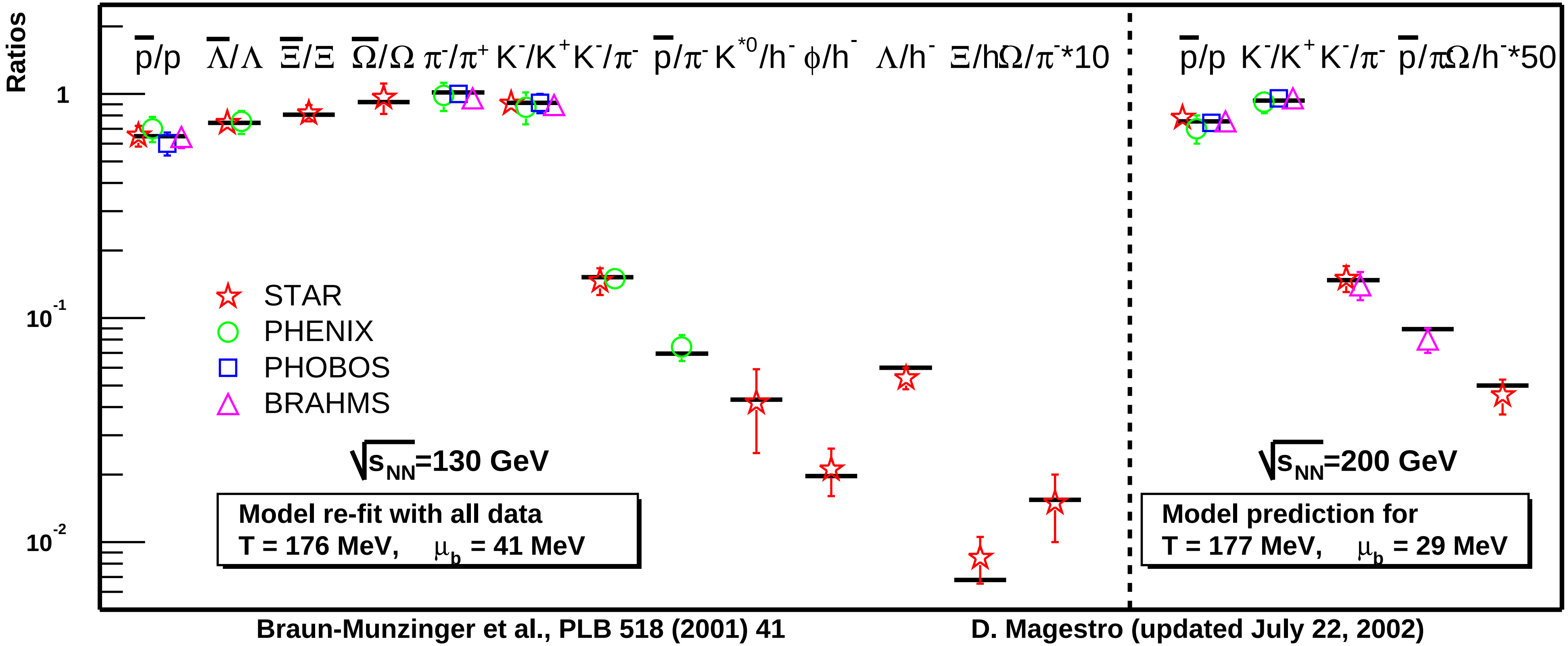
yields for 11.5 A GeV/c Au + Au are very similar

RHIC Data and Thermal Model

P.Braun-Munzinger, D. Magestro, K. Redlich, J.Stachel, Phys. Lett. B518 (2001) 41

central Au + Au collisions, data from all experiments combined

$\chi_r^2 = 0.8$ $\chi_r^2 = 1.1$

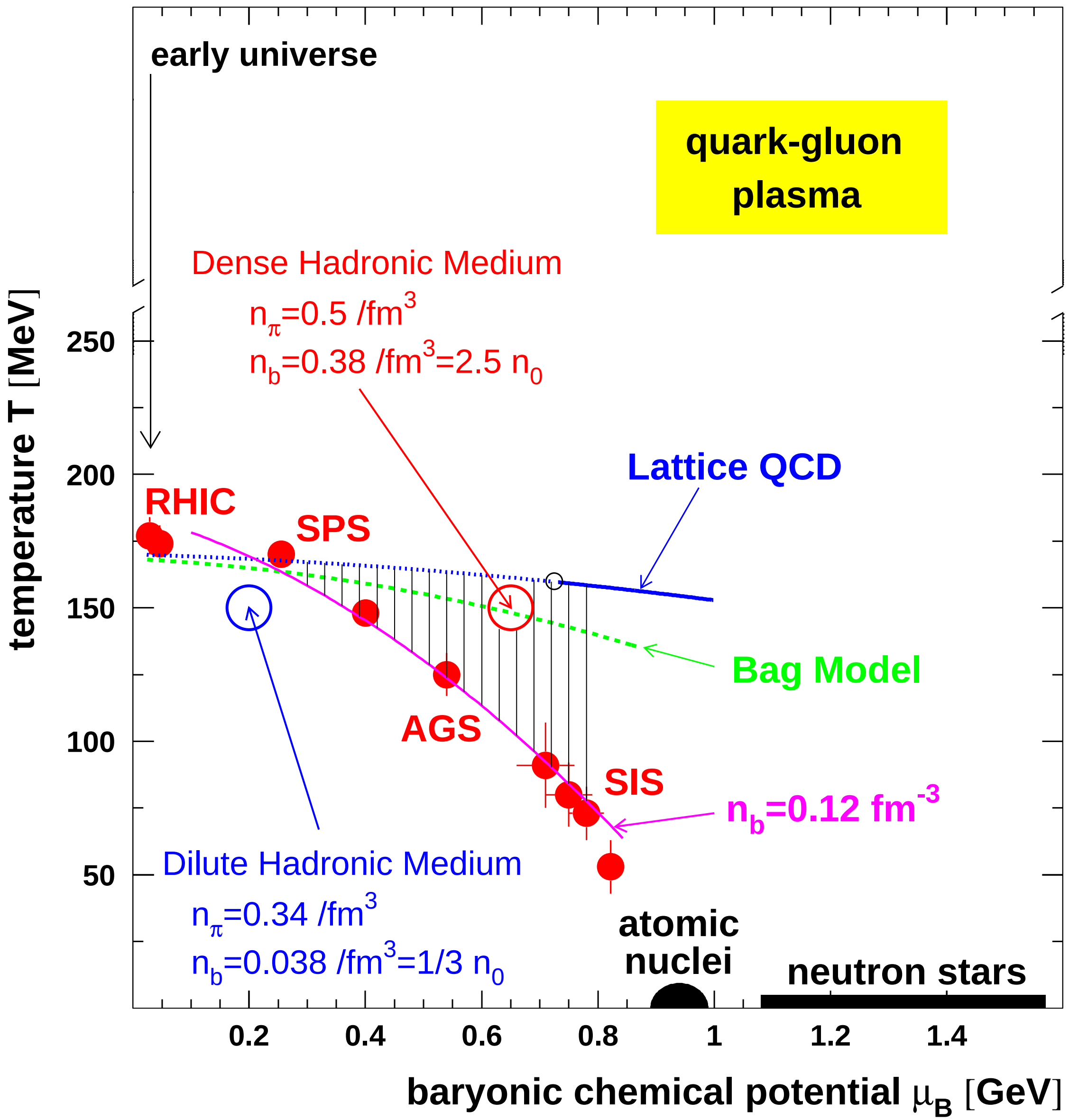


fit result confirmed by Becattini and Kaneta/Xu

interesting questions about resonances

Phase Diagram of Nuclear Matter

- hadron yields equilibrated
- for full SPS energy and above: hadron yields frozen at phase boundary



how is equilibrium achieved?

Mass Changes Close to T_c?

repeat fit of RHIC data with several hypotheses:

- change all masses by constant factor $\longrightarrow$ similar fit quality if variation $\leq$ 20 %
  (see also Michalec, Florkowski, Broniowski, nucl-th/0103029)
- reduce m_ϕ by 5 % $\longrightarrow$ 3 σ discrepancy with data
- reduce $m_{K^{0*}}$ by 10 % $\longrightarrow$ 2.5 σ discrepancy with data

no room for very significant changes

Longitudinal Expansion

from pion interferometry:

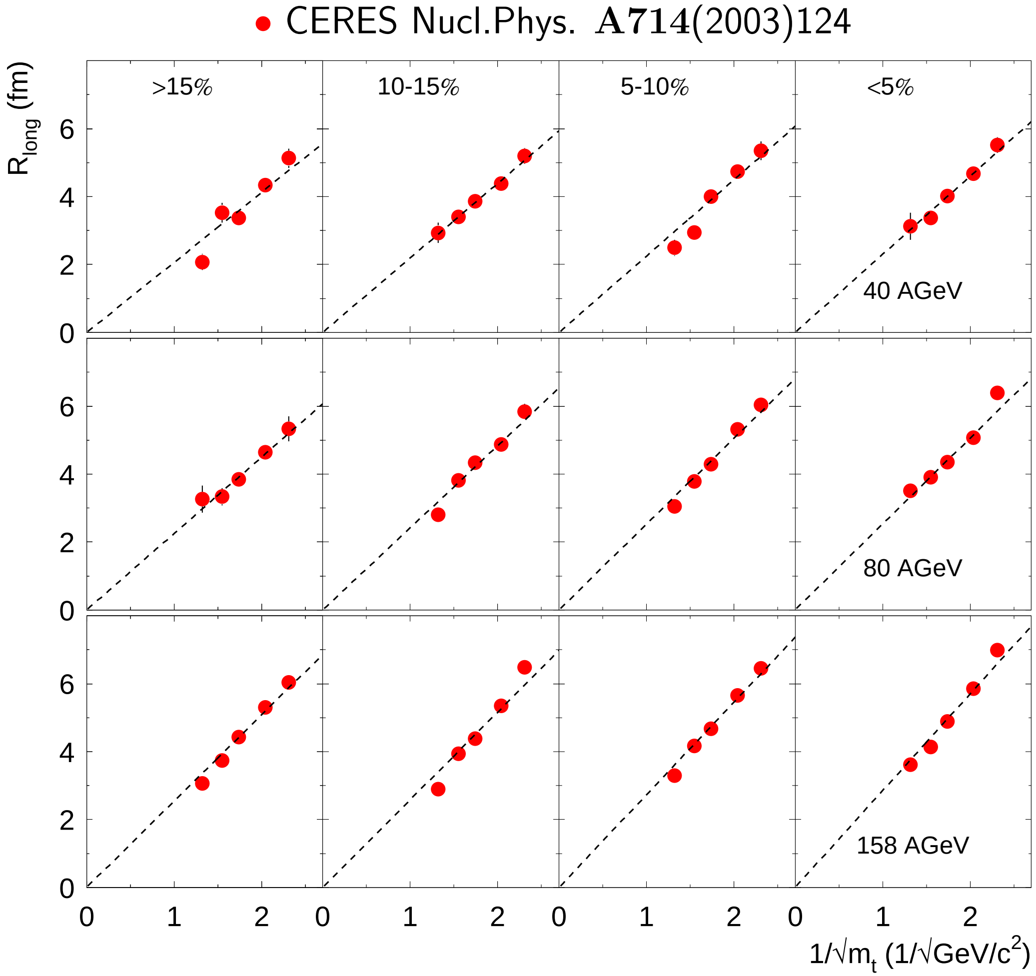
Duration of expansion (lifetime) τ of the system can be estimated from the transverse momentum dependence of R_{long}:

$$R_{\text{long}} \approx \tau \cdot \sqrt{\frac{T_f}{m_t}} \quad \textbf{Y. Sinyukov}$$

$\Rightarrow$

$$\tau = 6.5\text{-}8 \text{ fm/c} \quad \textbf{for } T_f = 120 \text{ MeV}$$

(13 % less for T_f = 160 MeV)



Transverse Expansion

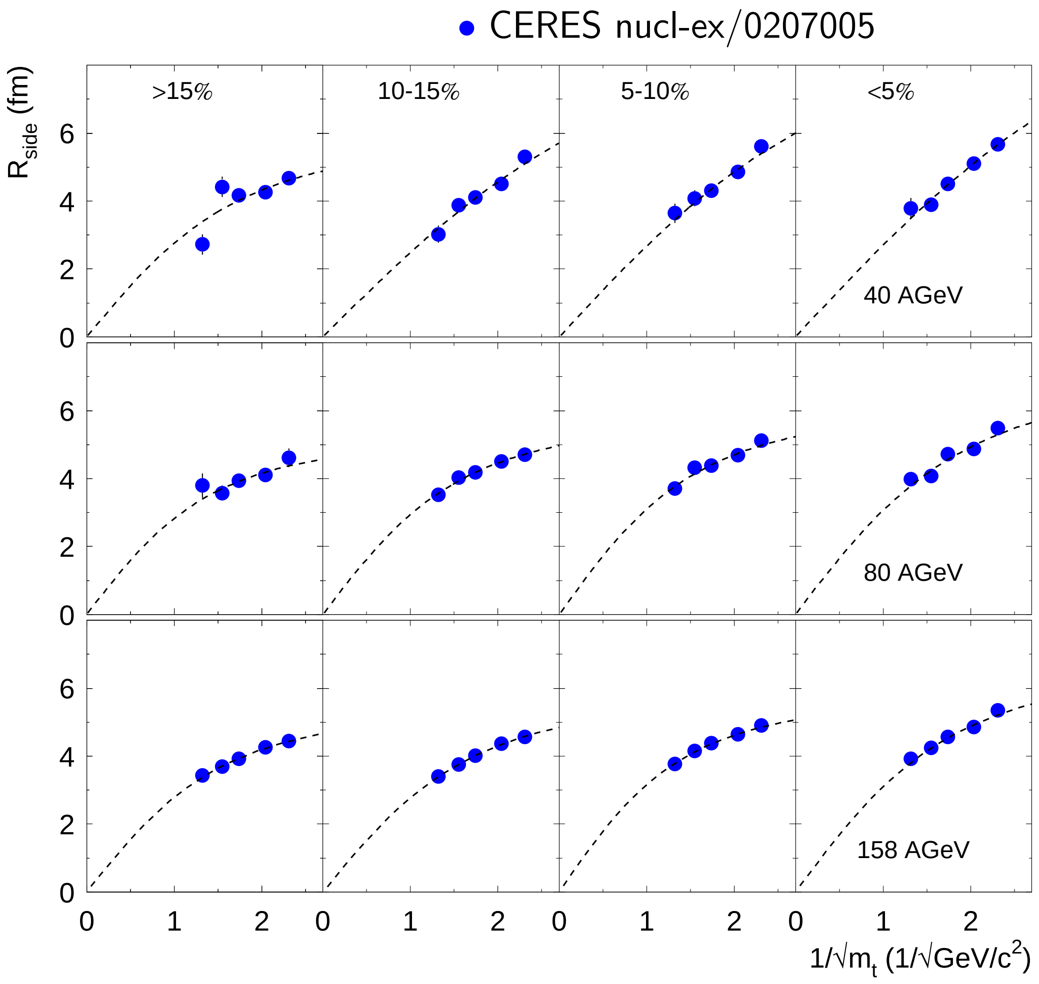
Transverse momentum dependence of R_{side} allows determination of geometric source size R_{geo} and
average transverse flow velocity β_t

$$R_{\text{side}} \approx R_{\text{geo}}/(1 + m_t \cdot F(T_f, \beta_t))^{\frac{1}{2}}$$

U. Heinz *et al.*

$\Rightarrow$

$\beta_t \approx 0.55$ **for T$_b$=120 MeV**



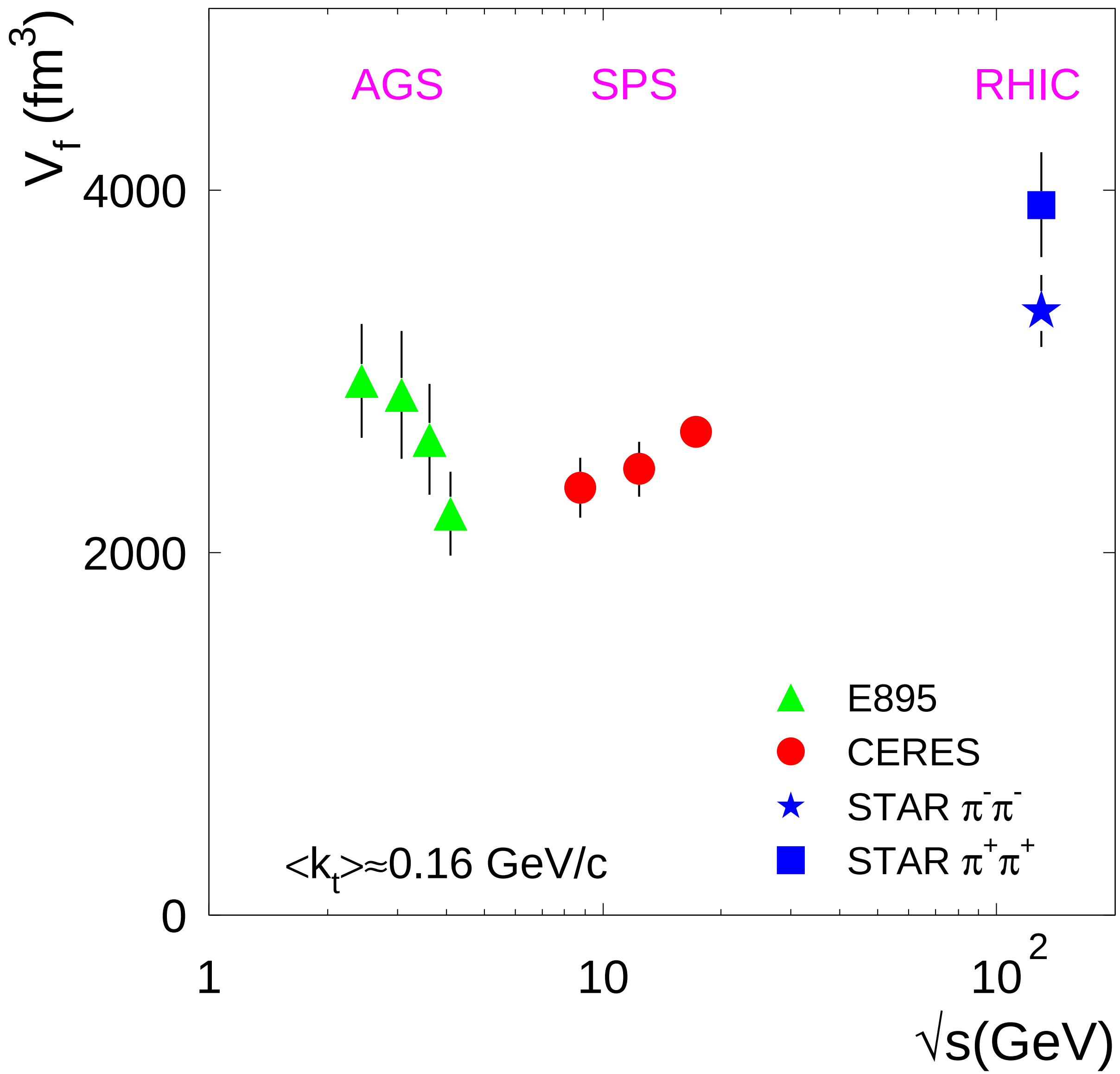
Freeze-out Volume

estimate for volume:

$$\mathbf{V} = (\mathbf{2}\pi)^{3/2}\ R_{\text{side}}^2\ R_{\text{long}}$$

note: this is volume of 0.88 units y

⇒ grows non-monotonically with $\sqrt{s}$

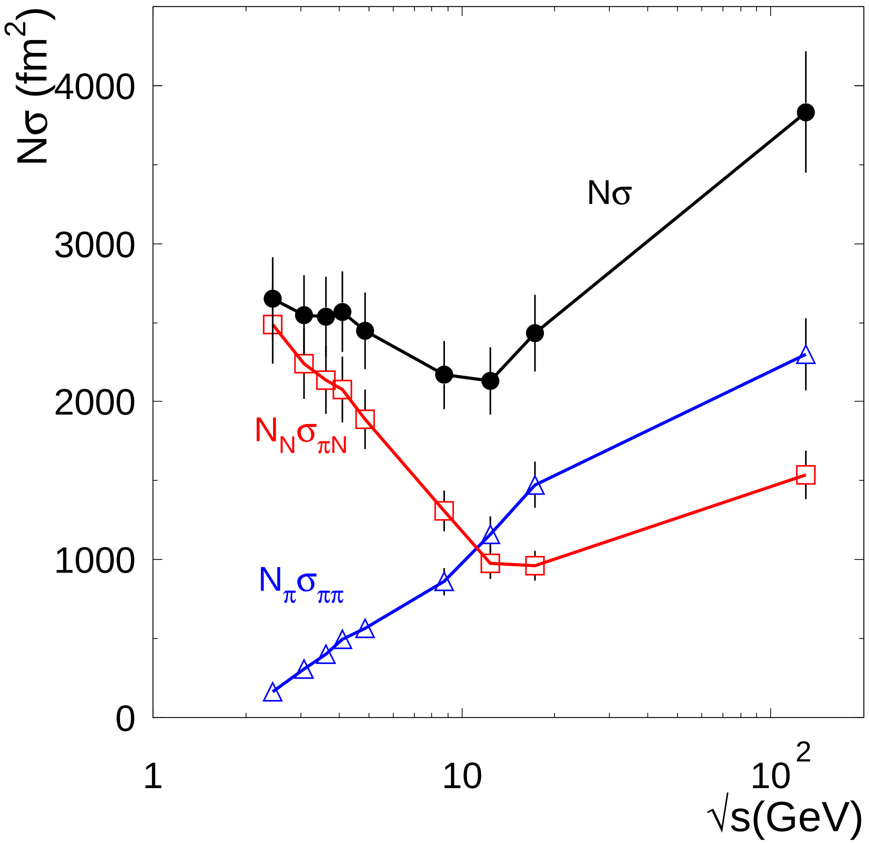


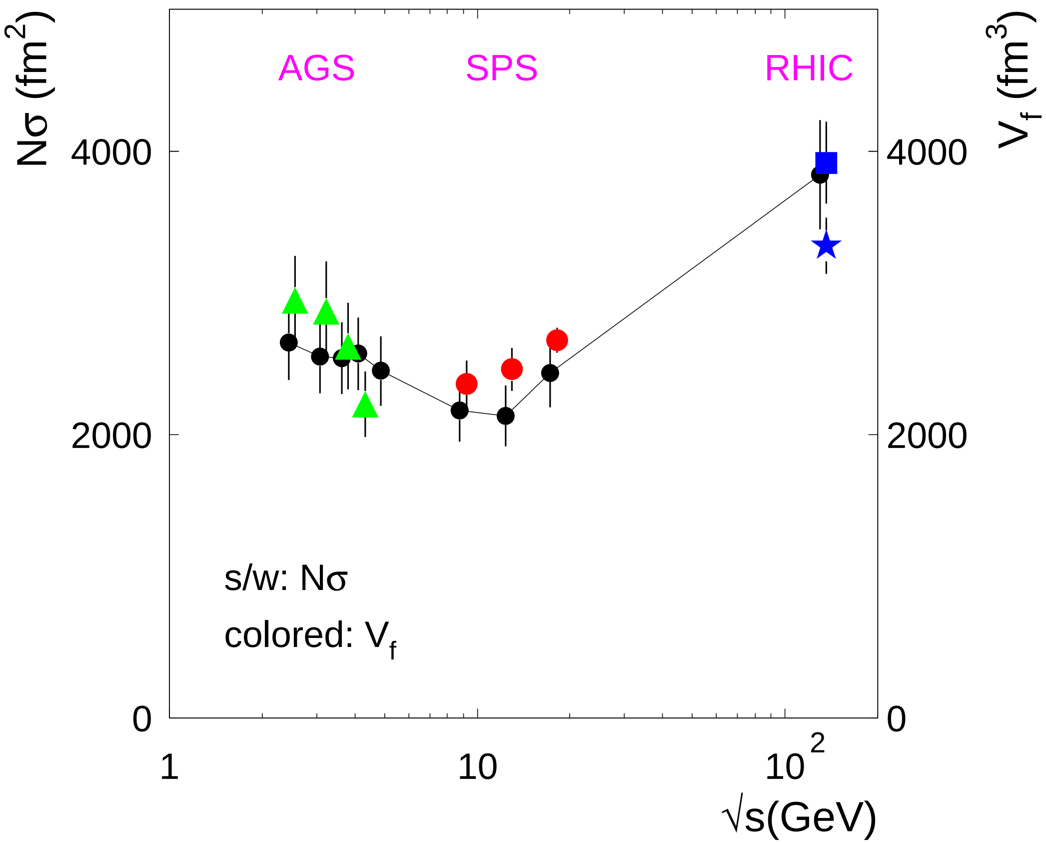
What Governs Thermal Freeze-Out?

H.Appelshäuser, CERES, PRL**90** (2003) 023001

Pion Mean Free Path: $\lambda_f = 1/(\rho_f \cdot \sigma) = V_f/(N \cdot \sigma)$

$N \cdot \sigma \approx N_N \cdot \sigma_{\pi N} + N_\pi \cdot \sigma_{\pi\pi}$

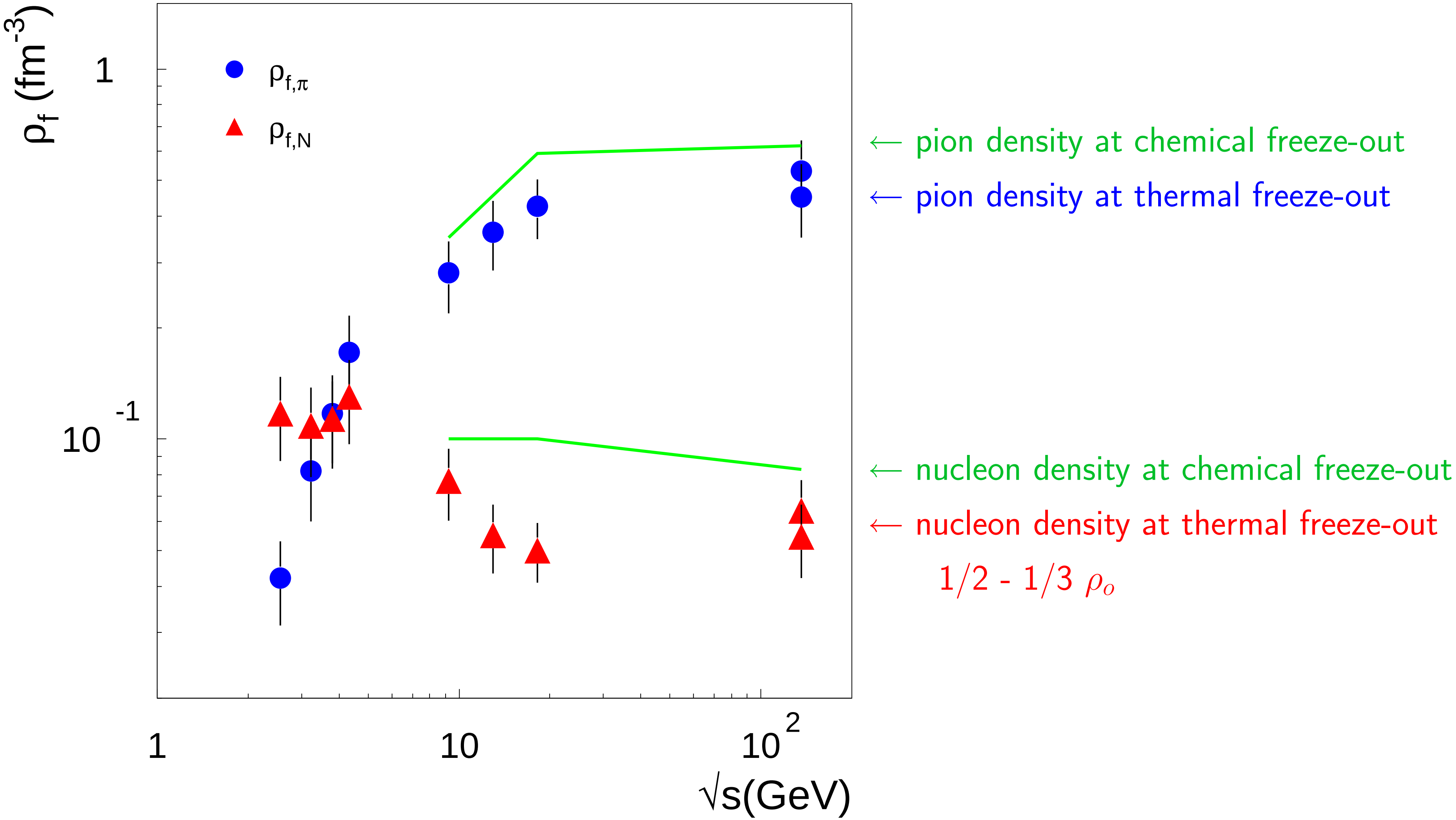




Universal freeze-out at mean free path of 1 fm - small vs system size!

Freeze-Out Density from Pion HBT

HBT gives density at thermal freeze-out



Volume appears to only grow 30 % between chemical and thermal freeze-out!

Duration of Pion Emission

Duration of pion emission τ_h from fireball can be estimated from difference in R_{out} and R_{side}:

$$R^2_{\text{out}} - R^2_{\text{side}} \approx \tau^2_h$$

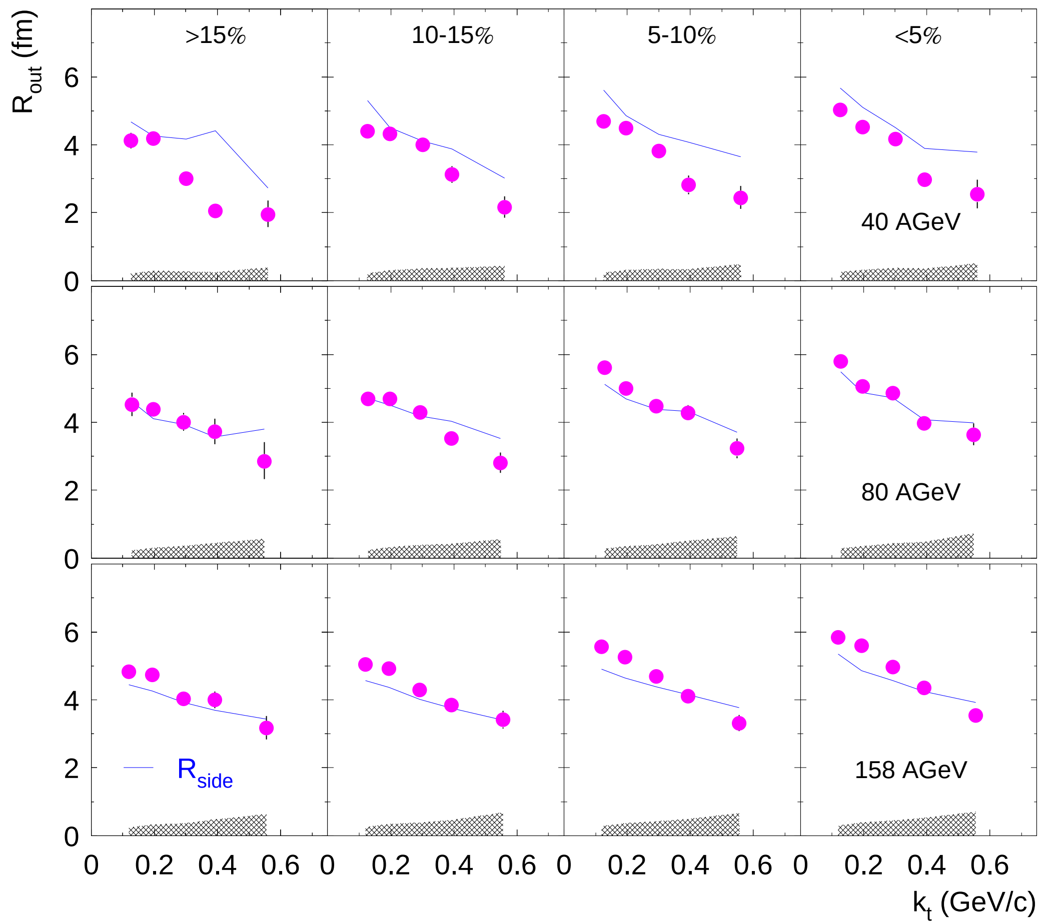
$\Rightarrow$

$\tau_f \leq$ **2 fm/c**

similar results at RHIC

sudden freeze-out!

CERES H.Appelshäuser, Nucl.Phys. **A714** (2003) 124



Hadronic Phase after Chemical Freeze-out?

- Survival of objects w. large cross section:
  - light nuclei d, ^{3}He, ^{4}He, ...
  - resonances Δ, Λ^*, K*, ρ, ...
  - $\bar{\text{p}}$/p and $\bar{\text{d}}$/d ratios
- duration of pion emission from HBT:
  - $R_o^2 - R_s^2 = \tau_h^2$ for SPS and RHIC $\tau_h \leq 2$ fm/c
- Densities at thermal freeze-out from HBT as compared to chemical freeze-out

$\Rightarrow$ Not much room for extended lifetime

4. Model for Rapid Equilibration $T_{ch} \approx T_c$

Arguments follow

P.Braun-Munzinger, J.S., C. Wetterich,
nucl-th/0311005, Phys. Lett. B, in print

Chemical Equilibration must take place in Hadronic Phase

- Hadron yields determined by Boltzmann factors using free hadronic masses
- Why would QGP have memory of free hadronic masses?
- yields scale not with strange quark but with strange hadron masses
- But large strangeness enhancement must come from QGP and/or hadronization

Scenario for Hadronic Expansion between T_{ch} and T_f

Values chosen appropriate for RHIC Au + Au collisions

- Assume: T_{ch}=176 MeV
  density decrease between chemical and thermal freeze-out: 30 %
- Two-pion correlation data: $R_{\rm side}$ =5.75 fm, $R_{\rm long}$=7.0 fm, mean β_t =0.5, $\beta_{\rm long}$=1
- Isentropic expansion $\rightarrow$ τ_f = 0.9 - 2.3 fm, T_f = 158 - 132 MeV
  (uncertainty due to variation in density profile)
- Near T_c: rate of decrease in temperature $|\dot{\rm T}/{\rm T}| = \tau_{\rm T}^{-1} = (13 \pm 1)$ % /fm

Can 2-Body Collisions maintain or even achieve Equilibrium?

typical densities at T_{ch}: ρ_π =0.174/fm^3 (incl. res.) ρ_{K} =0.030/fm^3 ρ_Ω = 0.0003/fm^3

- To maintain equilibrium even for 5 MeV below T_{ch} need relative rate change

$$\left|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}\right| = \tau_\Omega^{-1} - \tau_{\mathrm{K}}^{-1} = (1.10 - 0.55)/\mathrm{fm} = 0.55/\mathrm{fm}.$$

So, Ω density needs to change by 100 % within 1 fm/c

- Typical reactions with large cross sections of 10 mb and relative velocity of 0.6 give

$\Omega + \pi \to \Xi$+K $\longrightarrow$ $\bar{r}_\Omega / n_\Omega = n_{\bar{\pi}} \langle v_r \sigma \rangle$=0.086/fm

$\pi + \pi \to K + \bar{K}$ (σ=3mb) $\longrightarrow$ $\bar{r}_K / n_K = 0.18/$fm

i.e. much too slow to maintain equilibrium even over Δ T = 5 MeV!

- Even much more difficult: to produce large Ω abundancy
assume hadronization like in pp, factor 8 too few Ωs, to produce them within 1 fm/c
need reactions that provide $\bar{r}_\Omega / n_\Omega$=1.0 $\Rightarrow$ not with 2-body reactions

- Consensus in the literature: Koch, Müller, Rafelski, Phys. Rep. **142**(1986), C. Greiner, S. Leupold, J.Phys.**G27**(2001)L95; P. Huovinen, J. Kapusta, nucl-th/0310051

Multi-particle Reactions

consider situation at T_{ch}=176 MeV first

- rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with

$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

- The phase space factor ϕ depends on $\sqrt{s}$
  needs to be weighted by the probability $f(s)$ that multiparticle scattering occurs at a given value of $\sqrt{s}$
  evaluate numerically in Monte-Carlo using thermal momentum distribution
- typical reaction: $\Omega + \bar{N} \rightarrow 2\pi + 3K$
  assume cross section equal to measured value for $p + \bar{p} \rightarrow 5\pi$
  relevant $\sqrt{\mathrm{s}} = 3.25$ GeV $\rightarrow \sigma$=6.4 mb
- compute matrix element and use for rate of $2\pi + 3K \rightarrow \Omega + \bar{N}$

Multi-particle Reactions continued

reaction $2\pi + 3K \rightarrow \Omega + \bar{N}$ leads to

$$\mathrm{r}_\Omega = 0.00014 \text{ fm}^{-4} \text{ or } \mathrm{r}_\Omega/\mathrm{n}_\Omega = 1/\tau_\Omega = 0.46/\text{fm}$$

$\Rightarrow$ can achieve final density starting from 0 in 2.2 fm/c!

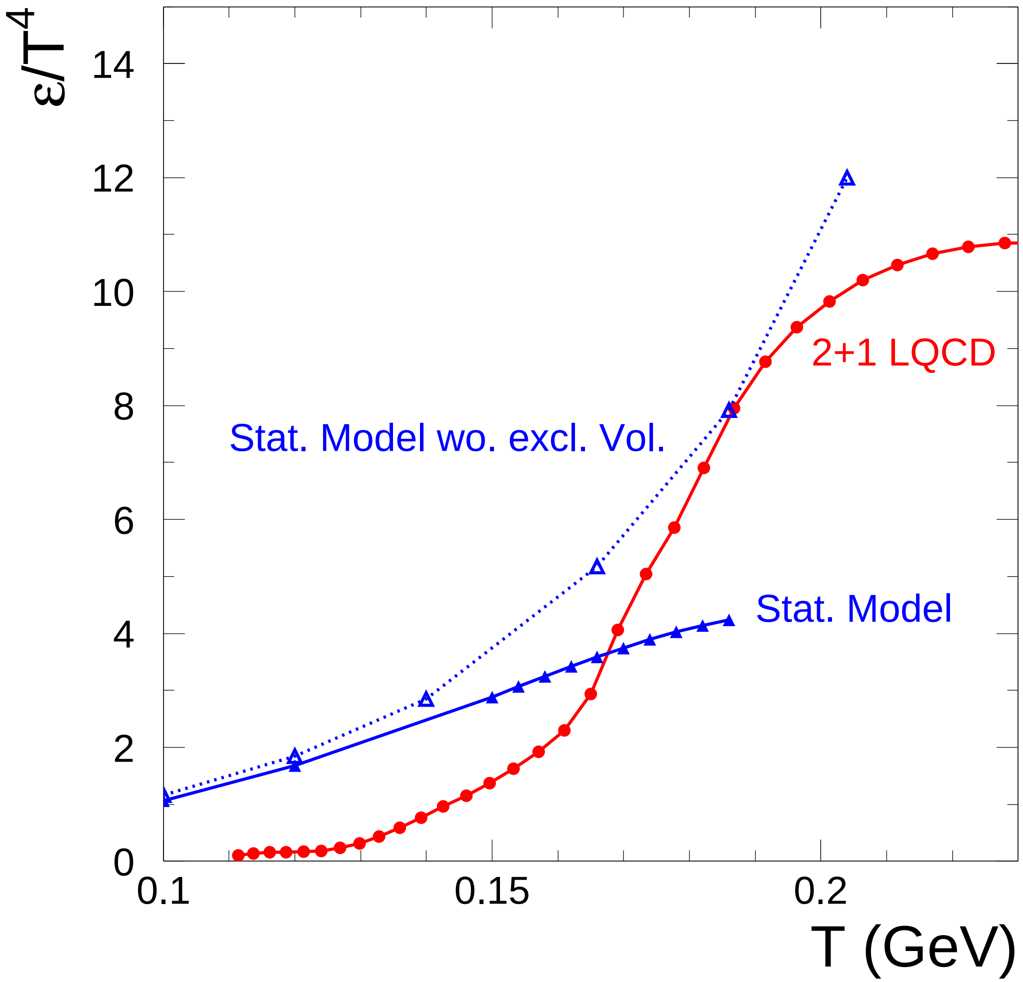
similarly one obtains

for $\quad 3\pi + 2K \rightarrow \Xi + \bar{N} \qquad \tau_\Xi = 0.71$ fm/c

and

for $\quad 4\pi + K \rightarrow \Lambda + \bar{N} \qquad \tau_\Lambda = 0.66$ fm/c

Rapid Density Growth near T_c

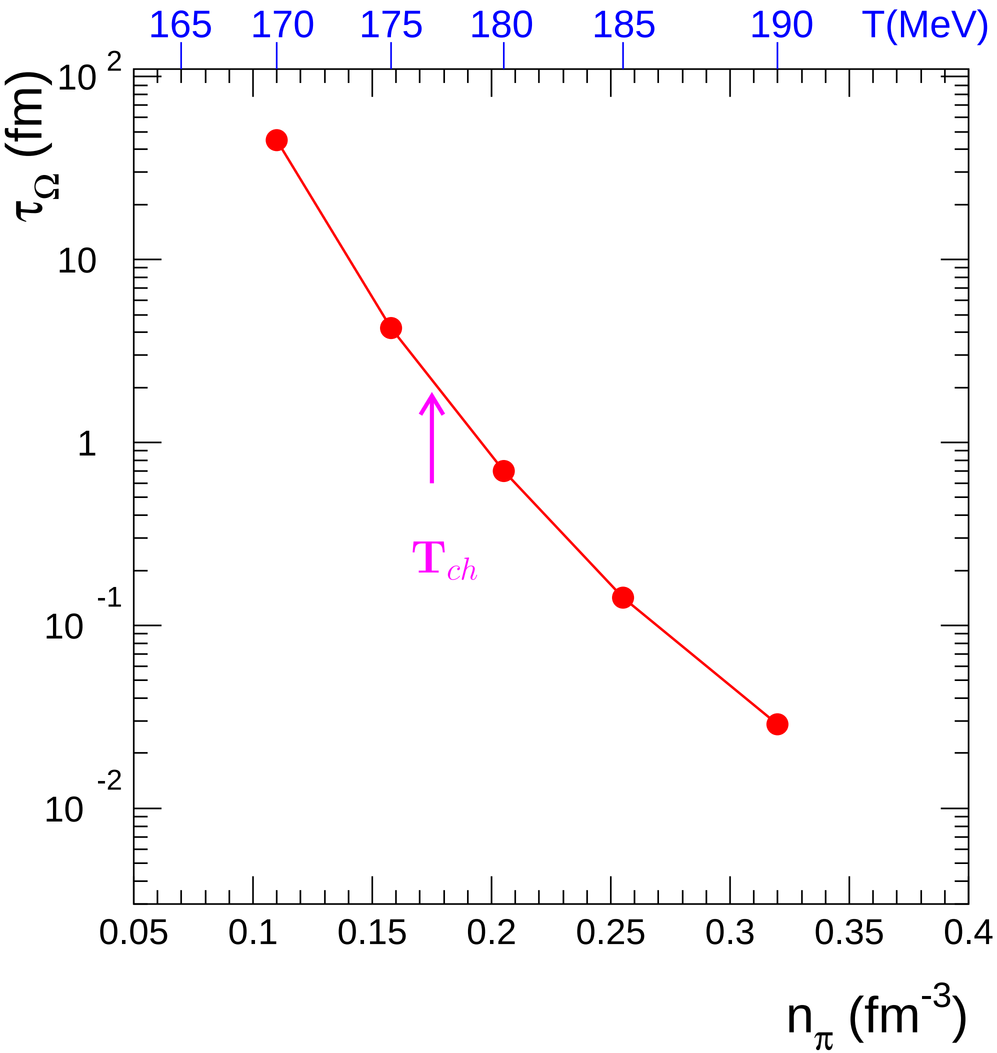


Karsch et al.

at T_c very large increase in energy density and particle density due to increase in degrees of freedom in QGP

Super-rapid Equilibration near T_c

both densities and phasespace very sensitive to T



increase ρ_π by 1/3: $\tau_\Omega = 0.2$ fm/c

decrease ρ_π by 1/3: $\tau_\Omega = 27$ fm/c

$\tau_\Omega \propto \mathbf{T}^{-60}$**!**

within very narrow interval of T all hadron yields can thermalize

New Scenario of Equilibration

- 2-body collisions too slow to bring multistrange hadrons into equilibrium
- near T_c new dynamics associated with collective excitations takes place typical for the vicinity of a phase transition
- propagation and scattering of these excitations is expressed in the form of multi-hadron scattering
- near T_c these multi-particle scatterings dominate and lead to rapid equilibration

Natural association between T_{ch} and T_c

Test of Detailed Balance

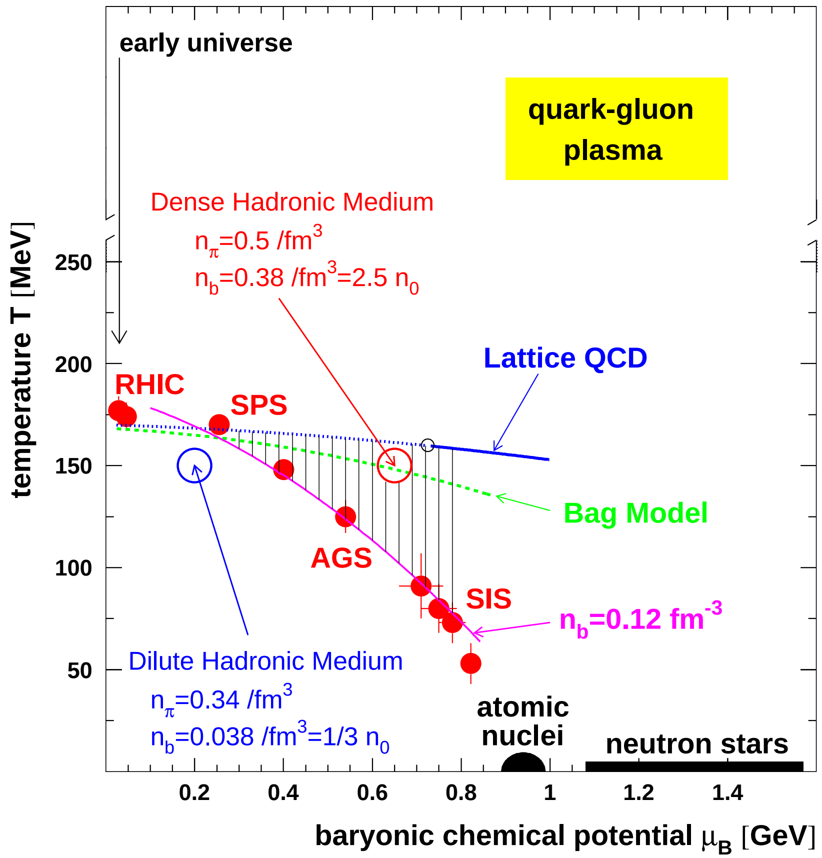
- Initially manifestly nonequilibrium situation - start with practially zero Ω density
- As equilibrium is approached
  rates $3\mathrm{K} + 2\pi \rightarrow \Omega + \bar{N}$ and $\Omega + \bar{N} \rightarrow 3\mathrm{K} + 2\pi$ have to become equal
- back and forth reactions scale very differently with pion density
  $\rightarrow$ only at one density can they be equal
- to explicitly check these rates now use pion, kaon, nucleon densities before strong decays, i.e. without resonance feeding
  (for all resonances corresponding rates have to be calculated accordingly)
- find: creation of Ω with $\mathrm{r}_\Omega/\mathrm{n}_\Omega = 3.4\ 10^{-3}/\mathrm{fm}$
  and annihilation of Ω with $\mathrm{r}_\Omega/\mathrm{n}_\Omega = 1.4\ 10^{-3}/\mathrm{fm}$

for equal rates reduce density by 25 %
reduce T by 2-3 MeV or excluded volume a bit larger

How Low in Beam Energy does this Work?

- at top SPS energy numbers work out nearly the same as at RHIC
- at 40 A GeV/c densities lower by 1/3 $\longrightarrow \tau_\Omega$ increases by factor 12

  other reactions involving baryons?



5. Summary

- **Knowlegde about the QCD Phase Boundary and T_c**
  vastly improved due to progress in LQCD; non-quenched calculations and absolute value of T_c; Question of order of phase transition and of critical point
- **Hadrochemical Equilibration and T_{ch}:**
  for top SPS energy and above apparently at or very close to T_c
- **Space-Time Dynamics and T_f:**
  give scenario with relatively shortlived hadronic phase, freeze-out governed by common mean free path
- **Model for Rapid Equilibration $T_{ch} \approx T_c$:**
  due to collective modes or multi-particle reactions in the vicinity of phase transition for top SPS energy and above