An overview of Cosmology

CERN Student Summer School

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What is Cosmology?

- Astrophysics ⇨ detailed description of « small » structures
- Cosmology ⇨ Universe as a whole
  - Is it static? Expanding ?
  - Is it flat, open or closed ?
  - What is it composed of ?
  - What about its past and future ?

- **Part I :** the expanding Universe
  - Hubble law
  - Newtonian gravity
  - General relativity
  - Friedmann-Lemaître model

Geometry and abstraction …

- **Part II :** the standard cosmological model
  - Hot Big bang scenario
  - Cosmological perturbations
  - Cosmological parameters
  - Inflation & Quintessence

Concrete predictions, results, observations !!

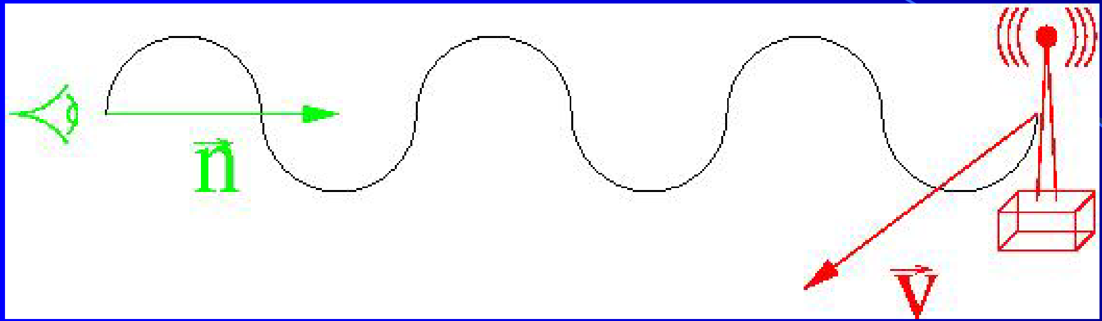
Part I :
The Expanding Universe

Part I : (1) - The Hubble Law

- First step in understanding the Universe…
  - first telescopes: observation of *nebulae*
  - 1750 : T. Wright : Milky way = *thin plate of stars*?
  - 1752 : E. Kant : nebulae = *other galaxies*?

- Galactic structure not tested before… 1923!

- 1842 : Doppler effect for sound and light



redshift :

$z = \Delta\lambda / \lambda = \mathbf{v} \cdot \mathbf{n} / c$

- 1868 : Huggins finds redshift of *star spectral lines*
- 1868 – 1920 : observation of many redshift of stars and nebulae
  - random distribution
  - late observations : *excess of z>0 for nebulae*

- 1920's : Leavitt & Shapley :
  - cepheids ⇨ period / absolute luminosity relation

$l = dL/ds = L/(4\pi r^2)$

  - measurement of distances of stars inside the Milky Way (~ 80.000 lightyear)

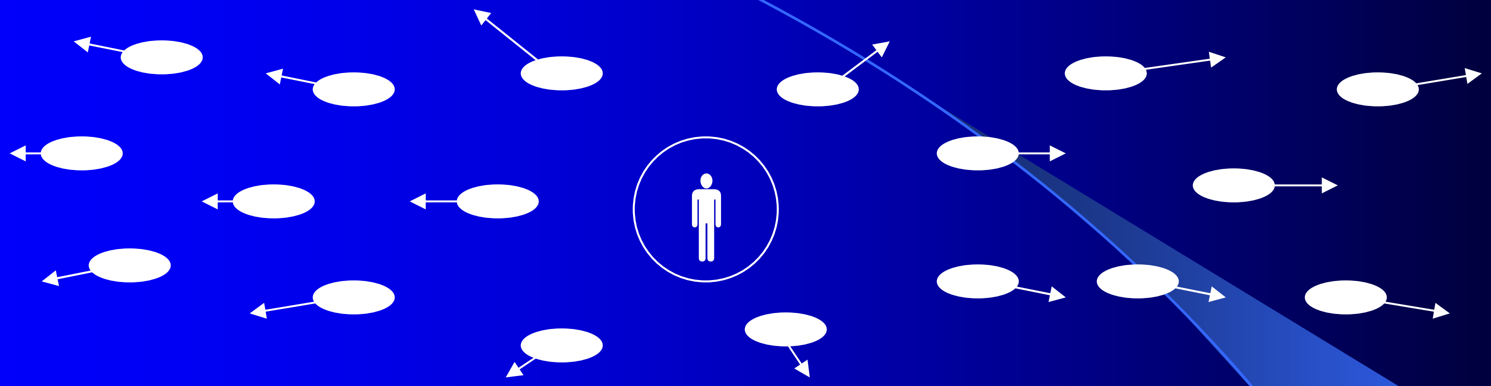
- 1923 : Edwin Hubble :
  - 2,50 m telescope at *Mount Wilson (CA)*
  - cepheids in *Andromeda*
  - distance of nearest galaxy = 900.000 lyr (in fact 2 Mlyr)

    ⇨ *first probe of galactic structure* !!!
  - so : excess of redshifted galaxies

⇩

Universe expansion ???

- IN GENERAL : expansion ⇨ center



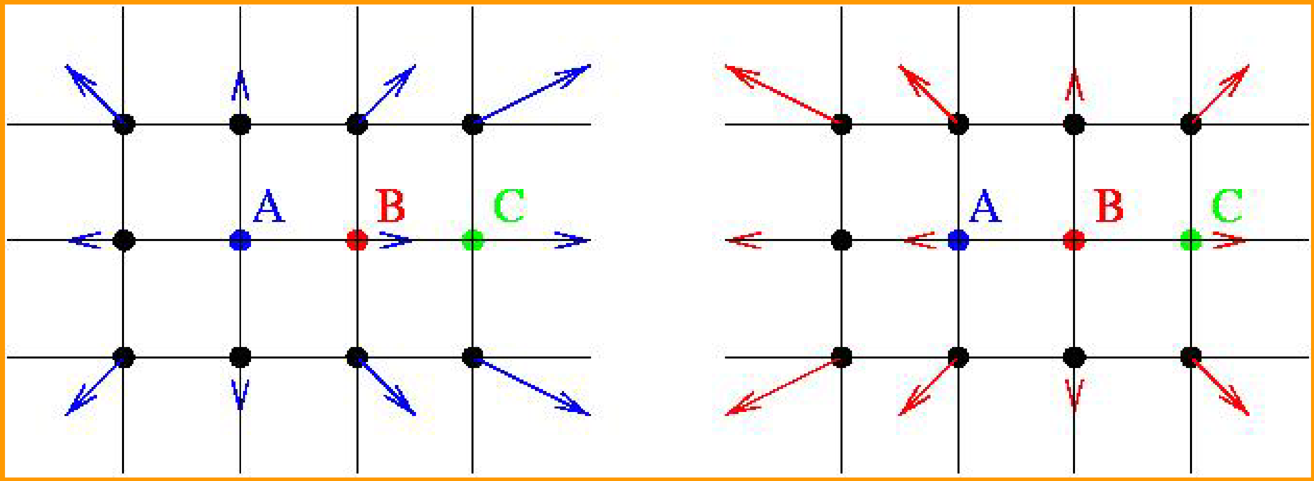
- Against « cosmological principle » (Milne):
  - Universe homogeneous …
  - no privileged point!

QUESTION : is any expansion a proof against homogeneity?

ANSWER : not if $\mathbf{v} = H\,\mathbf{r}$ ⇦ *linear expansion*

… like infinite rubber grid stretched in all directions …

Proof that *linear expansion* is the only possible homogeneous expansion :



- $\mathbf{v}_{B/A} = \mathbf{v}_{C/B}$ ⇦ homogeneity
- $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A} = 2\,\mathbf{v}_{B/A}$ ⇨ linearity

- 1929 : Hubble gives the first *velocity / distance* diagram :

1000 $km.s^{-1}$ →

0 $km.s^{-1}$ →

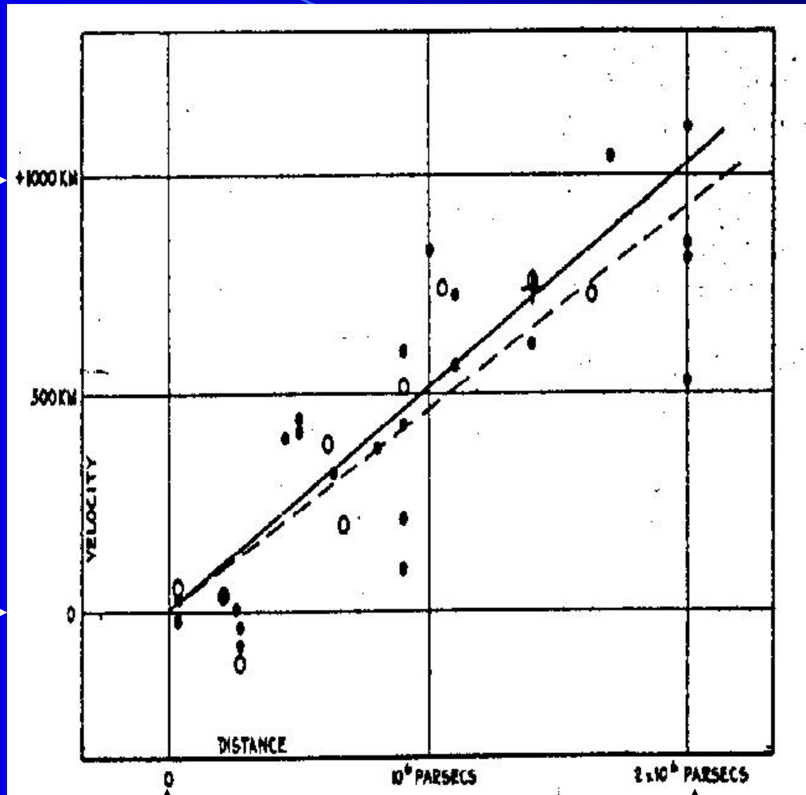


FIGURE 1

0 Mpc 2 Mpc

$H = v / r = 500\ km.s^{-1}.Mpc^{-1}$ for Hubble ($\cong$ 70 $km.s^{-1}.Mpc^{-1}$ for us)

1 Mpc = 3.10^{6} lyr = 3.10^{22} m

⇨ THE UNIVERSE IS IN HOMOGENEOUS EXPANSION

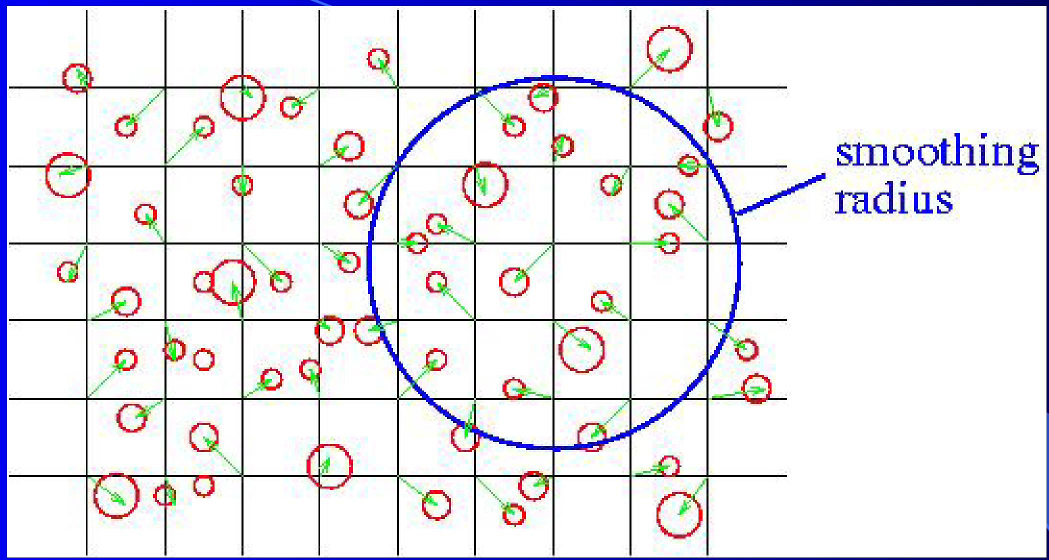
- 1929 : starting of modern cosmology …

Remark : what do we mean by

« the Universe is homogeneous »

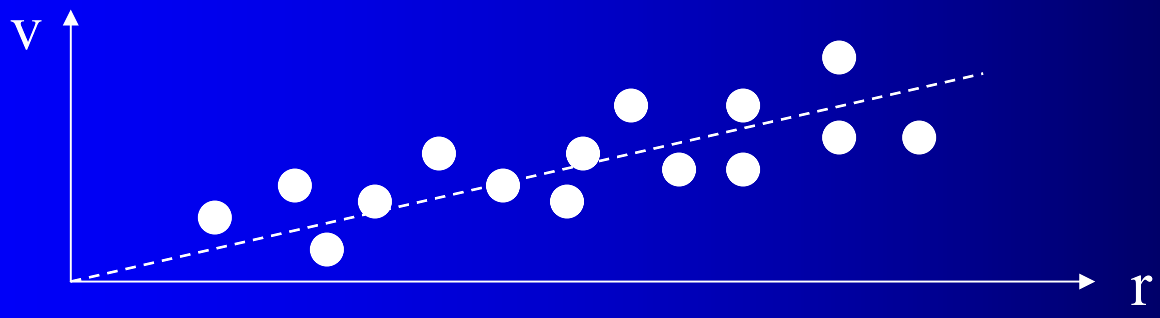
(*cosmological principle*) ?

- example of structure homogeneous *after smoothing:*



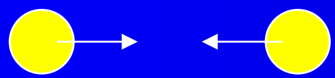
- today : data on *very large scales* ⇨ confirmation of homogeneity beyond ~ 30 – 40 Mpc

- local inhomogeneities ⇨ *scattering*



Part I : (2) – Universe expansion from Newtonian gravity

- on cosmic scales, only *gravitation*
- *Newton's law* = limit of *General Relativity* (GR)



$F = G\, m_1\, m_2 / r^2$

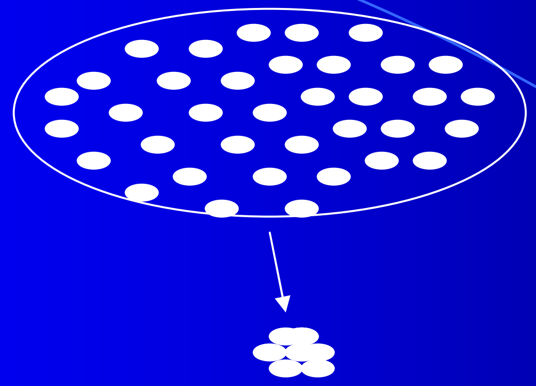
when v << c

OR | speed of object / speed of liberation

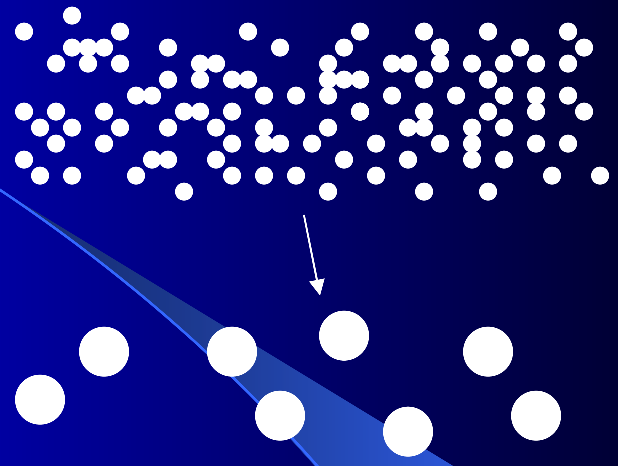
  - *Newton's law* should describe expansion at small distances with v = H r << c …
  - but historically, GR proposed the first predictions / explanations !!!

- **Newton:**

finite Universe

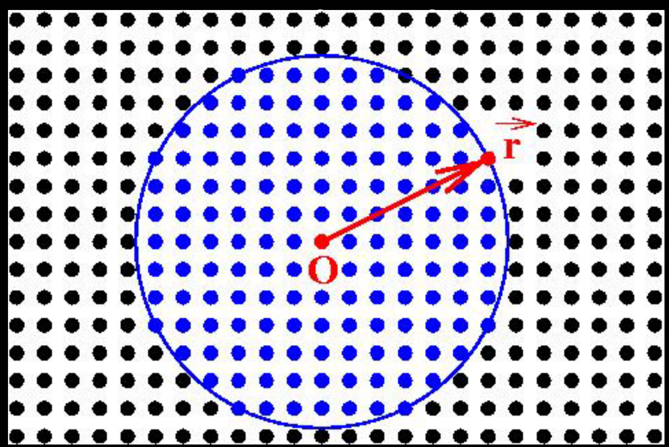


infinite Universe



  - but how to deal with infinity?

- **Gauss theorem :**



$$\ddot{r} = -\,G\,M_r / r^2$$

$$M_r = constant = (4/3)\,\pi\, r^3\, \rho_{mass}$$

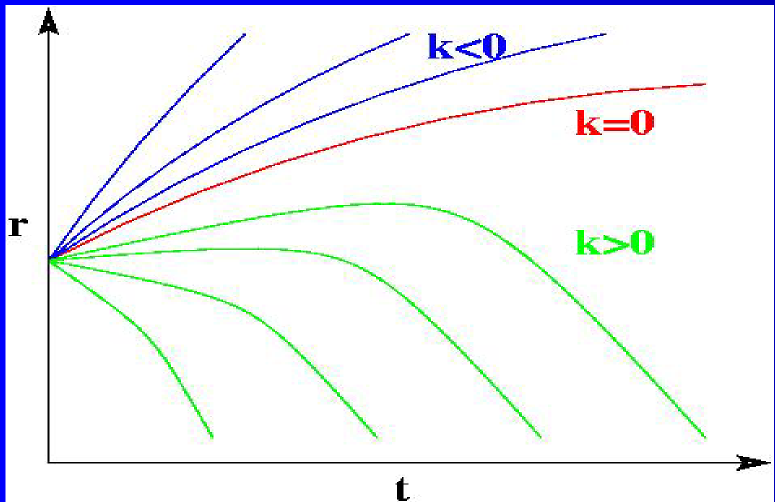
$$\Rightarrow \quad \dot{r}^2 = 2\,G\,M_r / r \; - \; k$$

$$= (8/3)\,\pi\, G\, \rho_{mass}\, r^2 \; - \; k$$

Newtonian expansion law : $(\dot{r}/r)^2 = (8\pi G/3)\,\rho_{mass} - k/r^2$

- $\rho_{mass}(t) \propto r(t)^{-3}$
- same motion as a *two-body problem* :

GRAVITY / INERTIA



$\rho_{mass} <$

$\rho_{mass} =$

$\rho_{mass} >$

$$\frac{3\,(\dot{r}/r)^2}{8\,\pi\,G}$$

- $k \neq 0 \;\Rightarrow\;$ non-homogeneous expansion ???
- $v = H\,r$ and $v << c \;\Rightarrow\; r < R_H \equiv c / H$

Part I : (3) – General Relativity and the Friedmann-Lemaître Universe

Newtonian gravity

$\oplus$

invariant speed of light

} General Relativity (Einstein 1916)

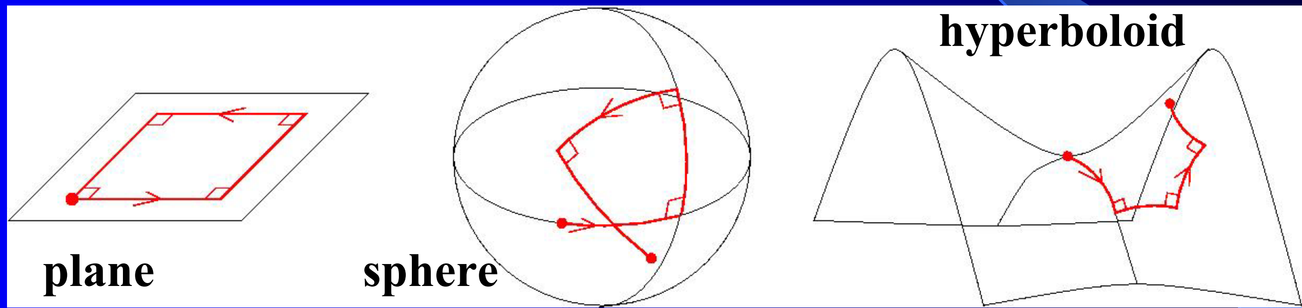
$\Rightarrow$ no more Φ_{grav} (*matter distribution* $\Leftrightarrow \Phi_{grav} \Leftrightarrow \mathbf{E} = \nabla\,\Phi_{grav}$)

$\Rightarrow$ three basic principles :

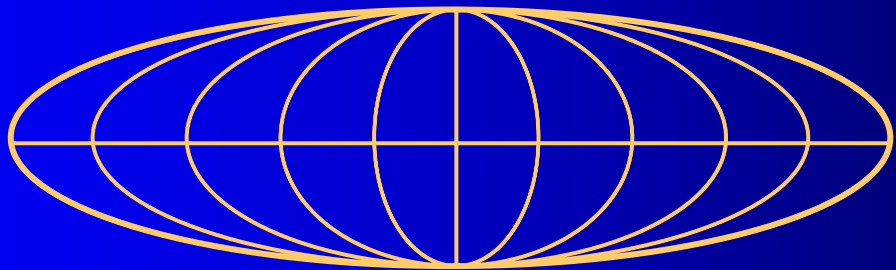
1) space-time (t,x,y,z) is curved
2) curvature $\Leftrightarrow$ matter
3) *free-falling bodies* follow *geodesics*

1) how can we define the *curvature* :

- of a 2-D surface ?
  - embedded in 3D
  - stay in 2D, and use angles :



  - stay in 2D, and use a scaling law : $dl(x_1,x_2)$



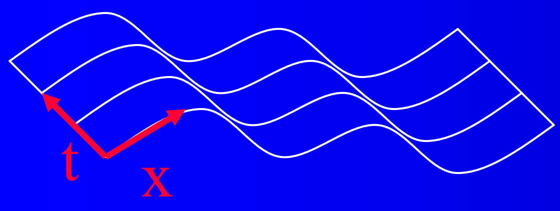
ex: sphere projected on ellipse, $dl(\theta)$

- of 3-D space ?
  - embedded in 4D : $x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2$
  - stay in 3D, but provide a scaling law, like on a planisphere : $dl(x_1, x_2, x_3)$

- of 4-D space-time ?
  - one more dimension
  - time different from space (*special relativity* : - + + +)

intuitive representations:

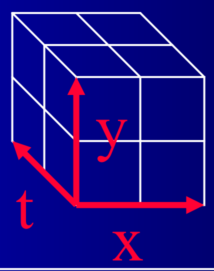
cuts (t,x), (x,y), etc., embedded in 3D



OR

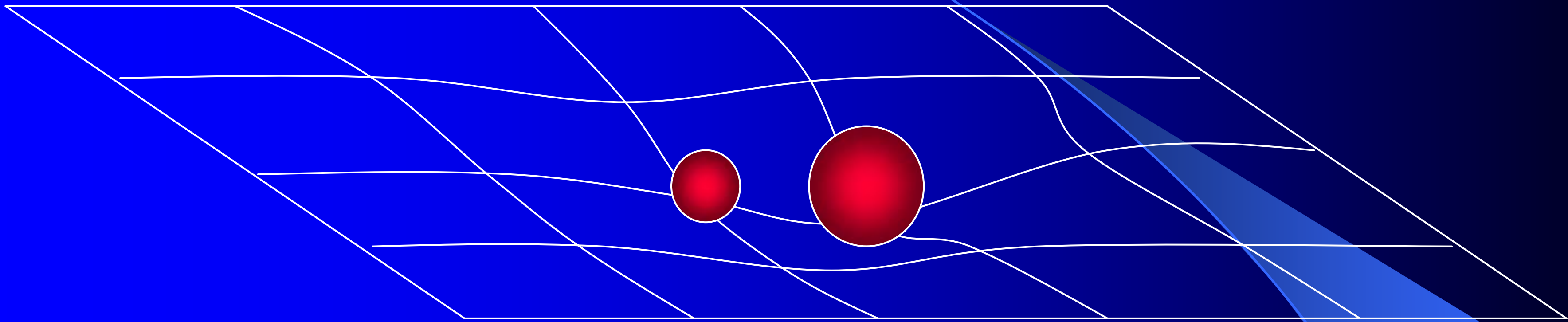
Euclidian coordinates

+ scaling law



+ scale(t,x,y,z)

2) curvature ⇔ matter :



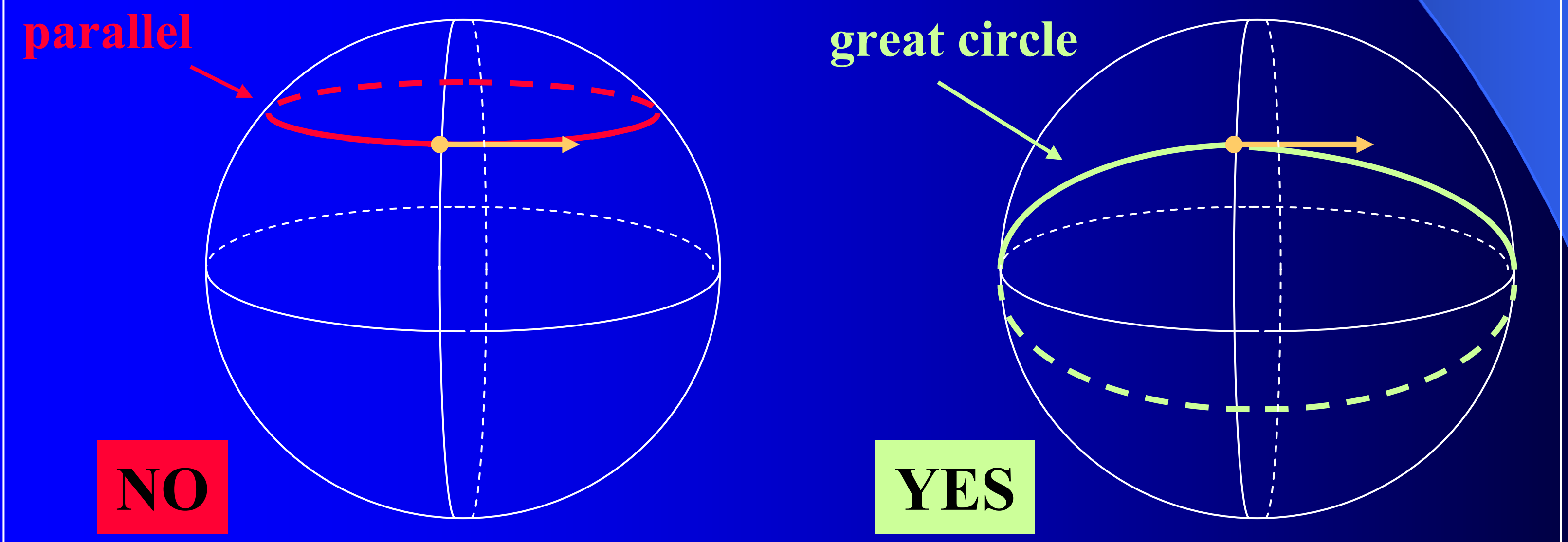
mathematical formulation = *Einstein equation*

3) *free-falling bodies* follow *geodesics*

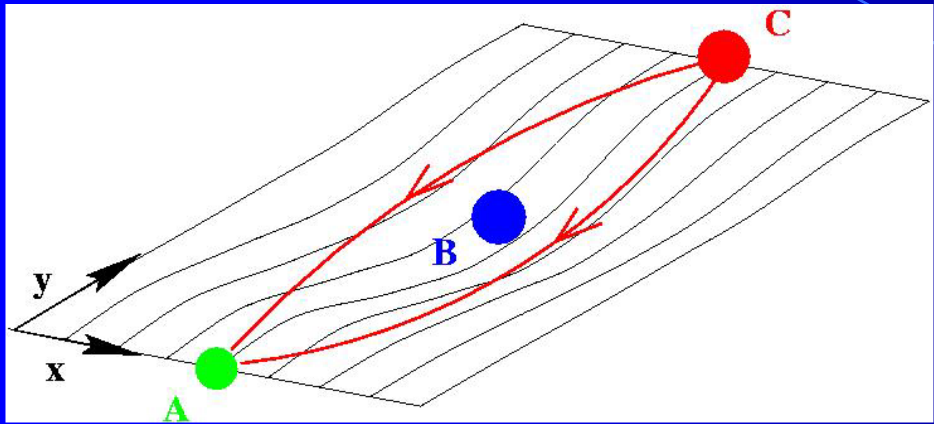
feel **only gravity**
(not E.M., etc.)
e.g. galaxies, light…

given **one point** and **one direction**
$\Rightarrow$ **one single line** such that :
$\forall$ A, B, [AB] = shortest trajectory

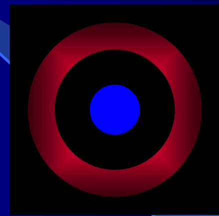
- example 1 : geodesics a sphere :

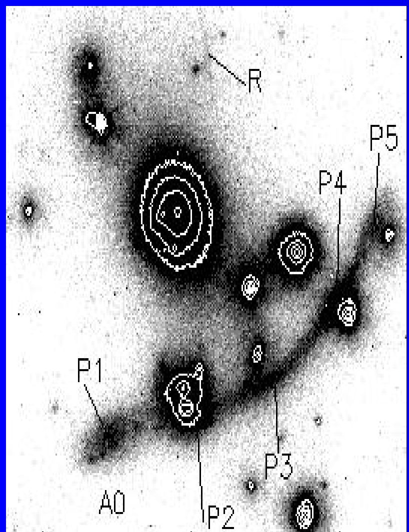


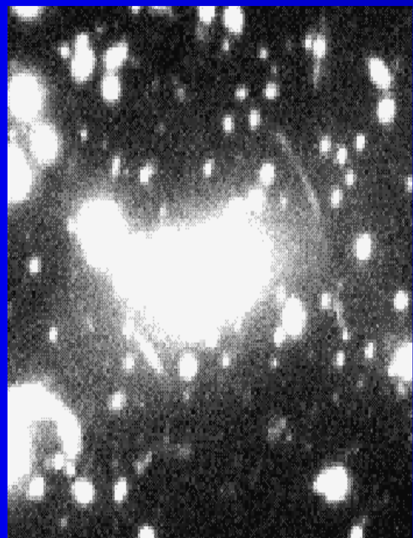
- example 2 : gravitational lensing :

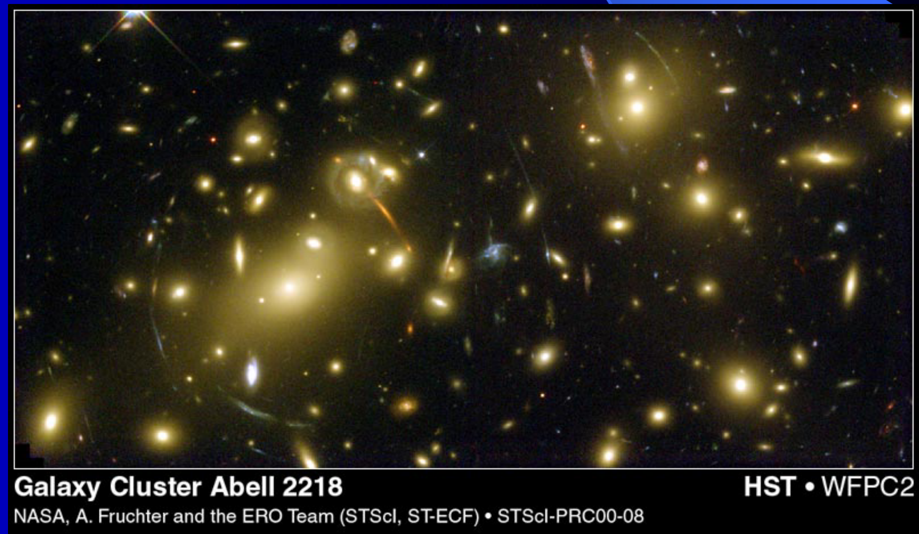


A sees an image of **C** lensed by **B** :



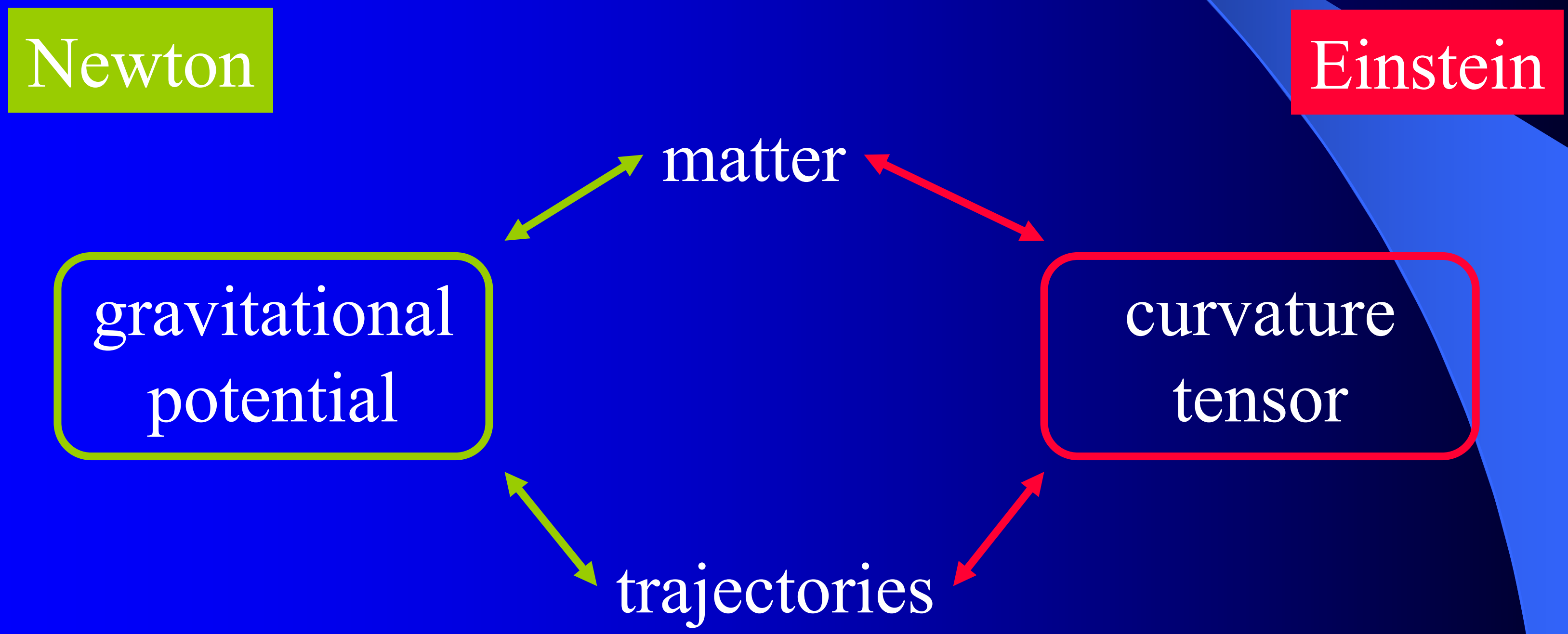






- Newtonian gravity *versus* G.R. :

two different theories of gravity, i.e. two ways of describing how the presence of matter affects the trajectories of surrounding bodies…



- applying G.R. to the Universe: *some history*
  - 1916 : Einstein has formulated G.R.
  - 1917 : Einstein, De Sitter try to build the first cosmological models (PREJUDICE : STATIC / STATIONNARY UNIVERSE)

  - 1922 : A. Friedmann (Ru)
  - 1927 : G. Lemaître (B)
  - 1933 : Robertson, Walker (USA)

  } investigate most general HOMOGENEOUS, ISOTROPIC, NON-STATIONNARY solutions of G.R. equations

  - 1929 : Hubble's law (first confirmation)
  - 1930-65 : accumulation of proofs in favour of FLRW
  - 1965 : CMB discovery : full confirmation

basic principles of G.R.	FLRW solution
space-time is *curved*	???????
free-falling objects follow *geodesics*	???????
curvature caused by / related to *matter*	???????

- summary of the situation :

NEWTON

⇒ matter distribution ⇔ Φ_{grav} ⇔ forces changing the trajectories

General Relativity (GR)

⇒ three basic principles :

1) space-time (t,x,y,z) is curved
2) *free-falling bodies* follow *geodesics*
3) curvature ⇔ matter

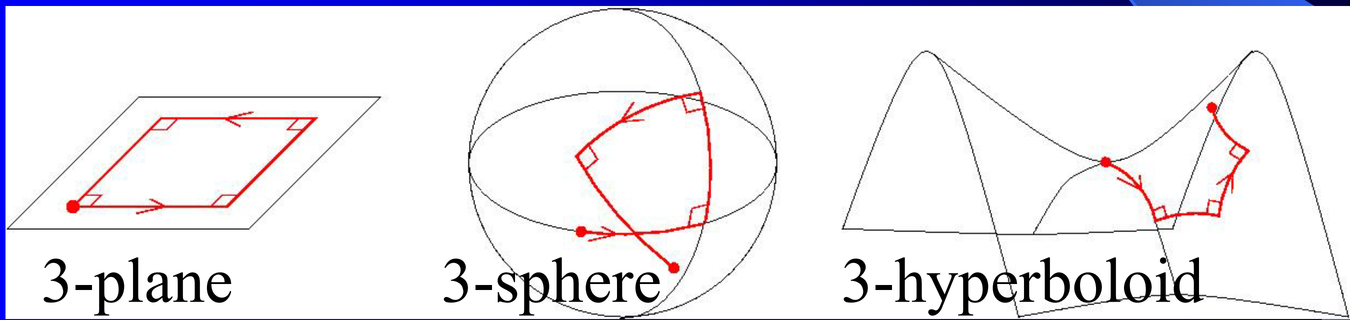
Friedmann-Lemaitre model = application of GR to homogeneous Universe

1) the *curvature* of the FLRW Universe :

- Universe space-time (t,x,y,z) curved by its *own* homogeneous matter density ρ(t)
- HOMOGENEITY ⇨ decomposition of curvature in :

1. spatial curvature of (x,y,z) at fixed t

3-D space is maximally symmetric :



FLAT

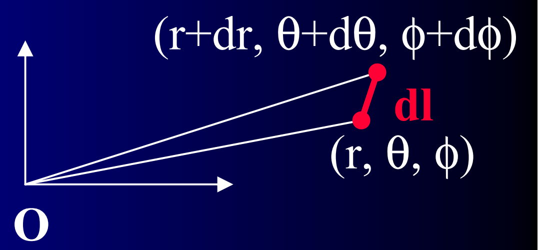
CLOSED ⇨ $R_C(t)$ ⇦ OPEN

2. 2-D space-time curvature of (t,x) ⇔ (t,y) ⇔ (t,z)

accounts for the expansion

- *scale* as a function of coordinates ?
  - COMOVING COORDINATES (t, r, θ, ϕ)
  - for *Euclidian space* :

$$dl^2 = dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$



for *FLRW* :

$$dl^2 = a^2(t) \left[\frac{dr^2}{(1-k\,r^2)} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) \right]$$

- $a(t) \equiv$ scale factor ⇨ 2-D space-time curvature
- k ⇨ spatial curvature
  - **k = 0 : FLAT**
  - **k > 0 : CLOSED,** $R_C(t) = a(t) / k^{1/2}$, $0 \le r < 1/ k^{1/2}$
  - **k < 0 : OPEN ,** $R_C(t) = a(t) / (-k)^{1/2}$

2) the *geodesics* in the FLRW Universe

- ≠ for
  - photons : $v = c$ **ULTRA-RELATIVISTIC**
  - ordinary matter : $v << c$ **NON-RELATIVISTIC**
    (e.g. galaxies)

1) non-relativistic matter:

$$dl = 0 \Rightarrow (r, \theta, \phi) = \textit{constant}$$

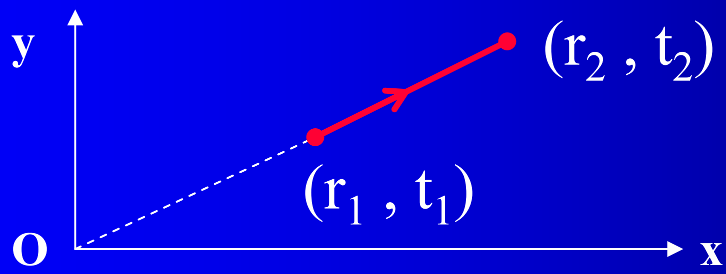
- galaxies are still in coordinate space …
- … but all distances are proportional to a(t)
  - a(t) gives the expansion between galaxies
  - (although they are still !!!)
- *like an inflated rubber balloon with points drawn on its surface …*

2) Relativistic matter : straight line in 3-D space, $dl = c\,dt$

$$\Rightarrow \quad c^2dt^2 = a^2(t)\left[\frac{dr^2}{(1-k\,r^2)} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2\,)\right]$$

EQUATION OF PROPAGATION OF LIGHT

- So $\Delta l = c\,\Delta t$ is *WRONG* :



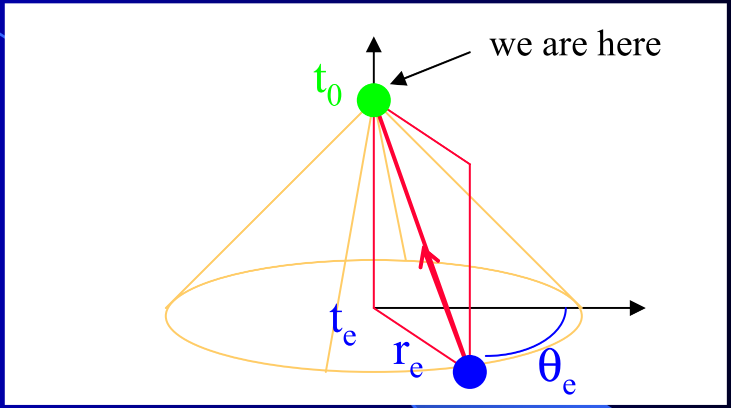
$$\int_{r_1}^{r_2} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t_2} \frac{c}{a(t)}dt.$$

⇨ *« bending of light in the Universe »*
(one of the two most fundamental equations in cosmology)

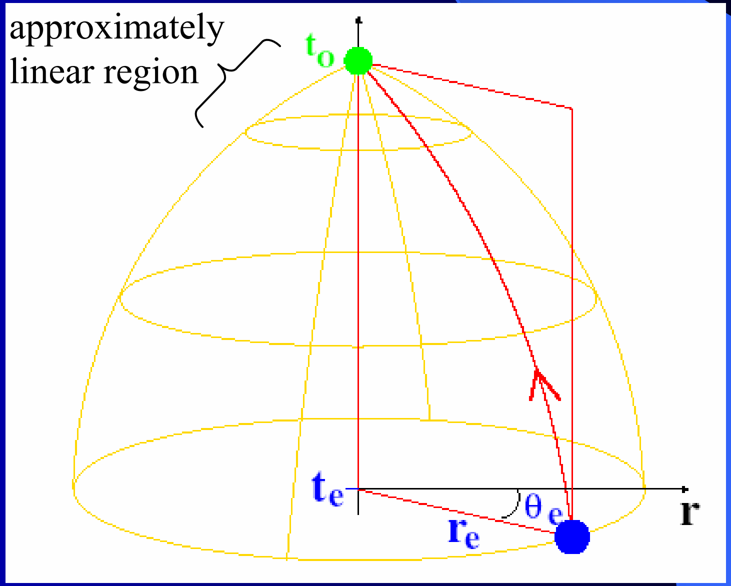
⇨ *various important consequences …*

❑ definition of the past light-cone :

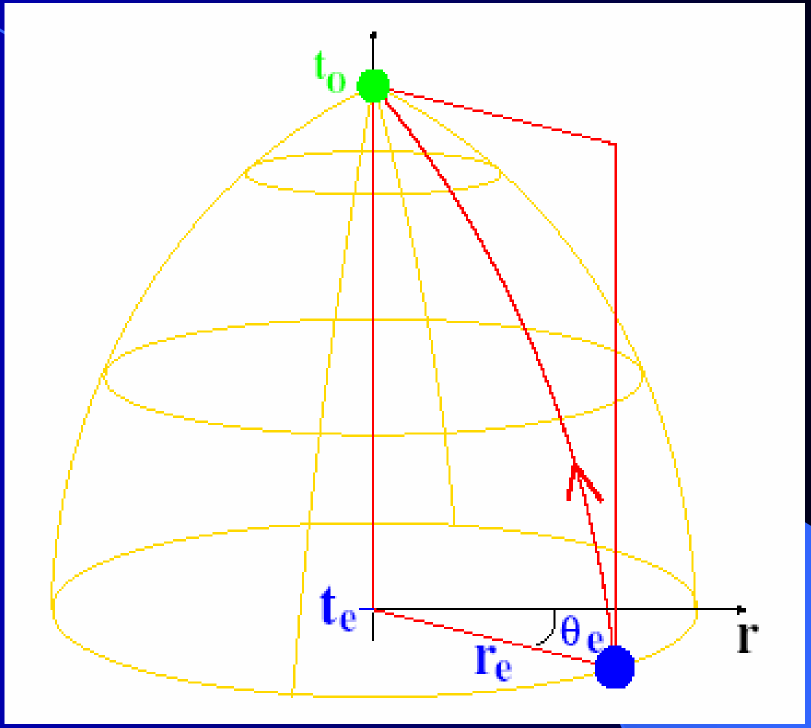
- *in Euclidian space :*
  - $\theta = constant$
  - $r_e = c\,(t_0 - t_e)$



- *in Friedmann universe :*
  - $\theta = constant$
  - $$\int_{r_e}^{0} \frac{-\,dr}{\sqrt{1-kr^2}} = \int_{t_e}^{t_0} \frac{c}{a(t)}\,dt$$
  - $$\frac{dr}{dt} = -\,\frac{c\ (1-kr)^{1/2}}{a(t)}$$



- observable consequences of *propagation of light equation:*



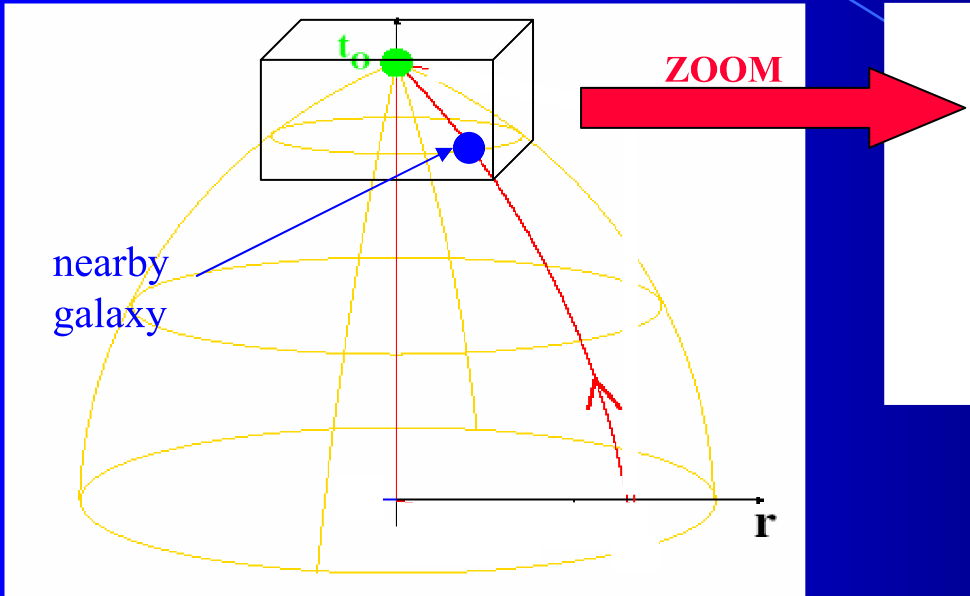
- <u>the redshift :</u>

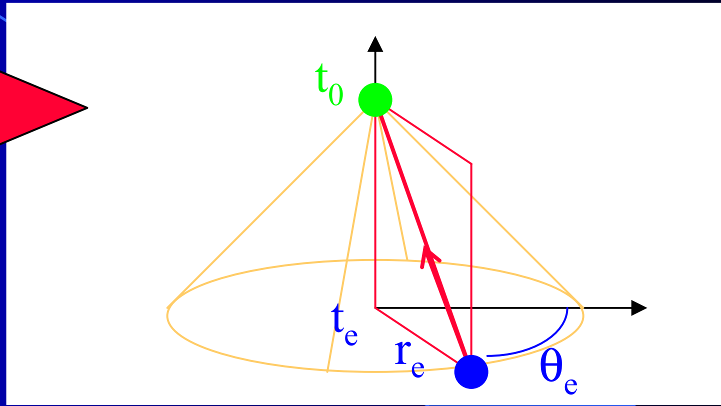
$$z = \Delta\lambda / \lambda = \lambda_0 / \lambda_e - 1$$

$$z = a(t_0) / a(t_e) - 1$$

- remark 1 :
  - *Newtonian :* $z = v / c \leq 1$
  - *G.R.* : no limit, as observed …

- remark 2 : at short distance, we can recover the *Hubble law* (z = v / c = H r / c)



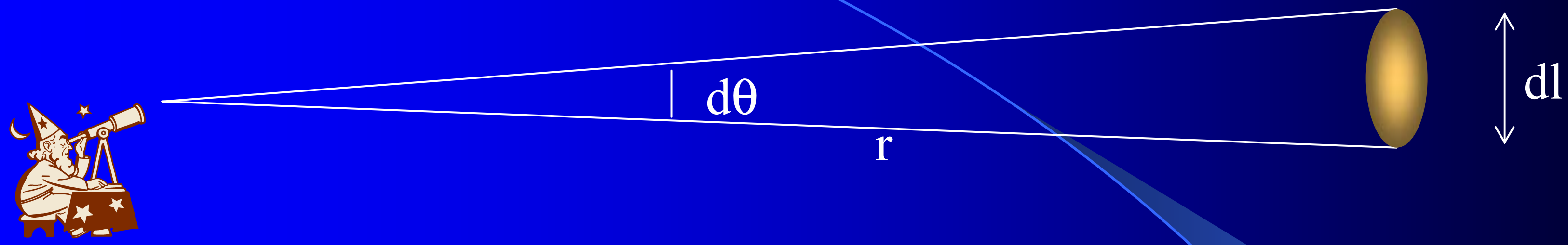


$t_0 - t_e = dt = dl / c$

- then : $$z = \frac{a(t_0)}{a(t_0 - dt)} - 1 = \frac{1}{1 - \frac{\dot a(t_0)}{a(t_0)}dt} - 1 = \frac{\dot a(t_0)}{a(t_0)}dt = \frac{\dot a(t_0)}{a(t_0)}\frac{dl}{c}$$

- so : **Hubble parameter =** $H(t) = \frac{\dot a(t)}{a(t)}$ **(H_0 = 70 km.s^{-1}.Mpc $^{-1}$)**

2) the angular diameter-redshift relation



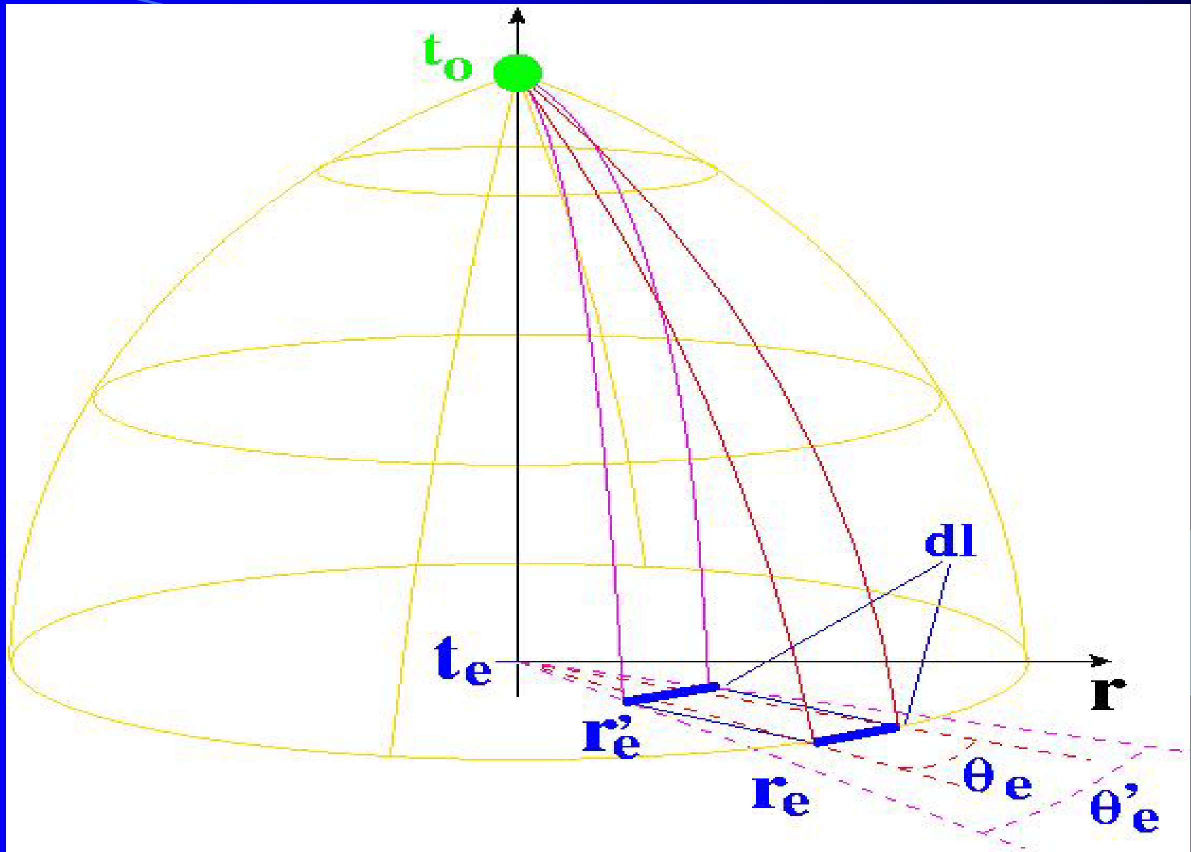
- *Euclidian space :*

$$dl = r\, d\theta \qquad \text{with} \qquad r = v / H = z\, c / H$$

- *G.R. :*

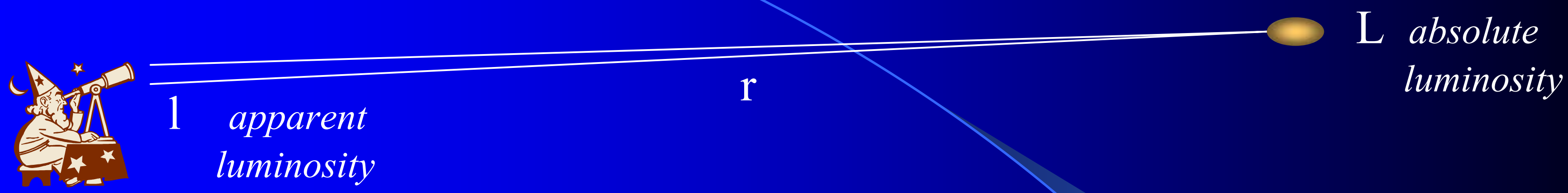
$$dl = a(t_e)\, r_e\, d\theta \qquad \text{with } r_e \text{ from} \qquad \int_{r_e}^{0} \frac{-dr}{\sqrt{1-kr^2}} = \int_{t_e}^{t_0} \frac{c}{a(t)} dt$$

↳ *if dl is known, measurement of $(d\theta, z)$ ⇨ $k, a(t)$*



- **CLOSED UNIVERSE :** objects seen under larger angle
- **OPEN UNIVERSE :** objects seen under smaller angle

3) the luminosity distance-redshift relation



- *Euclidian space :* $$l = \frac{L}{4\pi r^2}$$ with $r = z\, c\, /\, H$

- *G.R. :* $$l = \frac{L}{4\pi\, a^2(t_0)\, r_e^2 (1+z)^2}$$ with r_e from prop. of light

↳ *if L is known, measurement of (l, z) ⇨ k, a(t)*

3) relation between matter and curvature :

FRIEDMANN LAW

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3}\frac{\rho}{c^2} - \frac{kc^2}{a^2}$$

Remark : for non-relativistic matter, $E = m\,c^2 \Rightarrow \rho\,/\,c^2 = \rho_{mass}$

⇨ then *Friedmann law* looks similar to *Newtonian expansion law,* but CRUCIAL DIFFERENCES :

1) $a(t) \neq r(t)$: *very different interpretation*
2) $k \neq 0$ *not in contradiction with homogeneity*
3) accounts for *non-relativistic and relativistic matter*

NON-RELATIVISTIC

$v \ll c$

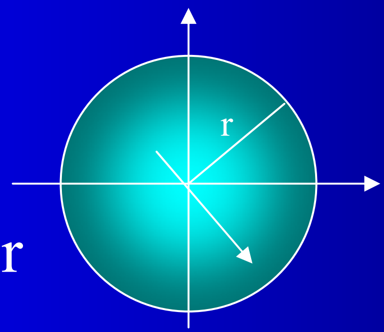
$E = m c^2$

ULTRA-RELATIVISTIC

$v = c$

$E = \hbar \nu = \hbar c / \lambda$

sphere with
- fixed comobile r
- fixed particle number



$V = 4/3 \, \pi \, r^3 \, a(t)^3$

Non-relativistic:

$E_V = \text{constant}$

$\rho = E_V / V \propto a(t)^{-3}$

Ultra-relativistic:

$E_V \propto 1 / \lambda \propto 1 / a(t)$

$\rho = E_V / V \propto a(t)^{-4}$

FRIEDMANN LAW is the same but DILUTION RATE is different

- in fact, in G.R., *curvature* $\Leftrightarrow$ *matter relation* given by

EINSTEIN EQUATION $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$

- in the FLRW solution :

EINSTEIN EQUATION $\Rightarrow$

- Friedmann law
- conservation equation : $\dot{\rho} = -3\,(\dot{a}/a)\,(\rho + p)$

1) non-relativistic :

$v \ll c \;\Rightarrow\; p \cong 0 \;\Rightarrow\; \dot{\rho}/\rho = -3\,\dot{a}/a \;\Rightarrow\; \rho \propto a(t)^{-3}$

2) ultra-relativistic :

$v = c \;\Rightarrow\; p = \rho/3 \;\Rightarrow\; \dot{\rho}/\rho = -4\,\dot{a}/a \;\Rightarrow\; \rho \propto a(t)^{-4}$

3) in QFT : vacuum with

$p = -\rho \;\Rightarrow\; \dot{\rho} = 0 \;\Rightarrow\; \rho =$ *constant*

« cosmological constant »

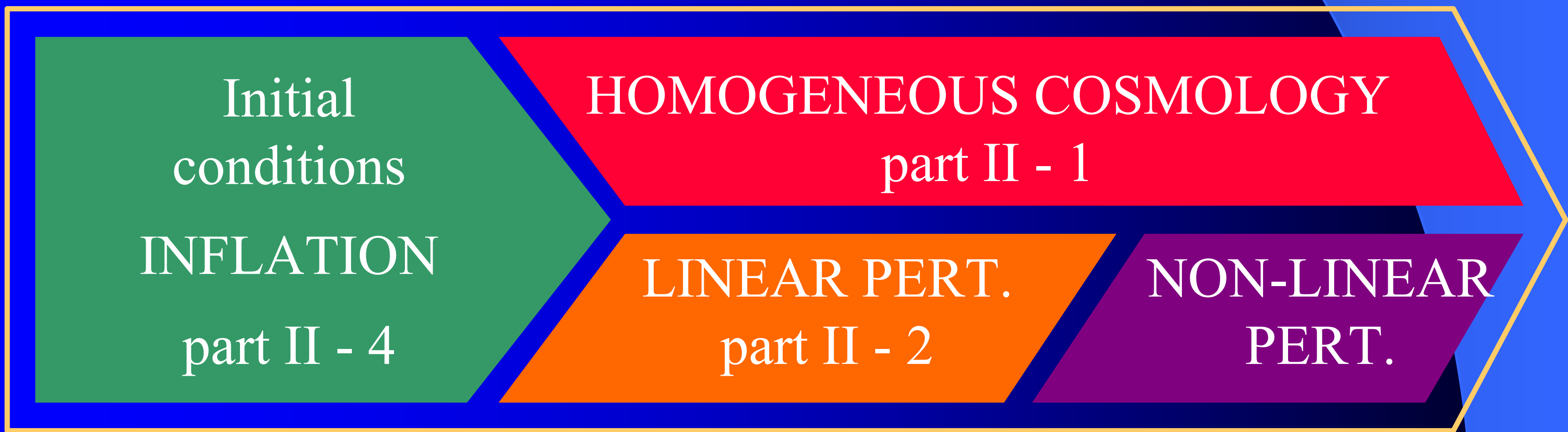
❑ summary of the situation :

basic principles of G.R.	FLRW solution
space-time is *curved*	2 types of curvatures : {a(t), k} $$dl^2 = a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right]$$
free-falling objects follow *geodesics*	• non-relativistic matter : dl = 0 • ultra-relativistic matter : dl = c dt ↳ BENDING OF LIGHT EQUATION
curvature caused by / related to *matter*	relation {a(t), k} ⇔ ρ(t) ↳ FRIEDMANN LAW + conservation equations

Part II : The Standard Cosmological Model

- decomposition of quantities:
  - $\rho(t,\mathbf{r}) = \bar{\rho}(t) + \delta\rho(t,\mathbf{r})$
  - $p(t,\mathbf{r}) = \bar{p}(t) + \delta p(t,\mathbf{r})$
  - *curvature* $= \underbrace{\{a(t), k\}}_{\text{HOMOGENEOUS BACKGROUND}} + \underbrace{\{\Phi(t,\mathbf{r}), \text{etc.}\}}_{\text{PERTURBATIONS}}$

HOMOGENEOUS BACKGROUND + PERTURBATIONS → **THEORY OF LINEAR PERTURBATIONS**



Part II : (1) – Homogeneous cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3}\frac{\rho}{c^2} - \frac{kc^2}{a^2}$$

- the evolution of the Universe depends :
  - *on* SPATIAL CURVATURE
  - *on the density of* :
    - RADIATION : ultra-relativistic particles

      $p = \rho / 3$ $\rho \propto a^{-4}$

      (photons, massless ν's, …)
    - MATTER : non-relativistic bodies

      $p = 0$ $\rho \propto a^{-3}$

      (galaxies, gas clouds, …)
    - COSMOLOGICAL CONSTANT Λ ← $[t^{-2}]$

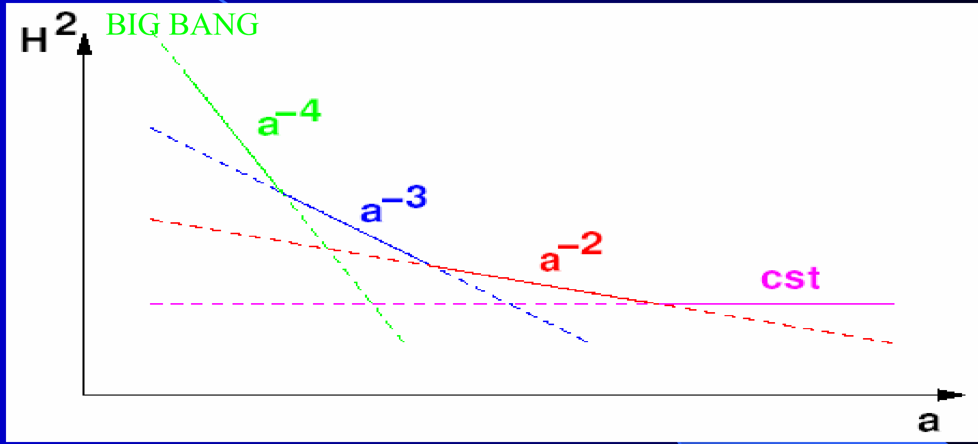
      $p = - \rho$ $\rho = \textit{constant} = \Lambda\, c^2 / (8\pi G)$

      (vacuum ? … ?)
    - …

Friedmann law:

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm M} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

- most « complete » scenario :
- phases can be skipped, but order cannot change



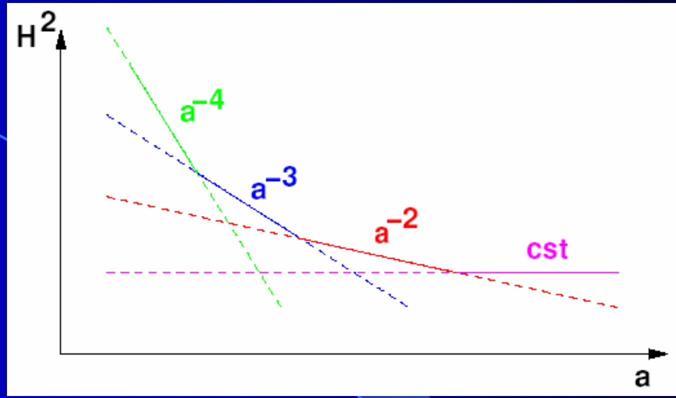
- RADIATION DOMINATION : $a \propto t^{1/2}$ $H = 1 / 2\,t$
- MATTER DOMINATION : $a \propto t^{2/3}$ $H = 2 / 3\,t$
- CURVATURE DOMINATION :
  - $k < 0$ (open) : $a \propto t$ $H = 1 / t$
  - $k > 0$ (closed) : $\dot a \to 0$, then $\dot a < 0$ or ⇨
- VACUUM DOMINATION : $a \propto \exp(\Lambda/3t)^{1/2}$ $H \to$ *constant*

Future of the Universe:

- if $\Lambda = 0$:
  - if $k < 0$ or $k = 0$ $\rightarrow$ indefinite *decelerated* expansion
  - if $k > 0$ $\rightarrow$ recollapse (BIG CRUNCH)
- if $\Lambda \neq 0$:
  - $\forall$ k $\rightarrow$ indefinite *accelerated* expansion

- RADIATION DOMINATION : $a \propto t^{1/2}$ $H = 1 / 2t$
- MATTER DOMINATION : $a \propto t^{2/3}$ $H = 2 / 3t$
- CURVATURE DOMINATION :
  - $k < 0$ (open) : $a \propto t$ $H = 1 / t$
  - $k > 0$ (closed) : $\dot{a} \rightarrow 0$, then $\dot{a} < 0$ or ⇘
- VACUUM DOMINATION : $a \propto exp(\Lambda/3t)^{1/2}$ $H \rightarrow$ *constant*

❑ the matter budget :



- if we can measure $\{\rho_R, \rho_M, k, \Lambda\}$ today, we can extrapolate back …

- today :

$$1 = \frac{8\pi\mathcal{G}}{3H_0^2c^2}(\rho_{R0} + \rho_{M0}) - \frac{kc^2}{a_0^2H_0^2} + \frac{\Lambda}{3H_0^2} = \Omega_R + \Omega_M - \Omega_k + \Omega_\Lambda$$

MATTER BUDGET EQUATION

- flatness condition : $\Omega_0 \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1$

- then : $\rho_R{}^0 + \rho_M{}^0 + \rho_\Lambda{}^0 = \dfrac{3\,H_0{}^2\,c^2}{8\pi G} \equiv \rho_c{}^0 \quad \Rightarrow \quad \Omega_X = \rho_X / \rho_c{}^0$

- so far : COSMOLOGICAL SCENARIOS ⟺ 4 independent parameters $\{\Omega_R , \Omega_M , \Omega_\Lambda , H_0\}$

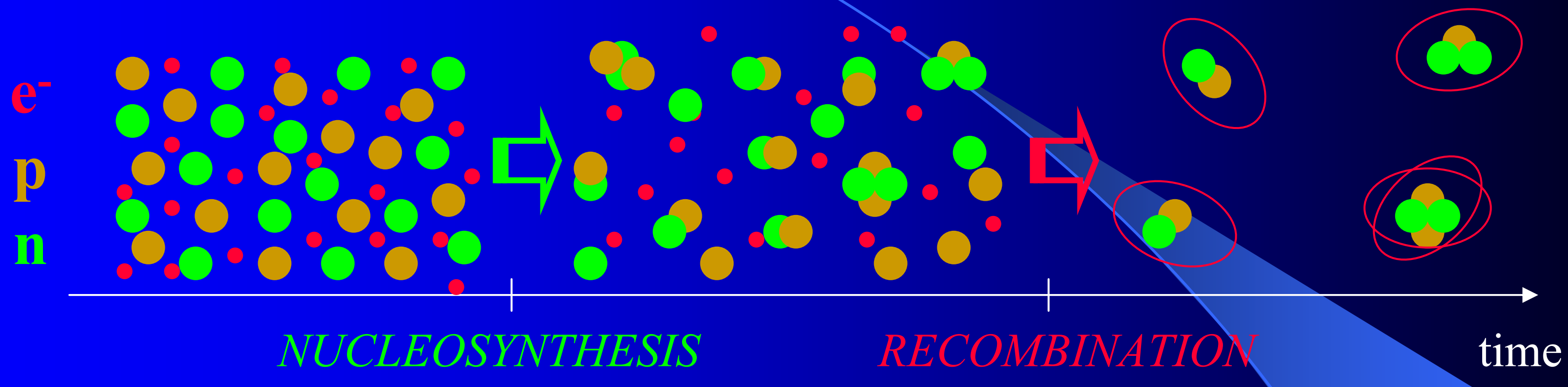
❑ COLD or HOT BIG BANG ???

- 1929–65 : no decisive observation in favour of Friedmann model (apart from accumulation of redshifts)
  ⇨ *works in cosmology remain marginal*
- but spectacular progress in particle physics…
- studies based on the *most simple* possible scenario:
  - Universe contains only non-relativistic matter
  - evolution under the laws of nuclear physics between Big Bang and today

↳ COLD BIG BANG SCENARIO

❑ COLD BIG BANG :

- $H^2 = (8\pi G/3c^2)\,\rho_M \quad \Rightarrow \quad \rho \propto a^{-3} \propto t^{-2}$



- *NUCLEOSYNTHESIS :*
  - ensemble of *nuclear reactions*

$$\left.\begin{array}{l} p \\ n \\ e^- \\ \nu \end{array}\right\} \longrightarrow \left\{\begin{array}{l} \mathrm{H} \\ \mathrm{D} \\ {}^3\mathrm{He}\ ,\ {}^4\mathrm{He} \\ \mathrm{Li}\ ,\ \textit{etc.} \end{array}\right.$$

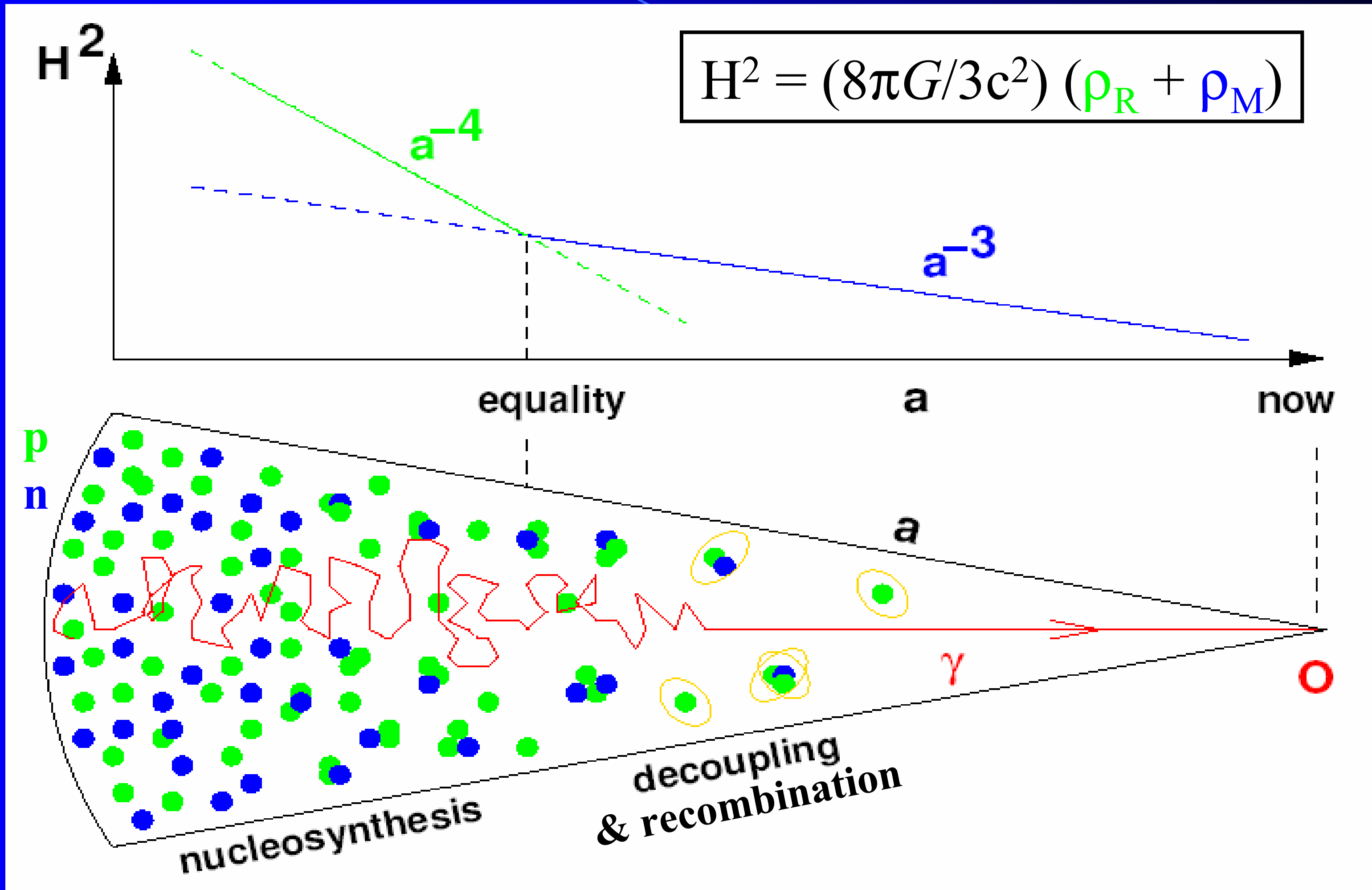
  - freeze-out due to *expansion*

- pioneering works on nucleosynthesis :

  - 1940 : Gamow et al. (USSR → USA)
    1964 : Zel'dovitch et al. (USSR)
    1965 : Hoyle & Taylor (UK)
    1965 : Peebles et al. (USA)

  - **COLD BIG BANG**
    - ↳ no hydrogen
      - ↳ need to change $H(t_{nucleo})$
        - ↳ add relativistic matter (photons) with $\rho_R >> \rho_M$
          - ↳ HOT BIG BANG !!!

HOT BIG BANG :

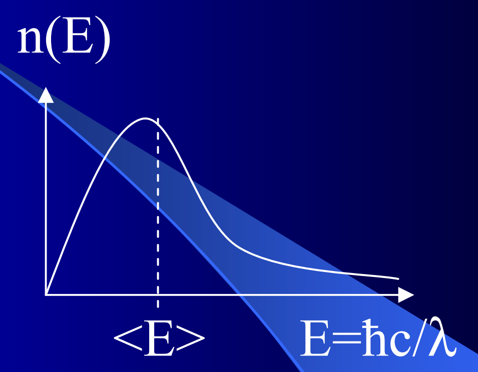


$H^2 = (8\pi G/3c^2)\ (\rho_R + \rho_M)$
H²
a⁻⁴
a⁻³
equality
a
now
p
n
a
γ
O
nucleosynthesis
decoupling
& recombination

- photon spectrum :
  - before recombination, thermal equilibrium

    ↳ blackbody spectrum :

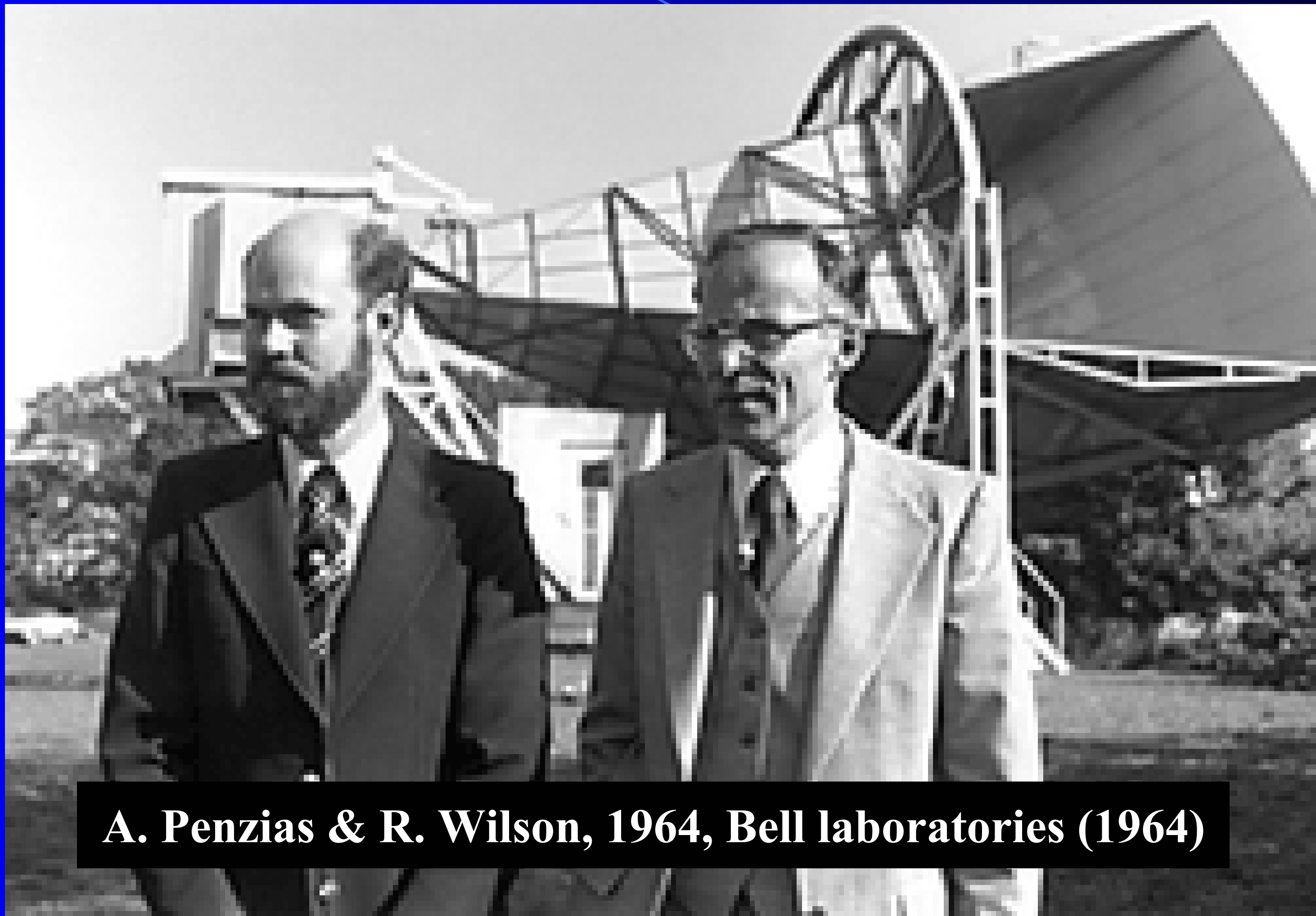
    ↳ **$T \propto a^{-1}$**



  - after recombination, Planck spectrum *frozen* and *redshifted*
  - so $T_0 \, a_0 = T_{nucleo} \, a_{nucleo}$
  - Gamow, Peebles et al. :

    nucleosynthesis $\Rightarrow$ $T_0 \approx$ 1-10 K $\Rightarrow$ $\lambda_0 \approx$ 1-10 mm

discovery of the Cosmic Microwave Background



A. Penzias & R. Wilson, 1964, Bell laboratories (1964)

❑ discovery of the Cosmic Microwave Background

A. Penzias (Bell) → B. Burke (MIT)

J. Peebles (Princeton) → Ken Turner (MIT) → B. Burke (MIT)

- publications by Penzias & Wilson and Peebles
- Nobel Prize for Penzias & Wilson …
- confirmation of **HOT BIG BANG !!!**

☺ CMB = 25% of TV set noise…

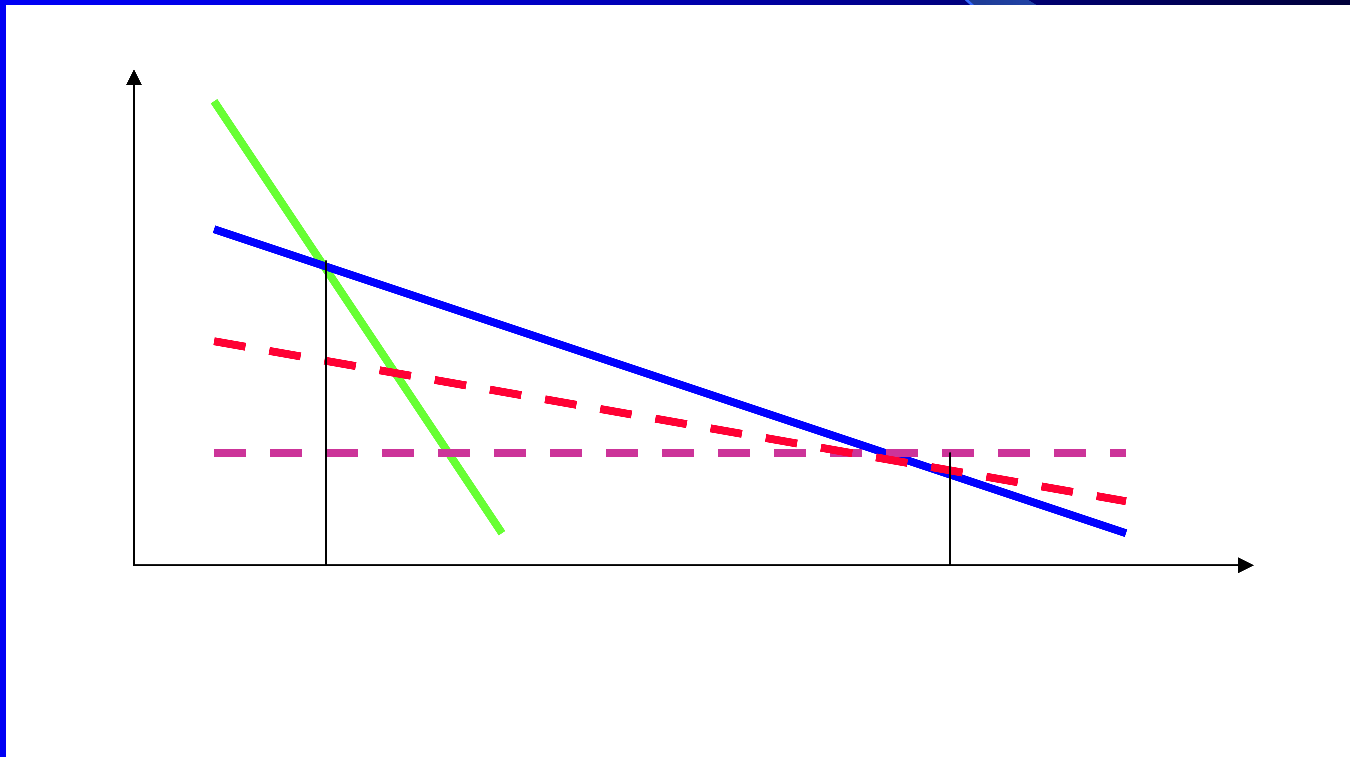
listen to it at http//:www.bell-labs.com/project/feature/archives/cosmology/

❑ thermal evolution of the Universe

	t	T	$\rho^{1/4}$
Planck time ↑ inflation ?	10^{-36} s		10^{18} GeV
GUT ↑ ↓	10^{-32} s		10^{16} GeV
EW ↓ bayogenesis ?	10^{-6} s	10^{15} K	100 GeV
quark-hadron	10^{-4} s	10^{12} K	100 MeV
nucleosynthesis	1-1000 s	10^{9-10} K	0.1 - 1 MeV
equality	10^{4} yr	10^{4} K	1 eV
decoupling	10^{5} yr	2500 K	0.1 eV
structure formation	10^{5} yr → t_0		
today → curvature / Λ ?	$t_0 \approx 13$ Gyr	≈ 3 K	$\approx 3.10^{-4}$ eV

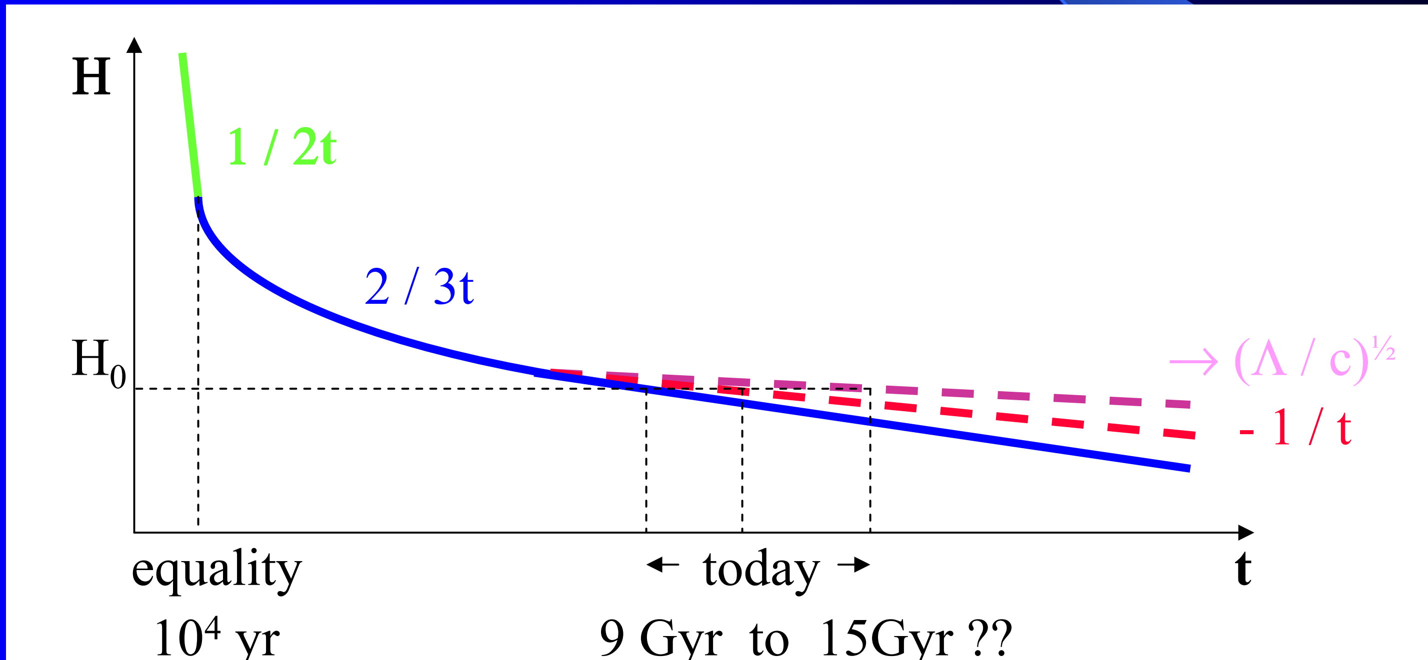
❑ is there a curvature / Λ domination today ?

- structure formation $\Rightarrow$ long-enough M.D. $\Rightarrow$ $\Omega_m \geq 0.2$
- if $\Omega_k \sim 1$ or $\Omega_\Lambda \sim 1$, curvature / Λ domination *started recently :*



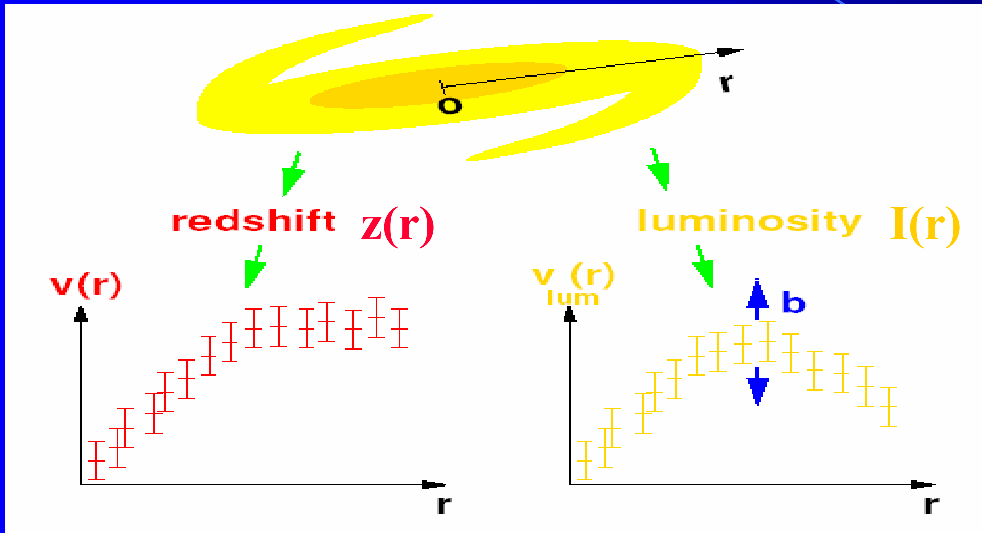
is there a curvature / Λ domination today ?

- HOW CAN WE KNOW ?
  - $\left.\begin{matrix}\Omega_k \\ \Omega_\Lambda\end{matrix}\right\} \Rightarrow$ change $\begin{bmatrix} k \\ a(t) \end{bmatrix} \Rightarrow \begin{cases} \textit{apparent luminosity / z} \text{ relation} \Rightarrow \text{SNIa} \\ \textit{angular diameter / z} \text{ relation} \Rightarrow \text{CMB} \end{cases}$
  - age of the Universe (measured with *redshift of quasars*)



DARK MATTER :

- galaxy rotation curves :



$\rho_{mass}(r) = b\, I(r)$

$\Delta\Phi_{grav}(r) = 8\pi G\, \rho_{mass}$

$v^2(r) = r\,(\partial\Phi_{grav}/\partial r)$

$\Rightarrow$ DM halo

- other compelling evidences
- nature of DM : { ~~non-luminous~~ baryons ? / Hot ~~dark matter~~ (neutrinos) ? / CDM : WIMPS (neutralinos) ? axions ?

- so far :

COSMOGICAL SCENARIO

⇕

5 INDEPENDENT PARAMETERS

$\{\Omega_R , \Omega_B , \Omega_{CDM} , \Omega_\Lambda , H_0\}$

Part II : (2) – cosmological perturbations

- Universe not completely homogeneous *even on large scales* ...
  - description of matter inhomogeneities ?? (clusters of galaxies, superclusters …)
  - description of CMB temperature anisotropies ??

  ⇒ { information on cosmological scenario
  measurement of cosmological parameters

- this section :
  - overview of *mathematical framework*
  - intuitive description of *main phenomena*

linear perturbation theory :

- $\forall x, \quad \rho_x(t, \mathbf{r}) = \bar{\rho}_x(t) + \delta\rho_x(t, \mathbf{r})$

HOMOGENEOUS BACKGROUND — PERTURBATION

- CMB temperature homogeneity

  $\Rightarrow$ perturbations linear at least for $t < t_{dec}$

$$\delta_x(t, \mathbf{r}) \equiv \delta\rho_x(t, \mathbf{r}) \,/\, \bar{\rho}_x(t) \quad \ll \quad 1$$

Einstein equations (matter ⇔ curvature)

- background equations :
  - Friedman law
  - conservation equations

$\{\bar{\rho}_\gamma , \bar{\rho}_\nu , \bar{\rho}_{CDM} , \bar{\rho}_B\} \Leftrightarrow \{a(t) , k\}$

- perturbation equations :

$\{\delta_\gamma , \delta_\nu , \delta_{CDM} , \delta_B\} \Leftrightarrow$ {curvature perturbations}

$\approx \Phi(t, \mathbf{r}) \rightarrow \Phi_{grav}$ inside R_H

partial derivative equations

$\Rightarrow$ *Fourier transformation …*

❑ comoving Fourier space

- $\delta_x^{\mathbf{k}}(t) \equiv \int dr^3\, e^{-i\,\mathbf{k}.\mathbf{r}}\, \delta_x(t, \mathbf{r})$
  - comoving Fourier wavenumber k
  - comoving wavelength $2\pi / k$
  - physical wavelength $\lambda(t) = 2\pi\, a(t) / k$

- independent perturbation equations :

$$\{\delta_\gamma^\kappa, \delta_\nu^\kappa, \delta_{CDM}^\kappa, \delta_B^\kappa\} \Leftrightarrow \Phi^\kappa$$

  - linear system of ordinary differential equations…
  - even at equilibrium, each wavelength redshifted

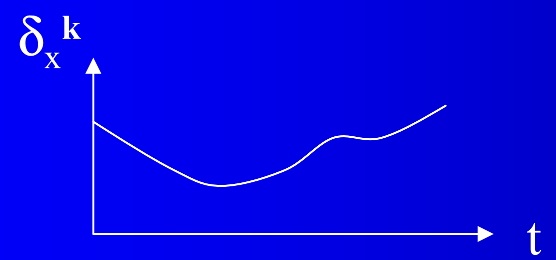
stochastic theory :

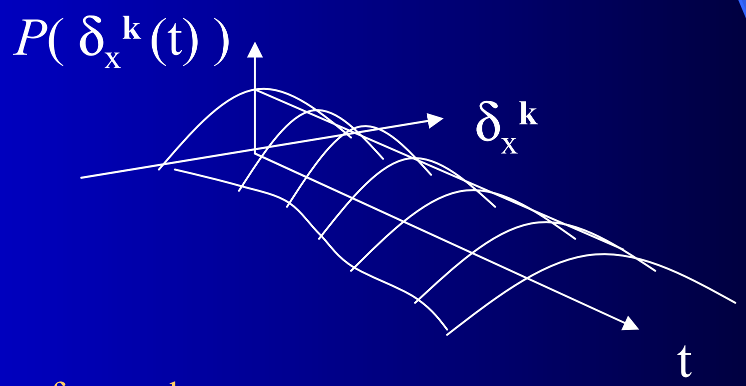
- random initial conditions : $P(\,\delta_x^{\mathbf{k}}(t_0)\,)$
- evolution under differential equation :

$$(d^2/dt^2)\,\delta_x^{\mathbf{k}} + \dots (d/dt)\,\delta_x^{\mathbf{k}} + \dots \delta_x^{\mathbf{k}} = \dots \delta_Y^{\mathbf{k}} + \dots \Phi^{\kappa}$$

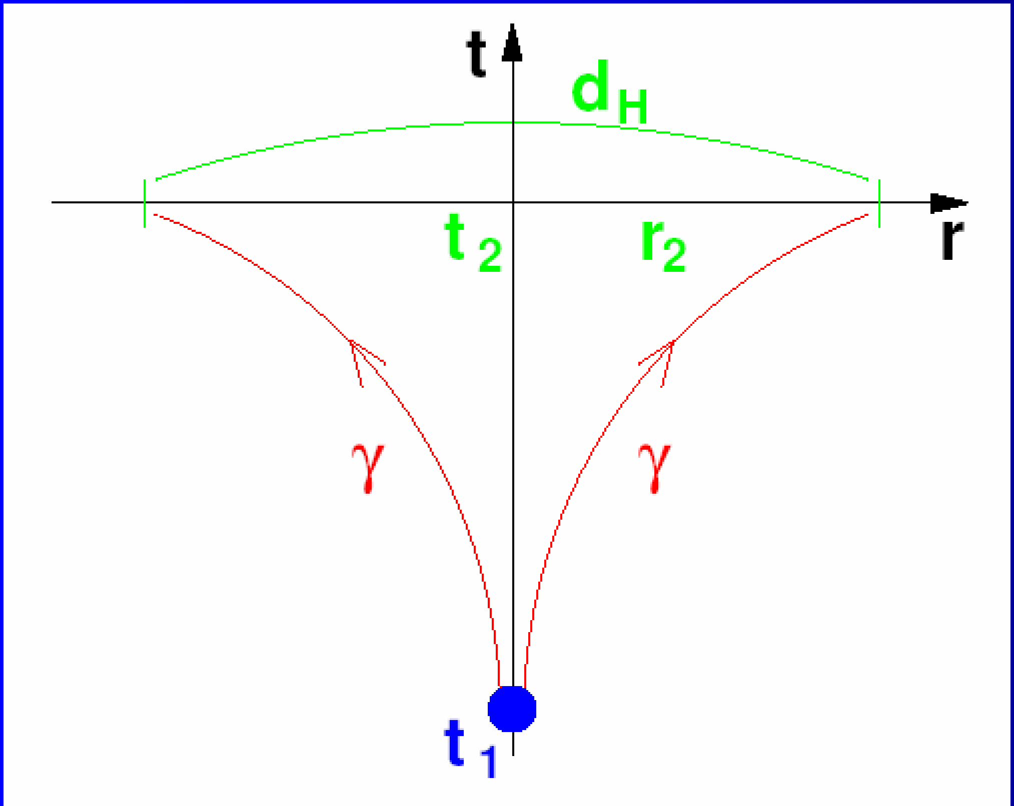
  - Early Universe $\Rightarrow$ gaussian distributions
  - linearity : shape $P(\,\delta_x^{\mathbf{k}}(t)\,)$ is preserved
  - differential equation = evolution of r.m.s.

e.g.:





- intuitive description of the evolution :
  - definition of the HORIZON :



$$d_H(t_1,t_2) = 2\int_0^{r_2} dl = 2\int_0^{r_2} a(t_2)\frac{dr}{\sqrt{1-kr^2}}$$

Propagation of light :

$$\int_{t_1}^{t_2} \frac{c\,dt}{a(t)} = \int_0^{r_2} \frac{dr}{\sqrt{1-kr^2}}$$

$$\Downarrow$$

$$d_H(t_1,t_2) = 2a(t_2)\int_{t_1}^{t_2} \frac{c\,dt}{a(t)}$$

- during radiation domination : $a(t) \propto t^{1/2}$, $R_H = 2\ c\ t$

$$d_H(t_1, t_2) = 4\ c\ t_2^{1/2}[t_2^{1/2} - t_1^{1/2}] \rightarrow 4\ c\ t_2 = 2\ R_H(t_2)$$

- during matter domination : $a(t) \propto t^{2/3}$, $R_H = 3/2\ c\ t$

$$d_H(t_1, t_2) = 6\ c\ t_2^{2/3}[t_2^{1/3} - t_1^{1/3}] \rightarrow 6\ c\ t_2 = 4\ R_H(t_2)$$

- physical process starting during RD, MD cannot affect

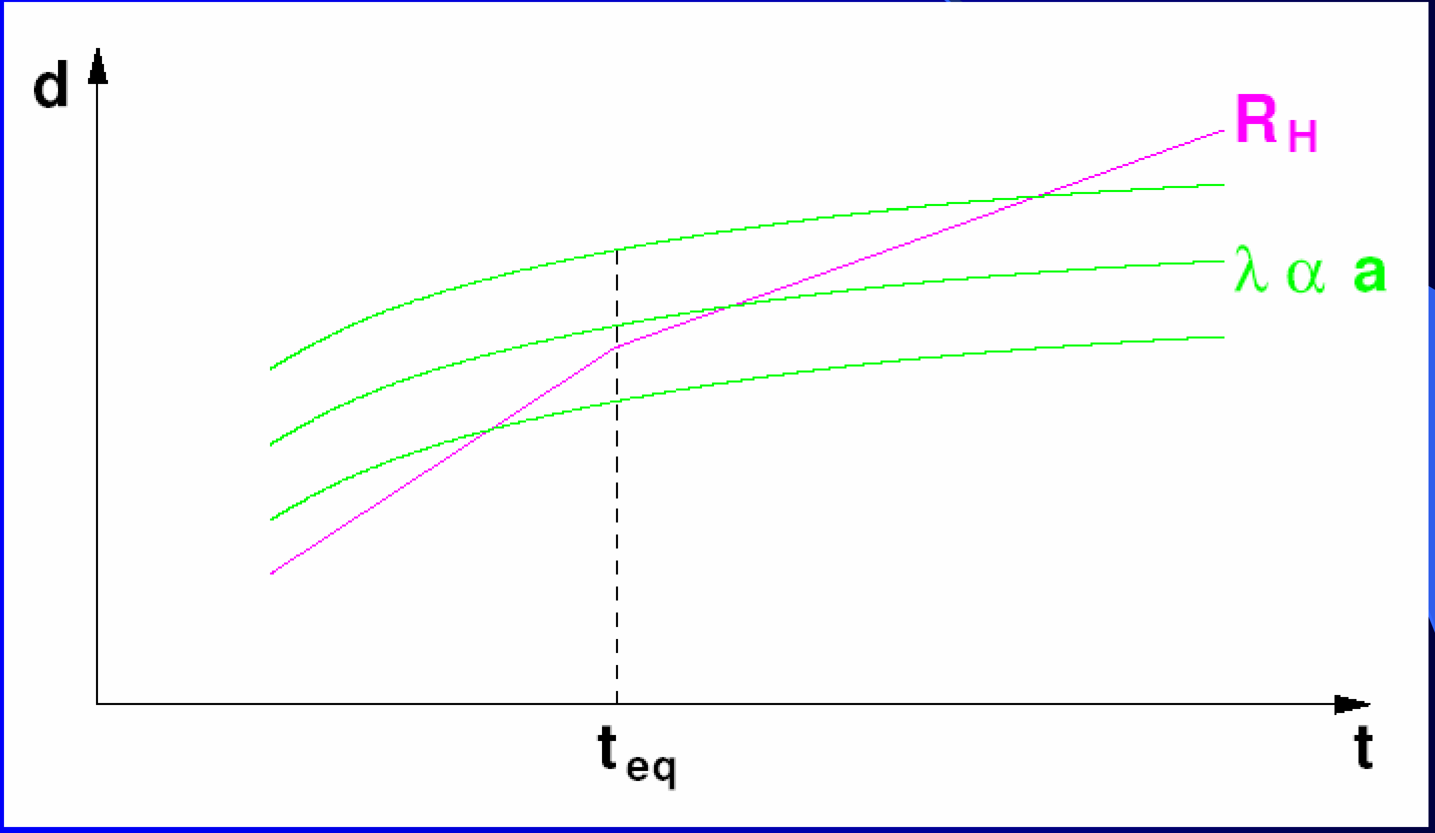
$$\lambda(t) \geq R_H(t)$$

without violating causality…

$R_H(t) \equiv$ causal horizon for RD / MD

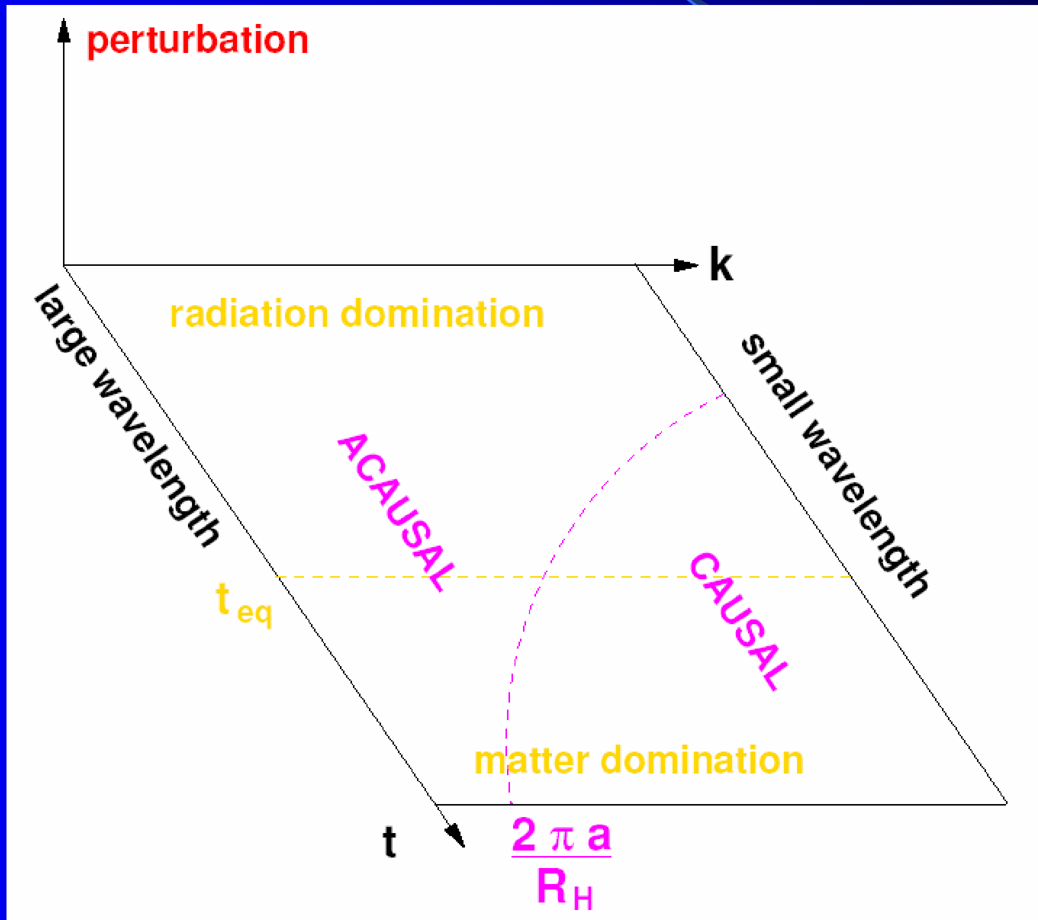
- evolution of wavelengths versus R_H :

$$\lambda(t) = R_H(t) \quad \Leftrightarrow \quad k = 2\pi\, a(t) / R_H(t)$$



- evolution of wavelengths versus R_H :

$$\lambda(t) = R_H(t) \quad \Leftrightarrow \quad k = 2\pi\, a(t) / R_H(t)$$



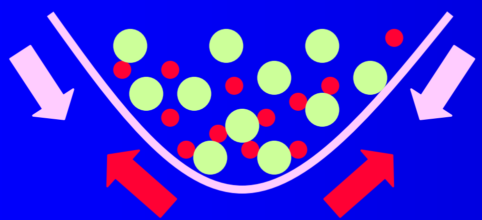
PHOTON PERTURBATIONS :

- Planck spectrum $\Rightarrow$ $\delta_\gamma(t, \mathbf{r}) \Leftrightarrow \delta T/T(t, \mathbf{r})$

1) ***before recombination*** (= decoupling ≈ equality) :

photons $\overset{\text{E.M.}}{\leftrightarrow}$ baryons $\overset{\text{gravity}}{\leftrightarrow}$ grav. pot.

(photons ↔ baryons:) tighly coupled fluid



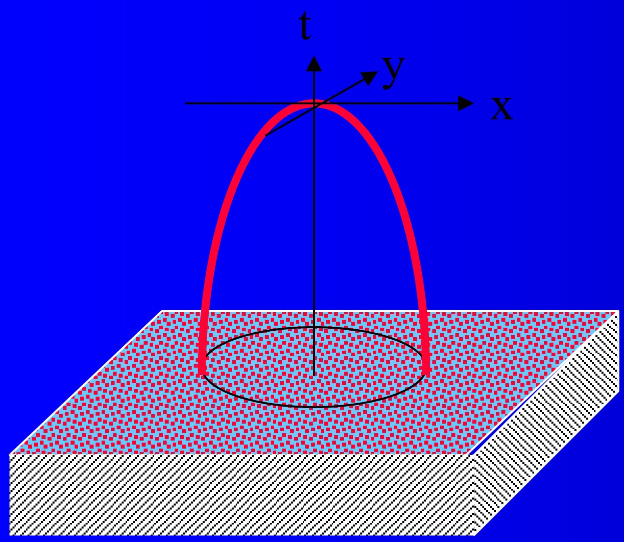
gravity, pressure } « acoustic oscillations »

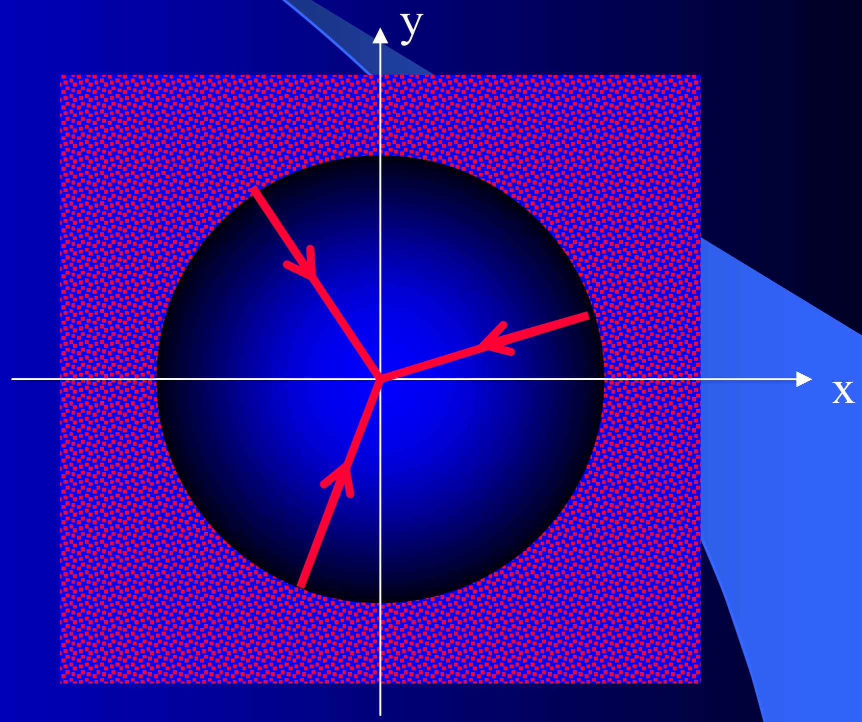
2) ***after recombination :***

photon decoupled $\rightarrow$ « free streaming »

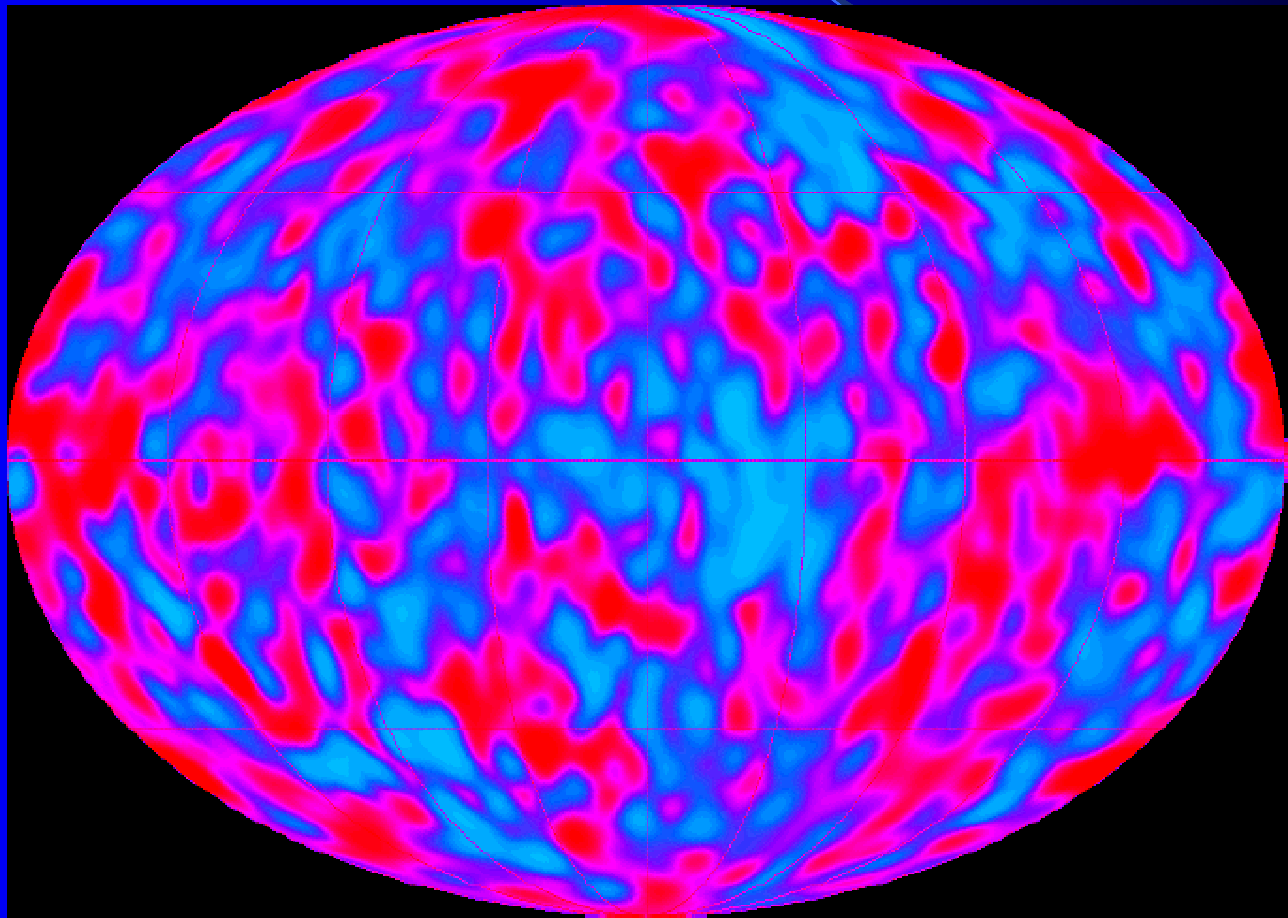
PHOTON PERTURBATIONS :

- last scattering surface :



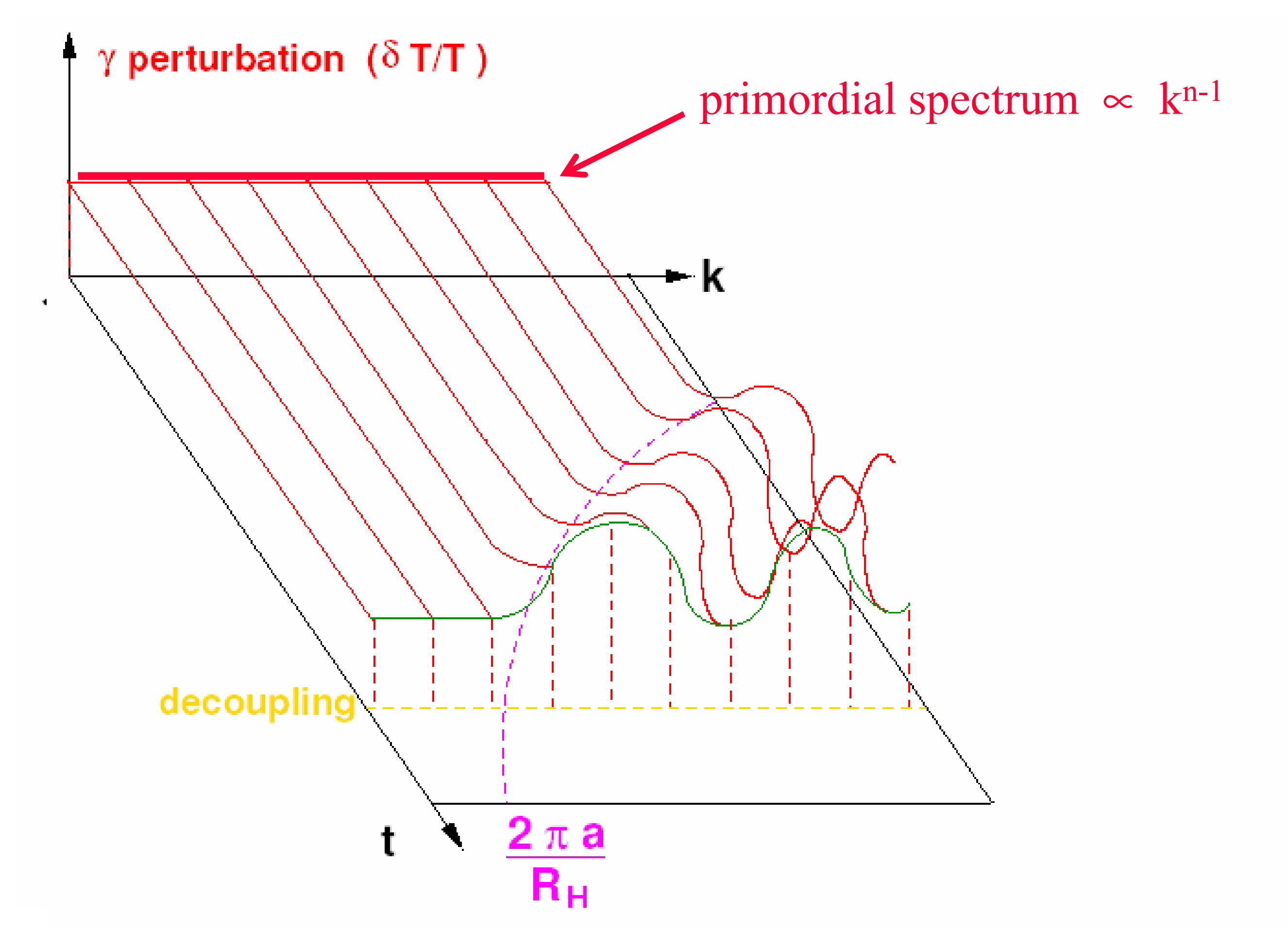


- PHOTON PERTURBATIONS :
  - last scattering surface *mapped by COBE DMR (1994) :*



PHOTON PERTURBATIONS :

- evolution of perturbations :



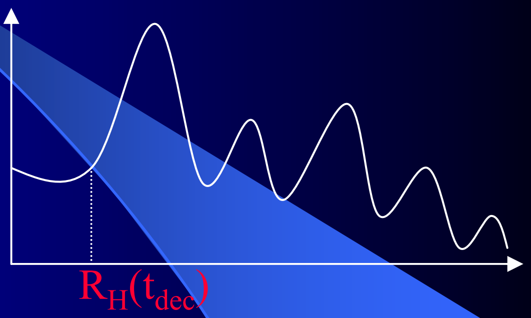
PHOTON PERTURBATIONS :

- spectrum of CMB anisotropies:

observation of CMB

↳ $\delta T/T$ map of last scattering surface

↳ Fourier spectrum with accoustic peaks



amplitude of the peaks ⇨ Ω_B , Ω_{CDM} , n … !!!

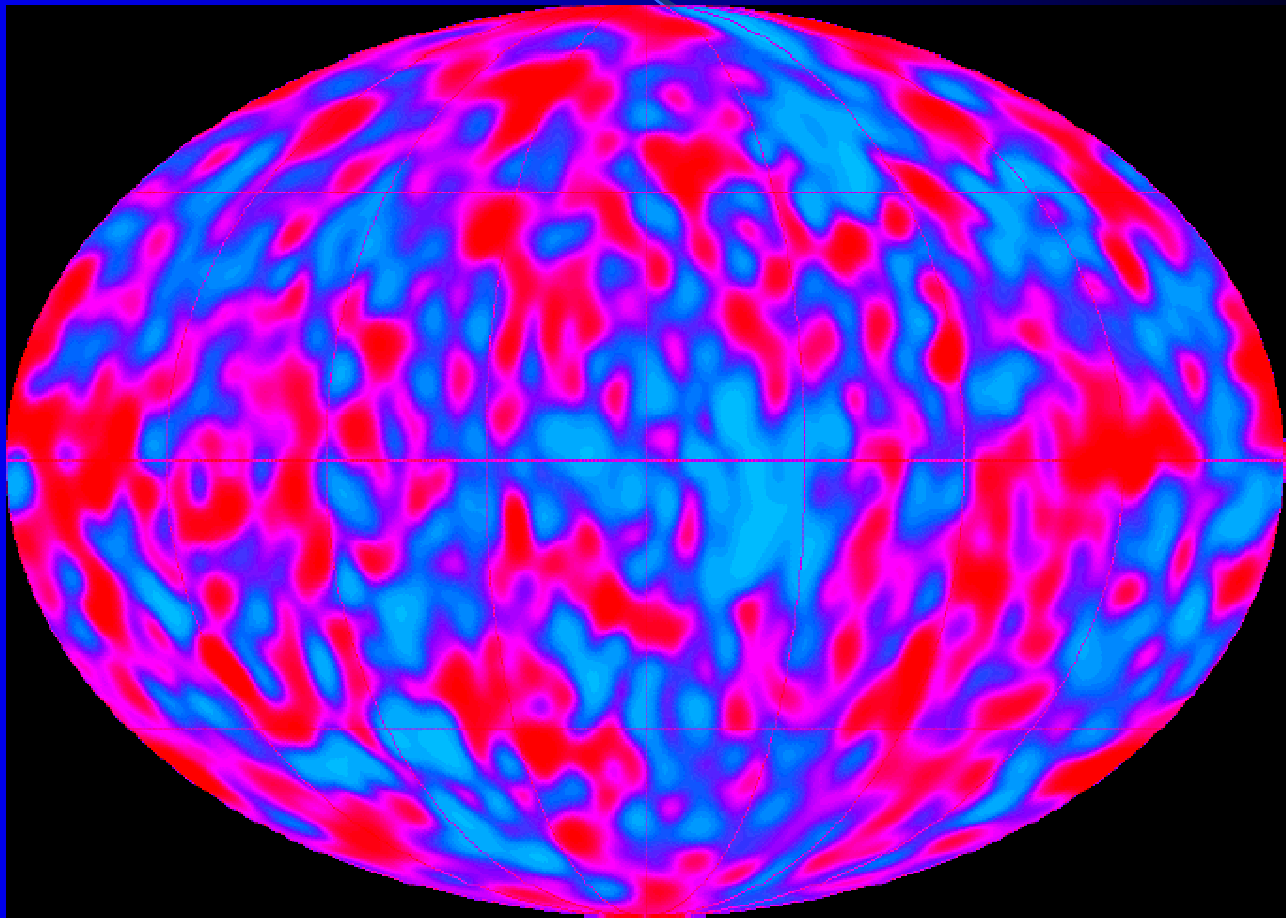
position of the peaks ⇨ angle under which $R_H(t_{dec})$ is seen

⇓ *angular diameter – redshift relation*

spatial curvature k !!!

PHOTON PERTURBATIONS :

- main observations : *COBE DMR (1994)*

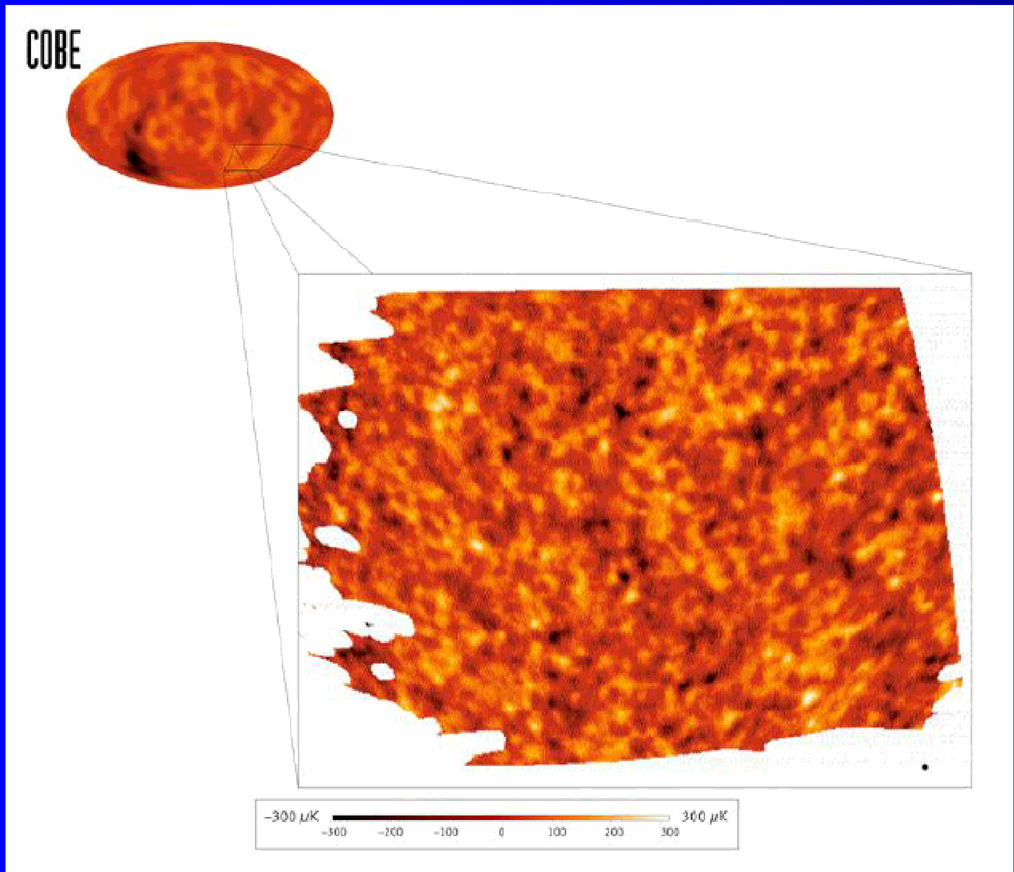


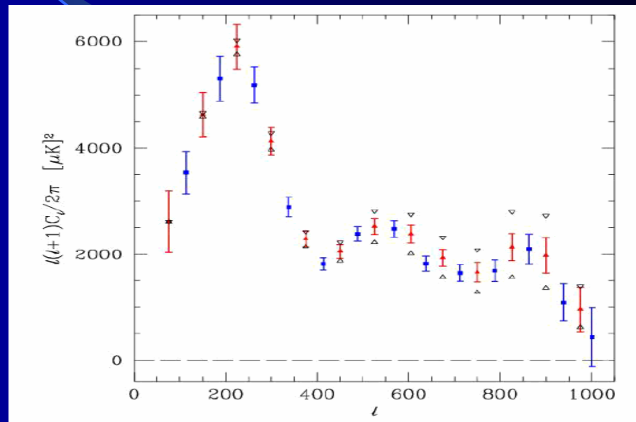
resolution ~ 10º

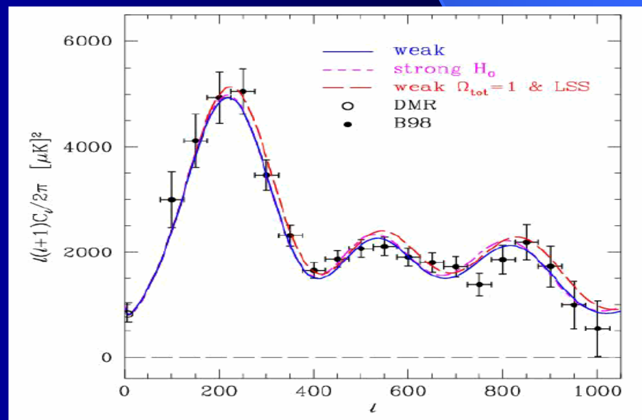
$\lambda(t) \geq R_H(t_{dec})$

PHOTON PERTURBATIONS :

- main observations : *Boomerang (2000)*

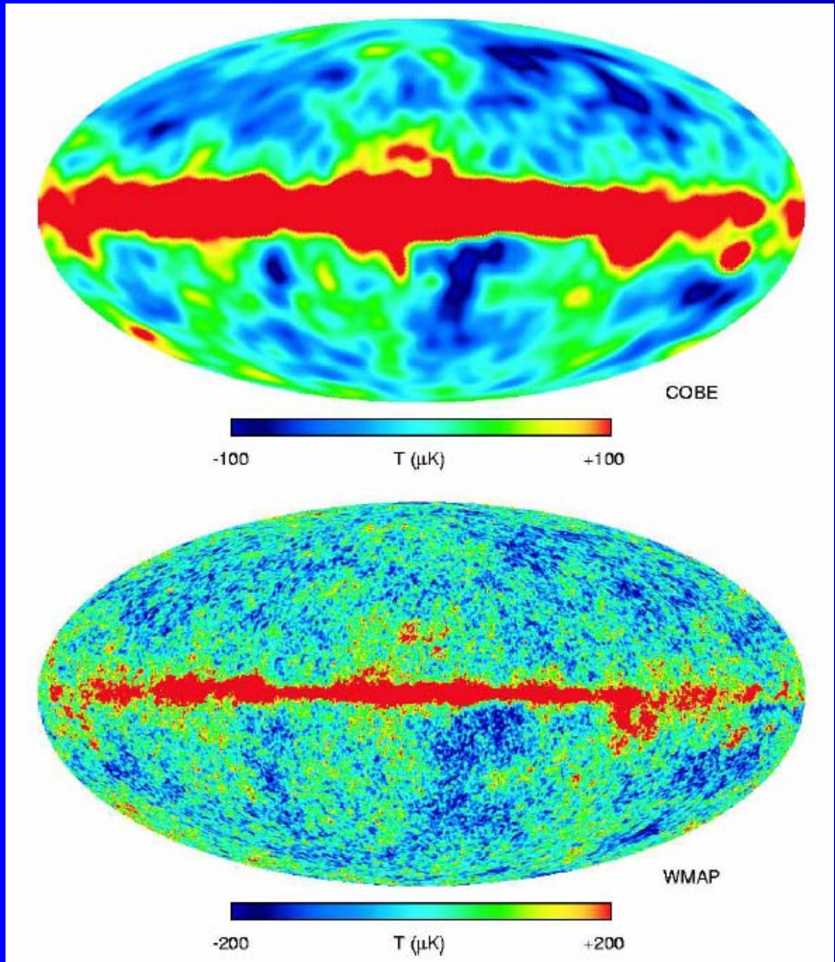


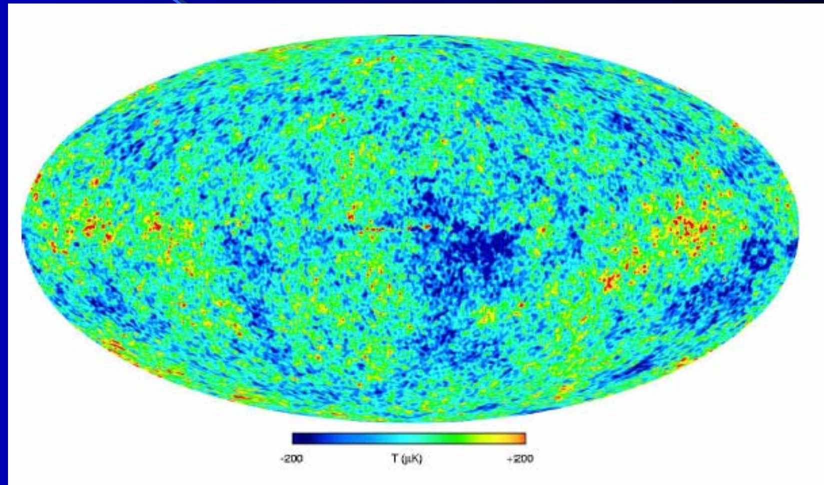


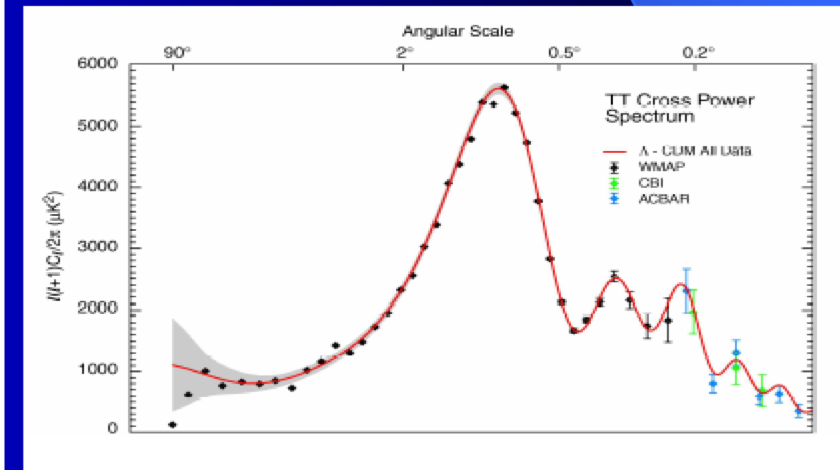


PHOTON PERTURBATIONS :

- main observations : *WMAP (February 2003)*

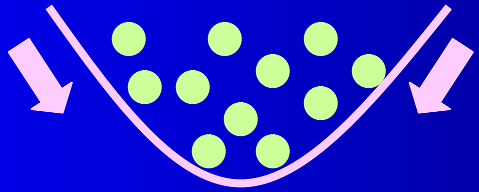






❑ MATTER PERTURBATIONS :

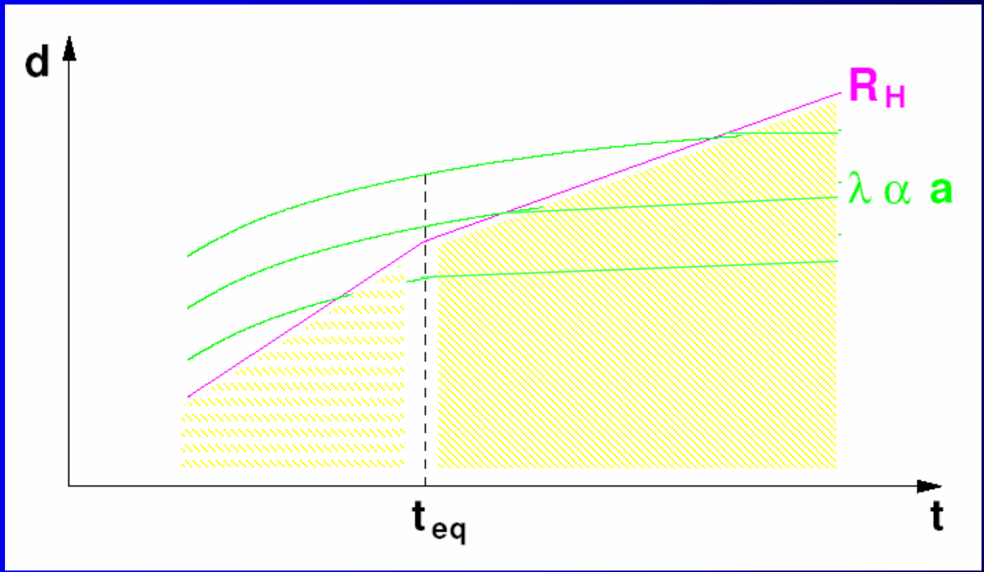
➢ CDM :



gravitationnal collapse efficient during MD

(RD : Φ_{grav} follows γ)

➢ baryons : follow
- γ during RD
- CDM during MD



❑ MATTER PERTURBATIONS :

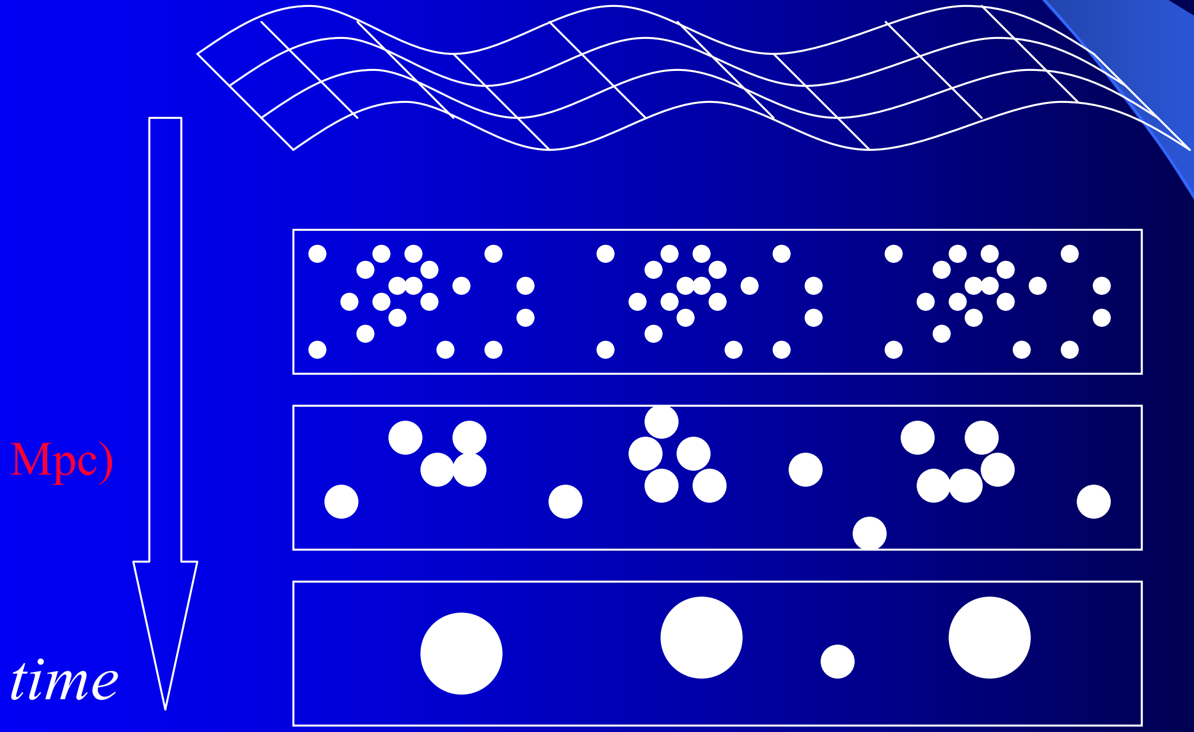
- non-linear evolution :

  $\delta_{CDM}^{\mathbf{k}}(t)$, $\delta_{B}^{\mathbf{k}}(t)$ ~ 1 first for large k / small λ

  ↳ *hierarchical structure formation :*

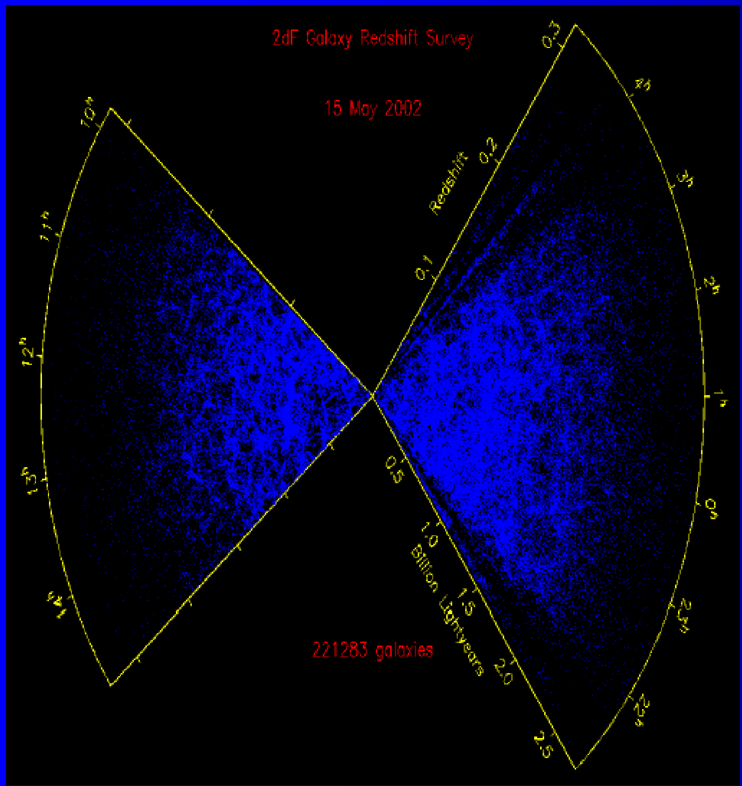
linear theory recovered by smoothing

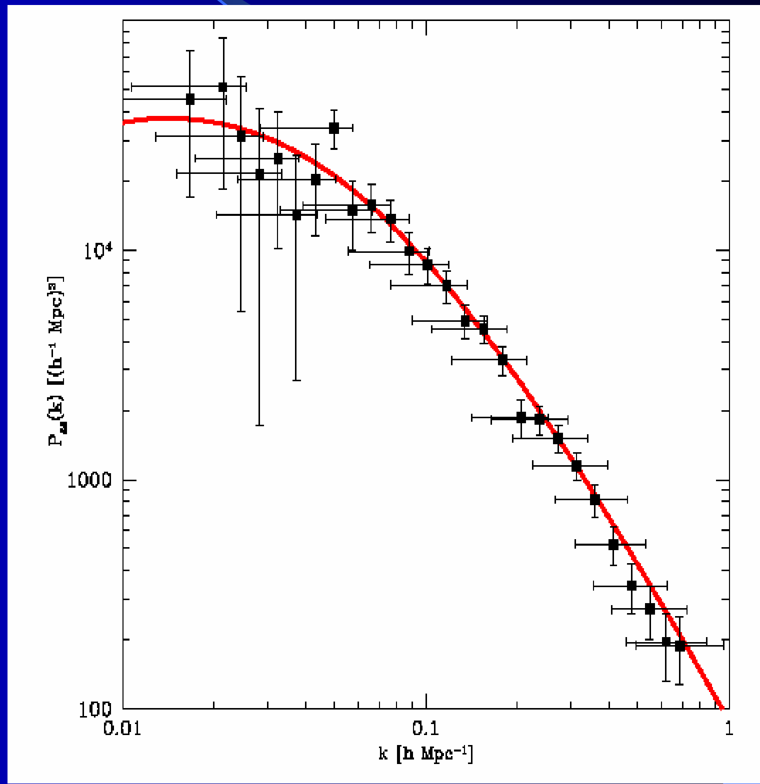
(today: over 30 Mpc)



MATTER PERTURBATIONS :

- observations : *2dF redshift survey*





Part II : (3) – cosmological parameters

COSMOGICAL SCENARIO

⇕

7 INDEPENDENT PARAMETERS …at least !!!

$\{\Omega_R , \Omega_B , \Omega_{CDM} , \Omega_\Lambda , H_0 , A , n\}$

$$h \equiv \frac{H_0}{100 \text{ km.s}^{-1}.\text{Mpc}^{-1}}$$

A short selection of cosmo

1) nucleosynthesis

- $\rho_B \Leftrightarrow \rho_{H, D, He, Li}$

2) CMB anisotropies

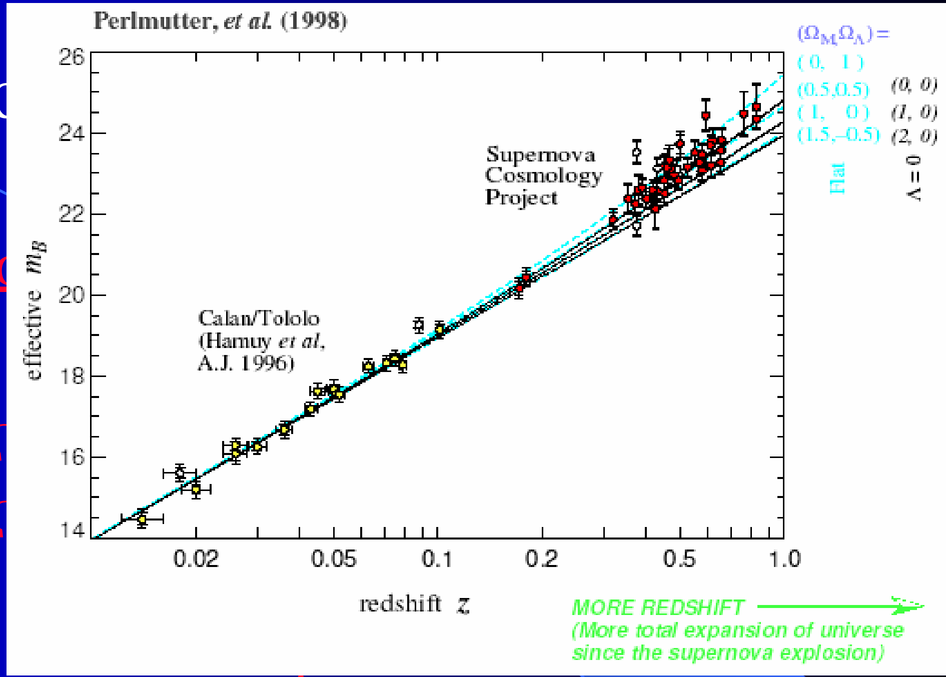
- position :
- amplitude :

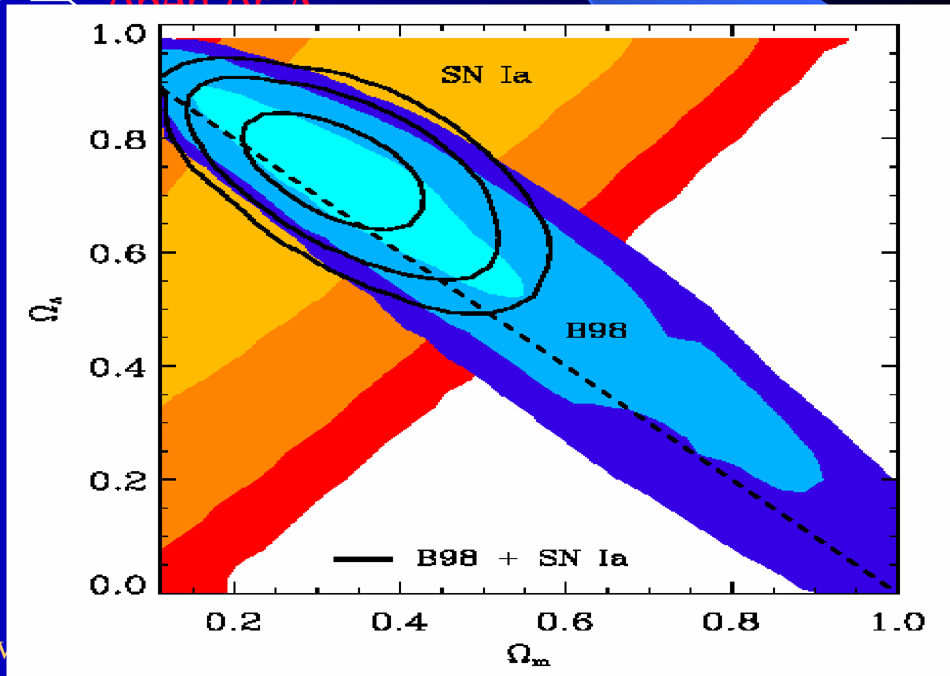
3) age

- quasars of age ≥ 11 Gyr ⇒ open or Λ

4) supernovae

- $\Omega_\Lambda - \Omega_M = 0.5 \pm 0.5$





A short selection of cosmological tests :

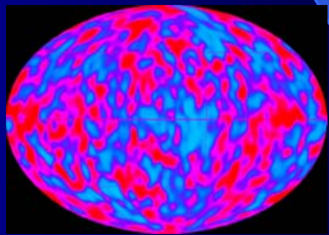
1) nucleosynthesis
   - $\rho_B \Leftrightarrow \rho_{H, D, He, Li}$ $\Omega_B h^2 = 0.020 \pm 0.002$, $\Omega_R \leftarrow \gamma + 3\ \nu$'s
2) CMB anisotropies
   - position : $\Omega_0 = \Omega_M + \Omega_\Lambda = 1.03 \pm 0.05$
   - amplitude : $\Omega_B h^2 = 0.024 \pm 0.001$, $n = 0.99 \pm 0.04$
3) age
   - quasars of age $\geq$ 11 Gyr $\Rightarrow$ open or Λ
4) supernovae
   - $\Omega_\Lambda - \Omega_M = 0.5 \pm 0.5$

combined : $\Omega_B \cong 0.044$, $\Omega_{CDM} \cong 0.23$, $\Omega_\Lambda \cong 0.73$, $h \cong 0.71$

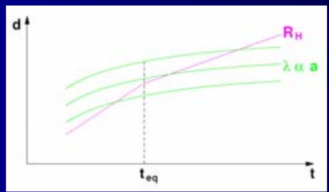
5) large scale structure : perfect agreement

Part II : (4) – Inflation & Quintessence

- « early problems » in the Hot Big Bang scenario :
  - flatness problem :
    - $$|\Omega_k(t)| = |\rho_0(t)/\rho_c(t) - 1| = \frac{c^2|k|}{a^2H^2} = \frac{c^2|k|}{\dot{a}^2}$$
    - $|\Omega_k|$ grows like t *(RD)* or $t^{2/3}$ *(MD)*
    - $|\Omega_k| \leq 0.1$ at t_0 $\Rightarrow$ $|\Omega_k| \leq 10^{-60}$ at t_P
  - horizon problem :
    - causal horizon on CMB maps ~ 1°
      $\Rightarrow$ 10^3 causally disconnected regions



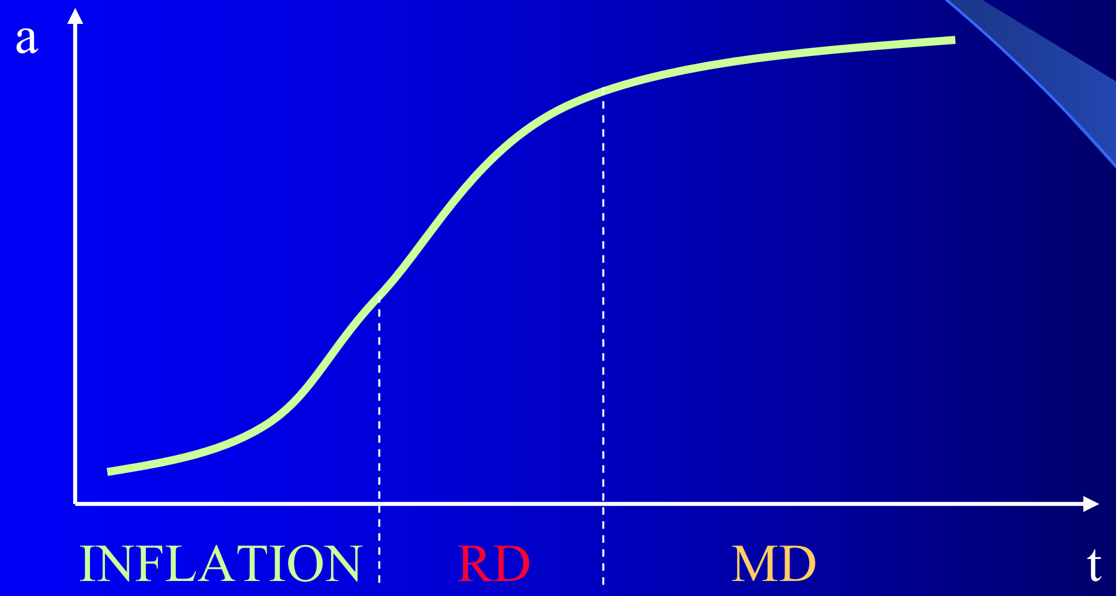
  - origin of fluctuations :
    - initially, $\lambda >> R_H$ …



❑ INFLATION :

(Guth 79; Starobinsky 79)

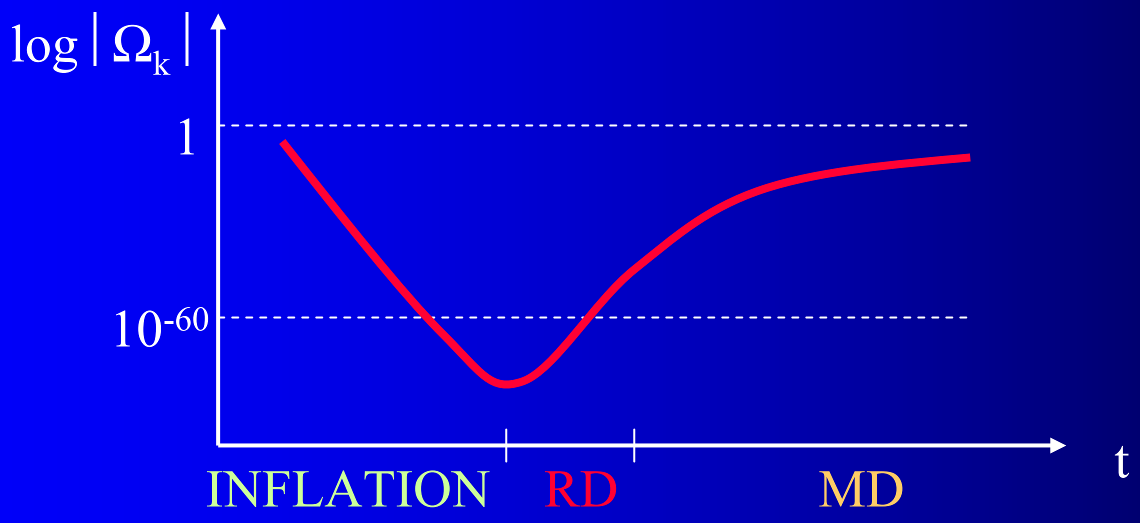
- defined as an initial *accelerated expansion* stage :



INFLATION :

- solves flatness problem :

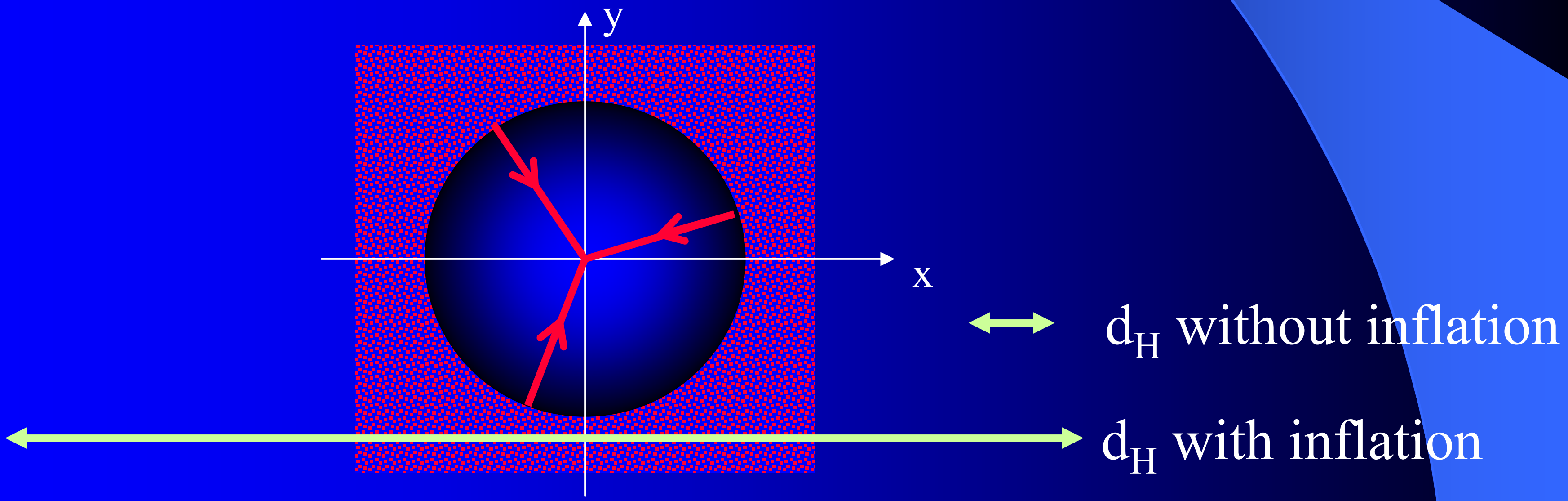
$$|\Omega_k(t)| = |\rho_0(t)/\rho_c(t) - 1| = \frac{c^2|k|}{a^2H^2} = \frac{c^2|k|}{\dot{a}^2} \searrow$$



❑ INFLATION :

- solves horizon problem :

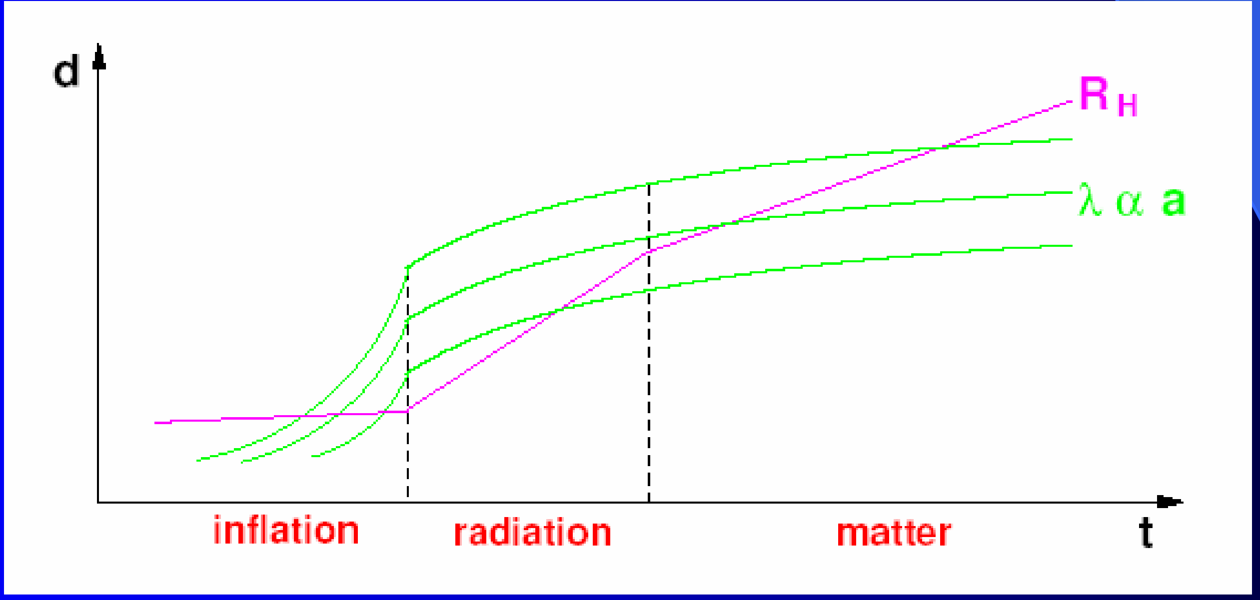
$$\begin{cases} d_H(t_1, t_2) \cong R_H(t_2) & \text{for } t_1 \ll t_2 \text{ and } t_1 \in \text{RD, MD} \\ d_H(t_1, t_2) \gg R_H(t_2) & \text{for } t_1 \ll t_2 \text{ and } t_1 \in \text{INFLATION} \end{cases}$$



INFLATION :

- solves generation of fluctuations :

$$\frac{\lambda(t)}{R_H(t)} = \frac{2\pi a(t)}{k}\frac{\dot{a}(t)}{c\ a(t)} = \frac{2\pi \dot{a}(t)}{c\ k}$$



❑ INFLATION :

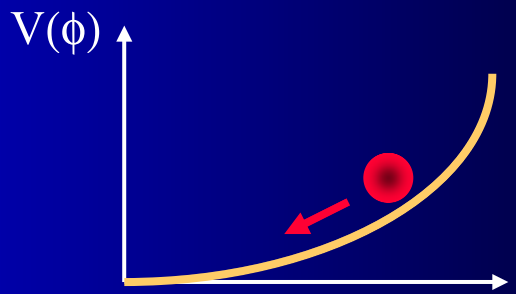
➢ requires :

Friedman law
conservation equation
} → $a(t) > 0 \iff \rho + 3\,p < 0$

➢ candidates :

- Λ : $\rho + 3\,p = -2\,p < 0$ but inflation forever …
- slow-rolling scalar field :

$$\begin{cases} \rho = \dot{\phi}^2 / 2 + V(\phi) \\ p = \dot{\phi}^2 / 2 - V(\phi) \end{cases}$$



❑ INFLATION with slow-rolling scalar field :

2 BONUS !!!

- mechanism for generation of cosmological perturbations

quantum fluctuations of scalar field — *wavelength amplification* → stochastic background of curvature perturbations

predictions : coherent, gaussian, adiabatic, scale-invariant ⟸ VALIDATED BY OBSERVATIONS

- mechanism for generation of first particles : PREHEATING

end of inflation → oscillations of scalar field → particle production

« late problems » in the Hot Big Bang scenario :

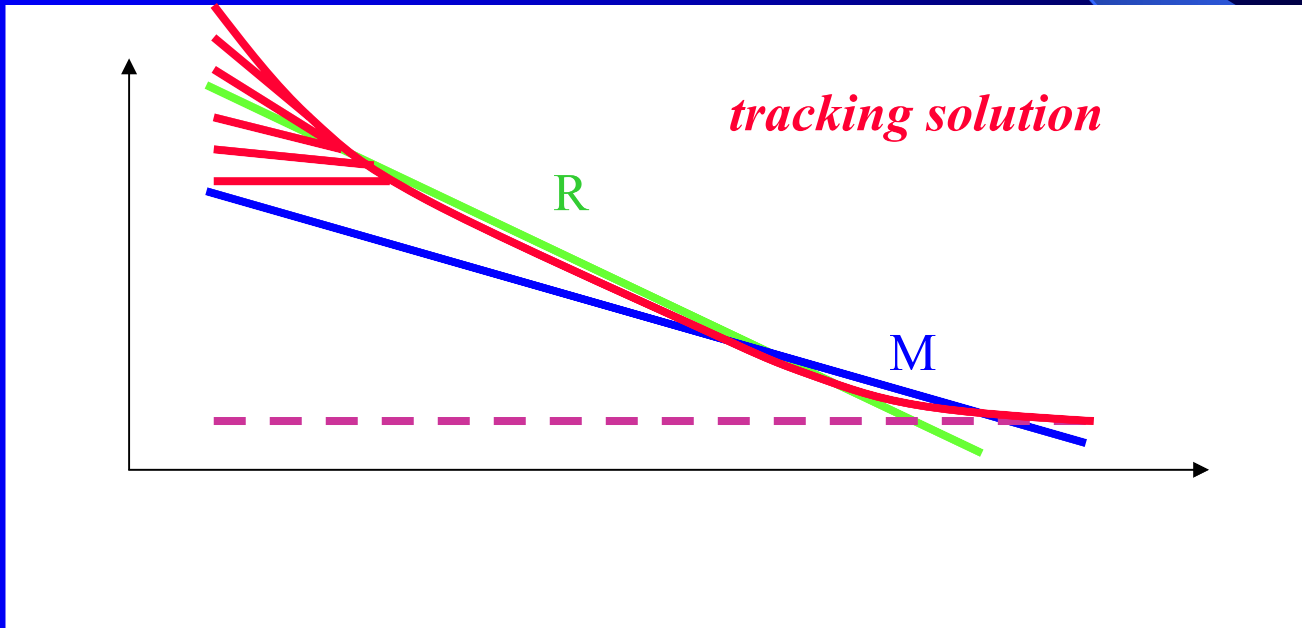
- magnitude of Λ :

$\rho_\Lambda^{1/4} \sim 10^{-3}$ eV !!!

- problem for particle physicists :
  - ↳ *killing* $\rho_\Lambda^{1/4} \sim$ MeV / TeV ...
- problem for cosmologists :
  - ↳ *generating such small number with respect to* $\rho_P^{1/4} \sim 10^{18}$ GeV ...
- « Cosmic coincidence problem » :
  - ↳ *why Λ domination today ?*

- many… unappealing proposals for Dark Energy, e.g. :
  - slow-rolling scalar field : « quintessence »

$$\begin{cases} \rho = \dot{\phi}^2/2 + V(\phi) \\ p = \dot{\phi}^2/2 - V(\phi) \end{cases}$$



❑ « late problems » in the Hot Big Bang scenario :

➢ magnitude of Λ : $\rho_\Lambda^{1/4} \sim 10^{-3}$ eV !!!

unsolved

- problem for particle physicists :

  ↳ *killing* $\rho_\Lambda^{1/4} \sim$ MeV / TeV …

- problem for cosmologists :

  ↳ *generating such small number*

  with respect to $\rho_P^{1/4} \sim 10^{18}$ GeV …

solved but m ~ 10^{-33} eV

improved

- « Cosmic coincidence problem » :

  ↳ *why Λ domination today ?*

CONCLUSION 1

- INFLATION is a convincing, predictive theory… but need to relate to particle physics models (GUT, Susy, strings…)

- no convincing theory for « DARK ENERGY »

CONCLUSION 2

- cosmology has remarkable control on :
  - cosmological parameters
  - nucleosynthesis
  - decoupling
  - structure formation, lensing, etc., etc.

- …but 23 + 73 = 96 % remains MYSTERIOUS !!!

$$dl^2 = a^2(t) \left[\frac{dr^2}{(1-k\,r^2)} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right) \right]$$

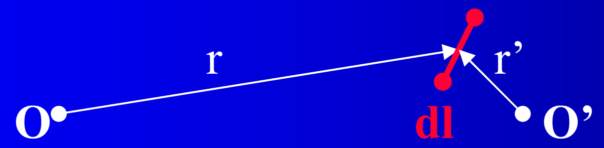
- Remark 1: if $\{ k = 0$ AND $a(t) \equiv$ *constant* $\}$,

  $\bar{r} = a\,r \Rightarrow$ *Euclidian space* $\Rightarrow$ Newton

$$dl^2 = a^2(t) \left[\frac{dr^2}{(1-k\,r^2)} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2\,) \right]$$

- <u>Remark 2 :</u>
  - *Euclidian* : $dl^2 = dr^2 + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2\,) = dx^2 + dy^2 + dz^2$

    ⇨ *does not depend on the choice of origin …*
  - *FLRW* : $k \neq 0$ ⇨ dl *seems to depend on the choice of origin …*



⇨ do solutions with $k \neq 0$ violate the assumption of homogeneity ??

NO, $k \neq 0$ respects homogeneity !!!

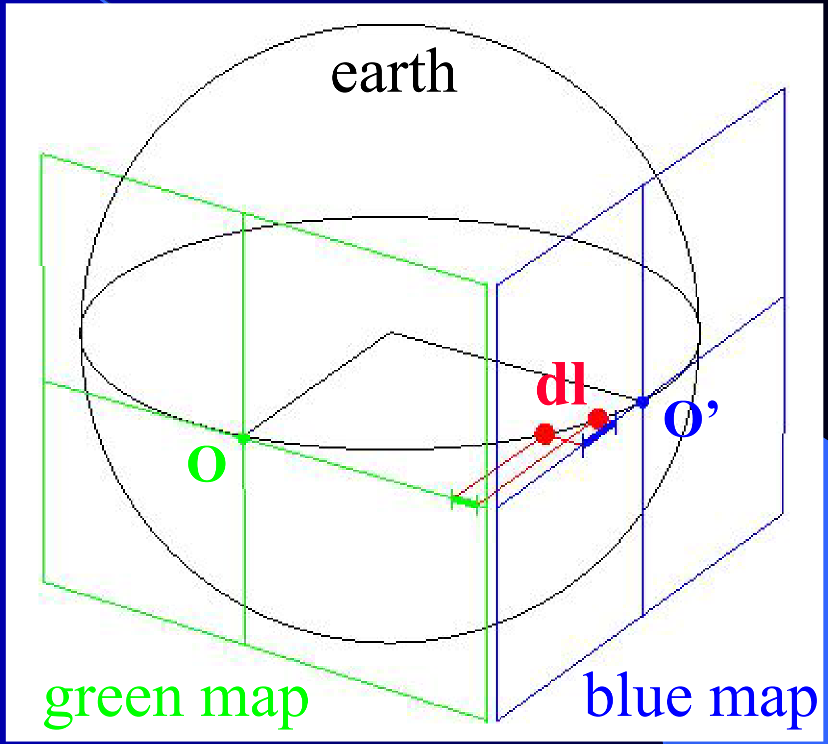
r changes, but also dr, θ, $d\theta$ and $d\phi$: in fact dl is the same

no particular point is priviledged

$$dl^2 = a^2(t) \left[\frac{dr^2}{(1-k\,r^2)} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2\,) \right]$$

- Remark 2 :
  analogy with the 2-D mapping of the earth by axial projection:

  - on each map, the scale *is a function of* r
  - but all points on the sphere are equivalent



… the FLRW model is completely homogeneous !!!

summary of the situation :

basic principles of G.R.	FLRW solution
space-time is *curved*	2 types of curvatures : {a(t), k} $dl^2 = a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\ d\phi^2)\right]$
free-falling objects follow *geodesics*	??????????
curvature caused by / related to *matter*	??????????