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**Resummed Higgs Boson  
Transverse Momentum Distribution at the LHC**

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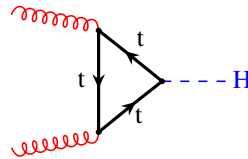


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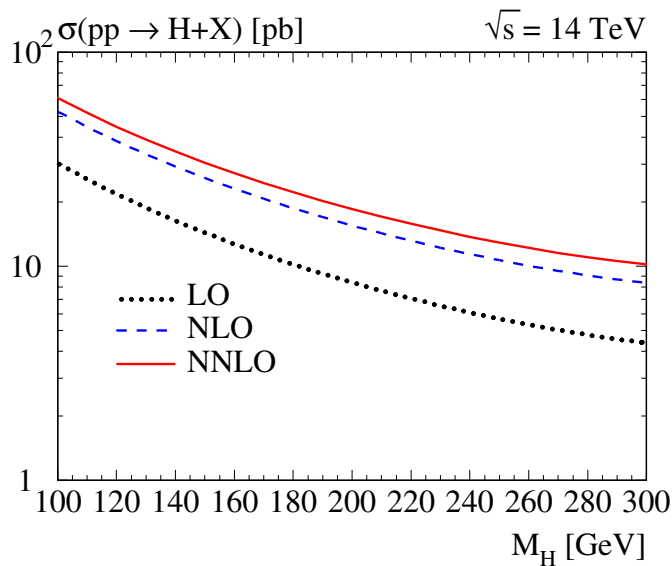
based on work in collaboration with  
G. Stermann and W. Vogelsang,  
Phys.Rev. D69 (2004) 014012 & Phys.Rev. D66 (2002) 014011

# 1. Introduction

- \* Light Higgs at the LHC – main production mechanism is **gluon-gluon fusion**  $gg \rightarrow HX$
- \* LO cross section  $O(\alpha_s^2)$  mediated via a heavy quark loop (SM: only top loop relevant)



for  $m_H \leq 2m_t$  Higgs-gluon interaction described using effective Lagrangian  $\rightarrow$  **effective  $ggH$  vertex**



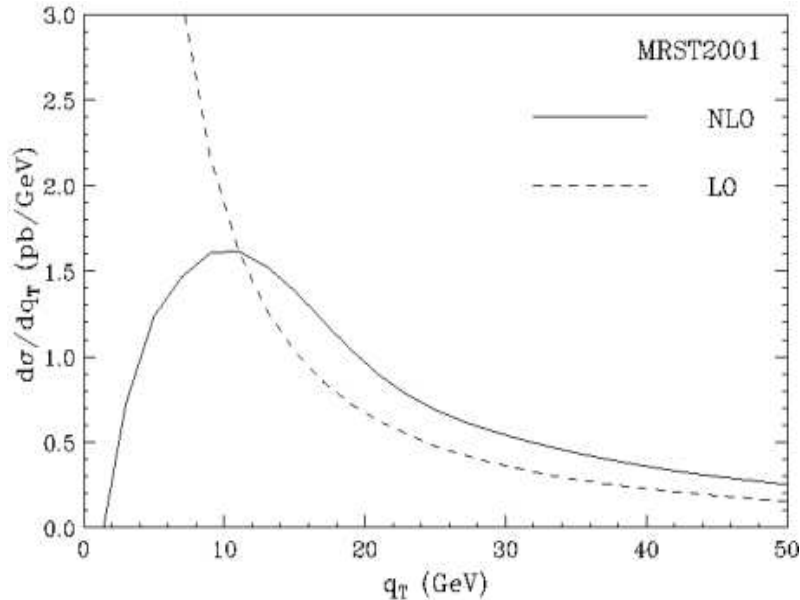
- NLO QCD corrections to the total cross section large  $\sim 70\%$  [Dawson'91] [Djouadi, Spira, Zerwas'91] [Spira, Djouadi, Graudenz, Zerwas'95]
- NNLO QCD corrections also substantial  $\sim 30\%$  [Harlander, Kilgore'02] [Anastasiou, Melnikov'02]
- Dominant part of these contributions comes from **soft & virtual** gluon contributions [Catani, de Florian, Grazzini'01] [Harlander, Kilgore'01]

[from R. Harlander et al. PRL88 (2002) 201801]

## Introduction

Search and analysis strategies for the Higgs boson at the LHC rely on precise knowledge of **production characteristics**

→ transverse momentum ( $Q_T$ ) distribution



[from M. Grazzini, talk at Loops and Legs'04]

\* Small  $Q_T$  regime related to **soft and collinear** gluon emission

## Introduction

Processes with two hadrons in the initial state – predictions based on perturbative calculations and **factorization theorems**

$$\sigma = \sum_q \int dx_1 dx_2 f_q(x_1, Q^2) f_{\bar{q}}(x_2, Q^2) \hat{\sigma}_{q\bar{q} \rightarrow l+l^-}$$

Collinear factorization: separation of short-distance IR-safe  $\sigma_{q\bar{q} \rightarrow l+l^-}$  and long-distance universal parton distribution functions  $f_q$

**However**, for short-distance quantities which are not inclusive enough, perturbation theory may not be applicable in certain kinematic regions (most often near phase-space boundary)

- **real gluon emission suppressed** ( $\Rightarrow$  **soft & collinear gluons**), **virtual emission always allowed**
- Real and virtual corrections cancellation leaves **large logarithmic terms** at each order in perturbation theory

## Introduction

Classes of logarithmic corrections related to soft gluon emission:

- **Explicit corrections** e.g. recoil (transverse momentum distributions)

$$\textit{leading} : \quad \alpha_s^N \frac{\ln^{2N-1} (Q^2/Q_T^2)}{Q_T^2}$$

- **Implicit corrections** e.g. threshold (hadronic collisions at threshold)  $\rightarrow$  integrated over

$$\textit{leading} : \quad \alpha_s^N \frac{\ln^{2N-1} (1-z)}{1-z}$$

inelasticity variable  $z$ , measures distance from the threshold, here  $z = Q^2/\hat{s}$

$\Rightarrow$  large logarithmic corrections close to the phase space edge, in the limit

$$z \rightarrow 1 \text{ (threshold)} \quad \text{and} \quad Q_T \rightarrow 0 \text{ (recoil)}$$

## 2. Resummation

Reorganization of the standard perturbative expansion in  $\alpha_s$  by selecting and summing classes of logarithmic terms to all orders

$$\hat{\sigma} \sim \hat{\sigma}_0 [T_1(\alpha_s L^2) + \alpha_s L T_2(\alpha_s L^2) + \alpha_s^2 L^2 T_3(\alpha_s L^2) + \dots]$$

where e.g.  $T_1$  (first 'Tower' of logarithms) collects terms of the form  $\alpha_s^N \ln^{2N-1}(\dots)$

Resummed cross sections exponentiate

⇒ In general, resummation follows from (re)factorization

(factorization ⇒ evolution eq. ⇒ exponentiation)

- \* **dynamical factorization** (process independent)
  - in the soft limit gluon emission factorizes [*Ermolaev, Fadin '81*] [*Bassetto, Ciafaloni, Marchesini '83*]
- \* **phase-space factorization** (process dependent, often performed in a conjugate space)

## Resummation

Resummed cross sections exist for both:

→ **recoil** resummation (in Fourier conjugate to  $Q_T$ ,  $\mathbf{b}$  space) [Collins, Soper '83] [Collins, Soper, Sterman '85]

$$\delta\left(\mathbf{Q}_T - \sum_i \mathbf{k}_T^i\right) = \frac{1}{2\pi^2} \int d^2b e^{i\mathbf{b}(\mathbf{Q}_T - \sum_i \mathbf{k}_T^i)} \quad \ln(Q/Q_T) \leftrightarrow \ln(Qb)$$

→ **threshold** resummation (in Mellin  $N$  space) [Sterman '87] [Catani, Trentadue '89]

$$\delta\left(1 - z - \sum_i z_i\right) = \frac{1}{2\pi i} \int_C dN e^{-N(1-z-\sum_i z_i)} \quad \ln(1-z) \leftrightarrow \ln N$$

and are of the form

$$\hat{\sigma} = \hat{\sigma}_0 \int_{\text{inv}} C \exp(\mathcal{S})$$

where

$$\mathcal{S} = \underbrace{L f_1(\alpha_s L)}_{LL} + \underbrace{f_2(\alpha_s L)}_{NLL} + \underbrace{\alpha_s f_3(\alpha_s L)}_{NNLL} + \dots$$

$L = \ln(bQ), \ln(N)$

$C$  contains finite contributions

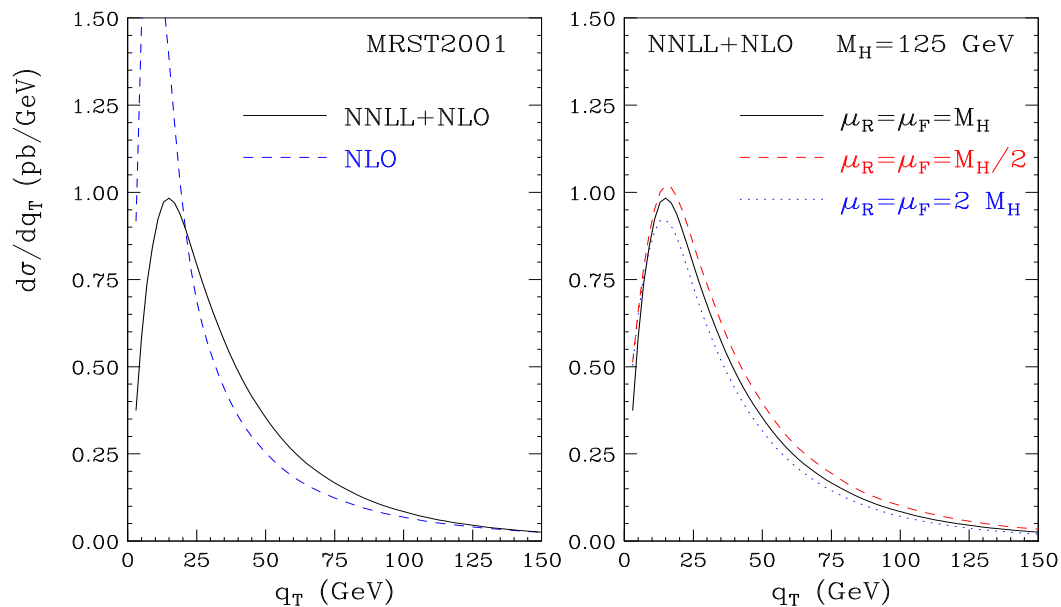
All singular dependence on  $L$  exponentiates

## Resummation for Higgs production

### Transverse momentum resummation

Many recent applications & developments of the “standard” Collins-Soper-Sterman ( $b$ -space) formalism [Bozzi, Catani, de Florian, Grazzini '03] [AK, Stirling '03] [Berger, Qiu '02] [Balazs, Yuan '00]

Comparison (also with PYTHIA and HERWIG predictions) can be found in the latest Les Houches workshop proceedings (hep-ph/0403052)



[from Bozzi et al. PLB564 (2003) 65]

→ NNLL matched to NLO at high  $Q_T$

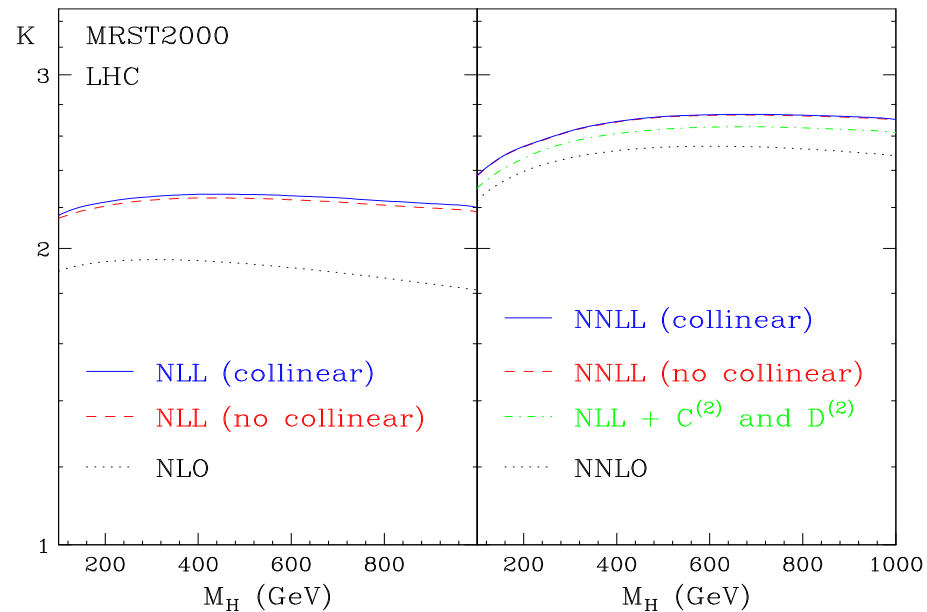
→ unitarity constraint imposed such that the total cross section recovered after integration over  $Q_T$



## Resummation for Higgs production

### Threshold resummation

[Catani, de Florian, Grazzini, Nason '02]



- better convergence to fixed order at NNLL  $\Rightarrow$  6-9% wrt. NNLO (13-20% at NLL)
- dominant subleading collinear terms taken into account
- colour charge  $C_A$  for gluons  $\rightarrow$  increased sensitivity to Sudakov logarithms
- gluon induced processes sensitive to radiation at low  $x$

### 3. Joint resummation

Joint resummation: **combined resummation of threshold and recoil** corrections [Laenen, Sterman, Vogelsang '00]

⇒ refactorization of short- and long-distance physics at fixed transverse momentum and energy

$$\frac{d\sigma^{\text{res}}}{dQ^2 dQ_T^2} = \sum_a \sigma_a^{(0)} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} C_a(Q, b, N, \mu, \mu_F) \exp \left[ S_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) \right] C_{\bar{a}}(Q, b, N, \mu, \mu_F)$$

where  $S_{a\bar{a}}^{\text{PT}}$  summarizes QCD dynamics in the transverse momentum range  $Q/\chi \leq k_T \leq Q$  and up to NLL accuracy in  $N$  and  $b$  [AK, Sterman, Vogelsang '02]

$$S_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) = - \int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[ A_a(\alpha_s(k_T)) \ln \left( \frac{Q^2}{k_T^2} \right) + B_a(\alpha_s(k_T)) \right]$$

$$A(\alpha_s) = \sum_{i=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^i A^{(i)}, \quad B(\alpha_s) = \sum_{i=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^i B^{(i)}$$

$$C_a(Q, b, N, \mu, \mu_F) = \sum_{j,k} C_{a/j}(N, \alpha_s(\mu)) \mathcal{E}_{jk}(N, Q/\chi, \mu_F) f_k(N, \mu_F), \quad C(\alpha_s) = \sum_{i=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^i C^{(i)}$$

$\mathcal{E}$  - evolution matrix from  $\mu_F$  to  $Q/\chi$

at LO  $\mathcal{E} = \exp \left( \frac{\gamma_N^0}{2\pi} \int_{\mu_F^2}^{Q^2/\chi^2} \frac{dk_T^2}{k_T^2} \alpha_s(k_T^2) \right)$

## Joint resummation

⇒ sums threshold and recoil corrections to NLL accuracy

⇒ recovers threshold and recoil resummation formalisms up to NLL in the appropriate limits,

$N \rightarrow \infty$  (at fixed  $b$ )

$$\text{Threshold} \rightarrow \sigma(N) \sim \sum_q \exp \left\{ 2 \int_{Q^2/\bar{N}^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu}^2)) \ln \left( \frac{\bar{\mu}\bar{N}}{Q} \right) \right\} f_q^N(Q) f_{\bar{q}}^N(Q),$$

$b \rightarrow \infty$  (at fixed  $N$ )

$$\begin{aligned} \text{Recoil} \rightarrow \sigma(b) &\sim \sum_q \exp \left\{ - \int_{Q^2/\bar{b}^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln \left( \frac{Q^2}{\bar{\mu}^2} \right) A(\alpha_s(\bar{\mu}^2)) + B(\alpha_s(\bar{\mu}^2)) \right] \right\} f_q^N(Q/\bar{b}) f_{\bar{q}}^N(Q/\bar{b}) \\ &\rightarrow \sum_q \exp \left\{ 2 \int_{Q^2/\bar{b}^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} A(\alpha_s(\bar{\mu}^2)) \ln \left( \frac{\bar{\mu}\bar{N}}{Q} \right) \right\} f_q^N(Q) f_{\bar{q}}^N(Q) \end{aligned}$$

$\chi$  is chosen to be

$$\chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \eta \bar{b}/\bar{N}}; \quad \bar{N} \equiv N e^{\gamma_E}; \quad \bar{b} \equiv b Q e^{\gamma_E}/2; \quad \eta = \text{const.}$$

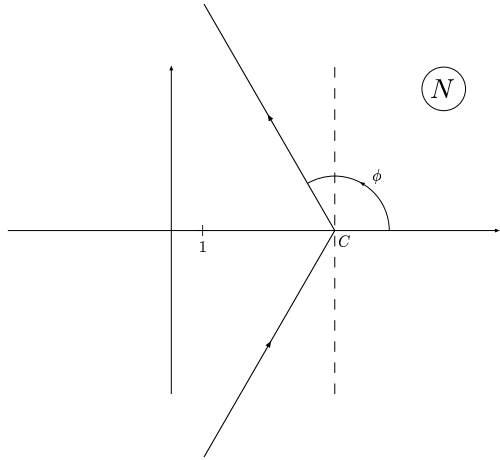
⇒ includes the leading  $\alpha_s^N \ln^{2N-1} N/N$  collinear non-soft terms to all orders

## Joint resummation: transforms

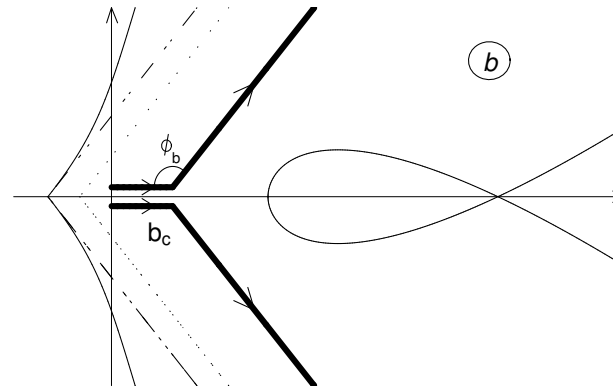
\* Inverse  $\bar{N}$  and  $b$  transforms – integrals along contours in  $(\bar{N}, b)$  complex space

\* Regularization of running coupling singularity – ‘Minimal Prescription’ [Catani et al.’96]

N contour:



b contour:



Landau pole:

$$\beta = \frac{1}{2} \Rightarrow \chi(\bar{N}, \bar{b}) = \exp[1/(2b_0\alpha_s(\mu))] \equiv \rho_L$$

*rightmost pdf singularity  $< C < \rho_L$*

$$\chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \frac{\bar{b}}{4\bar{N}}} = \exp[1/(2b_0\alpha_s(\mu))] \equiv \rho_L$$

$$\rightarrow \chi = 0 \Rightarrow \bar{b} = -2\bar{N}$$

$$\rightarrow 1 + \frac{\bar{b}}{4\bar{N}} = 0 \Rightarrow \bar{b} = -4\bar{N}$$

\* unambiguous definition of resummed perturbation theory without introducing any additional dimensional scales

\* functional form of non-perturbative corrections implied

## Joint resummation for Higgs production

In a specific resummation scheme [Catani, de Florian, Grazzini '00]

$$\begin{aligned} \frac{d\sigma^{\text{res}}}{dQ^2 dQ_T^2} &= \pi\tau\sigma_0^H \delta(Q^2 - M_H^2) H(\alpha_s(Q^2)) \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \\ &\times C_{g/A}(Q, b, N, \mu, \mu_F) \exp \left[ S_{gg}^{\text{PT}}(N, b, Q, \mu) \right] C_{g/B}(Q, b, N, \mu, \mu_F) \end{aligned}$$

where  $H$  collects the effect of hard virtual corrections

$$H(\alpha_s) = 1 + \frac{\alpha_s}{2\pi} (2\pi^2 + 11)$$

and [Kodaira, Trentadue '82] [Catani, D'Emilio, Trentadue '88] [de Florian, Grazzini '00]

$$C_{g/g}^{(1)}(N, \alpha_s) = 1 + \frac{\alpha_s}{4\pi} \pi^2, \quad C_{g/q}^{(1)}(N, \alpha_s) = \frac{\alpha_s}{2\pi} C_F \frac{1}{N+1} = C_{g/\bar{q}}(N, \alpha_s),$$

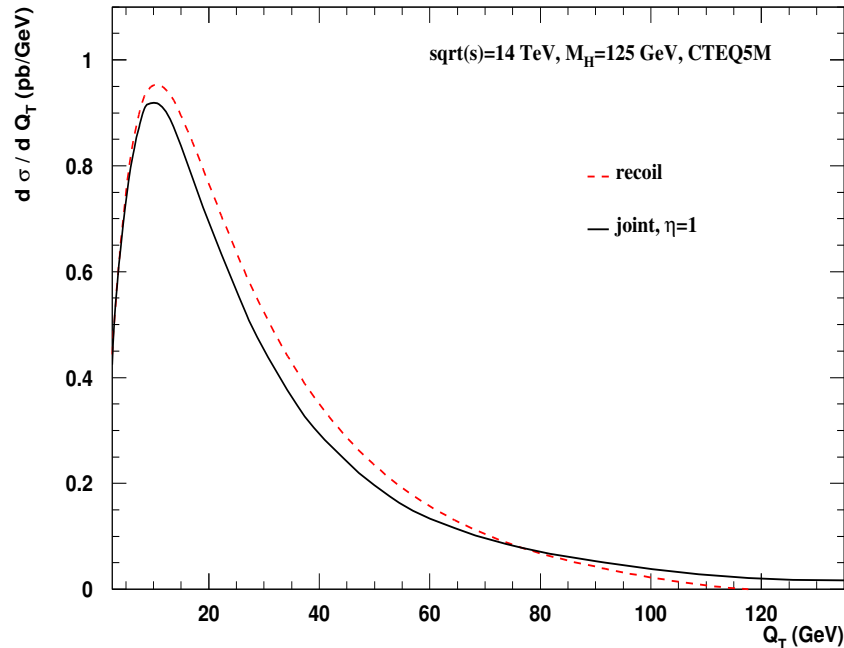
$$A_g^{(1)} = C_A, \quad B_g^{(1)} = -\frac{1}{6}(11C_A - 4T_R N_F), \quad A_g^{(2)} = \frac{C_A}{2} \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R N_F \right],$$

$$B_g^{(2)} = C_A^2 \left( -\frac{4}{3} + \frac{11}{36} \pi^2 - \frac{3}{2} \zeta_3 \right) + \frac{1}{2} C_F T_R N_F + C_A N_F T_R \left( \frac{2}{3} - \frac{\pi^2}{9} \right).$$

In principle, joint resummation valid only up to NLL (formal extension to NNLL – work in progress)

Contribution from NNLL terms with  $B^{(2)}$ ,  $C^{(1)}$  and  $H$  numerically significant – but easy to include

## Joint resummation [AK, Sterman, Vogelsang '04]



- NLL with the NNLL coefficients  $B^{(2)}$ ,  $C^{(1)}$  and  $H$  included
- matched to LO  $Q_T$  distribution
- $\mu = \mu_F = Q = m_H$
- non-perturbative term  $-gb^2$  added to the exponent,  $g = 1.67\text{GeV}^2$ .

- joint resummation comparable with recoil resummation in the low  $Q_T$  region
- relatively low influence of threshold effects at low  $Q_T$ ; applicability of recoil approach confirmed
- integrated distribution contains non-leading contributions beyond threshold resummation  
→ associated with *small*  $-x$  terms from parton evolution and decreasing the distribution

## 4. Conclusions

### Resummation:

- recoil and threshold resummation methods well established
- results for Higgs production at the LHC available; many new developments

### Joint resummation:

- successfully sums threshold and recoil corrections to next-to-leading log accuracy
- recovers threshold and recoil resummation formalisms in the appropriate limits
- provides result for Higgs  $Q_T$  distribution confirming applicability of recoil resummation at low  $Q_T$
- shows corrections due to threshold effects small at low  $Q_T$