Particle Physics in one page

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\bar{\psi} \not{D} \psi$$ *The gauge sector* (1)

$$+ \psi_i \lambda_{ij} \psi_j h + h.c.$$ *The flavor sector* (2)

$$+ |D_\mu h|^2 - V(h)$$ *The EWSB sector* (3)

$$+ N_i M_{ij} N_j$$ *The ν-mass sector* (4)
(if Majorana)

(1) : best tested, at least to per-mille accuracy

(2) + (4) : main developments of last 5 years, different in nature, both highly significant

(3): the most elusive, so far

The history (I)

1860's	Maxwell's theory of electromagnetism	
1896	Discovery of radioactivity	Becquerel, P. Curie, M. Curie
1897	Discovery of the electron	Thompson
1930	The neutrino hypothesis	Pauli
1934	The theory of □-decay	Fermi
1940's	Formulation of QED	Feynman, Schwinger, Tomonaga
1957	Discovery of parity violation	Lee, Yang
1964	Discovery of CP violation	Cronin, Fitch

The history (II)

1960's The SM formulated — Glashow, Weinberg, Salam;

1960/70's Discovery of matter triplication — Richter, Ting
Lederman, Schwartz, Steinberger
Perl

1960/70's Discovery of the quarks and formulation of QCD
Gell-mann
Friedman, Kendall, Taylor
Gross, Politzer, Wilczek

1971-72 The SM prooved renormalizable — 't Hooft, Veltman

1973 Discovery of the neutral current

1983 Discovery of the weak bosons — Rubbia, Van der Meer

1990/2000's Discovery of neutrino masses — Davis, Koshiba

Program

1. The basic structure
2. The gauge sector
3. Flavor and CP
4. The neutrino-mass sector
5. ElectroWeak Symmetry Breaking

Notes:

1. A symbol (!?) means need of work/thought by the interested student
2. "One cannot teach what is worth learning" (Oscar Wilde?)
3. Factors of 2 and π await confirmation

Lecture 1

The basic structure of the theory

From the Fermi theory of β-decay and QED to the Standard Model

1. There was the Fermi theory of $n \to p + e + \square$

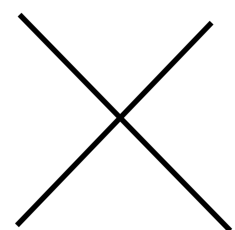
$$\mathcal{L}_{\mathcal{F}} = \frac{G}{\sqrt{2}} \cos\square_C (\bar{p}\square_\mu(1+\square\square_5)n)(\bar{e}\square_\mu(1+\square_5)\square)$$

$$\Rightarrow \quad \square = \frac{G^2\square^5}{60\square^3}\cos^2\square_C(1+3\square^2)0.47 \qquad \square = 1.29MeV$$

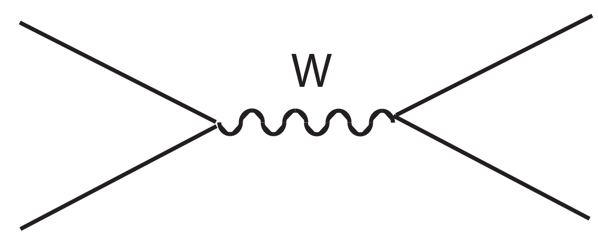
$$\square_n = 1/\square = 885.7 \pm 0.8 sec; \qquad \square = -1.2695 \pm 0.0029$$

$$\Rightarrow \quad G^{-1/2} \approx 250GeV$$

2. Mimicking QED



$\Rightarrow$



$$\mathcal{L}_I = \frac{g}{\sqrt{2}} W_\mu^+ J_\mu^- + h.c.; \quad J_\mu^- = (\bar{u}\square_\mu(1+\square_5)d + \bar{e}\square_\mu(1+\square_5)\square)$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

(switching to quarks and setting $\square$ = 1)

A field theory interlude: A Gauge theory

Defined by:

1. The gauge group G with its generators $T^A; \quad A = 1, \ldots, N,$
   their Lee algebra $[T^A, T^B] = i f^{ABC} T^C$
   and one gauge boson for every $T^A, \quad A_\mu^A$

2. The matter fields:
   - the fermions $\square^a, \quad a = 1, \ldots, n,$
     transforming as a rep $r_\square$ with generators $t^A \quad (n \times n)$
   - the scalars $\square^\square, \quad \square = 1, \ldots, m,$
     transforming as a rep $r_\square$ with generators $\square^A \quad (m \times m)$

⇒ The Gauge- and Lorentz- invariant Lagrangian

$$\mathcal{L} = \mathcal{L}(F^A_{\mu\nu}, \psi, D_\mu\psi, \phi, D_\mu\phi)$$

involving:

the field strengths $F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + f^{ABC} A^B_\mu A^C_\nu$

the covariant derivatives $D_\mu = \partial_\mu - i A^A_\mu t^A(\theta^A)$

⇒ The minimal Gauge Lagrangian

$$\mathcal{L}^g = -\frac{1}{4} F^A_{\mu\nu} F^A_{\mu\nu} - i\bar{\psi} \not{D}\psi - |D_\mu\phi|^2$$

$$\mathcal{L}_I = A^A_\mu \bar{\psi}\gamma_\mu t^A \psi$$

Properties of $\mathcal{L}^g$

1. It has only one relevant (!?) dimensionless parameter per simple group factor
2. It is renormalizable (if and only if it has no anomaly: $D_{ABC} = Tr(\{t^A, t^B\} t^C) = 0$ for every A,B,C (with $t^{A,B,C}$ on □ all left-handed))

More in general, with $D_{ABC} = 0$, any gauge Lagrangian is renormalizable if it contains only monomials of mass dimension d≤4

(h = c = 1, so that $[A_\mu] = [M], \quad [\square] = [M^{3/2}], \quad [\square] = [M]$)

(More on renormalizability later on)

Back to the Fermi theory

$$\mathcal{L}_I^{(0)} = W_\mu^+ J_\mu^- + h.c.; \quad J_\mu^\pm = \bar{Q}\square_\mu \frac{\square^\pm}{2} Q + \bar{L}\square_\mu \frac{\square^\pm}{2} L$$

where (a matter of notation only)

$$Q = (1+\square_5)\begin{pmatrix} p \\ n \end{pmatrix} \qquad L = (1+\square_5)\begin{pmatrix} \square \\ e \end{pmatrix} \qquad \square^\pm = \frac{1}{\sqrt{2}}(\square_1 \pm \square_2)$$

In a general gauge theory (see above)

$$\mathcal{L}_I = A_\mu^A J_\mu^A \quad \text{with} \quad J_\mu^A = \bar{\square}\,\square_\mu t^A \square$$

□ To close the algebra, need to add $\square_3$ to $\square^\pm$ (SU(2))

□ A new interaction $\square\mathcal{L}_I^{(1)} = W_\mu^3 J_\mu^3; \quad J_\mu^3 = \bar{Q}\square_\mu \frac{\square^3}{2} Q + \bar{L}\square_\mu \frac{\square^3}{2} L$

Almost, but not quite, QED because the charges are wrong: ± 1/2

⇒ Need an extra neutral interaction

$$\Delta\mathcal{L}_I^{(2)} = B_\mu J_\mu^B; \quad J_\mu^B = Y_Q \bar{Q}\gamma_\mu Q + Y_L \bar{L}\gamma_\mu L$$

with Y_Q, Y_L chosen (!?) so that $T_3 + Y = Q_{em}$

$$\begin{pmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{pmatrix} + \begin{pmatrix} Y_Q \\ Y_Q \\ Y_L \\ Y_L \end{pmatrix} = \begin{pmatrix} 2/3 \\ -1/3 \\ 0 \\ -1 \end{pmatrix} \qquad Y_Q = \frac{1}{6} \qquad Y_L = -\frac{1}{2}$$

QED not yet fully included, because of left handed-ness

$$\Rightarrow \quad \Delta\mathcal{L}_I^{(3)} = B_\mu \Delta J_\mu^B; \quad \Delta J_\mu^B = Y_{u_c}\bar{u}_c\gamma_\mu u_c + Y_{d_c}\bar{d}_c\gamma_\mu d_c + Y_{e_c}\bar{e}_c\gamma_\mu e_c$$

with $Y_{u_c} = -2/3, \quad Y_{d_c} = 1/3, \quad Y_{e_c} = 1$ to maintain $T_3 + Y = Q_{em}$

(Don't be surprised by change of sign: u_c, d_c, e_c are the charge conjugate of the standard right-handed fields)

Summary of the candidate gauge theory

$$G = SU(2) X U(1) \qquad T^A = (I^a, Y) \quad A = 1,\ldots,4 \quad a = 1,2,3$$

with $\quad [I^a, I^b] = i\epsilon^{abc} I^c, \quad [I^a, Y] = 0$

and matter multiplets $\quad \psi^T_{ri} = (Q^a, u^a_c, d^a_c, L, e_c, N)_i \qquad a = B,R,G \quad i = 1,2,3$

Altogether, this makes 16 Weyl spinors, $r = 1,\ldots,16$, transforming as

(**2**, 1/6)⊕(**1**, -2/3)⊕(**1**, 1/3)⊕(**2**, -1/2)⊕(**1**, 1)⊕(**1**, 0)

(Note that I have added an extra field, N, with no gauge int.s)

so that:

$$\mathcal{L}^g = -\frac{1}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{4g'^2} B_{\mu\nu} B_{\mu\nu}$$

$$+ i\bar{Q}\, \not{D} Q + i\bar{L}\, \not{D} L + i\bar{u}_c\, \not{D} u_c + i\bar{d}_c\, \not{D} d_c + i\bar{e}_c\, \not{D} e_c + i\bar{N}\, \not{\partial} N$$

$$\Leftarrow -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + i\bar{\psi}\, \not{D} \psi$$ of the first page

From $\mathcal{L}^g$ to a realistic Lagrangian

1. The gauge symmetry is certainly unrealistic.
   How about other (global) symmetries?

$Q \rightarrow \exp i\square_Q Q, \quad L \rightarrow \exp i\square_L L,$ etc. □ 6 conserved charges (U(1)'s)
one combination of which is Y itself
□ 5 conserved charges: B_L, B_R, L_L, L_R, N_N

In fact a much larger symmetry, since no distinction of flavor replicas, yet

$$G^{gl}(\mathcal{L}^g) = SU(3)^6 X U(1)^5$$

Insisting on renormalizability, an unavoidable conclusion since

$$\mathcal{L}^{ren} = \mathcal{L}^g + N_i M_{ij} N_j \qquad \Rightarrow \qquad SU(3)^5 X U(1)^4$$

From $\mathcal{L}^g$ to a realistic Lagrangian (continued)

2. A "minimal" addition:

Introduce a complex scalar doublet $\phi_a = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ = (**2**, 1/2)

and couple it in the most general (!?) renormalizable way:

$$\mathcal{L} = \mathcal{L}^g + \mathcal{L}^Y - V(\phi) + N^T M N$$

where

$$\mathcal{L}^g = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + i \bar{\psi} \not{D} \psi + |D_\mu \phi|^2$$

$$\mathcal{L}^Y = Q^T \lambda^u u_c \phi + Q^T \lambda^d d_c \phi^* + L^T \lambda^e e_c \phi^* + L^T \lambda^\nu N \phi$$

$$V(\phi) = \mu^2 |\phi|^2 + \lambda |\phi|^4$$

$\Rightarrow$ The Lagrangian of page 1

How about symmetries again?

1. Global symmetries

$$\mathcal{L}^Y = Q^T \Box^u u_c \Box + Q^T \Box^d d_c \Box^* + L^T \Box^e e_c \Box^* + L^T \Box^{\Box} N \Box$$

□ $Q_i \to \exp i\Box_B Q_i, \quad u_i^c \to \exp -i\Box_B u_i^c, \quad d_i^c \to \exp -i\Box_B d_i^c$

Baryon Number

□ (neglecting $N^T M N$ since related to neutrino masses, which are small, after all. See Lect.4)

$$L_i \to \exp i\Box_L L_i, \quad e_i^c \to \exp -i\Box_L e_i^c, \quad N_i \to \exp -i\Box_L N_i$$

Lepton Number

2. The gauge symmetry is still there, but a new force, the □ self-interaction in V(□), might break it in a spontaneous way

Gauge symmetry breaking (an anticipation)

The configuration of minimal energy for □ is $<□>= \begin{pmatrix} 0 \\ v \end{pmatrix}$

As a consequence: [Seen by replacing □ with <□> in $\mathcal{L}$ (!?)]

1. The $SU(2)XU(1)$ invariance is not there anymore, but only a residual $U(1)_{em}$: $(T_3+Y)<□>=0$ □ Electric charge defined

2. All vector bosons, but one, pick up a mass

$$A_\mu = \sin □ W^3_\mu + \cos □ B_\mu \quad m^2_A = 0$$

$$Z_\mu = \cos □ W^3_\mu - \sin □ B_\mu \quad m^2_Z = \frac{g^2 v^2}{2\cos^2 □}$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \pm W^2_\mu) \quad m^2_W = \frac{g^2 v^2}{2}$$

$$v^2 = \frac{1}{2\sqrt{2}G}$$

$$\tan □ = \frac{g'}{g}$$

3. Fermion masses appear as well

$$\mathcal{L}_m = u^T □^u u_c v + d^T □^d d_c v + e^T □^e e_c v (+ □^T □^□ N v + N^T M N)$$

4. Of the 4 real components in □, 3 are unphysical ($□_a$) and one is a physical scalar, the Higgs particle

Renormalizability (R) and Gauge Invariance (GI)

The complete Lagrangian enjoys 2 properties:

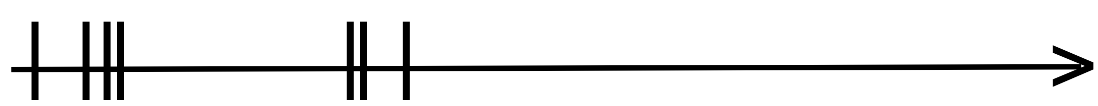
1. It is Renormalizable
2. It is the most general R and Gauge Invariant Lagrangian for the given $G = SU(3) X SU(2) X U(1)$ and particle content

GI is more important than R. The reasons:

A. Adding □$\mathcal{L} = m^2 B_\mu^2$ keeps R but destroys GI

□ The photon becomes massive (!?)

B. Any theory, R or nor R, but GI under G and with the same light spectrum as the SM one is undistinguishable from it (the SM) at sufficiently low energies □



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A note on charge quantization

Although $T_3 + Y = Q$ is a consequence of the theory, the Y's of the different multiplets (hence their charges) are not fixed by GI (!?)

However:

a - Pretend that QED conserves parity

□ $$Y_{u_c} = -1/2 - Y_Q, \quad Y_{d_c} = +1/2 - Y_Q,$$
$$Y_N = -1/2 - Y_L, \quad Y_{e_c} = +1/2 - Y_Q$$

b - Require no anomaly

□ (!?) $$Y_L = -3Y_Q$$

[c - Force $Q_{□} = Q_N = 0$ □ all Y's fixed as in the SM]