Lecture 3
Flavor and CP

3 Theorems

(in spite of the many parameters in $\mathcal{L}$)

Theorem 1: Neglecting neutrino masses, L_e, L_μ and L_τ are separately conserved (and CP is exact in the lepton sector)

Proof: $$\mathcal{L}^{(lept)} = i\bar{L}_i \not{D} L_i + i\bar{e}_i^c \not{D} e_i^c + e_i \lambda_{ij}^e e_j^c (v+h) + (N-terms)$$

Since $\lambda^e = V_L^T \lambda_d^e V_R$ with "d" for "diagonal"

can redefine

$$V_R e^c \Rightarrow e_{ph}^c \qquad V_L L = \begin{pmatrix} V_L \nu \\ V_L e \end{pmatrix} \Rightarrow \begin{pmatrix} \nu_{ph} \\ e_{ph} \end{pmatrix} \equiv L_{ph}$$

so that

$$\mathcal{L}^{(lept)} = i\bar{L}_{ph} \not{D} L_{ph} + i\bar{e}_{ph}^c \not{D} e_{ph}^c + e_{ph}^T \lambda_d^e e_{ph}^c (v+h) + (N-terms)$$

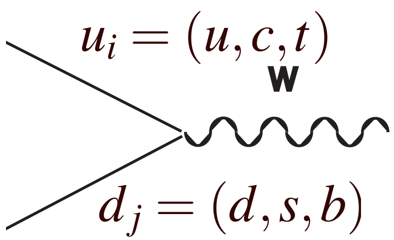
$$(m_e e e_c + m_\mu \mu \mu_c + m_\tau \tau \tau_c)(1 + h/v)$$

essential that ν and e are rotated simultaneously, since

$$Z_\mu \bar{e} \gamma_\mu e, \quad Z_\mu \bar{\nu} \gamma_\mu \nu$$

$$W_\mu \bar{e} \gamma_\mu \nu$$

Theorem 2: In the quarks, all flavor violations reside in the weak charged current amplitude proportional to a unitary matrix



$$= V_{ij}A \quad \text{with} \quad VV^{+} = \mathbf{1}$$

Proof:

$$\mathcal{L}^{(quarks)} = i\bar{Q}\,\not{D}Q + i\bar{u^c}\,\not{D}u^c + i\bar{d^c}\,\not{D}d^c$$

$$+u^T U_L^T \lambda_d^u U_R u^c (v+h) + d^T D_L^T \lambda_d^d D_R d^c (v+h)$$

hence, this time, by going to the physical basis

$$W_\mu \bar{u}\gamma_\mu d \Rightarrow W_\mu \bar{u}_{ph} U_L D_L^{+} \gamma_\mu d_{ph}$$

$$= W_\mu \bar{u}_{ph} V \gamma_\mu d_{ph} \quad \text{with} \quad V = U_L D_L^{+}$$

Theorem 3: Neglecting □-masses, CP is violated in as much as* V is "intrinsically" complex, i.e. a single phase □ is nonzero

Proof: Under a CP transformation, the overall $\mathcal{L}$ is unchanged except for (!?)

$$gW_\mu^+ \bar{u} □_\mu V d + gW_\mu^- \bar{d} □_\mu V^+ u \Rightarrow gW_\mu^- \bar{d} □_\mu V^T u + gW_\mu^+ \bar{u} □_\mu V^* d$$

Counting "intrinsic" phases:

$$N(V_{n\times n}) = n^2$$

$$N(O_{n\times n}) = \frac{n(n-1)}{2}$$

$$\Rightarrow N(phys.phases) = n^2 - \frac{n(n-1)}{2} - (2n-1) = \frac{1}{2}(n^2 - 3n + 2)$$

2n quark phases

$U(1)_B$

n	2	3	4
angles	1	3	6
phys. phases	0	1	3

(* up to the □-problem)

Testing the Theorems

Qualitative, but highly significant:

L_e, L_μ and L_τ -Violations: the benchmark $BR(\mu \to e + \gamma) < 1.2 \cdot 10^{-11}$

Quantitative: (highly interrelated)

$VV^+ = \mathbf{1}$

Calculable Flavour Changing Neutral Current processes

CP-asymmetries

(A major change in the 2000's)

$VV^{+} = \mathbf{1}$

$\sum_i |V_{ai}|^2 = 1 \quad a = 1,2,3$ 3 rel.s (Type I)

$\sum_i V_{ai} V_{ib}^* = 0 \quad a \neq b$ 6 rel.s (Type II)

Type I:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 =$$

$0.9487(10) \quad + \quad ? \quad + \quad 10^{-5}$

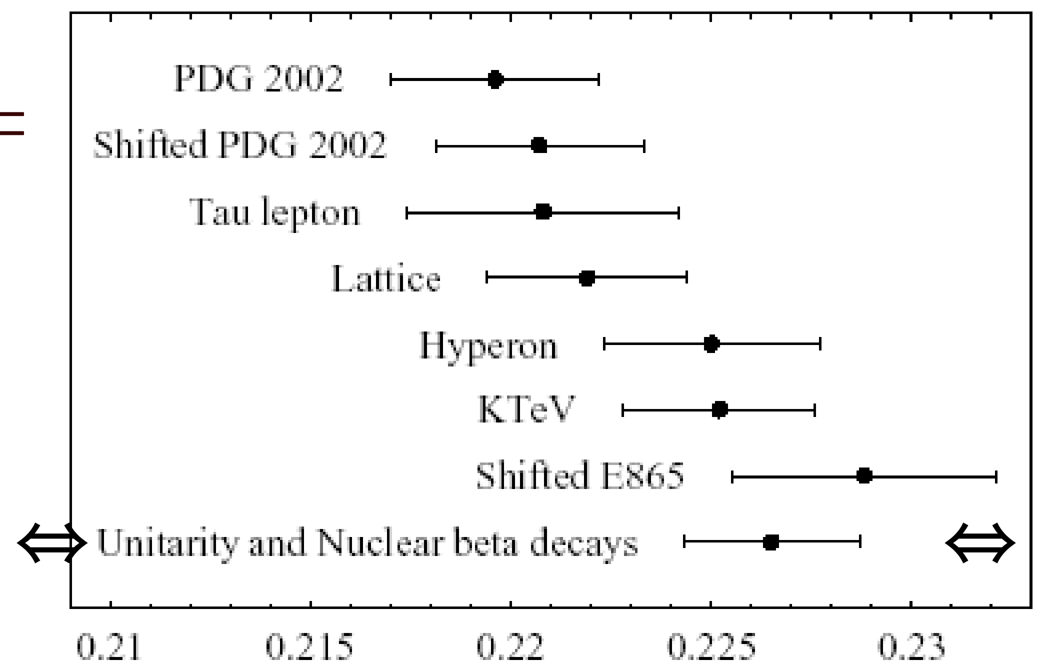
$N \rightarrow N' + e + \nu \Rightarrow |V_{ud} f^{ud}(0)|$

$K \rightarrow \pi + e + \nu \Rightarrow |V_{us} f^{us}(0)|$

$b \rightarrow u + X \Rightarrow |V_{ub}|$

(Czarnecki et al, 2004)

KLOE

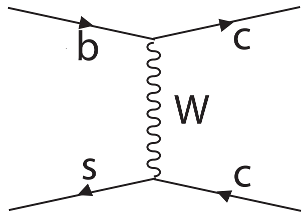


Summary of $|V_{us}|$ determinations

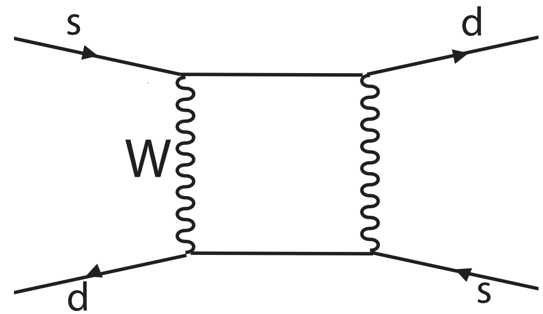
FCNC processes (genuine and calculable)

1. Interesting because absent at tree level
   (Theor. 2: only the W-int.s produce flavor change!)

2. Genuine? E.g.: $b\bar{s} \longrightarrow c\bar{c}$? No



3. Calculable? E.g.: $s\bar{d} \longrightarrow d\bar{s}$?

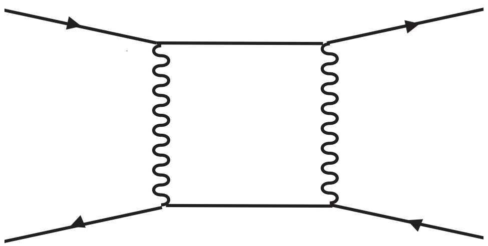


Yes this diagram, but how about its gluon dressing?
It depends on the typical momentum of the int. lines:
If small (≤ 1 GeV) no, if large yes.

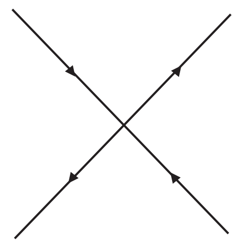
$\Rightarrow$ $\Delta m_{K\bar{K}}$ (the "real part") no (!?)
ϵ_K (the "imag. part") yes (!?)

The actual computation of a FCNC

1. The "short-distance" EW loop

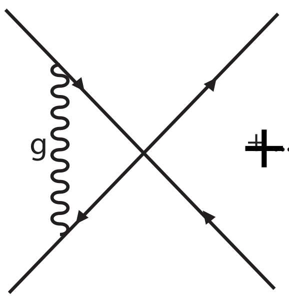


$\Rightarrow$



: an effective operator $\widehat{O}$ with a known coefficient C

2. The gluon dressing



+ ...

:generally divergent

$$\Rightarrow C(\alpha_S \log\frac{M}{m}, \alpha_S) \quad M = M_W, m_t \quad m = m_c, m_b$$

Need to resum all orders (RG)

3. The "matrix element" for the actual physical process

$$A_{i\to f} = C < f|\widehat{O}|i >$$

The Flavor Precision Test (FPT) program

(compare with the EWPT)

Genuine FCNC processes induced by a “calculable” loop

		Exp	Th
ϵ_K	$\bar{s}d \to \bar{d}s$	1%	5-10%
ϵ_K'	$\bar{s}d \to \bar{q}q$	10%	100%
$K^+ \to \pi^+ \nu\bar{\nu}$	$\bar{s}d \to \bar{\nu}\nu$	70%	5%
Δm_{B_d}	$\bar{b}d \to \bar{d}b$	1%	10%
$A_{CP}(B_d \to \psi K_S)$	$\bar{b}d \to \bar{d}b$	5%	<1%
$B_d \to X_s + \gamma$	$b \to s + \gamma$	10%	5-10%
$B_d \to X_s + l\bar{l}$	$b \to s + l\bar{l}$	20%	5-10%

Large room for improvements in precision, redundancy, new entries

An example of redundancy

(Ligeti 2004)

Dominant process	final state	SM upper limit on $\lvert\sin 2\beta_{\rm eff} - \sin 2\beta\rvert$	$\sin 2\beta_{\rm eff}$	C_f
$b \to c\bar{c}s$	ψK_S	< 0.01	$+0.726 \pm 0.037$	$+0.031 \pm 0.029$
$b \to c\bar{c}d$	$\psi\pi^0$	~ 0.2	$+0.40 \pm 0.33$	$+0.12 \pm 0.24$
	$D^{*+}D^{*-}$	~ 0.2	$+0.20 \pm 0.32$	$+0.28 \pm 0.17$
$b \to s\bar{q}q$	ϕK^0	~ 0.05	$+0.34 \pm 0.20$	-0.04 ± 0.17
	$\eta' K_S$	~ 0.1	$+0.41 \pm 0.11$	-0.04 ± 0.08
	$K^+K^-K_S$	~ 0.15	$+0.53 \pm 0.17$	$+0.09 \pm 0.10$
	$\pi^0 K_S$	~ 0.15	$+0.34 \pm 0.28$	$+0.09 \pm 0.14$
	$f_0 K_S$	~ 0.15	$+0.39 \pm 0.26$	$+0.14 \pm 0.22$
	ωK_S	~ 0.15	$+0.75 \pm 0.66$	-0.26 ± 0.50

CP-asymmetries $\sin 2\beta_{eff}$ for which the SM predicts $\sin 2\beta$, all equal to each other, with some process-dependent uncertainty

CP violation

Useful to "integrate out" the heavy particles (t, W, Z) to obtain an $\mathcal{L}^{eff}$. Which operators can give rise to CP-violation?

□ In order of increasing dimensionality (= decreasing relevance):$^{+}$

dim 5: quarks Electric Dipole Moments (!?)

$$\mu \bar{q}_L \square_{\mu\square} q_R F^{\mu\square} + m \bar{q}_L q_R$$ with □/m complex

$$\Rightarrow d_{neutron}(SM) \approx 10^{-31} e \cdot cm$$ against

$$d_{neutron}(exp) \leq 6 \cdot 10^{-26} e \cdot cm \approx 10^{-11} \frac{e}{2m_N}$$

[$d_e(SM) \approx 0$ against

$$d_e(exp) = (0.07 \pm 0.07) 10^{-26} e \cdot cm \approx 10^{-16} \mu_B$$]

$^{+}$neglecting $\square G^a_{\mu\square} G^a_{\square\square} \square^{\mu\square\square\square}$ since □ is a parameter which may be set to 0, maybe by a dynamical mechanism (the axion?)

CP violation (continued)

dim 6: FCNC $(\bar{q}q)(\bar{q}q)$ interactions (!?)

now clearly seen in:

$$\epsilon_K \quad \Rightarrow \Delta S = 2/\Delta S = 1$$

$$\epsilon'_K \quad \Rightarrow \Delta S = 1$$

$$A_{CP}(B_d \to \psi K_S) \quad \Rightarrow \Delta B = 2/\Delta B = 1$$

$$A_{CP}(B_d \to \pi^+ K^-) \quad \Rightarrow \Delta B = 1$$

Examples of theoretically clean asymmetries

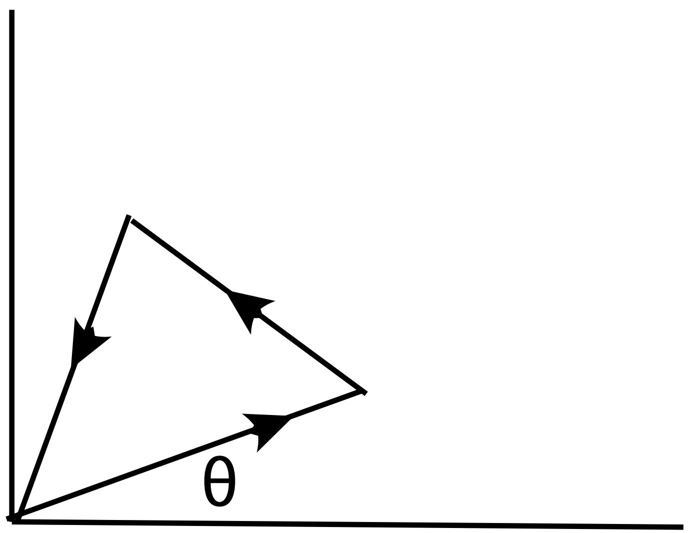
Measurement (in SM)	Theoretical limit	Present error
$B \to \psi K_S \quad (\beta)$	$\sim 0.2^\circ$	1.6°
$B \to \phi K_S,\ \eta^{(\prime)} K_S,\ \ldots\ (\beta)$	$\sim 2^\circ$	$\sim 10^\circ$
$B \to \pi\pi,\ \rho\rho,\ \rho\pi \quad (\alpha)$	$\sim 1^\circ$	$\sim 15^\circ$
$B \to DK \quad (\gamma)$	$\ll 1^\circ$	$\sim 25^\circ$
$B_s \to \psi\phi \quad (\beta_s)$	$\sim 0.2^\circ$	—
$B_s \to D_s K \quad (\gamma - 2\beta_s)$	$\ll 1^\circ$	—

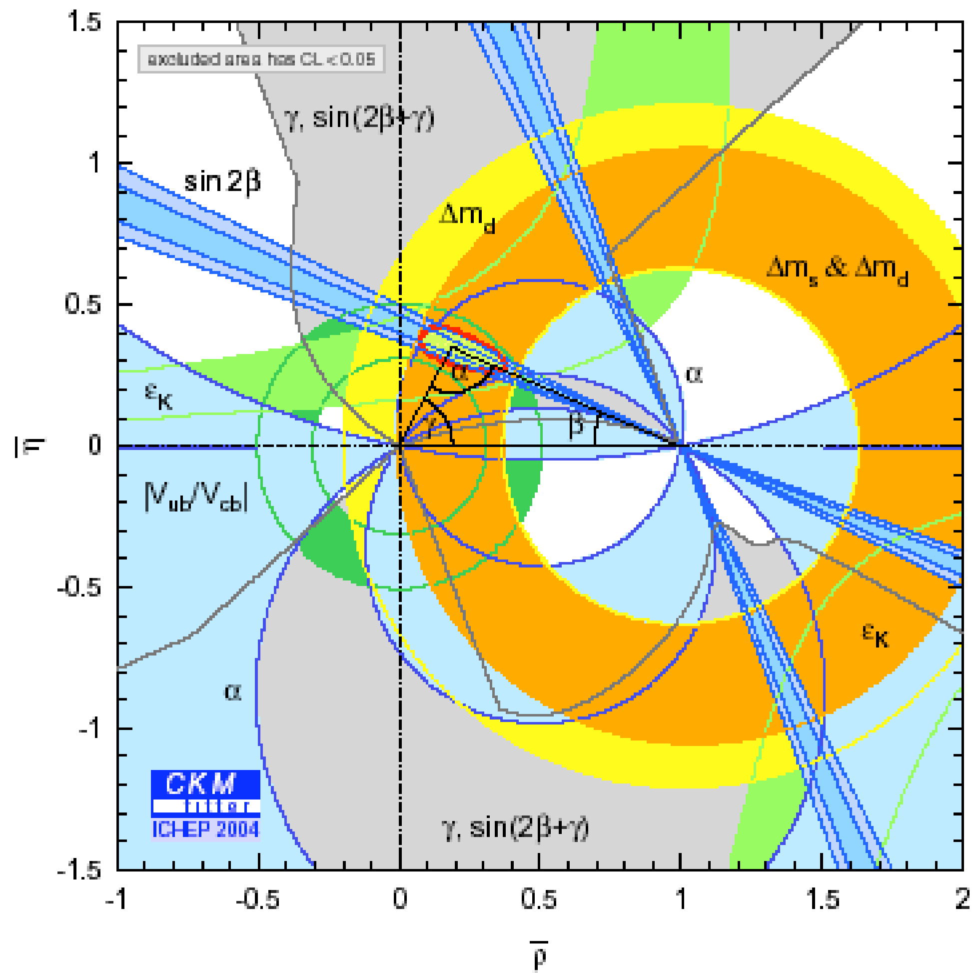
(Ligeti 2004)

The current comparison with data (2004)

type II $\quad V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$

represented as:





the angle □ has no physical meaning