Lecture 4

The neutrino-mass sector

$$\mathcal{L}^{(\nu-mass)} = L_i \lambda^{\nu}_{ij} N_j v + N_i M_{ij} N_j$$

1. $M_{ij} = 0$ DIRAC masses

- Neutrinos are Dirac spinors ($\nu_L, \nu_R \equiv N^C$), like charged fermions
- Lepton number is exactly conserved, like Baryon number
- In the mass-eigenstate basis $W_\mu \bar{e}_i \gamma_\mu V^{\dagger}_{ij} \nu_j$ with $V^{\dagger}$ like V
  - Physical parameters:

    3 masses, 3 angles and 1 phase

- A crucial question: Why m_ν's $<< m_e, m_q$'s given the symmetry between neutrinos and charged fermions?

Technically, by $\lambda^{\nu} << \lambda^{u}, \lambda^{d}, \lambda^{e}$

$$\mathcal{L}^{(\nu-mass)} = L_i \lambda_{ij} N_j v + N_i M_{ij} N_j$$

2. $M_{ij} \neq 0$ MAJORANA masses

⇒ A basic asymmetry between neutrinos and charged fermions

$$\mathcal{L}^{(\nu-mass)} = (\nu^T N^T) \begin{pmatrix} 0 & \lambda v \\ \lambda v & M \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}$$

with ν, N each 3-vectors and λ, M 3x3 matrices

⇒ a 6x6 mass matrix ⇒ 6 different eigenvalues (in general)

How come, with the same number of d.o.f. as for charged leptons? (e, e^c) ⇒ (ν, N)

A Dirac spinor: e (⇑⇓) + $\bar{e}$ (⇑⇓) : 4 d.o.f.

A Majorana spinor: ν_M (⇑⇓) ≡ $\bar{\nu}_M$ (⇑⇓) : 2 d.o.f.

Possible at all because Lepton number is violated

2. $M_{ij} \neq 0$ MAJORANA masses - continued

Don't we see 3 neutrinos in $Z \to \Box_i \bar{\Box}_i$? ($N_{\Box} = 2.9841(83)$)

We see 3 *interacting* neutrinos, in a left helicity state (*active*, as opposed to *sterile*)

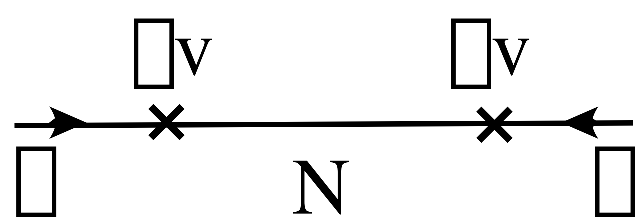
Why $m_{\Box} << m_e, m_q$? Because $M >> \Box v$

By diagonalizing the mass matrix

$$\Box_{light} = \cos\Box\,\Box + \sin\Box\, N \quad m_l \approx \frac{\Box^2 v^2}{M} \qquad \Box \approx \frac{\Box v}{M}$$

$$\Box_{heavy} = -\sin\Box\,\Box + \cos\Box\, N \quad m_h \approx M$$

By *integrating out* the heavy N's



$$= \Box^T (\Box^T v) \frac{1}{M} (\Box v) \Box$$

$$\mathcal{L}^{(\Box - mass)} \approx \Box_i m_{ij} \Box_j \qquad m = \Box^T M^{-1} \Box v^2$$

2. $M_{ij} \neq 0$ MAJORANA masses - continued

- Physical parameters (with 3 light neutrinos hereafter):

$$\mathcal{L}^{lept} = \bar{L}\, \not{D} L + \bar{e^c}\, \not{D} e^c + e^T U_L^T m_d^e U_R^T e^c + \square^T V_\square^T m_d^\square V_\square \square$$

By going to the physical basis:

$$W_\mu \bar{e} \square_\mu \square \Rightarrow W_\mu \bar{e}_{ph} U_L V_\square^+ \square_\mu \square_{ph} \qquad V = U_L V_\square^+$$

Counting intrinsic phases again

$$N(phys.phases) = n^2 - \frac{n(n-1)}{2} - n^{(*)} = \frac{1}{2} n(n-1)$$

= 3 (instead of 1) for n=3

(*unlike the (2n -1) of the quark case, because the $\square$-phases defined by $\square^T m_d^\square \square$)

The physical parameters precisely defined

3 masses: m_1, m_2, m_3 , not by the order (as for charged fermion)

but by

$$|\Box m_{32}^2|, |\Box m_{31}^2| > \Box m_{21}^2 > 0 \quad \left\{ \begin{array}{ll} m_3 > m_2 > m_1 & \textit{normal} \\ m_2 > m_1 > m_3 & \textit{inverted} \end{array} \right.$$

3 angles and 1(+2) phases:

$$V = R_{23}(\Box_{23}) \begin{pmatrix} 1 & & \\ & e^{i\Box} & \\ & & 1 \end{pmatrix} R_{13}(\Box_{13}) R_{12}(\Box_{12}) \begin{pmatrix} 1 & & \\ & e^{i\Box} & \\ & & e^{i\Box} \end{pmatrix}$$

Majorana phases

in $W_\mu \bar{e} V \Box_\mu \Box$

Means to determine the physical parameters

A. Absolute spectrum

1. Endpoint of □-decay spectrum $(Z,A) \rightarrow (Z+1,A) + e + \square_i$

phase space: $d\square_i \quad p_{\square_i}\square(\square - m_i - E_e)\,dE_e$

$$\square \equiv M_i - M_f$$

$$\Rightarrow \frac{d\square}{dE_e} \quad \square_i |V_{ei}|^2 p_{\square_i}\square(\square - m_i - E_e)$$

$$p_{\square_i} = [(\square - E_e)^2 - m_i^2]^{1/2}$$

Currently $\square_i |V_{ei}|^2 m_i^2 \lesssim (2eV)^2$ (+ info from osc.s: $m_i|_{Max} \lesssim 2eV$)

2. Cosmological observations

a - Density power spectrum distorted at small scales

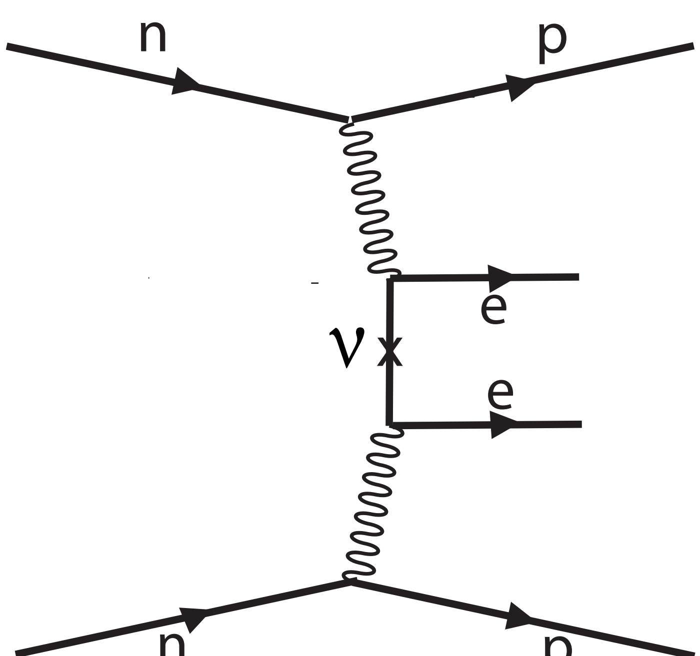
b - CMB distorsions

Currently, mostly from a, $\square_i m_i \lesssim 0.5eV$ ($m_i|_{Max} \lesssim 0.15eV$)

Means to determine the physical parameters
(continued)

3. $\beta\beta_{0\nu}$- decay $\quad (Z,A) \rightarrow (Z+2,A) + 2e$

only if Majorana, since L violated!



$$A \propto \sum_i V_{ei}^2 m_i M_{nucl} \equiv m_{ee} M_{nucl}$$

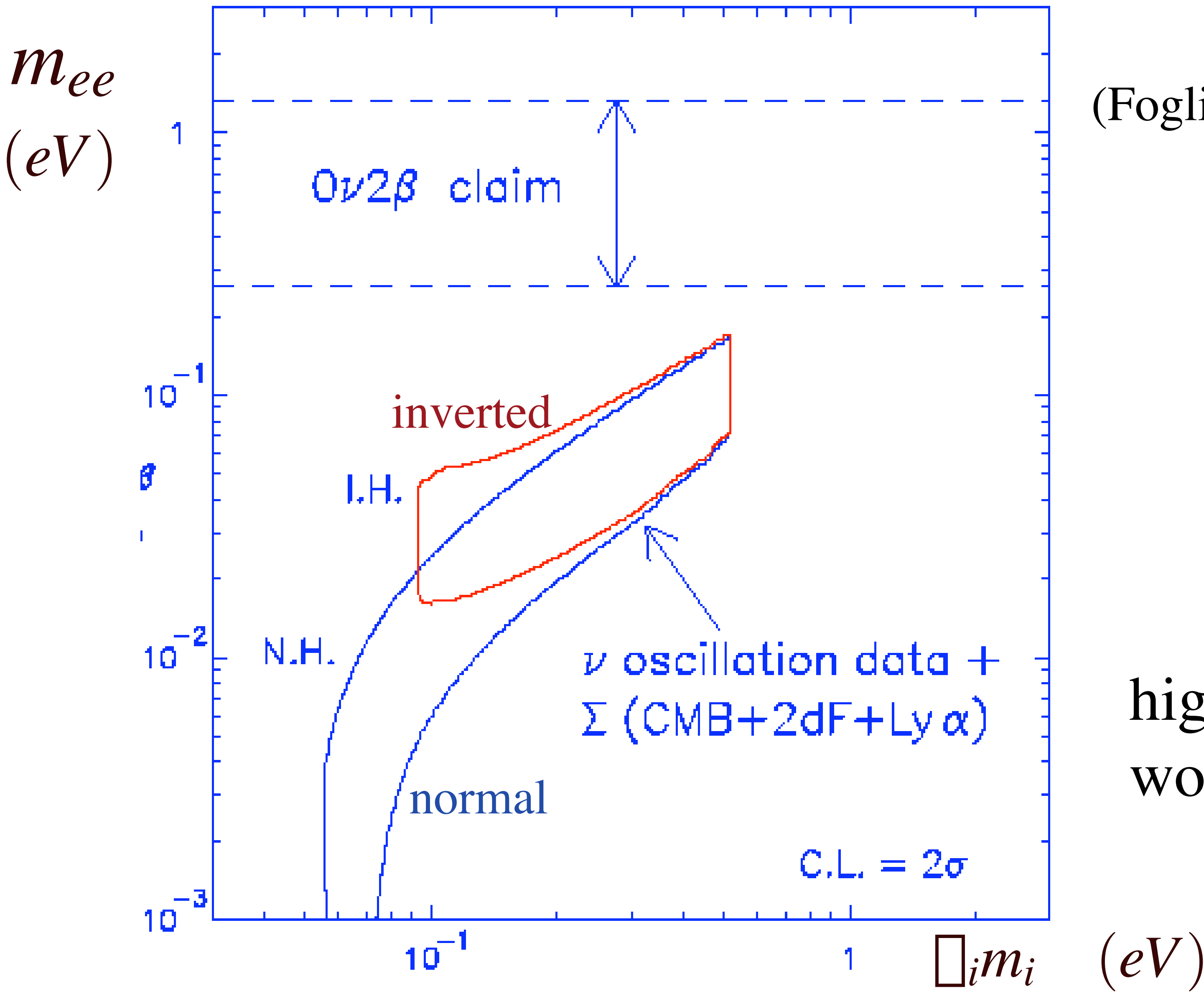
$$T_{1/2} \propto \frac{1}{|m_{ee}|^2 |M_{nucl}|^2}$$

$$m_{ee} = c_{13}^2 (c_{12}^2 m_1 + s_{12} e^{2i\alpha} m_2) + s_{13}^2 m_3 e^{2i\beta} \quad (!?)$$

$$\delta_{th}(M_{nucl}) = O(M_{nucl})$$

Currently: a claimed observation at $0.17 eV < |m_{ee}| < 2.0 eV$

3. $\beta\beta_{0\nu}$ - decay with oscillation and cosmological data (current 2004)



(Fogli et al)

highly non trivial but worth trying

Means to determine the physical parameters
(continued)

B. Mixing parameters from neutrino oscillations

The standard picture: Calculate the amplitude for a $|\Box_{l_\Box} >$ at t=0 to become a $|\Box_{l_\Box} >$ at time t

$|\Box_{l_\Box} >$ $\underset{t=0}{\vdash}$ ——— $R = ct$ ——— $\underset{t}{\dashv}$ $|\Box_{l_\Box} >$ $\qquad l_{\Box,\Box} = e, \mu, \Box$

$$|\Box_{l_\Box}, t >= \Box_\Box \Box_\Box(t) |\Box_{l_\Box} >$$

$$i\frac{d}{dt}\Box = V\mathbf{E}V^+\Box \qquad \mathbf{E}_{ij} = (p + \frac{m_i^2}{2p})\Box_{ij}$$

$$< \Box_\Box | \Box_{l_\Box}, t >= \Box_{i=1,2,3} V_{i\Box}^* V_{i\Box} e^{-i\frac{m_i^2 t}{2p}} \qquad (!?)$$

up to an irrelevant phase factor

Note: Majorana phases and absolute m_i^2 ‘s irrelevant to $| < \Box_{l_\Box} | \Box_{l_\Box}, t > |$

(Apparent) Paradoxes of the standard picture

1. Why is the □-state given a definite p rather then E?

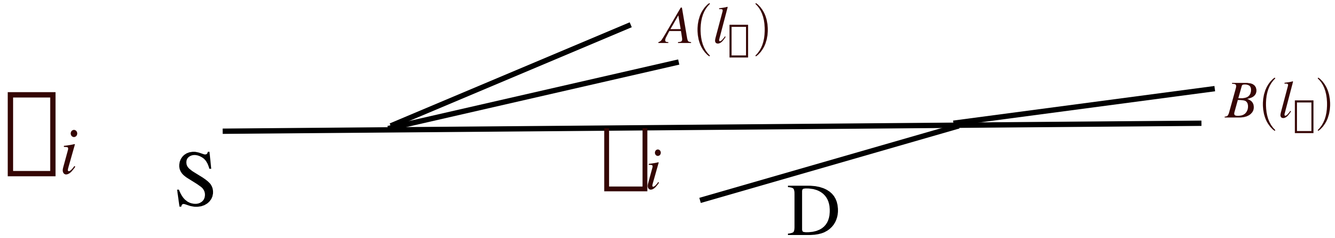
2. If p defined, how about coherence? $v_1 = p/E_1 \neq v_2 = p/E_2$

3. If p defined, □$x =$. Why then t = R/c?

etc...

Suggested framework to address them:

The amplitude of a physical process =



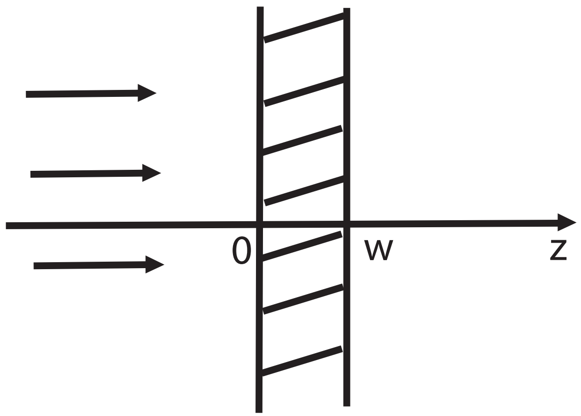
S = source

D = detector

treated in second-order time-dependent perturbation theory (!?)

Matter effects in ν-oscillations

ν-propagation in matter: like in optics



$$e^{ipz} \Rightarrow e^{ipz}(1 + iwNf(0)\frac{2\pi}{p})$$

$$e^{ipz} \Rightarrow e^{(inpw+p(z-w))} \approx e^{ipz}(1 + iwp(n-1))$$

$$\Rightarrow \quad n - 1 = Nf(0)\frac{2\pi}{p^2}$$

• A modified Schroedinger equation in a medium of electron number density N_e

$$i\frac{d}{dt}\psi = (V\mathbf{E}V^{+} + \mathbf{A})\psi$$

with $\mathbf{A} = 0$ except for $\mathbf{A}_{ee} = \pm\sqrt{2}GN_e \quad (+_{\nu}, -_{\bar{\nu}})$ (!?)

Nucleons do not contribute to $\mathbf{A}$ (!?)

Solving the equation (!?)

(in a simple, but relevant, 2x2 case)

$$E_{matter} = \pm\frac{1}{2}[(\Delta\cos 2\theta_v - \sqrt{2}GN_e)^2 + \Delta^2 \sin^2 2\theta_v]^{-1/2}$$

$$\Delta \equiv \frac{\Delta m^2}{2p}$$

$$\tan 2\theta_m = \frac{\Delta \sin 2\theta_v}{\Delta\cos 2\theta_v - \sqrt{2}GN_e}$$

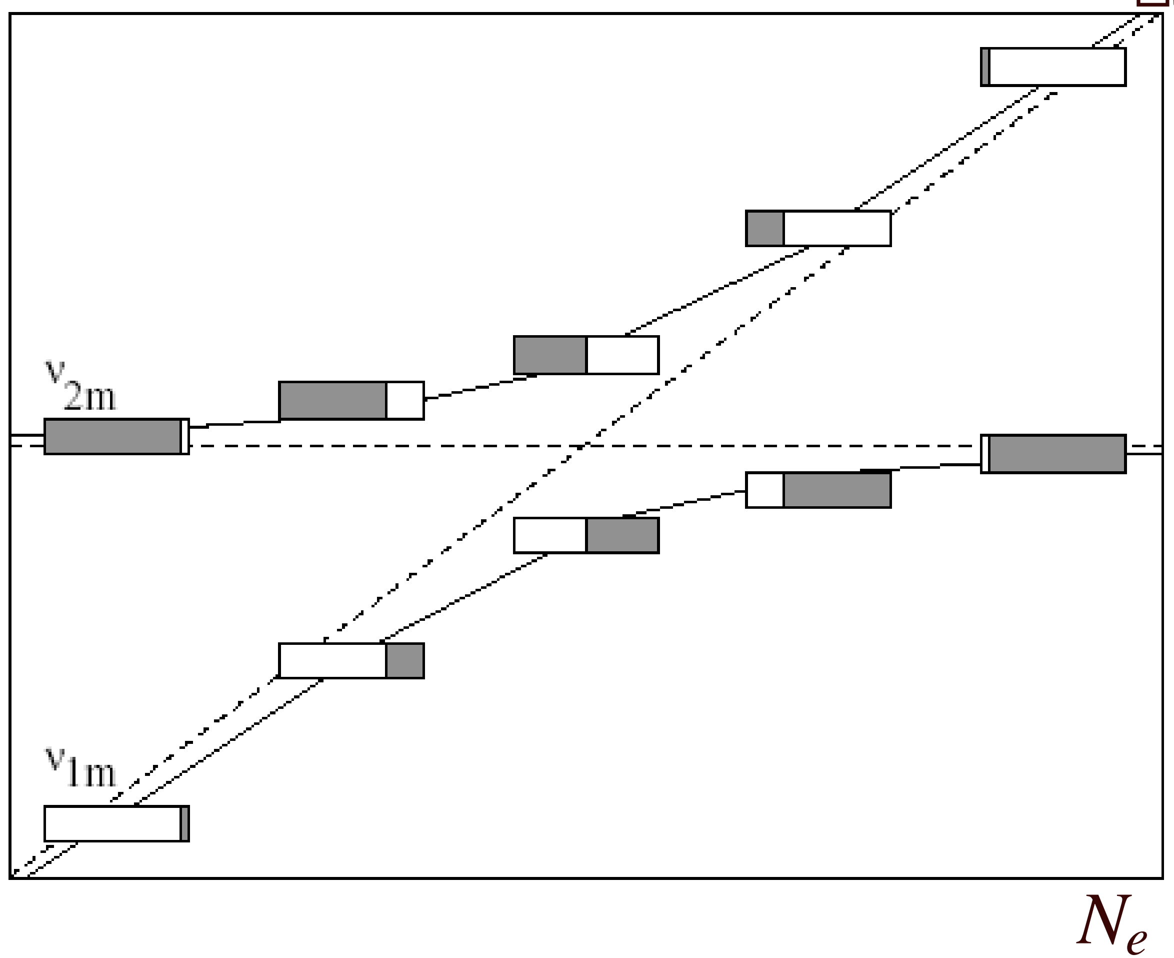
θ_v = vacuum angle

small θ_v

adiabatic conditions

$$\delta_{res} << \Delta R$$

$$\Delta R \frac{dV}{dr} = \Delta E_{min}$$



(Smirnov)

Current knowledge (2004)

$|\Delta m^2_{32}| \approx |\Delta m^2_{31}| \approx 2 \cdot 10^{-3} eV^2 \quad (\Delta m^2_{Atm})$

$\Delta m^2_{21} \approx 8 \cdot 10^{-5} eV^2 \quad (\Delta m^2_{Sun})$

$\theta_{23} = 36^0 \div 54^0$ (90% C.L.)

$\tan^2 \theta_{12} = 0.40^{+0.09}_{-0.07} \quad (\theta_{12} = 32^0 \pm 3^0)$

$\sin^2 \theta_{13} < 0.03$ (90% C.L.)

Current ignorance (2004)

0. Dirac or Majorana?

1. Absolute scale? $0.05 eV \lesssim m_i|_{Max} \lesssim 0.15 eV$ (if cosmo)

2. Sign of Δm^2_{32} (inverted or normal spectrum)

3. $\theta_{13} = ?$

4. $\delta = ?$ Need $\theta_{13} \neq 0$ to be defined at all