Lecture 5

The ElectroWeak Symmetry Breaking

$$\mathcal{L}^{(EWSB)} = |D_\mu \Box|^2 - \mu^2 |\Box|^2 - \Box|\Box|^4 \equiv |D_\mu \Box|^2 - V(\Box)$$

Suppose that $\mu^2 < 0$. Then $H(\Box) = |\partial_0 \Box|^2 + |\Box\Box|^2 + V(\Box)$ is minimized by □ homogeneous and constant in time, with $|\Box|^2 = -\frac{\mu^2}{2\Box}$

Physical interpretation: A BE condensation of scalars with $p_\mu = 0$, filling all space in a constant configuration □ <□>

Which configuration? It does not matter, since all the others can be reached by a $SU(2)XU(1)$ transformation.

□ Pick up one, e.g. $<\Box>= \begin{pmatrix} 0 \\ v \end{pmatrix}$ □ the *vacuum* configuration,

with $v^2 = -\frac{\mu^2}{2\Box}$

Physical consequences:

[Seen by replacing ϕ with $<\phi>$ in $\mathcal{L}$ (!?)]

1. The $SU(2)XU(1)$ invariance is not there anymore, but only a residual $U(1)_{em}$: $\quad (T_3+Y)<\phi>=0 \quad \Rightarrow$ Electric charge defined

2. All vector bosons, but one, pick up a mass

$$A_\mu = \sin\theta W_\mu^3 + \cos\theta B_\mu \quad m_A^2 = 0$$

$$Z_\mu = \cos\theta W_\mu^3 - \sin\theta B_\mu \quad m_Z^2 = \frac{g^2 v^2}{2\cos^2\theta}$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm W_\mu^2) \quad m_W^2 = \frac{g^2 v^2}{2}$$

3. Fermion masses appear as well

$$\mathcal{L}_m = u^T \lambda^u u_c v + d^T \lambda^d d_c v + e^T \lambda^e e_c v (+\nu^T \lambda^\nu N v + N^T M N)$$

4. The Higgs physics:

A. By setting $<\square>=\begin{pmatrix}\square_1+i\square_2\\ \square_3+i\square_4\end{pmatrix}\equiv e^{i\square\cdot\square}\begin{pmatrix}0\\ v+\frac{1}{\sqrt{2}}h\end{pmatrix}$ the $\vec{\square}$-fields:

a - disappear from $V(|\square|)$: massless *Goldstone bosons*

b - disappear at all from $\mathcal{L}$ by a gauge transformation:
eaten up Goldstone bosons

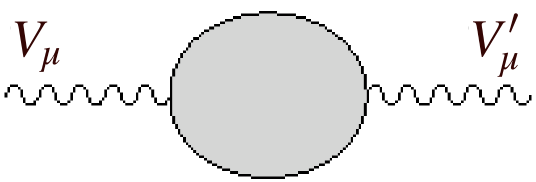
B. All the interactions of the physical h-field determined (!?) by v and:

- g (g') : with the gauge bosons = (schematically) $gvAAh+g^2AAh^2$
- m_h : with itself = $\frac{m_h^2}{\sqrt{2}v}h^3+\frac{m_h^2}{16v^2}h^4$
- m_f : with the fermions = $\frac{m_f}{v}hff^c$

Back to the EWPT fit (including LEP2)

What is its significance for the EWSB problem?

- Consider a theory characterized by a scale Λ_{SB} with its virtual effects likely significant in the vac. pol. amplitudes of the vector bosons. At $q^2 < \Lambda_{SB}^2$



$$\Pi_V(q^2) \approx \Pi_V(0) + q^2 \Pi'_V(0) + \frac{(q^2)^2}{2} \Pi''_V(0) + \ldots$$

where $V = W^+W^-, W_3W_3, BB, W_3B.$

Up to $O((q^2)^2)$ the number of coefficients is

$$3 \times 4 = 12 = 3(g, g', v) + 2(m_\gamma = 0, Q = T_3 + Y) + 7$$

predicted in the SM in terms of m_h

Their definition and symmetry properties

Adimensional form factors			operators			custodial	$SU(2)_L$
$g^{-2}\widehat{S}$	$=$	$\Pi'_{W_3B}(0)$	$\mathcal{O}_{WB}$	$=$	$(H^\dagger\tau^a H)W^a_{\mu\nu}B_{\mu\nu}/gg'$	$+$	$-$
$g^{-2}M_W^2\widehat{T}$	$=$	$\Pi_{W_3W_3}(0)-\Pi_{W^+W^-}(0)$	$\mathcal{O}_H$	$=$	$\lvert H^\dagger D_\mu H\rvert^2$	$-$	$-$
$-g^{-2}\widehat{U}$	$=$	$\Pi'_{W_3W_3}(0)-\Pi'_{W^+W^-}(0)$		$-$		$-$	$-$
$2g^{-2}M_W^{-2}V$	$=$	$\Pi''_{W_3W_3}(0)-\Pi''_{W^+W^-}(0)$		$-$		$-$	$-$
$2g^{-1}g'^{-1}M_W^{-2}X$	$=$	$\Pi''_{W_3B}(0)$		$-$		$+$	$-$
$2g'^{-2}M_W^{-2}Y$	$=$	$\Pi''_{BB}(0)$	$\mathcal{O}_{BB}$	$=$	$(\partial_\rho B_{\mu\nu})^2/2g'^2$	$+$	$+$
$2g^{-2}M_W^{-2}W$	$=$	$\Pi''_{W_3W_3}(0)$	$\mathcal{O}_{WW}$	$=$	$(D_\rho W^a_{\mu\nu})^2/2g^2$	$+$	$+$

- relation with *standard* S, T, U:

S$= 4s_W^2\widehat{S}/\square \approx 119\widehat{S},\ T=\widehat{T}/\square \approx 129\widehat{T},\ U=-4s_W^2\widehat{U}/\square.$

- "custodial": $SU(2)_V$ under which W^a_μ transform as a triplet

and $\quad \square = \begin{pmatrix} \square_0^* & \square_+ \\ -\square_+^* & \square_0 \end{pmatrix} \Rightarrow e^{i\vec{w}\square}\square e^{-i\vec{w}\square}$

- for the *operators*, see below

Their determination

- Data: the EWPT's and $e^+e^- \to f\bar{f}$ at LEP2

- Define the various coeff.s as *deviations* from the SM (hence the result is $\log m_h$- dependent)

- Limit the fit to the likely dominant terms, $\widehat{S}, \widehat{T}, W, Y$.

Type of fit	$10^3\,\widehat{S}$	$10^3\widehat{T}$	$10^3 Y$	$10^3 W$
One-by-one (light Higgs)	0.0 ± 0.5	0.1 ± 0.6	0.0 ± 0.6	-0.3 ± 0.6
One-by-one (heavy Higgs)	—	2.7 ± 0.6	—	—
All together (light Higgs)	0.0 ± 1.3	0.1 ± 0.9	0.1 ± 1.2	-0.4 ± 0.8
All together (heavy Higgs)	-0.9 ± 1.3	2.0 ± 1.0	0.0 ± 1.2	-0.2 ± 0.8

(B, Pomarol, Rattazzi, Strumia)

 The deviations from the SM pretty constrained

[A heavy Higgs (800 GeV) technically allowed. Significant?]

A matter of *naturalness*

Back to the Higgs potential. At tree level

$$v^2 = -\frac{\mu^2}{2\lambda}, \quad m_h^2 = -2\mu^2 \qquad\qquad (\mu,\lambda) \Rightarrow (v, m_h)$$

Including 1 loop corrections (!?)

$$m_h^2 = -2\mu^2 + \frac{6G}{\sqrt{2}\pi^2}\left(m_t^2 - \frac{M_W^2}{2} - \frac{M_Z^2}{4} - \frac{m_h^2}{4}\right)\int^{\Lambda} KdK\left(1 + O\left(\frac{m^2}{\Lambda^2}\right)\right)$$

where

$$K \equiv (k_E^2)^{1/2}$$

Λ = a *cut-off* = a change of regime of the SM

A matter of *naturalness* *(continued)*

- Attitude 1: Never mind this Λ^2- divergence. Absorb it in $\mu^2 \Rightarrow \mu^2_{Ren}$ and forget it

[Technically impeccable]

- Attitude 2: The Λ^2- divergence is highly significant. Barrying accidental cancellations in (from the previous formula with the masses replaced with their values)

$$m_h^2 = (115 GeV)^2 \left(\frac{\Lambda}{500 GeV}\right)^2 - 2\mu^2 - 0.01\left(\frac{m_h}{100 GeV}\right)^2 \Lambda^2$$

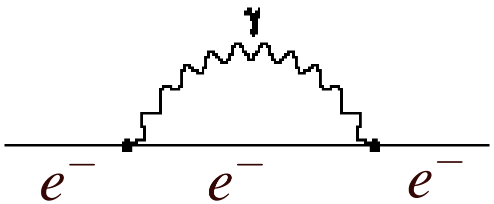
this implies a low cut-off of the SM, in the TeV range

[□ Promising for the LHC]

A matter of *naturalness* *(continued)*

2 examples that support this second view:

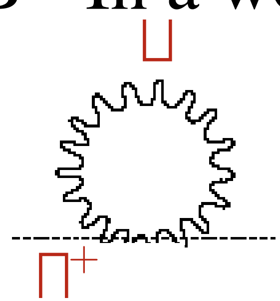
A - The electron self-energy, before QED

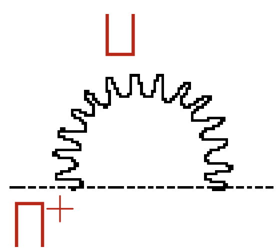


$$\square E \approx \frac{\square}{r_{\sqcap}} \approx m_e \frac{10^{-14} cm}{r_{\sqcap}} \approx m_e \frac{\square}{100 MeV}$$

Non relativistic ElectroDynamics is modified well before 100 MeV $(m_{e^+} \approx 0.5 MeV)$

B - In a world of pions, before QCD





$$m^2_{\square^+} - m^2_{\square^0} \approx \frac{\square}{\sqcap} \square^2 = \square m_{\square} 2 m_{\square} (\frac{\square}{400 MeV})^2$$

One might have guessed the scale of QCD

A matter of *naturalness* *(continued)*

Shouldn't have we seen already a sign of Λ_{SB}?

From Lecture 1: "Any theory, R or nor R, but GI under G and with the same light spectrum as the SM one is undistinguishable from it (the SM) at sufficiently low energies"

Let us try to parametrize it then (hard without knowing it!)

$$L_{eff}(E < \Lambda) = L_{SM} + \sum_{i,p} \frac{c_i}{\Lambda^p} O_i^{(4+p)}$$

where

$O_i^{(4+p)}$ = gauge invariant operators of dimension 4+p (in mass)

c_i = unknown dimensionless constants

A matter of *naturalness* *(continued)*

... and compare it with data (the EWPT once again)

Dimension six operator	$c_i = -1$	$c_i = +1$
$\mathcal{O}_{WB} = (H^+\sigma^a H)W^a_{\mu\nu}B_{\mu\nu}$	9.0	13
$\mathcal{O}_H = \lvert H^+ D_\mu H\rvert^2$	4.2	7.0
$\mathcal{O}_{LL} = \frac{1}{2}(\bar{L}\gamma_\mu\sigma^a L)^2$	8.2	8.8
$\mathcal{O}_{HL} = i(H^+ D_\mu H)(\bar{L}\gamma_\mu L)$	14	8.0

(B, Strumia)

95% lower bounds on □/TeV for the individual operators

$(m_h = 115 GeV)$

A clash between this lower bound and the upper bound from *naturalness*? This goes under the name of “little hierarchy problem”.

The Large Hadron Collider will tell

Particle Physics in one page

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\not{D}\psi$$
The gauge sector (1)

$$+\psi_i\lambda_{ij}\psi_j h + h.c.$$
The flavor sector (2)

$$+|D_\mu h|^2 - V(h)$$
The EWSB sector (3)

$$+N_i M_{ij} N_j$$
The ν-mass sector (4)
(if Majorana)

(1) : best tested, at least to per-mille accuracy

(2) + (4) : main developments of last 5 years, different in nature, both highly significant

(3): the most elusive, so far