Assorted NLL small-x comments
(with emphasis on preasymptotics)

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In collaboration with M. Ciafaloni, D. Colferai and A. Staśto

QCD at cosmic energies

Erice, August 30 – September 4, 2004

Contents inspired by discussion during workshop

- Relative importance of running coupling versus higher orders (cf. Mueller)
  Many features common to all small-x problems with $\perp$ cutoffs:
  - saturation
  - splitting function (will explain why $\equiv$ cutoff)

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  - saturation
  - splitting function (will explain why $\equiv$ cutoff)
- Give discussion in context of CCSS approach (cf. Ciafaloni)
  - NLL BFKL supplemented with DGLAP effects
  - numerical solutions of resulting equations (‘no approximations’)
  - extraction of splitting function

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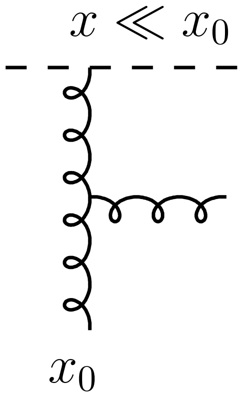
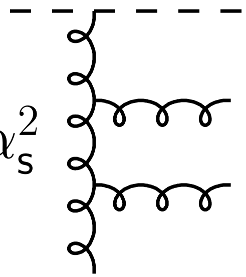
- Give discussion in context of CCSS approach (cf. Ciafaloni)
  - NLL BFKL supplemented with DGLAP effects
  - numerical solutions of resulting equations (‘no approximations’)
  - extraction of splitting function
- Characteristic result: significant preasymptotic effects
  - impact on phenomenology? (question by Strikman)
  - convolution of splitting function with CTEQ gluon

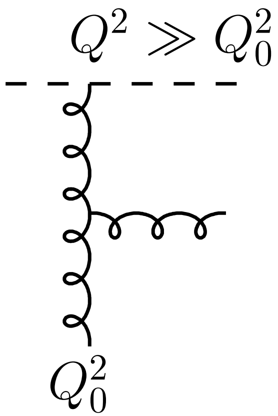
Improved NLLx? Start with kernel. . .

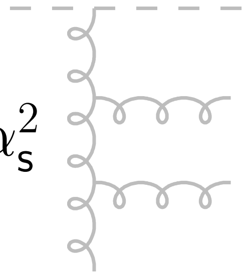
BFKL

α_s 

$+ \; \alpha_s^2$ 
$\times \; \ln \frac{x_0}{x}$

DGLAP

α_s 

$+ \; \alpha_s^2$ 
$\times \; \ln \frac{Q^2}{Q_0^2}$

$+ \; Q^2 \Leftrightarrow Q_0^2$

anti-DGLAP

Improved NLLx? Start with kernel…

BFKL

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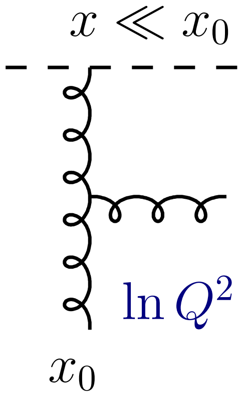

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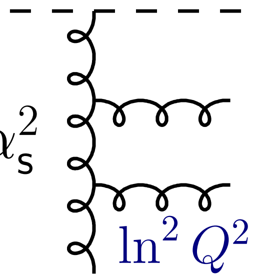

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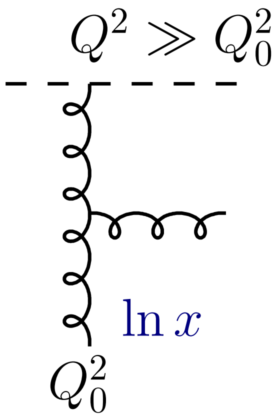

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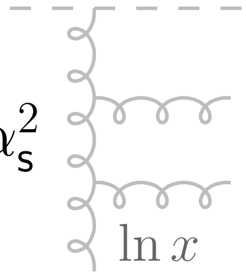

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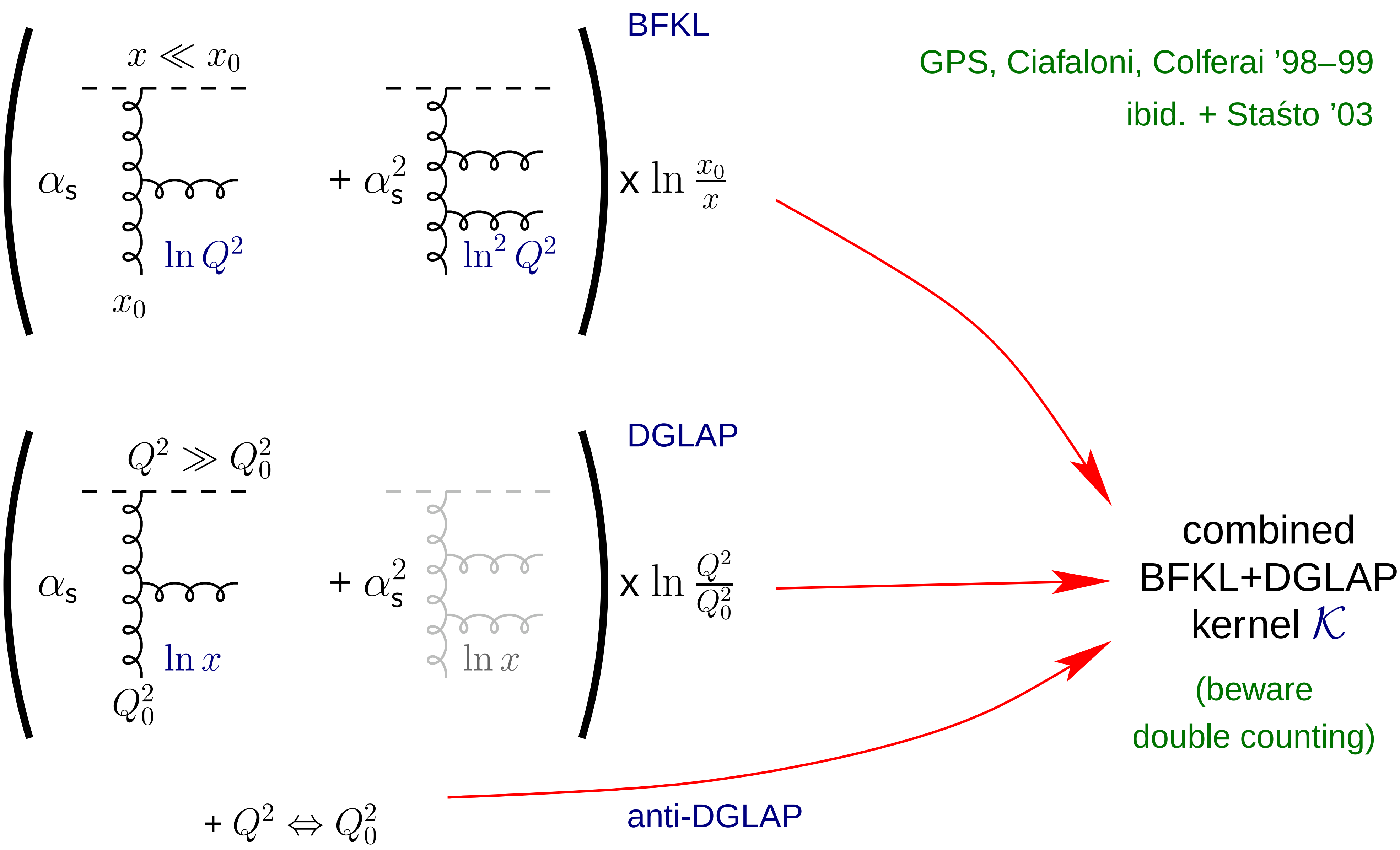
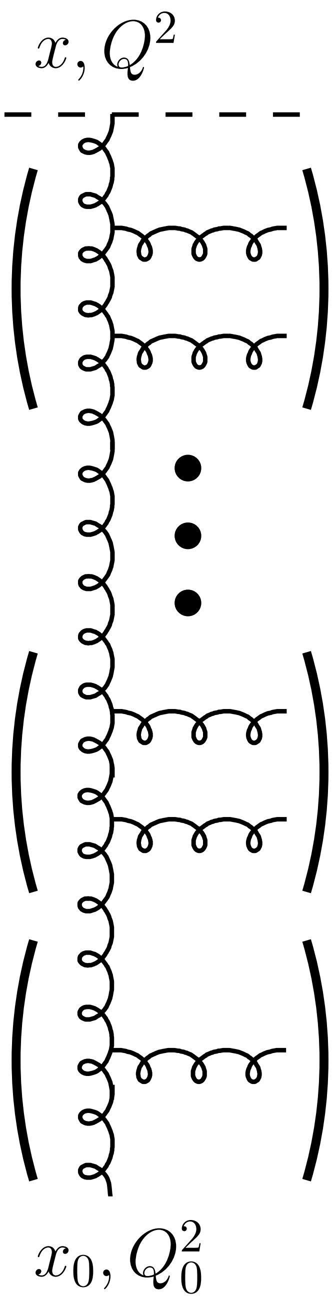


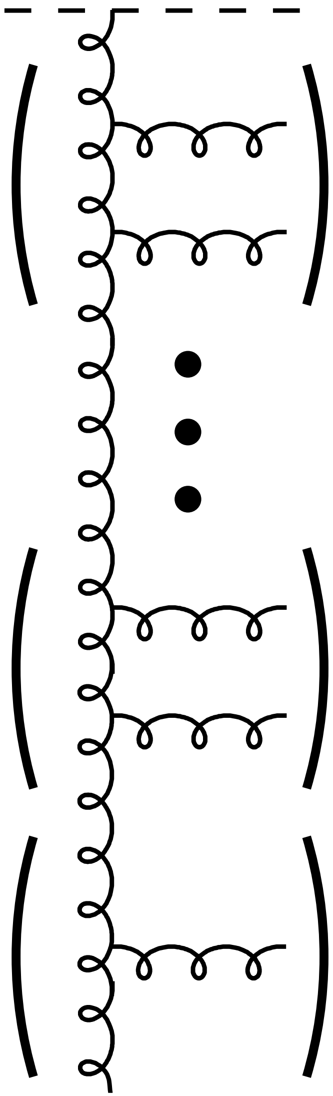
Iteration of kernel ⇒ Green function



Green function: $G\left(\ln \frac{x}{x_0}; Q_0, Q\right)$

Iteration of kernel $\Rightarrow$ Green function

x, Q^2



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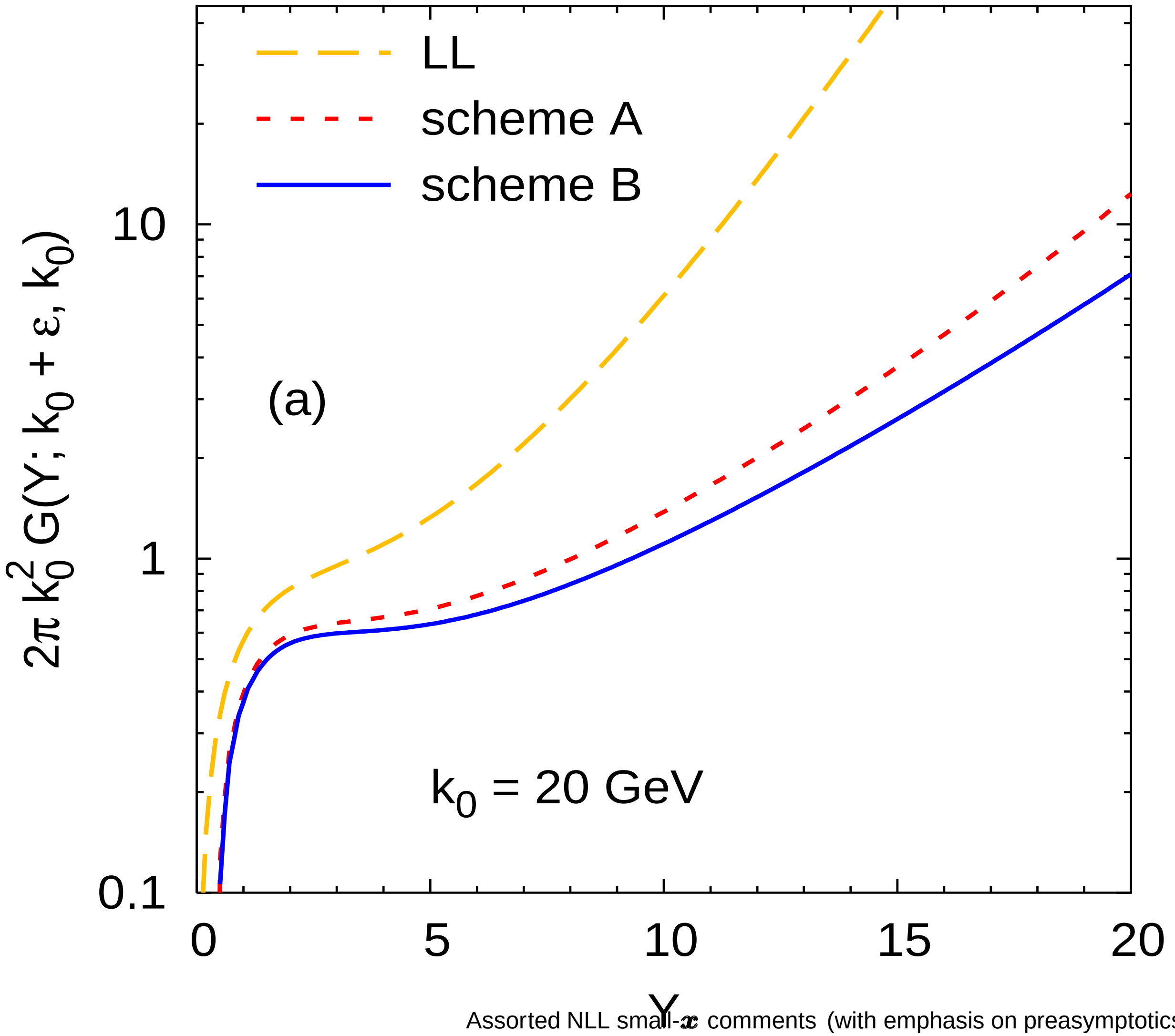

Green function $\Rightarrow$ effective DGLAP splitting function

Construct a gluon density from Green function (take $k \gg k_0$):

$$xg(x, Q^2) \equiv \int^Q d^2k \; G^{(\nu_0 = k^2)}(\ln 1/x, k, k_0)$$

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$$\frac{dg(x, Q^2)}{d \ln Q^2} = \int \frac{dz}{z}\, P_{gg,\text{eff}}(z, Q^2)\, g\left(\frac{x}{z}, Q^2\right)$$

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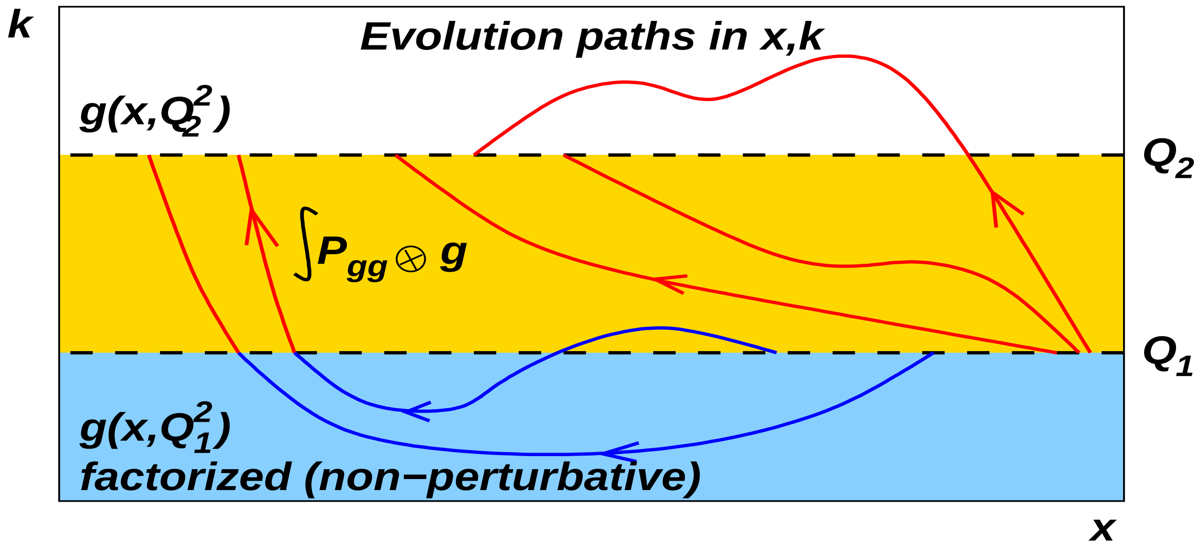
Factorisation

- Splitting function:
  red paths
- Green function:
  all paths



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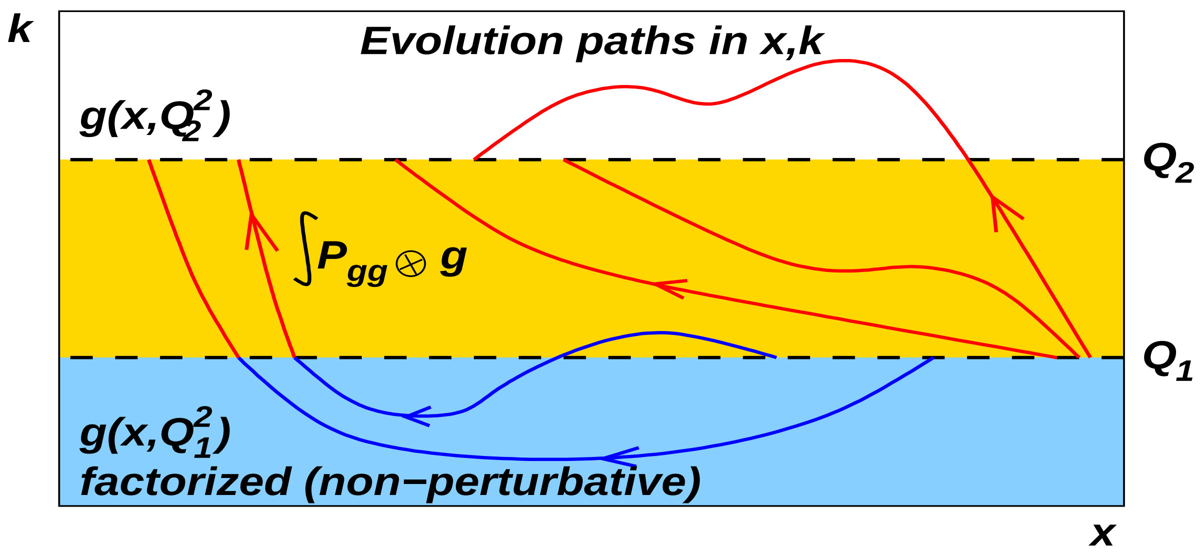
Splitting function ≡ evolution with cutoff



BFKL splitting function 'power'

Two classes of correction, to power growth ω:

$$\omega = 4 \ln 2 \, \bar{\alpha}_s(Q^2) \left(1 - \underbrace{6.5 \, \bar{\alpha}_s}_{NLL} - \underbrace{4.0 \, \bar{\alpha}_s^{2/3}}_{running} + \cdots \right)$$

$$\bar{\alpha}_s = \alpha_s N_c / \pi$$

- NLL piece is *universal*
- running piece appears only in problems with *cutoffs*
  - a consequence of *asymmetry* due to cutoff (only scales higher than cutoff contribute)

$$\alpha_s(Q^2) \longrightarrow \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

Hancock & Ross '92

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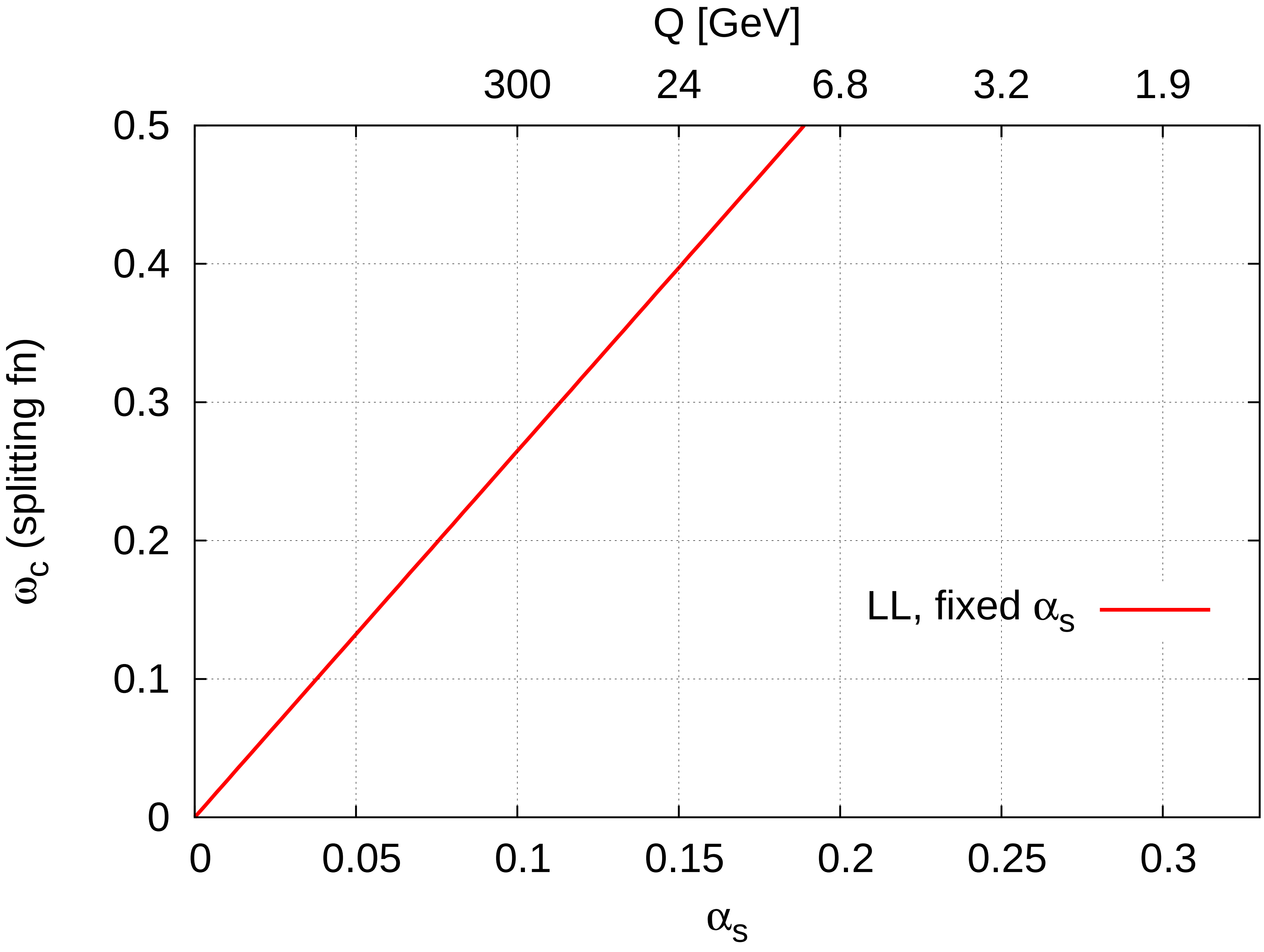
- Beyond first terms, not possible to separate effects of 'pure' higher orders & running coupling

BFKL splitting function power growth – results



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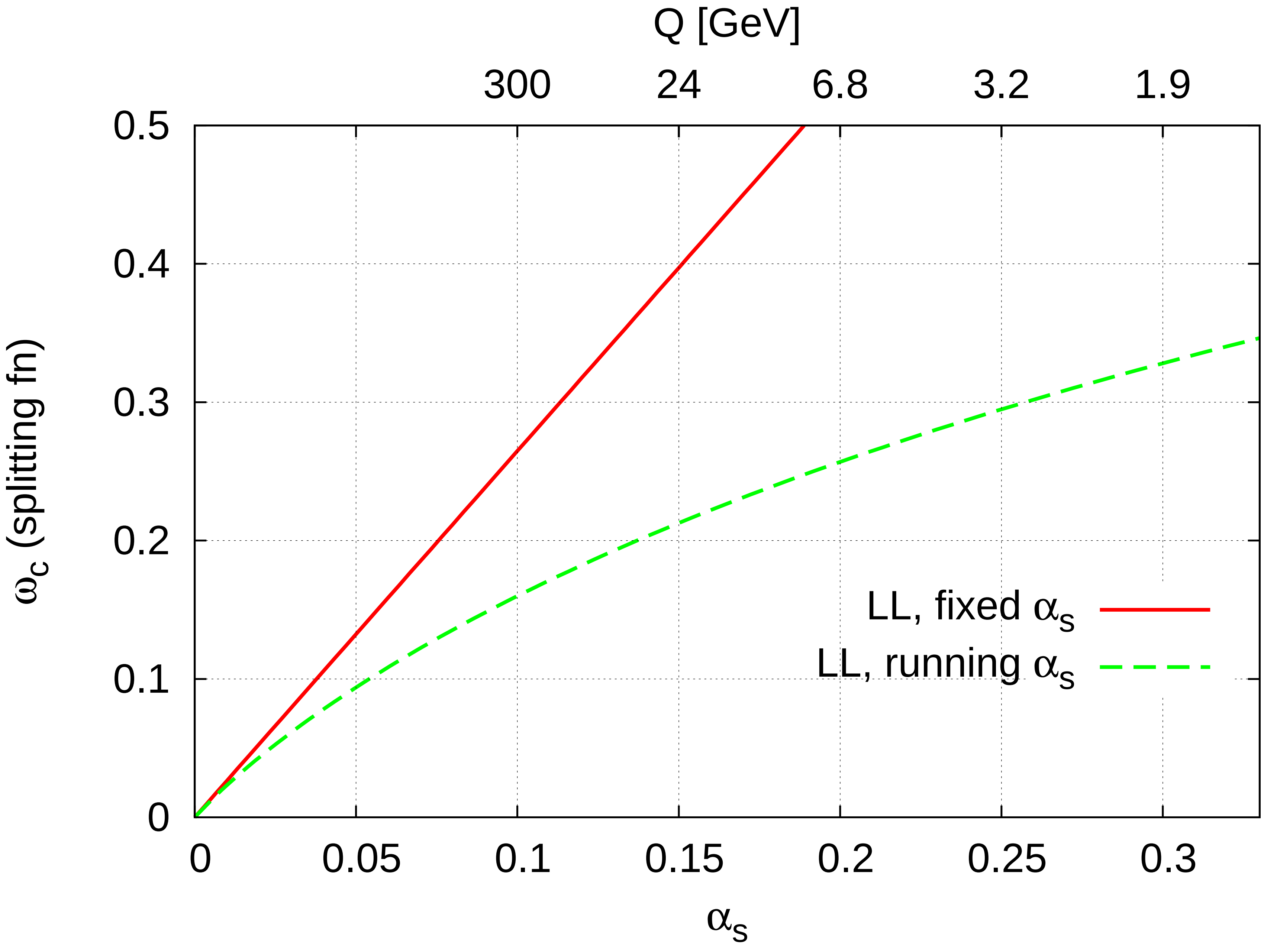

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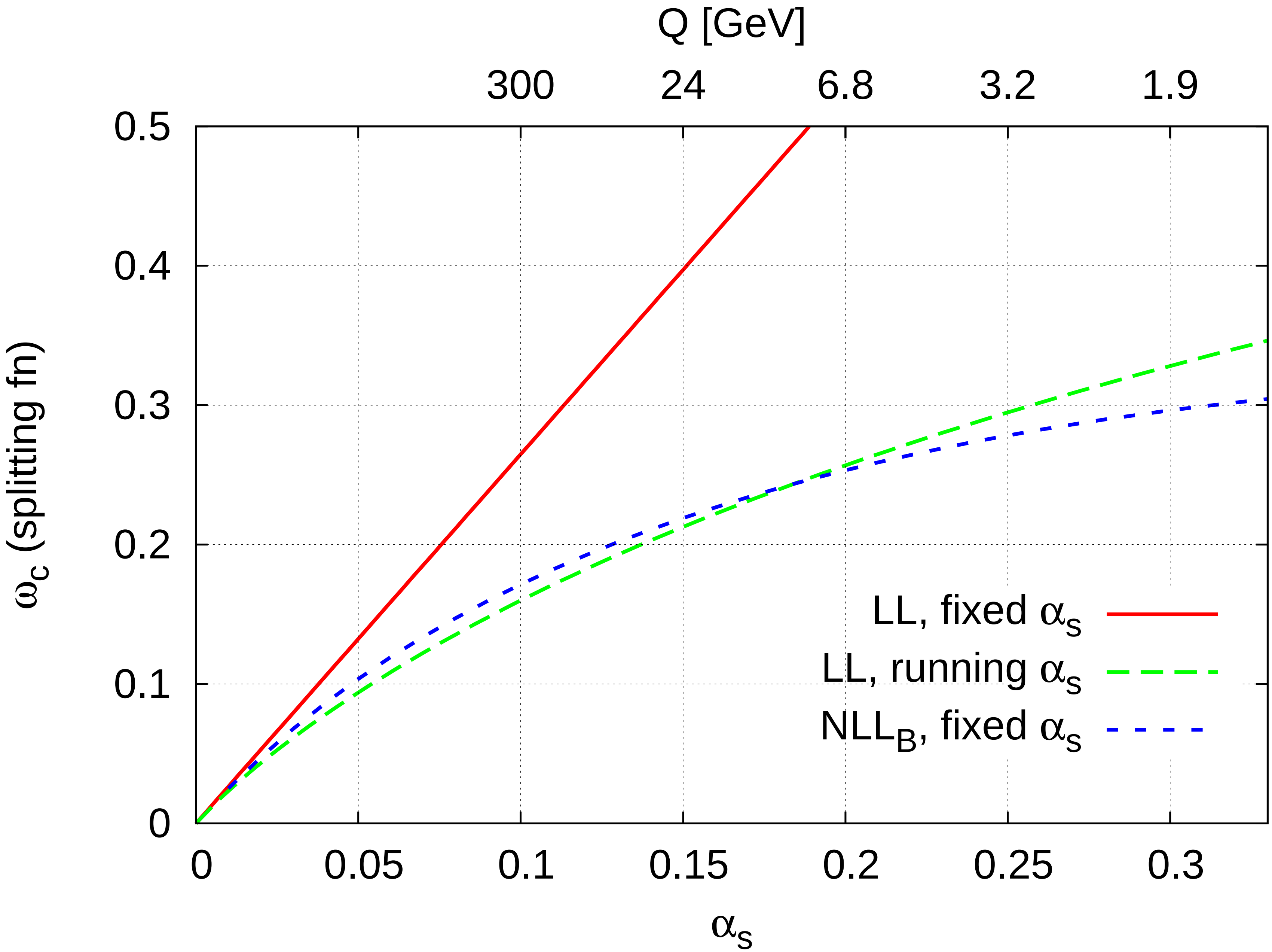

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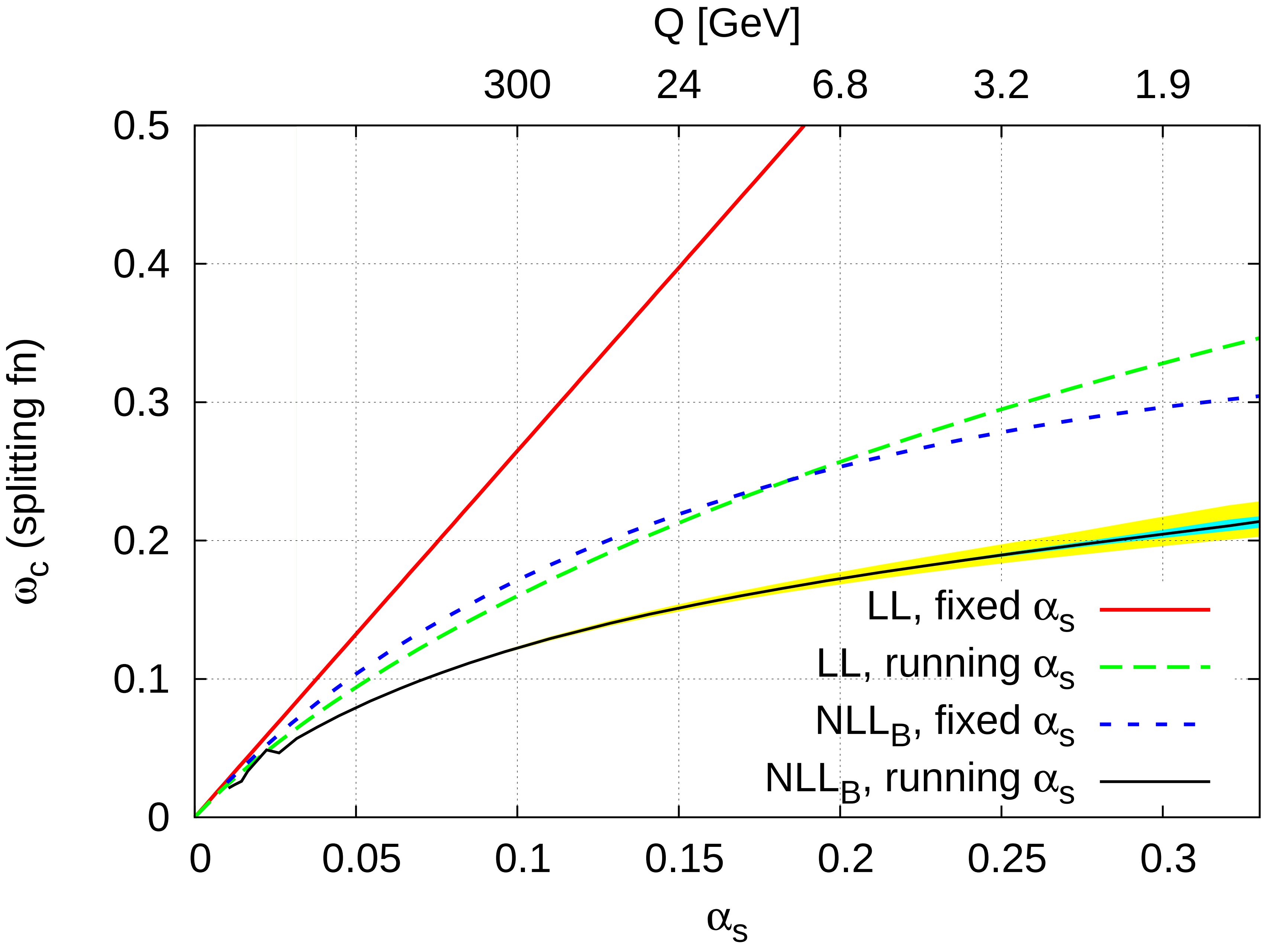
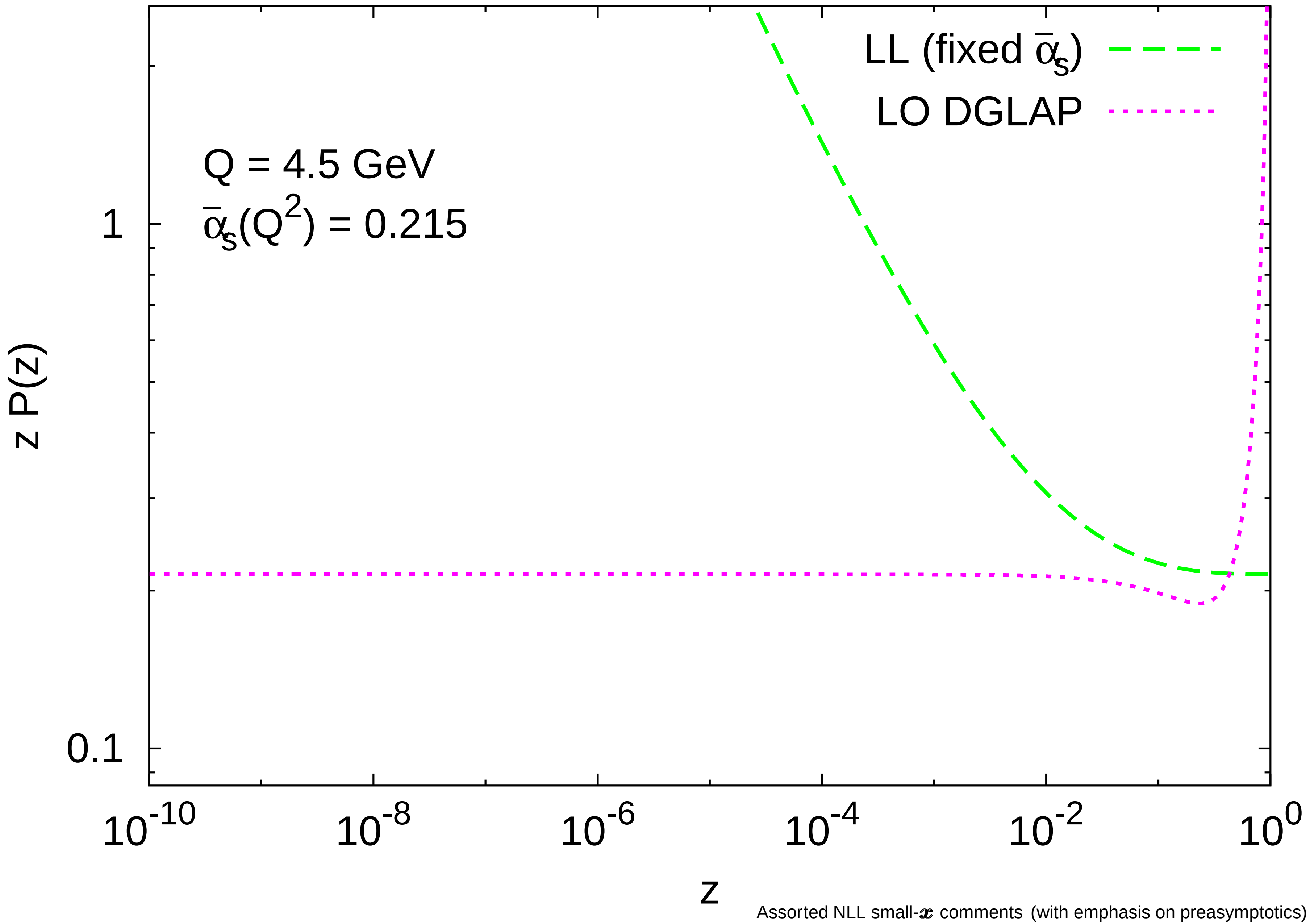


Can one neglect NLL terms? Examine full $P_{gg}(z)$ splitting fn



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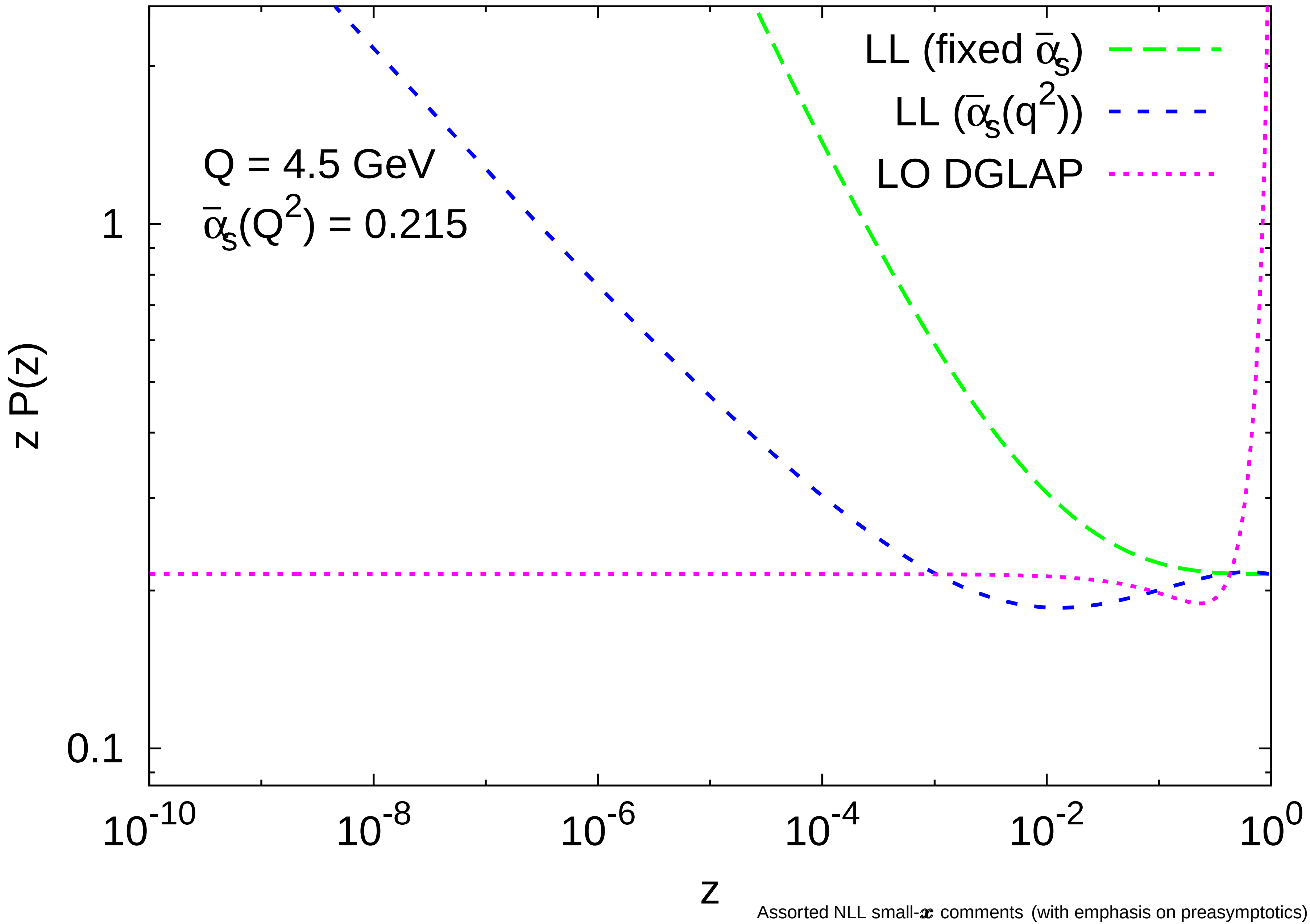

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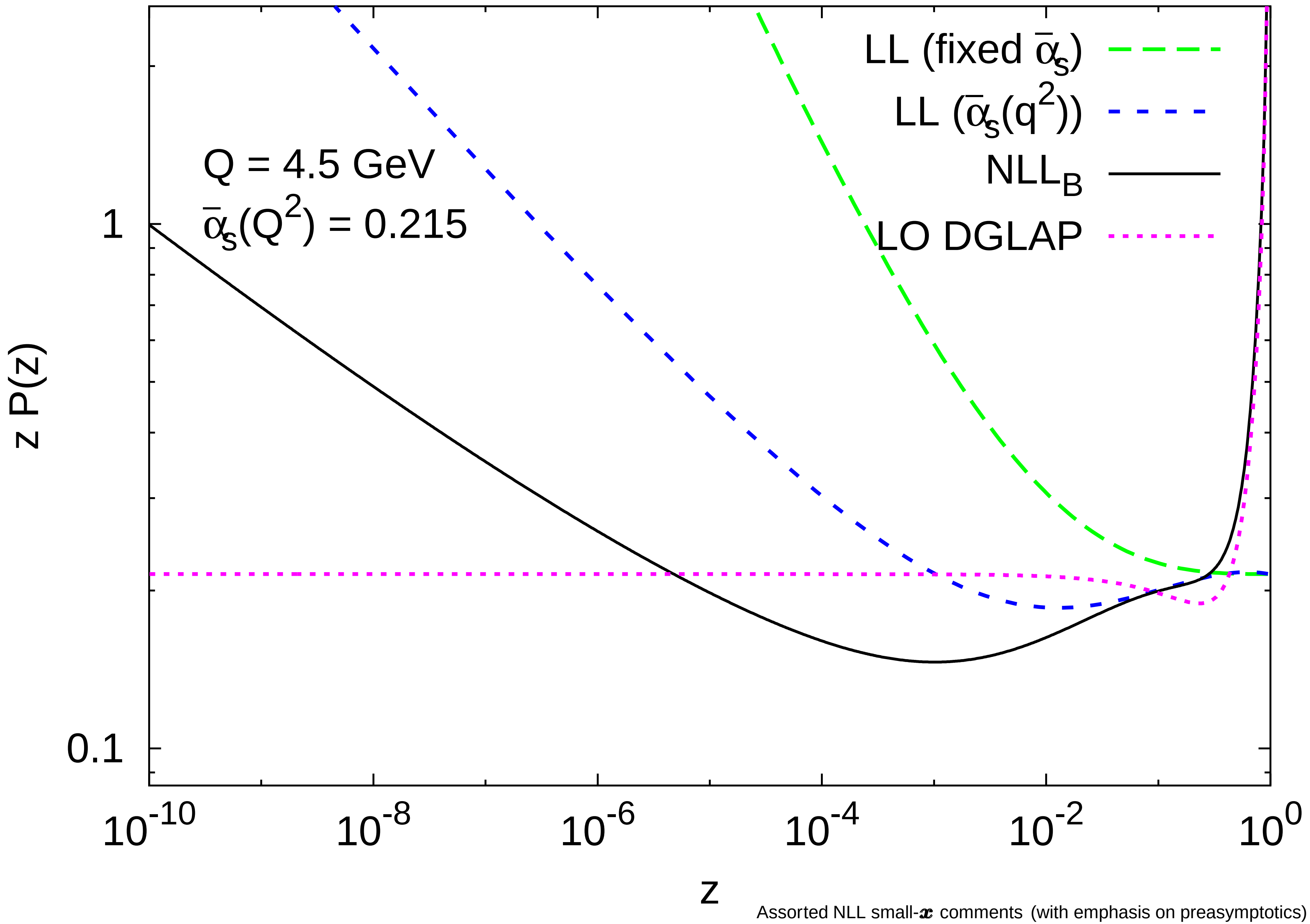

Interim conclusion

- Individually, running coupling and NLL effects are large
- BFKL 'power' has only moderate extra suppression when combining both

non-linearities between higher-orders and running coupling

$$\alpha_s(Q^2) \to \alpha_s(Q^2 e^{-X/(b\alpha_s)^{1/3}}) - C\alpha_s^2(Q^2 e^{-X/(b\alpha_s)^{1/3}})$$

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Likely to be true also for saturation scale $Q_s^2(x)\ldots$

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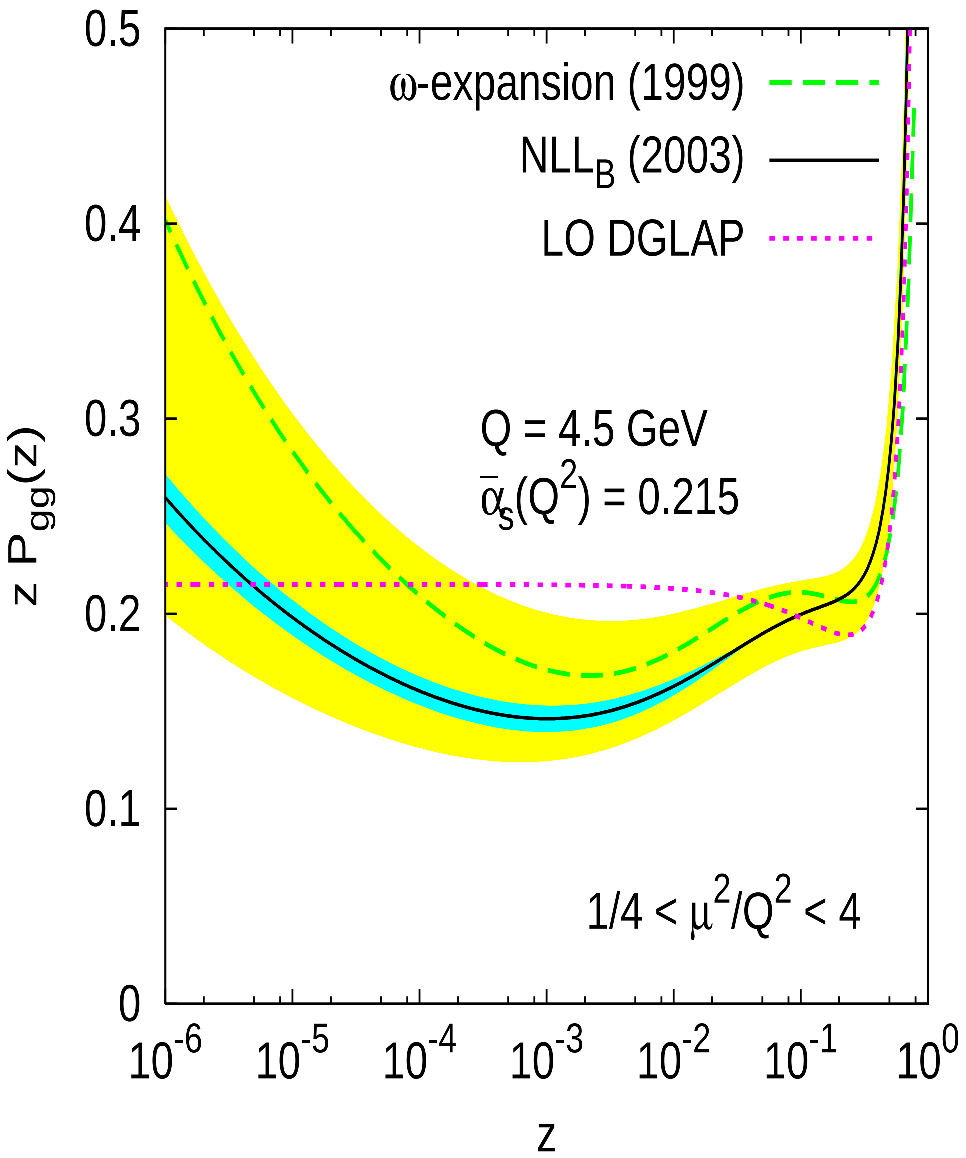
- Rapid rise in P_{gg} is not for today's energies!
- Main feature is a *dip at* $x \sim 10^{-3}$



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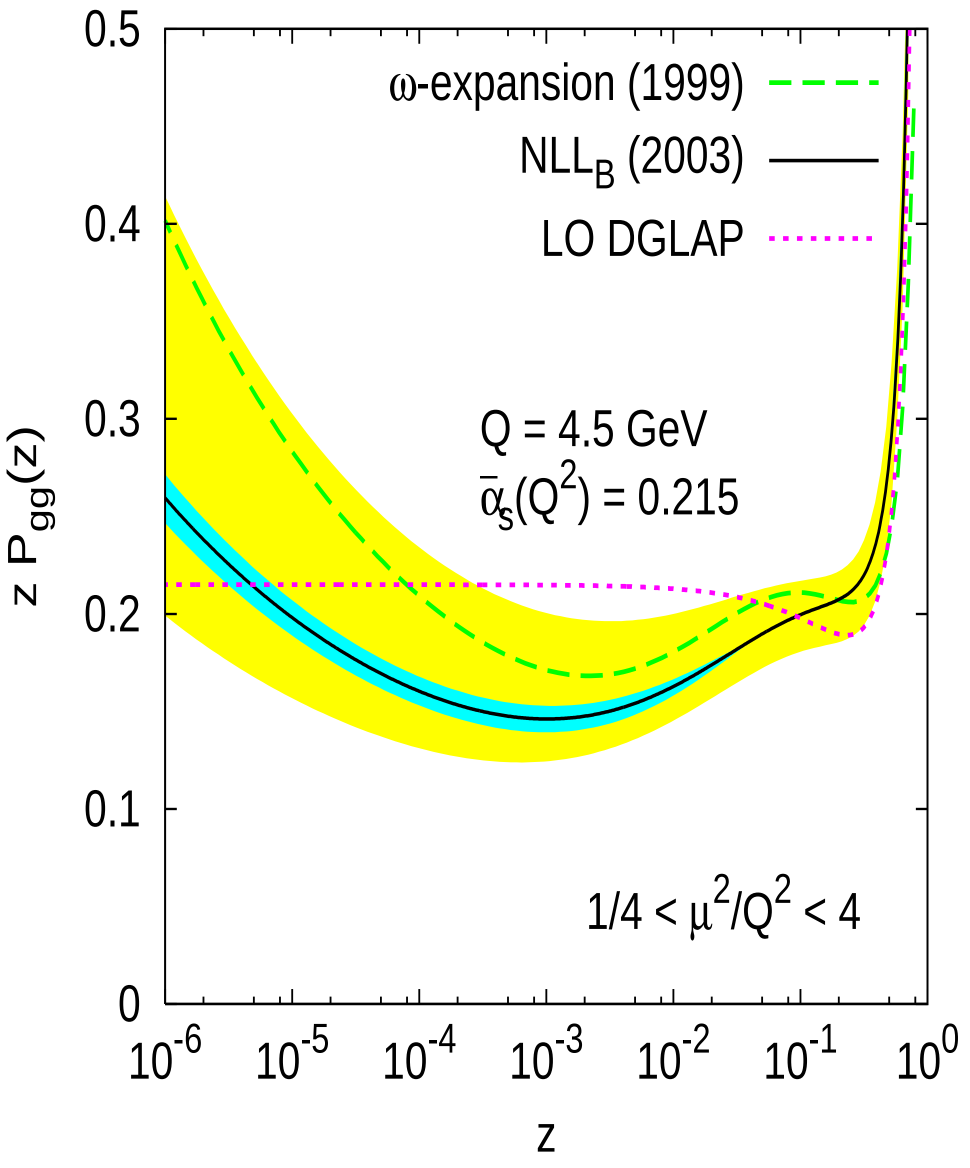

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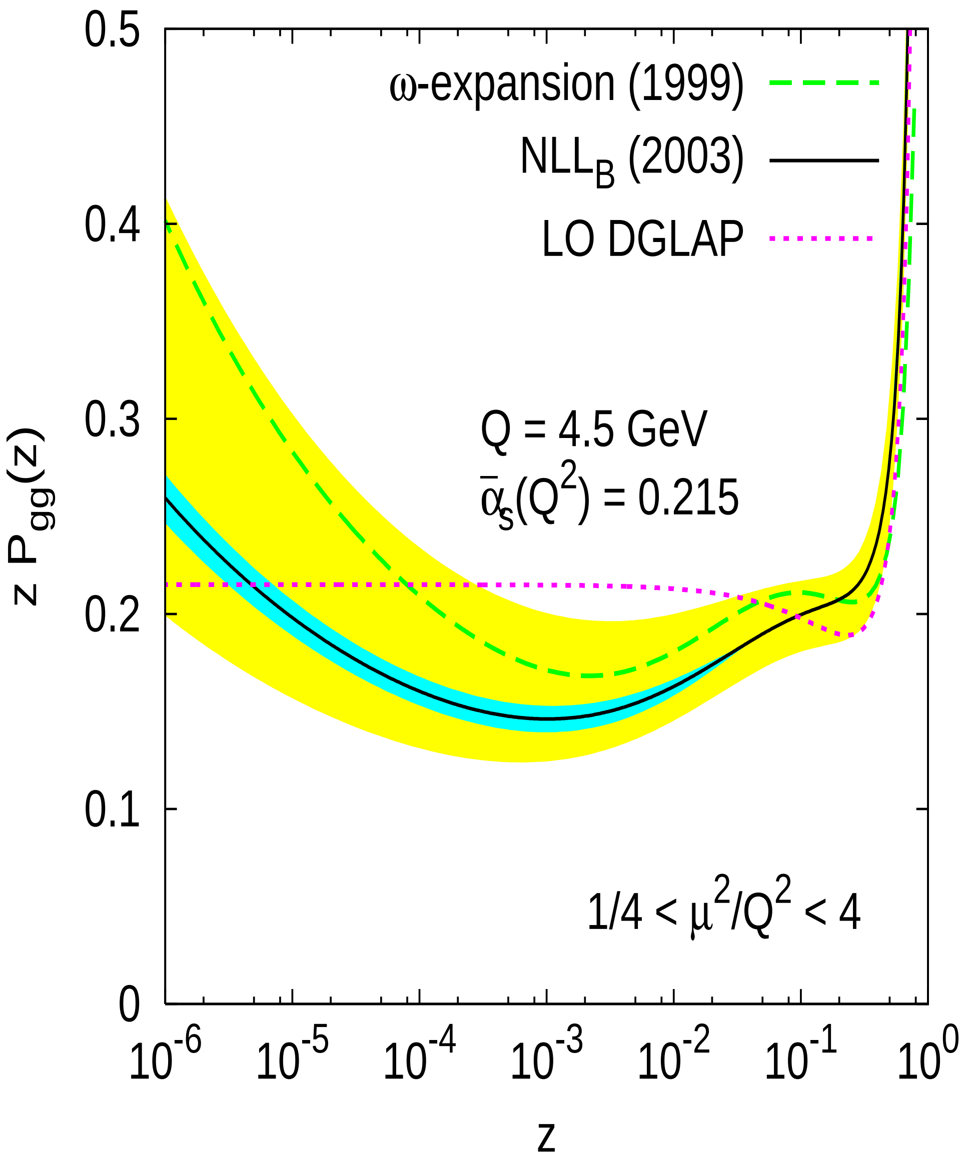

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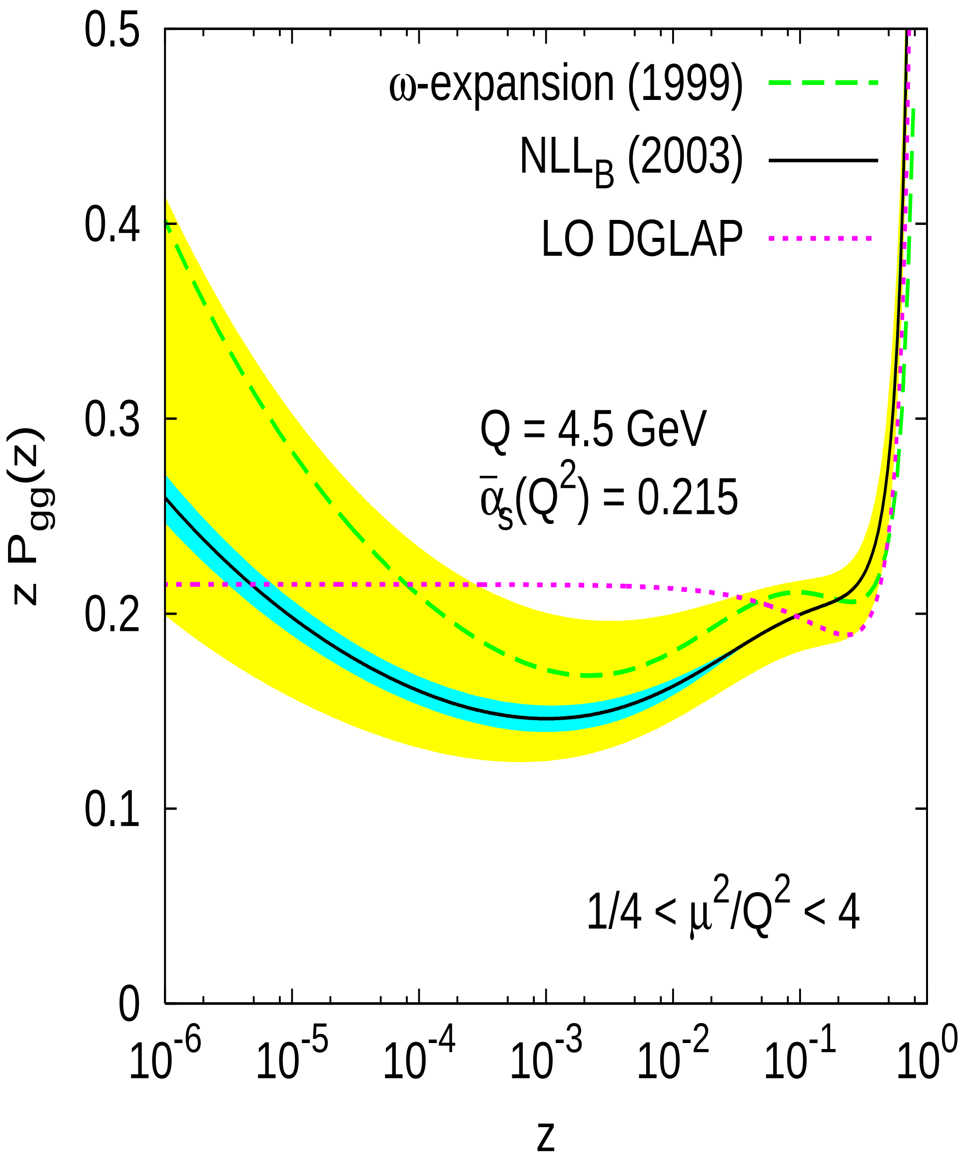

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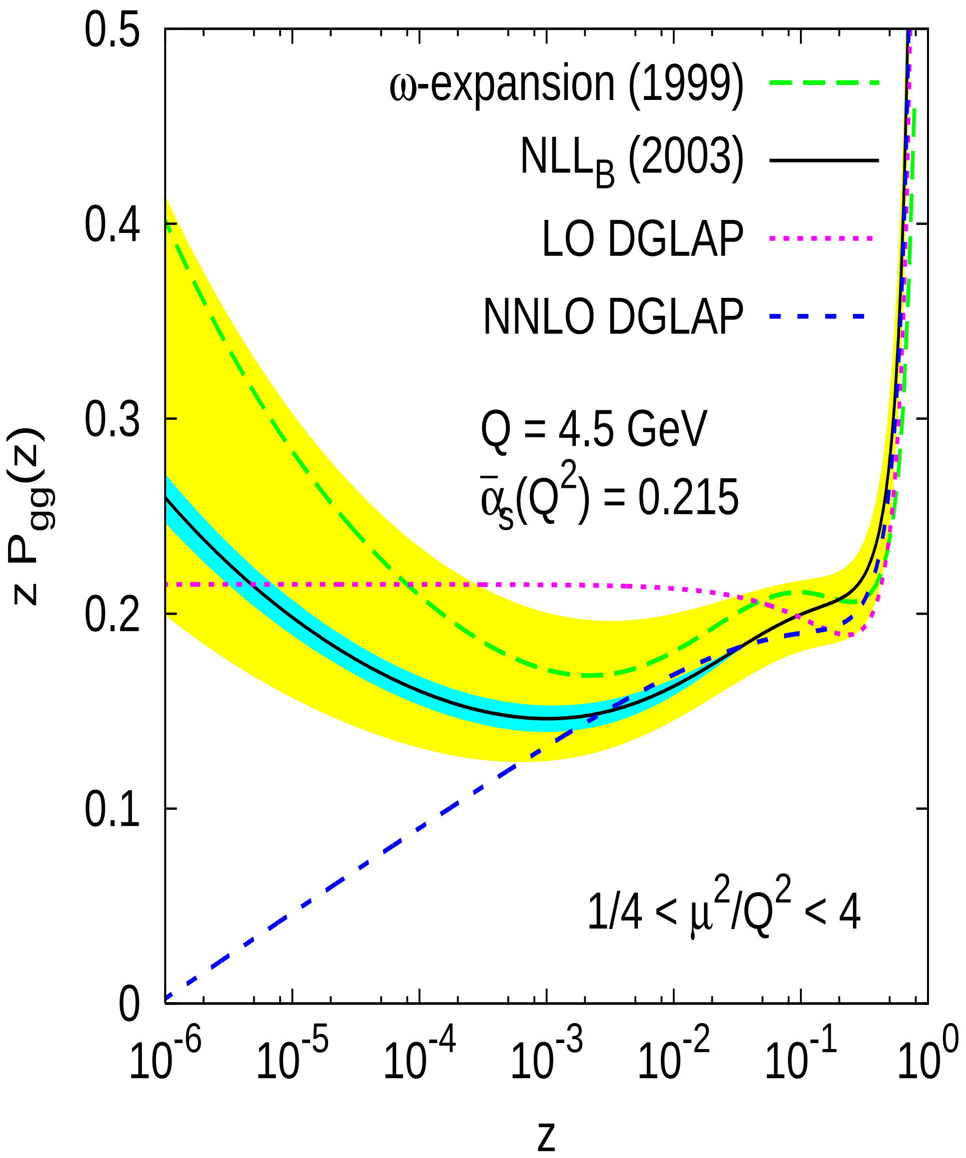
NNLO DGLAP gives a clue. . .

$$-1.54\,\bar{\alpha}_s^3 \ln \frac{1}{x}$$



Reorganise perturbative series

	LLx	NLLx	NNLLx	. . .
α_s	x	–	–	
α_s^2	0	n_f	–	
α_s^3	0	x	x	
α_s^4	x	x	x	
α_s^5	0	x	x	
$\vdots$				

const.

$\ln 1/x$

$\ln^2 1/x$

$\ln^3 1/x$

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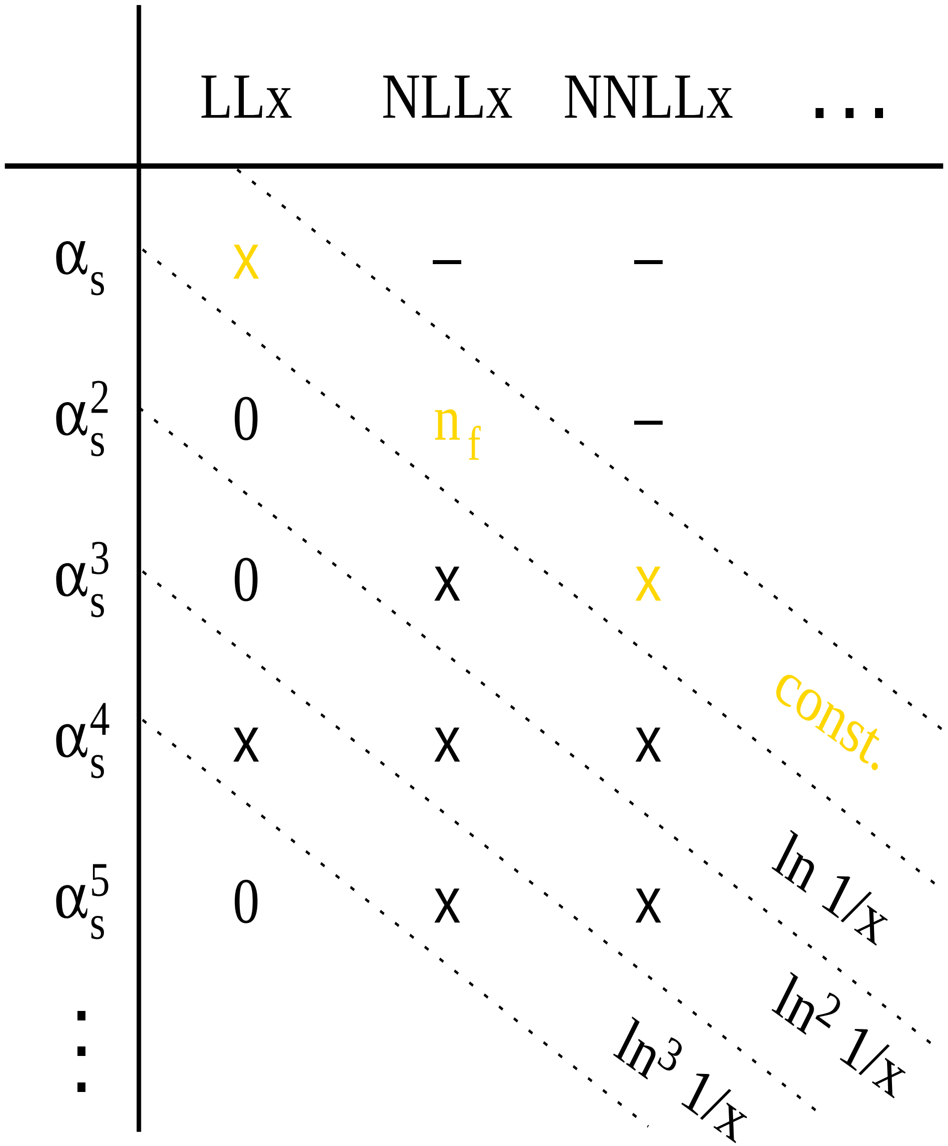

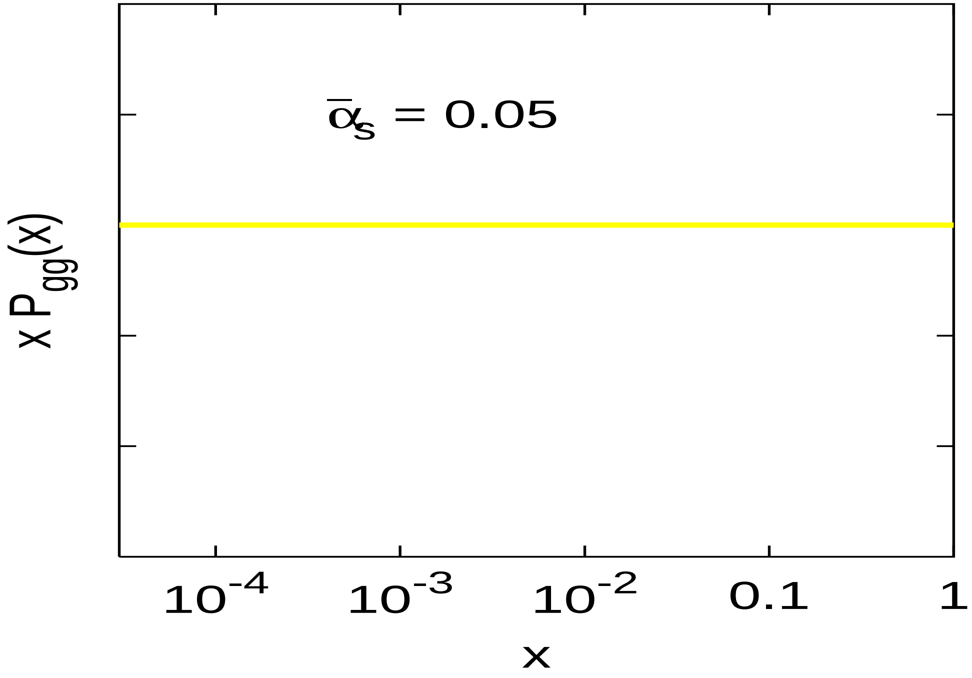

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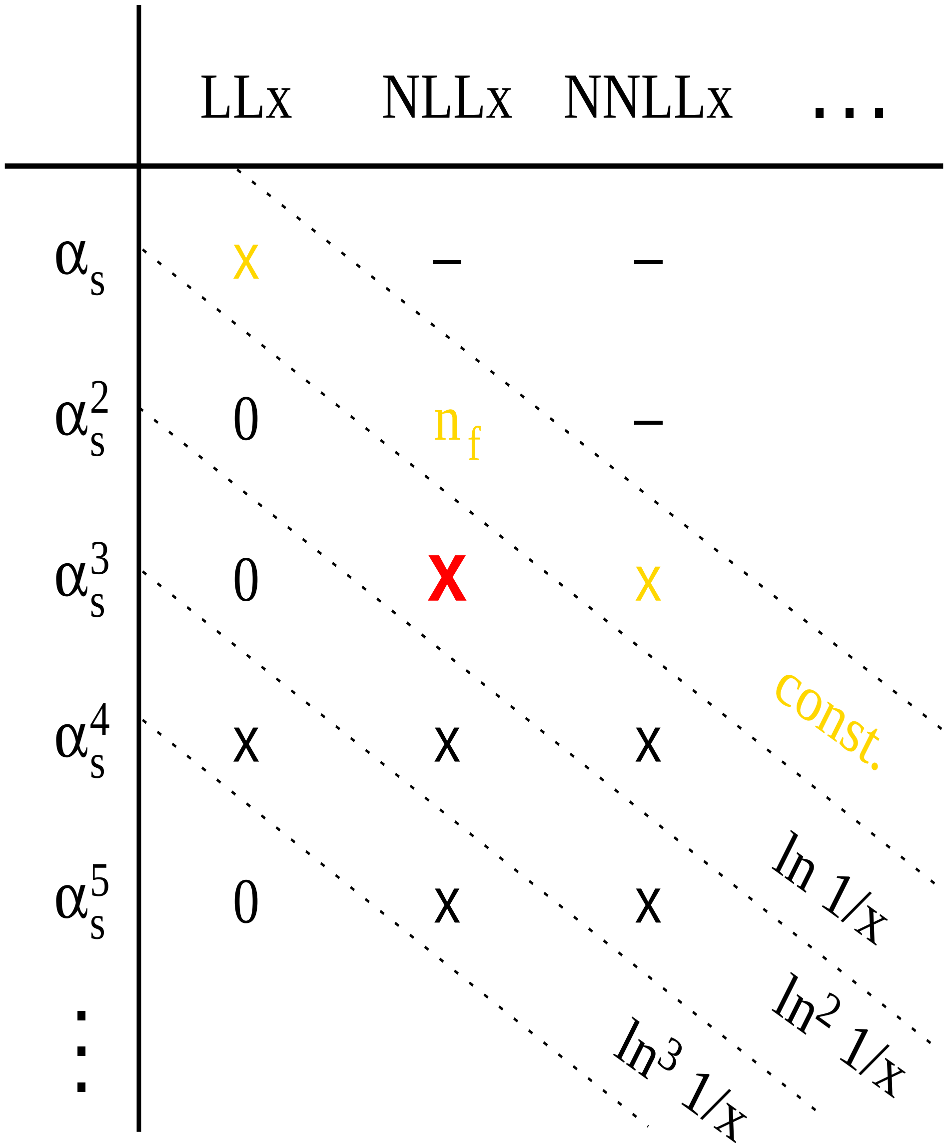
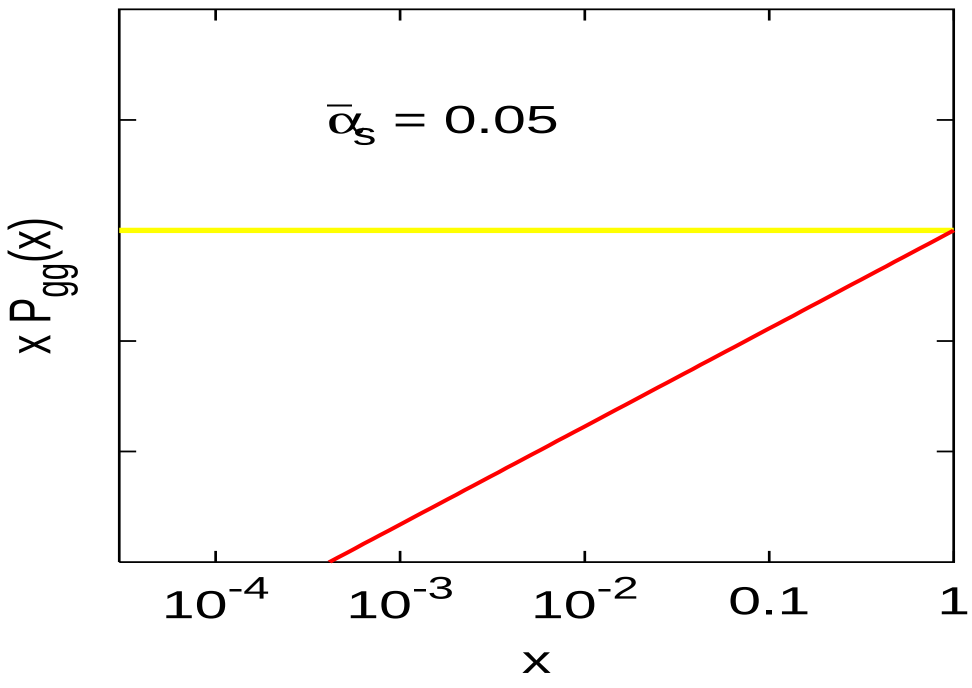


At moderately small x, first terms with x-dependence are

$$-1.54\,\bar{\alpha}_s^3 \ln\frac{1}{x}$$



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α_s^5	0	x	x	$\ln^2 1/x$
⋮			$\ln^3 1/x$	

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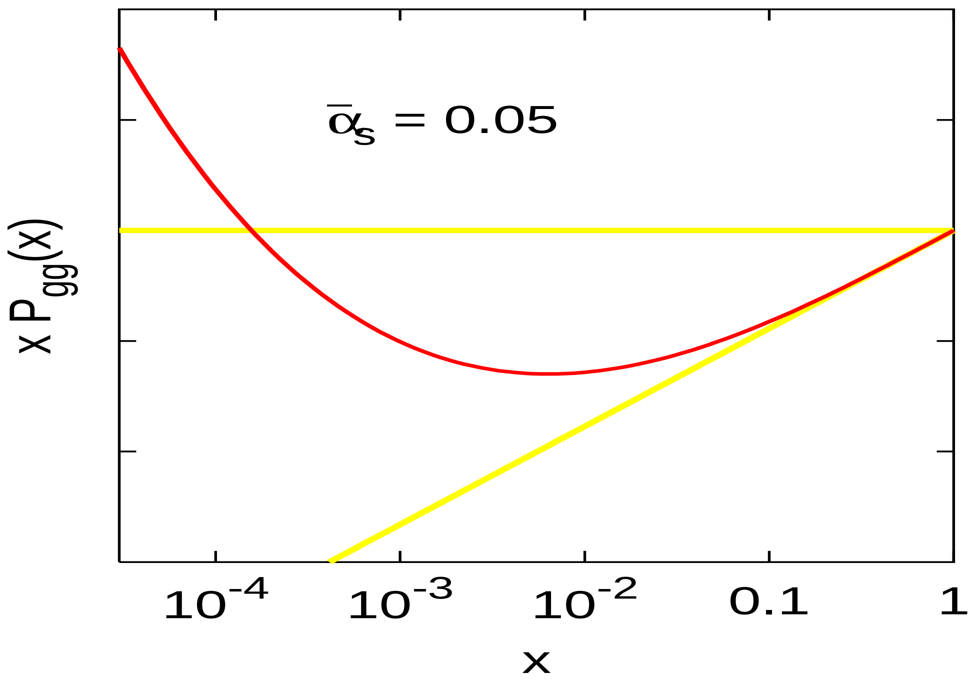

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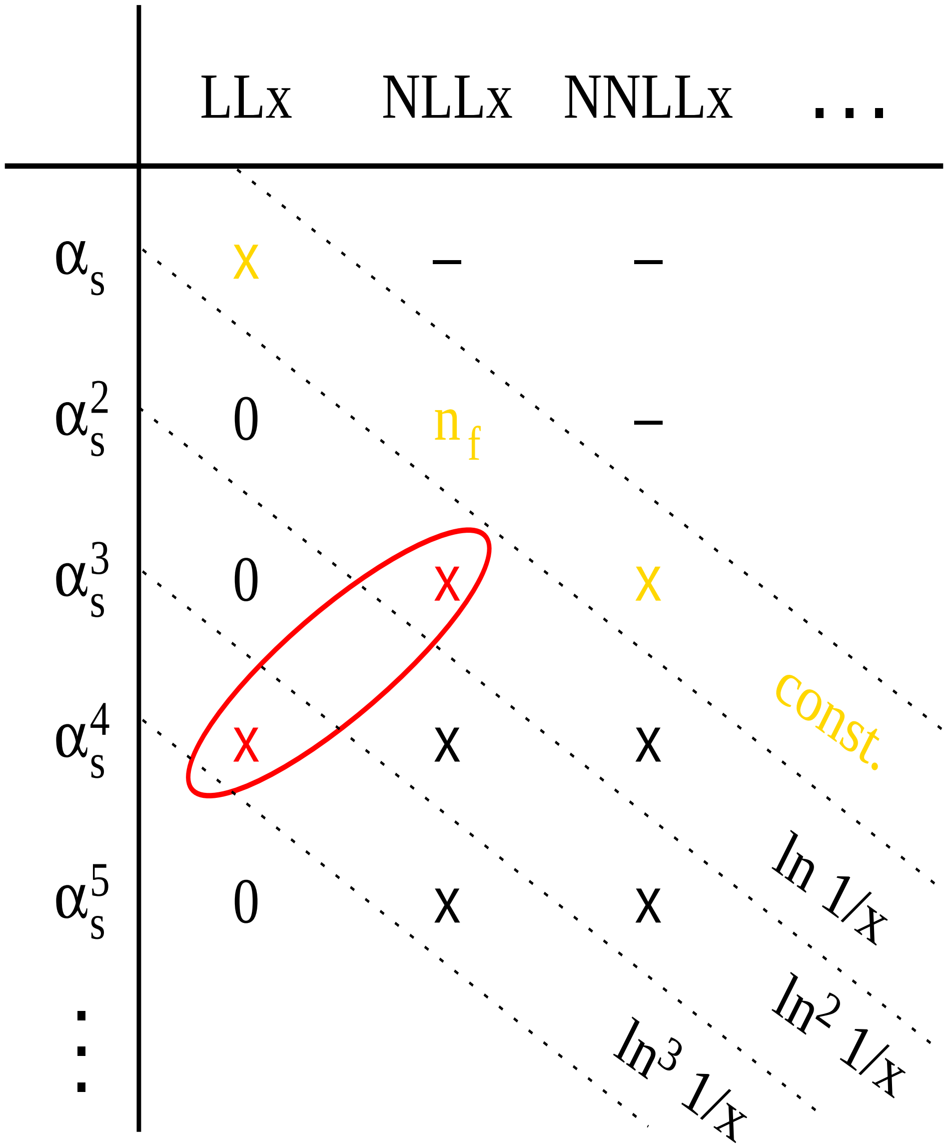

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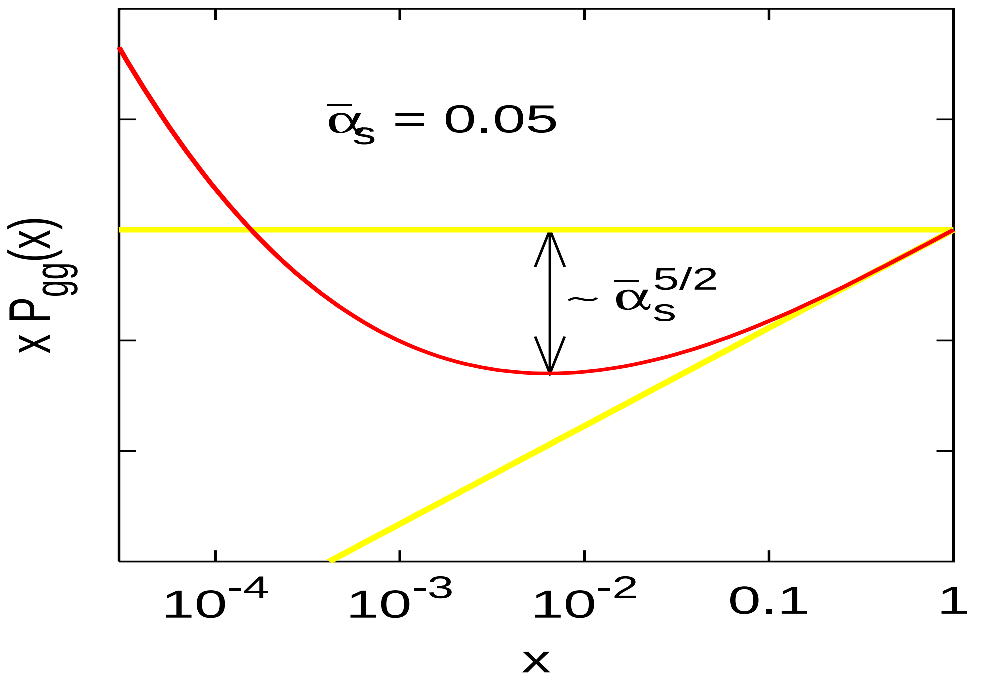
Minimum when

$$\alpha_s \ln^2 x \sim 1 \quad \equiv \quad \ln\frac{1}{x} \sim \frac{1}{\sqrt{\alpha_s}}$$



Systematic expansion in $\sqrt{\alpha_s}$

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ln 1/x

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Position of dip

$$\ln \frac{1}{x_{\min}} \simeq \frac{1.156}{\sqrt{\bar{\alpha}_s}}$$

Depth of dip

$$-d \simeq -1.237\bar{\alpha}_s^{5/2}$$

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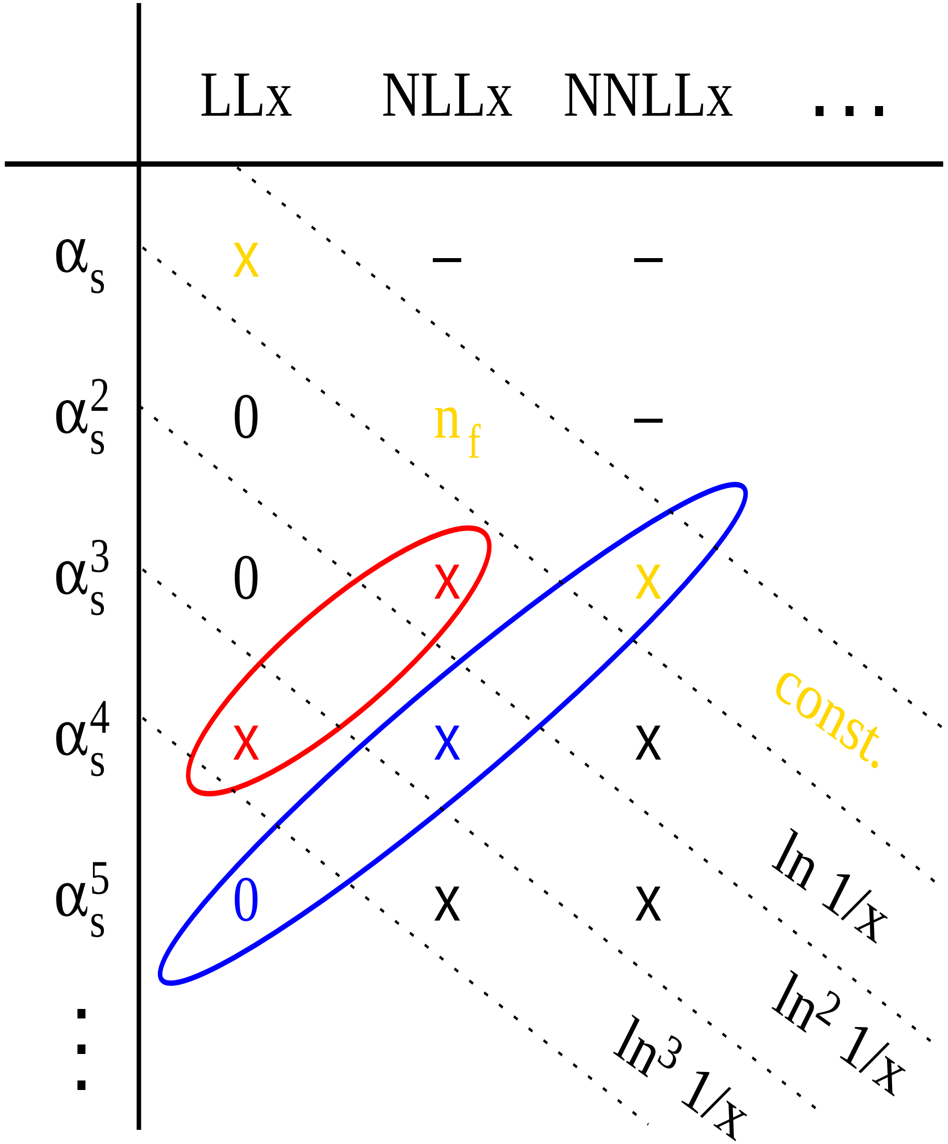

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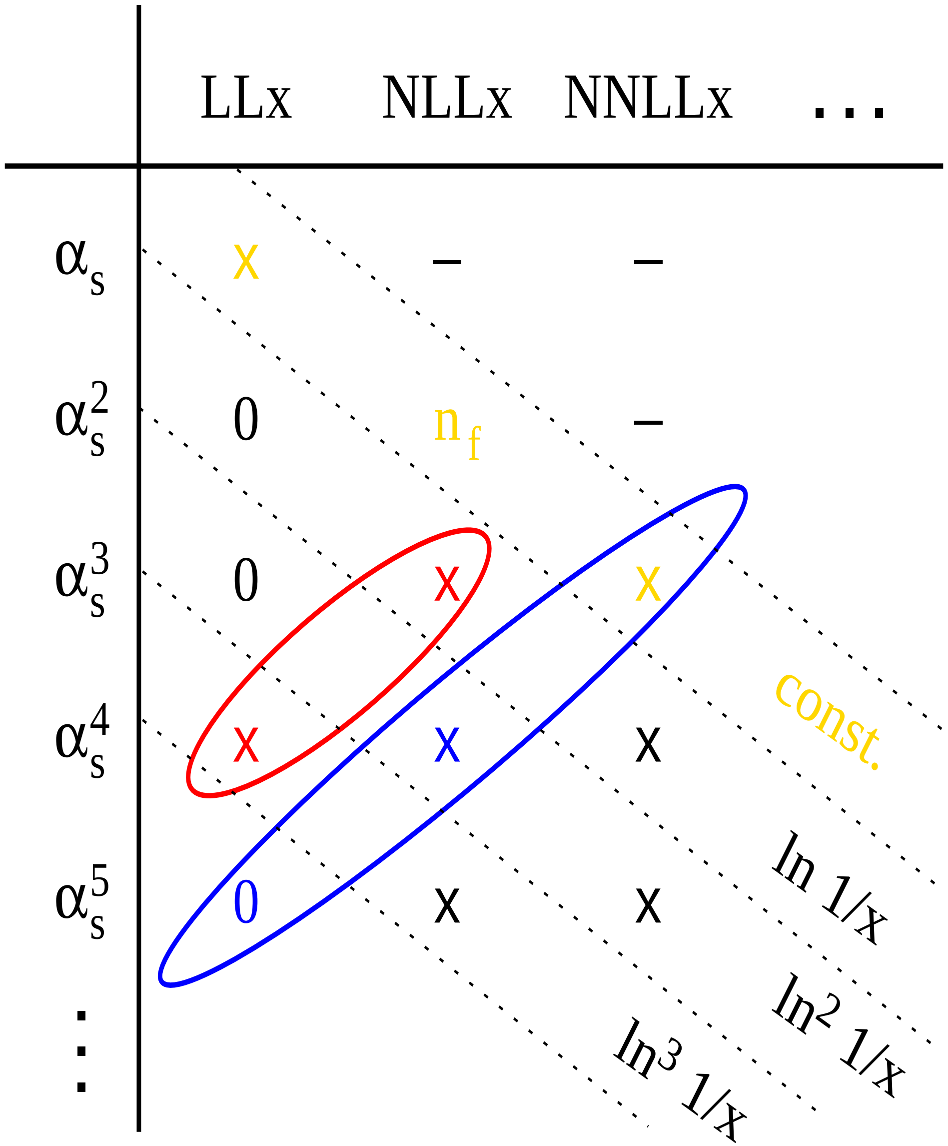

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NB:

- convergence is very poor

  As ever at small x!

- higher-order terms in expansion need NNLLx info

Phenomenological impact?

Phenomenological relevance comes through impact on growth of small-x gluon with Q^2.

$$\frac{\partial g(x,Q^2)}{d\ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

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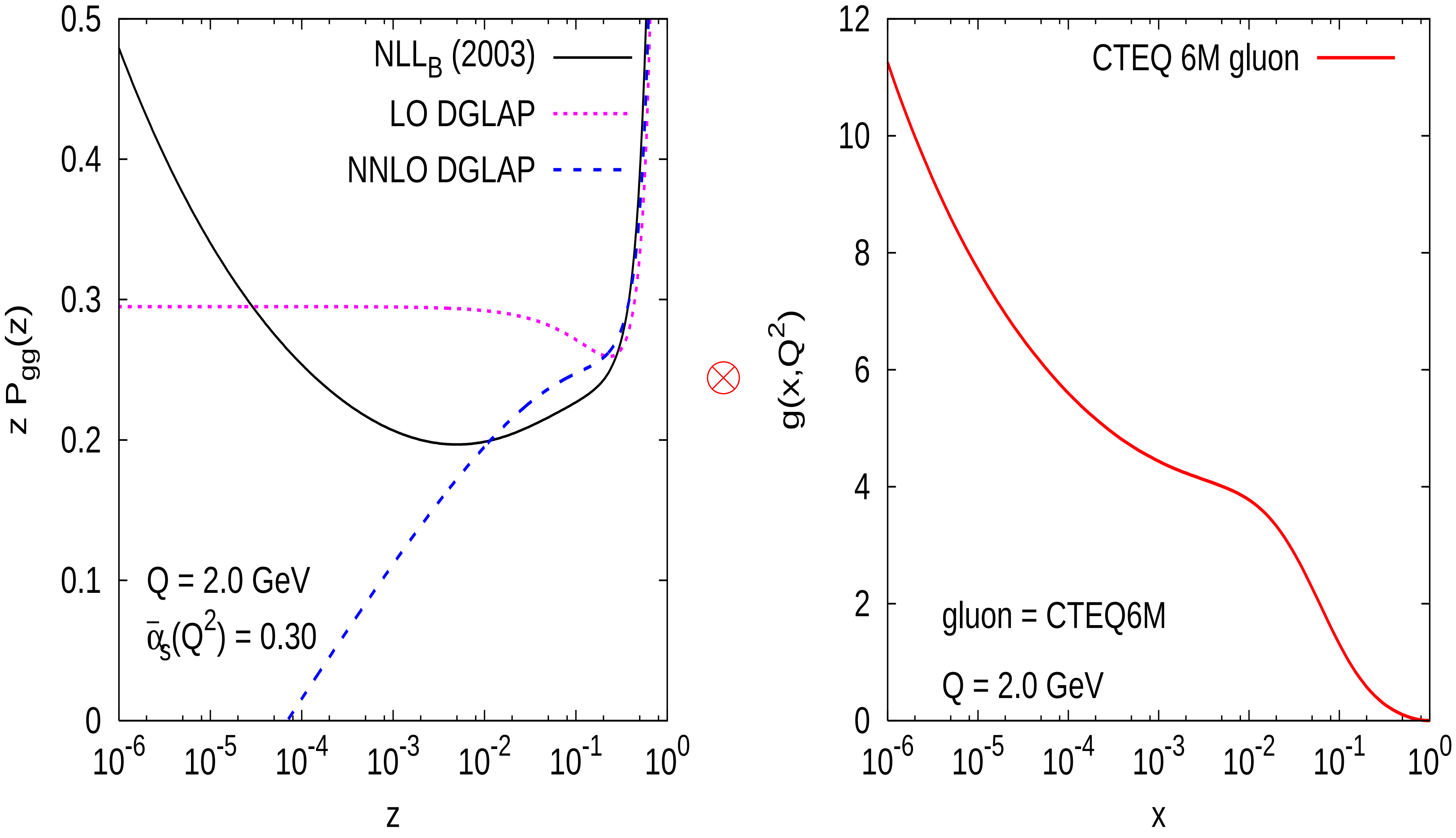
$$\frac{\partial g(x, Q^2)}{d \ln Q^2} = P_{gg} \otimes g + P_{gq} \otimes q$$

At small x, $P_{gg} \otimes g$ dominates.

Take CTEQ6M gluon as 'test' case for convolution.

Because it's nicely behaved at small-x

Phenomenological impact? $P_{gg} \otimes g(x)$



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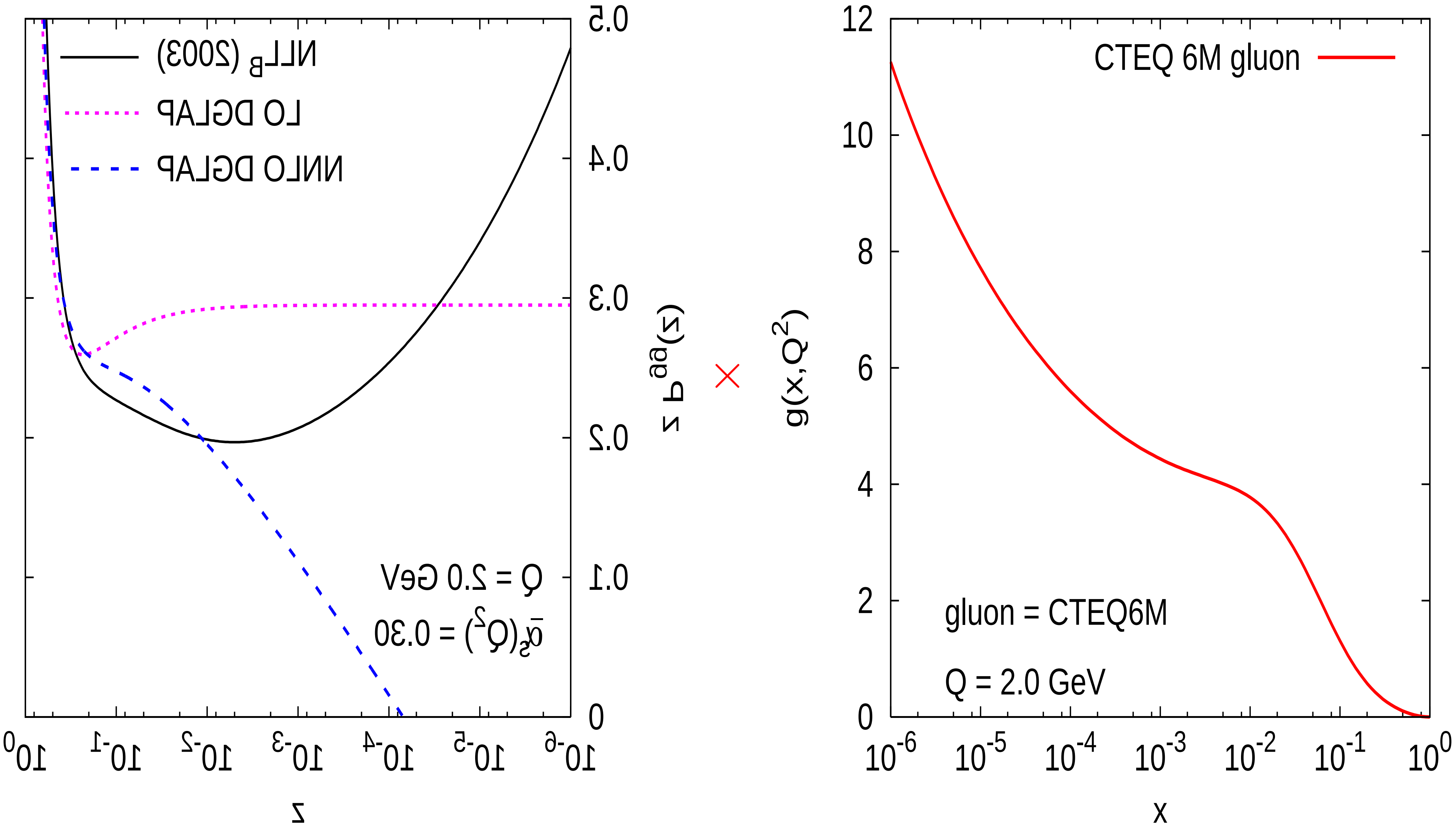

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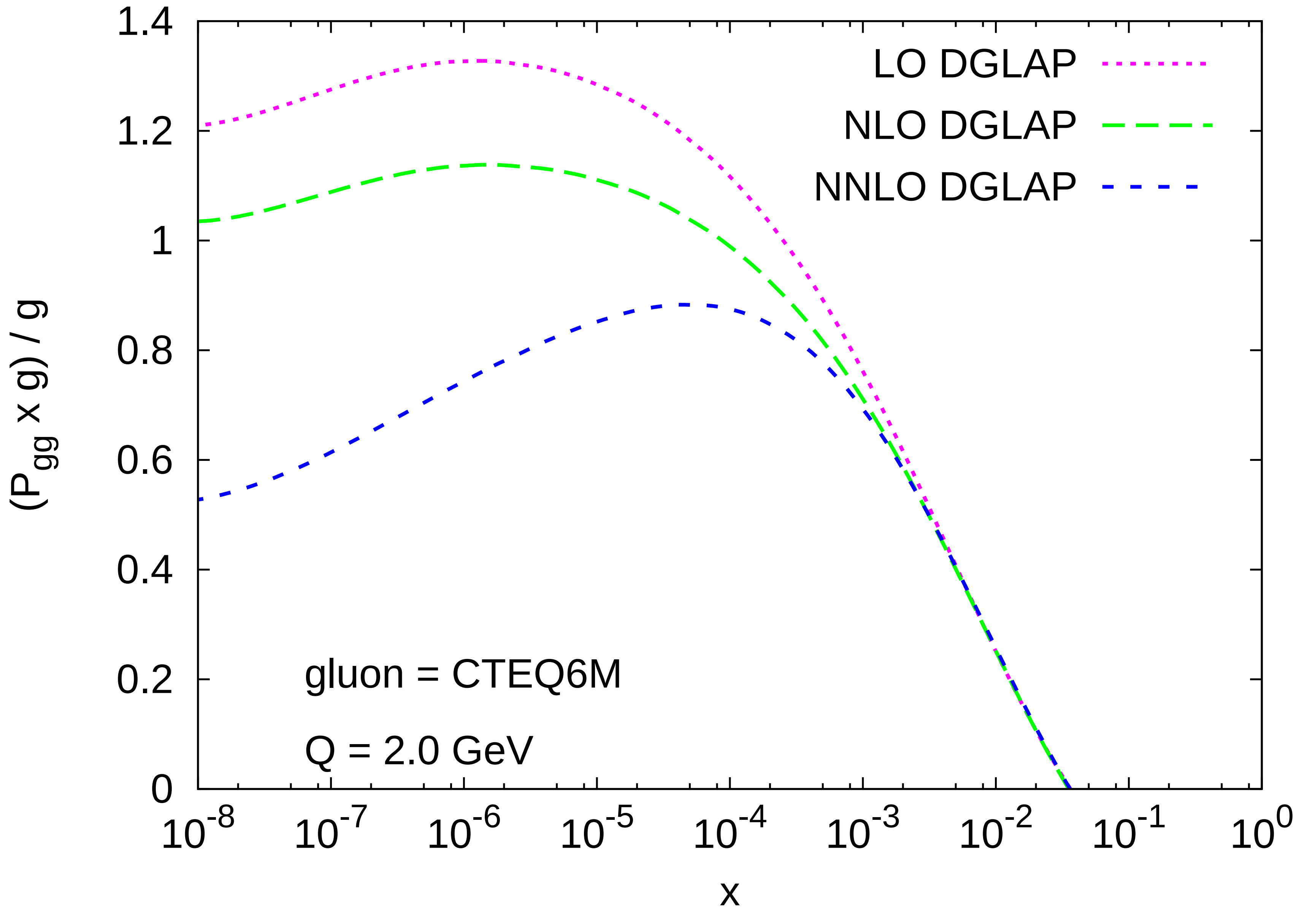
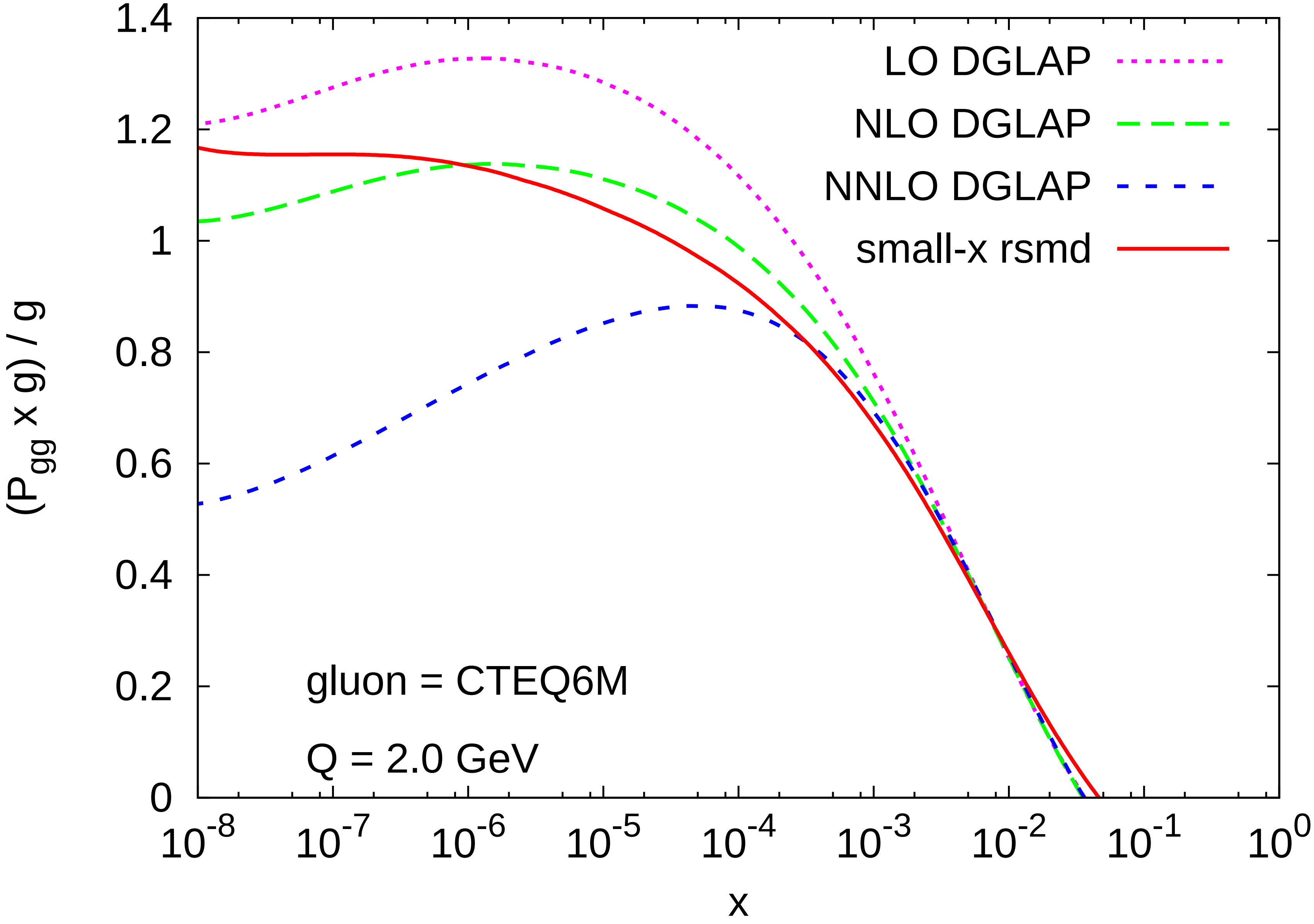


$P_{gg} \otimes g(x)$



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NB: detailed phenomenology still needs considerably more work