Threshold resummation for differential heavy quark structure functions

ERIC LAENEN

NIKHEF

Outline

1. Motivation
2. Heavy quark production at HERA
3. Threshold resummation for differential structure functions
4. Summary

Motivation

Heavy quark production processes at HERA teach us about

- (hard) QCD dynamics in ep collisions
- presence/absence of obvious other physics there

Many observables O to measure. Their QCD description all of the form:

$$O = \phi \otimes C_O \otimes F + P_0$$

- C_O: partonic version of observable
- ϕ/F: universal parton distribution/fragmentation functions
- P_O: power corrections

$$O = \phi \otimes C_O \otimes F + P_O$$

This relation can be used in various ways:

- take ϕ, F from WWW, assume $P_O = 0$, calculate C_O, compare with O_{exp}.
- calculate C_0 to certain approximation (assume $P_O = 0$), measure O, fit ϕ, F.

The game: find the *weakest link*, and update.
Quantities are not unrelated: if ϕ's are fit using N^{k}LO C_O, then those ϕ's also N^{k}LO, and should be used as such.

Purpose (also for LHC): do best possible calculation of C_0

- to mature calculational (resummation) tools
- to extract best possible PDF's etc

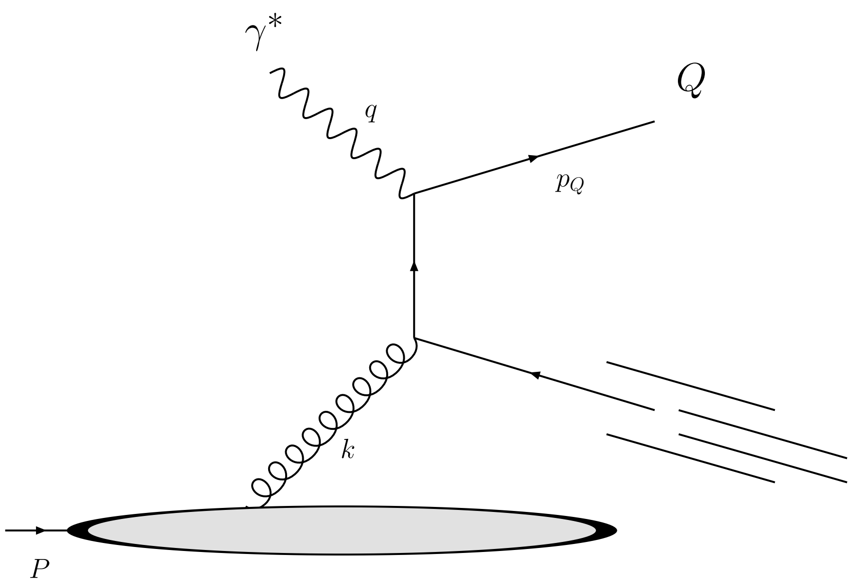
Inclusive vs. differential cross sections

In general, "best" O's: (if possible) differential cross sections

$$\frac{d^{3n}\,\sigma}{d^3p_1\ldots d^3p_n},\quad 2\rightarrow \mathrm{n\,scattering}$$

- flexible integration over regions of phase space where experiment X has a detector → better comparison, less theoretical uncertainty
- generation of different observables by partial integration over kinematic variables

Heavy quark electroproduction



Some kinematic variables, besides the usual DIS x, y, Q^2:

- $S = (q+P)^2 = S' - Q^2$: $\gamma^* P$ cm energy squared
- $T_1 = (P - p_Q)^2 - m^2$: hadronic momentum transfer squared
- $U_1 = (q - p_Q)^2 - m^2$

Partonic equivalents $(s(s'), t_1, u_1)$: $P \to k$.

Relevant for later:

$$S_4 = S' + T_1 + U_1, \qquad s_4 = s' + t_1 + u_1,$$

Meaning: extra invariant mass of hadronic/parton recoiling system. When $s_4 \to 0$: elastic kinematics.

From $T_1, U_1, \ldots$ can make p_T, y of heavy quark.

Factorization & HQ electroproduction

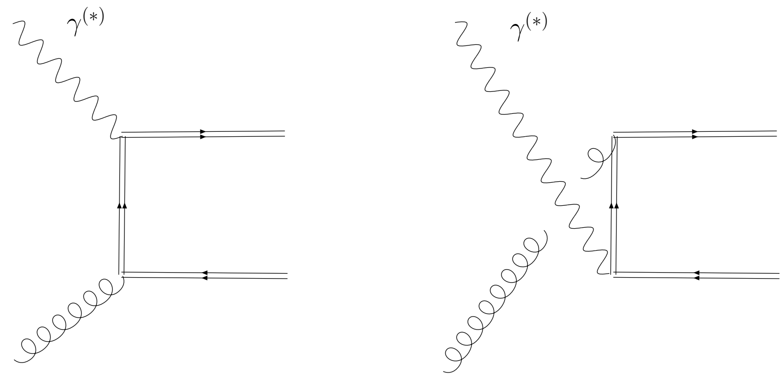
As before, now with a bit more detail

$$d\sigma_{\gamma^*+A\to Q+X} = \sum_a \int d\xi \;\; \phi_{a/A}(\xi,\mu)$$
$$\times\; d\hat{\sigma}_{\gamma^*+a\to Q+X}(\xi,\mu,m,Q^2,\alpha_s(m)) + \text{power corrections}$$

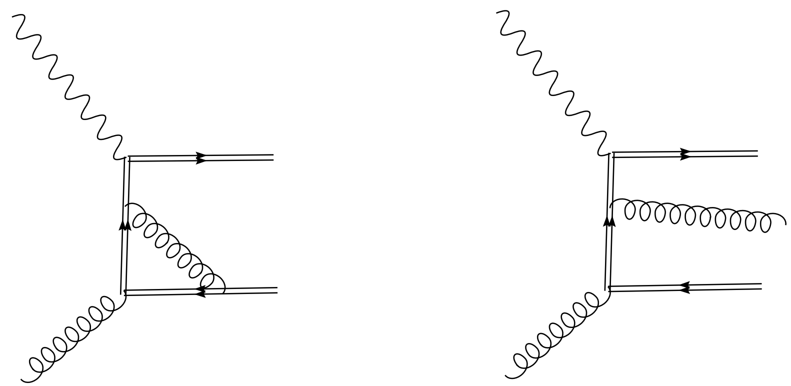
Again, to make precise predictions, one must

- compute $d\hat{\sigma}_{\gamma^*+a\to Q+X}$ to sufficiently high order
- know the $\phi_{a/A}(\xi,\mu)$ as accurately as possible

For LO accuracy for $d\hat{\sigma}_{\gamma^* + a \to Q + X}$, compute



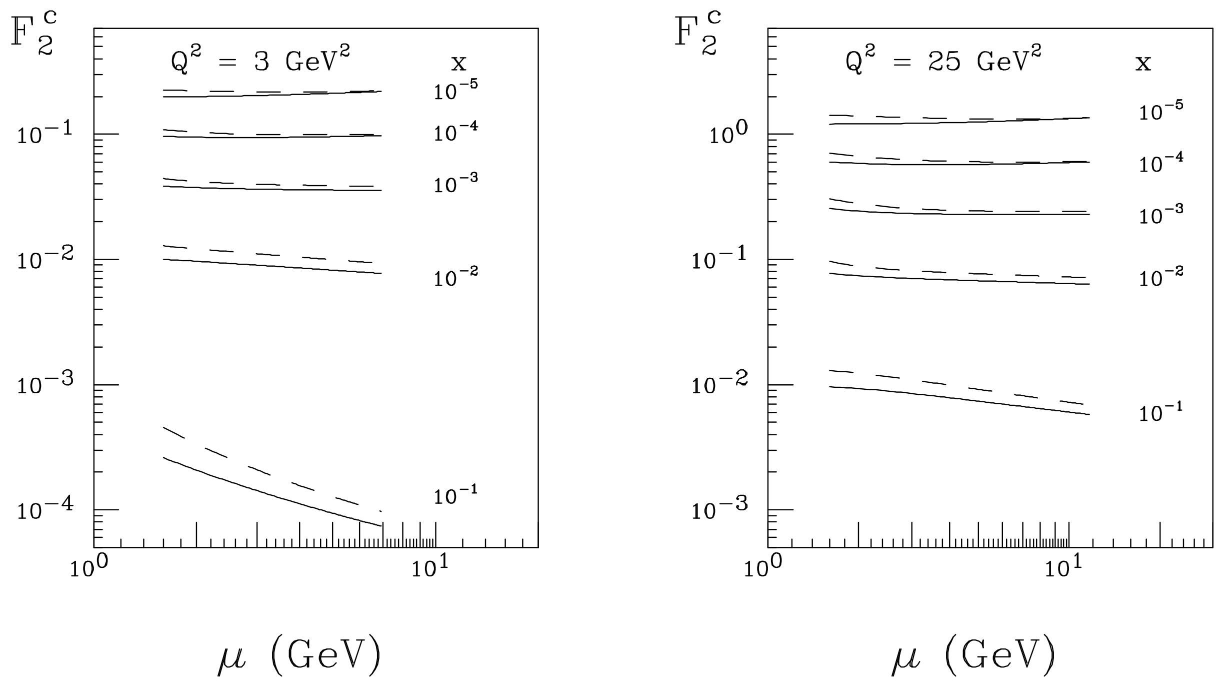
For NLO accuracy, add graphs like



(there are many more..) NNLO is unknown, for massive quarks, and pretty much hopeless.

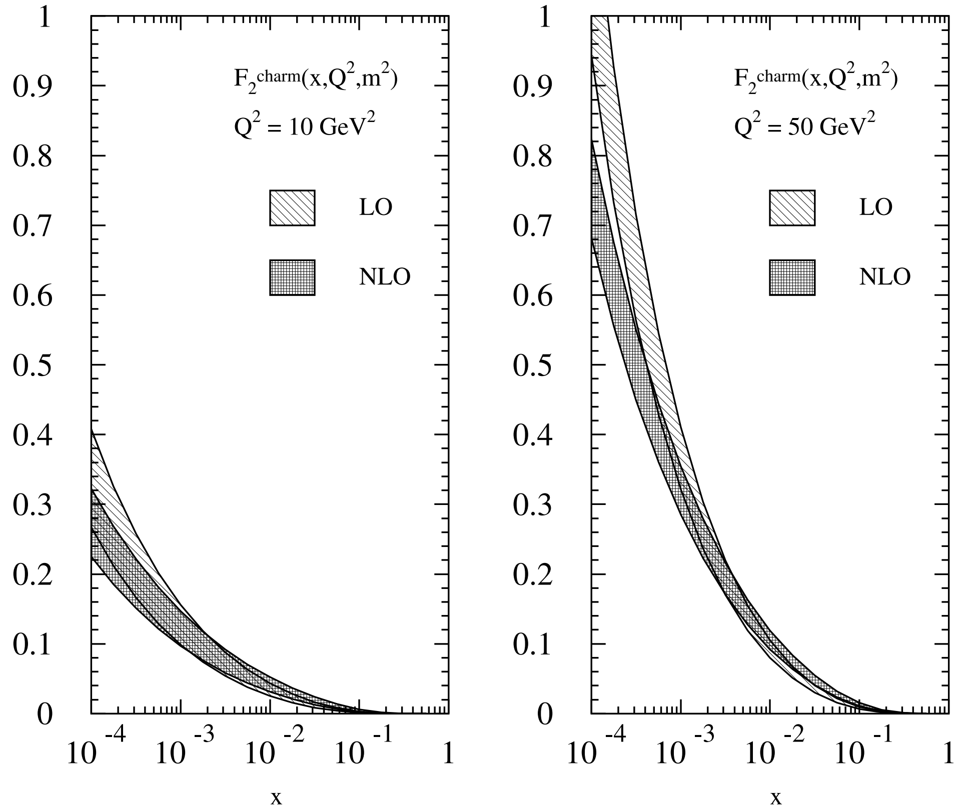
Finite order calculations

To remind you, the NLO *inclusive* (F_2^{charm}) and single-particle differential distributions EL, Riemersma, Smith, v. Neerven



show hardly any scale dependence: $\alpha_s(\mu)$ and $\phi_{g/P}(\xi, \mu)$ anti-correlated.

The dominant uncertainty is due to the charm mass (here 1.3-1.7 GeV)



HVQDIS

A fully differential NLO calculation exists Harris, Smith, and has been coded into a user-friendly program (HVQDIS), already much used by H1 and ZEUS.

It allows choice of various fragmentation functions, and also semi-leptonic decay of charmed hadron, if one wishes. Has already been used to extract F and, charm mass:

(Extraction of NLO PDF's (gluon especially) possible, will surely be done.)

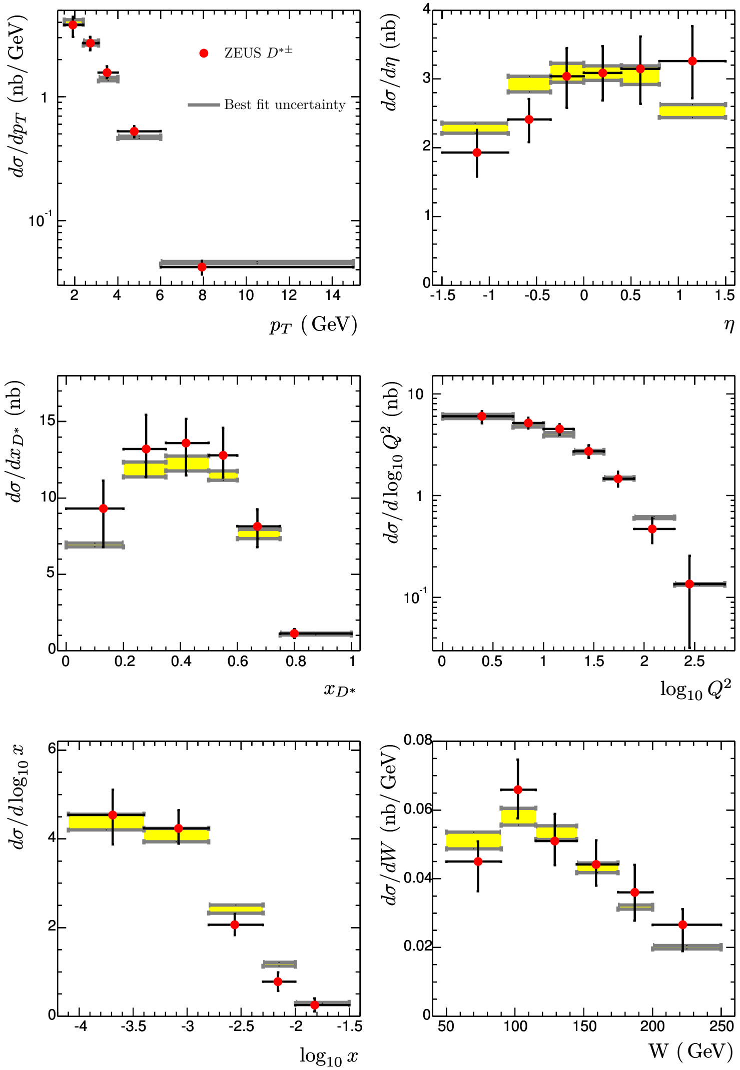
Extracting m_c and $\epsilon_{Peterson}$

Sven Schagen, '04.

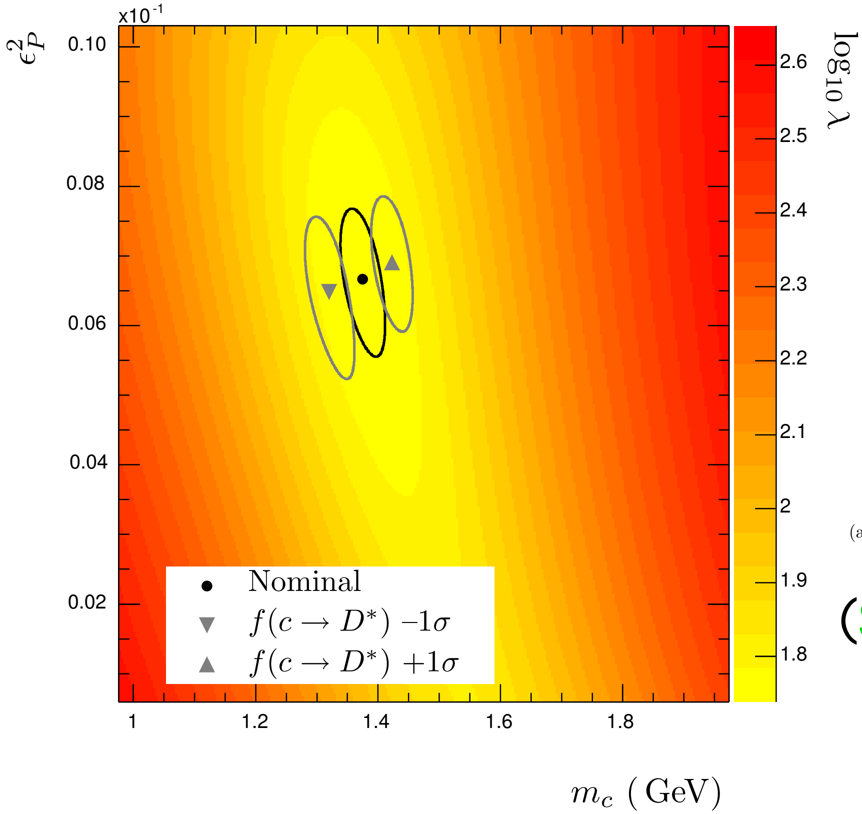
Because the charm mass has the dominant uncertainty in charm DIS differential cross sections, determine it by comparing to NLO theory.

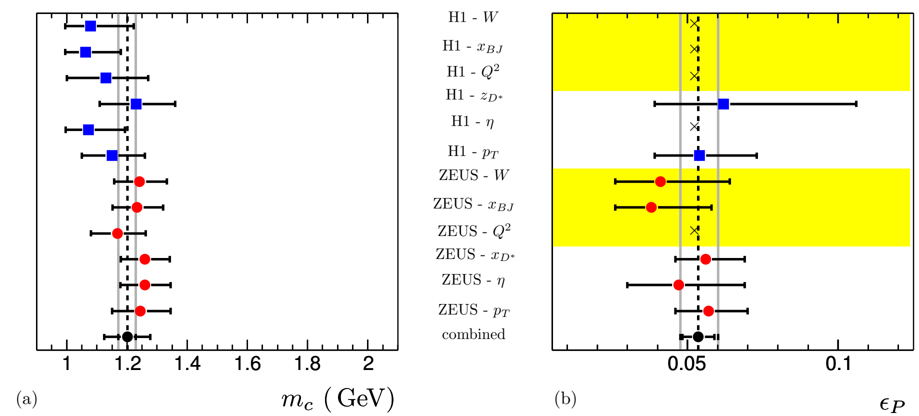
Use HVQDIS with Peterson fragmentation function, vary m_c and ϵ_P and fit to data.

Requires *a lot* of CPU (use GRID..)



Extracting m_c, ϵ_P , cont'd





(Sven Schagen, '04)

$$m_c = 1.37 \pm 0.04 \pm 0.08$$

$$\epsilon_P = 0.082 \pm 0.006 \pm 0.001$$

BNLO

How to go Beyond NLO? Finite order PT gathers systematically all terms

$$O = \sum_n \alpha_s^n O_n + C$$

With NNLO just not in the cards, there is at least one other way of systematically gathering terms

$$O = H \, \exp \left[\sum_n \sum_{m=0} \alpha_n L^{n+1-m} d_{n,m} \right] + C'$$

QCD resummation is a calculational framework just as systematic as normal PT. The two can be matched systematically.

Resummation

Resummation = organization of large logarithms (of IR origin) in perturbative expansions:

$$
\begin{aligned}
d\sigma &= 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \ldots \\
&= \exp\left(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L) + \ldots \right) \underbrace{C(\alpha_s)}_{\text{constants}} \\
&\quad + \text{ suppressed terms}
\end{aligned}
$$

$L = \ln(?)$. Argument differs per observable. Resummation

- can restore predictive power
- can increase theoretical accuracy (e.g. reduction of scale uncertainty)
- can teach about power corrections

Threshold resummation

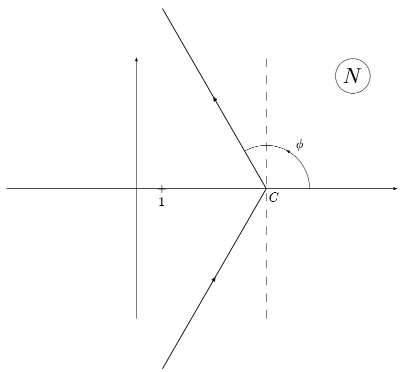
Threshold resummation begins with defining a threshold (elastic limit) $w = 0$. This depends on the observable, e.g. in heavy quark production

- inclusive : $w = s' - 4m^2 = 0 \Rightarrow$ Born process above threshold
- p_T distribution: $w = s' - 4(m^2 + p_T^2) = 0 \Rightarrow$ Born process above threshold
- double-differential distribution: $w = s_4 = s' + t_1 + u_1 = 0 \Rightarrow$ Born process *at* threshold

To resum $\ln(w)$ effects, it is best to first pass to a conjugate space via Laplace (or Mellin) transform $\int_0 dw \exp(-w\,N)$. Note: $w \to 0 \Rightarrow N \to \infty$. The large log is then $\ln N$. To go back:

$$\int_C dN \exp(w\,N) f(N)$$

with C an appropriately chosen contour:



Threshold res´n & HQ electroproduction

To making resummation ever more quantitative, we (T. Eynck, A. Vogt, EL) examined threshold effects systematically:

- derived NLL exponent
- examined issues related to differential cross section
- in expansion, we compared N space vs momentum space
- compared to fully inclusive F_2

Differential structure functions defined via

$$\frac{d^4\sigma^{eP\to eQX}}{dx\,dQ^2\,dT_1\,dU_1} = \frac{2\pi\alpha^2}{x\,Q^4}\left[\left(1+(1-y)^2\right)\frac{d^2F_2^Q(x,Q^2,m^2,T_1,U_1)}{dT_1\,dU_1} - y^2\frac{d^2F_L^Q(x,Q^2,m^2,T_1,U_1)}{dT_1\,dU_1}\right].$$

Concentrate on F_2, which is dominated by near-threshold processes (Moch, EL)

NIKHEF

Factorized forms

Near threshold

$$S'^2 \frac{d^2 F_2^Q(S_4, T_1, U_1)}{dT_1\, dU_1} \simeq \sum_g \int_{z^-}^{1} \frac{dz}{z}\, \phi_i(z, \mu_F^2) \quad \omega_i\Big(z, s_4, t_1, u_1, \mu_F^2, \mu_R^2\Big)$$

Normally, we can turn convolutions into nice simple products via Mellin/Laplace transforms: $F(N, Q^2) = \int_0^1 dx \exp[N(1-x)] F_2(x, Q^2) = C(N)\phi(N, Q^2)$. In inclusive DIS, recoil invariant mass: $(1-x)Q^2/x$, near-elastic for $x \to 1$.
Here, need invariant mass *recoiling* against heavy quark S_4

$$S_4 = \underbrace{s_4}_{\text{parton radiation}} + \underbrace{(1-z)(S' + T_1)}_{\text{proton remnant}}$$

Laplace transform $\int dS_4 \exp[N\, S_4] \ldots$

$$S'^2 \frac{d^2 F_2^Q(N, T_1, U_1)}{dT_1\, dU_1} \simeq \bar{\phi}_g\left(N \frac{S' + T_1}{m^2}\right) \omega\Big(N, z^{\mathrm{opt}}, T_1, U_1\Big)\,.$$

All singular dependence in N transform, non-singular z dependence chosen to approximate best at NLO: $z^{\mathrm{opt}} = 1 - S_4/(S' + T_1)$

Singular functions

At lowest order, in momentum space

$$\omega^{(0)}(w) = \sigma_B(t_1, u_1, Q^2)\delta(s_4)$$

(Similar to e.g. Drell-Yan $\propto \delta(s - M_Z^2)$ at lowest order.) At higher order

$$\omega = \sum_{l=0}^{2n-1} c^O_{n,k} \left[\frac{\ln^l(s_4)}{s_4}\right]_+ + c^O_{n,\delta}\delta(s_4) + f^O(s_4)$$

with the plus distributions

$$\mathcal{D}_{l+1} \equiv \left[\frac{\ln^l(s_4)}{s_4}\right]_+ = \lim_{\kappa\to 0}\left\{\frac{\ln^l(s_4)}{s_4}\theta(s_4-\kappa) + \frac{1}{l+1}\ln^{l+1}(\kappa)\,\delta(s_4)\right\}.$$

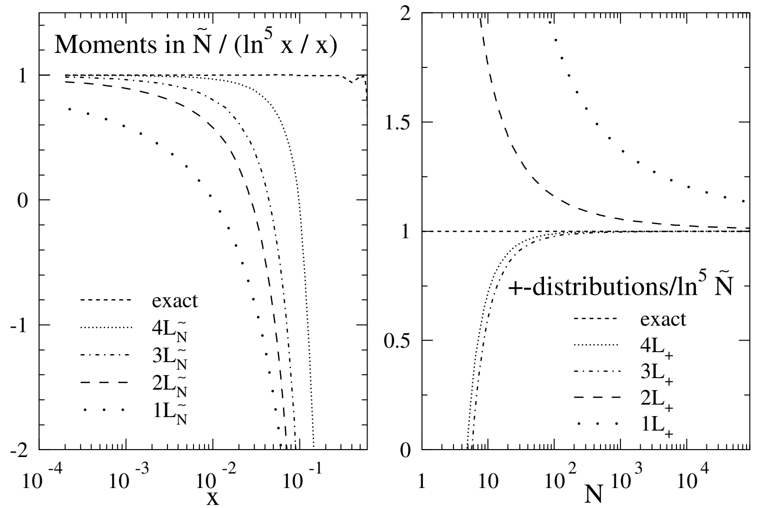
After Laplace transform (small $s_4 \sim$ large N)

$$\left[\frac{\ln^l(s_4)}{s_4}\right]_+ \leftrightarrow \frac{(-1)^{l+1}}{l}\left(\ln^{l+1} N \;+\; \text{subleading terms}\right)$$

How important are the subleading terms?

Subleading terms in N space

Subleading logs often come endowed with large coefficients

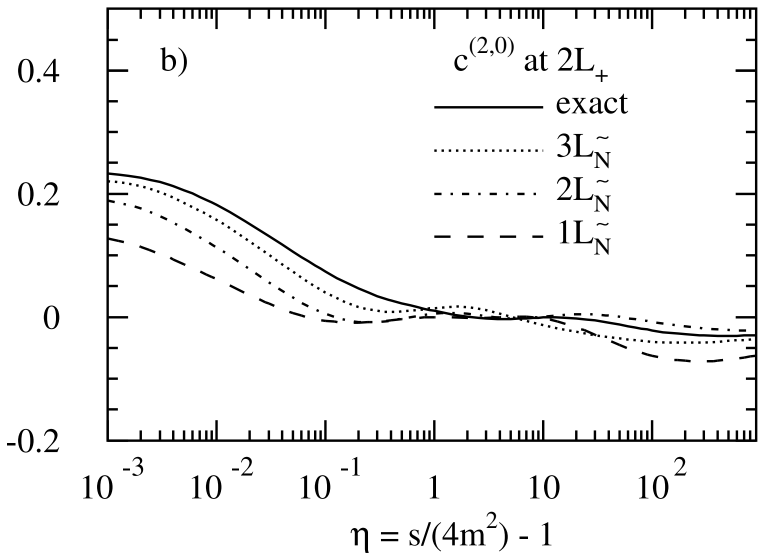


	$\ln^7(x)/x$	$\ln^5(x)/x$	$\ln^3(x)/x$
$\ln^8 \tilde{N}$	0.125	-	-
$\ln^7 \tilde{N}$	-	-	-
$\ln^6 \tilde{N}$	5.76	0.167	-
$\ln^5 \tilde{N}$	16.8	-	-
$\ln^4 \tilde{N}$	128	4.11	0.25
$\ln^3 \tilde{N}$	451	8.01	-
$\ln^2 \tilde{N}$	1424	36.5	2.47
$\ln \tilde{N}$	2815	64.4	2.4
const	2829	67.8	3.65

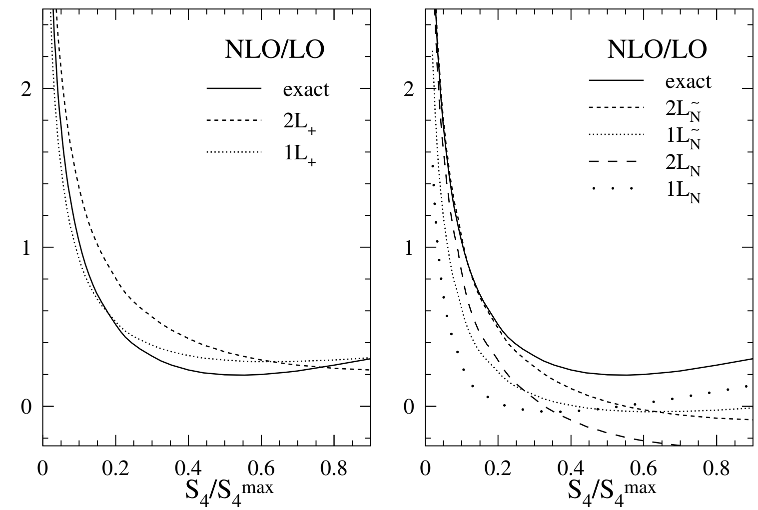
Has been seen before in various guises Catani, Mangano, Nason, Trentadue; Vogt; Kidonakis

Subleading terms in N space

For a more physical relevant *inclusive* quantity: approximate 2-loop partonic inclusive F_2^c



For $d^2F_2^c/dT_1dS_4$ as functions of $S_4/S_4^{\max}$ (T_1 chosen halfway in range)



Progression towards exact results somewhat more uniform in N space.

Under the resummation hood

A factorization analysis gives the formula

$$\omega_g(N) = \underbrace{\prod_i \psi_i(\hat{N}, Q)}_{ISR} \left[\underbrace{S(N)}_{Soft} \otimes H(Q) \right]$$

with $H(Q)$ a matching function, and

$$\hat{\psi} \sim \exp\left[-\int_0^1 \frac{z^{N-1}-1}{1-z} \left\{ \int_{(1-z)}^1 \frac{d\lambda}{\lambda} A(\alpha_s(\lambda Q)) + \frac{1}{2}\nu(\alpha_s((1-z)Q)) \right\} \right]$$

Probability of not emitting a soft-collinear gluon leading to recoil mass higher than Q/N

$$S(N) \sim P \exp\left[\int_Q^{Q/N} \frac{d\mu}{\mu} 2\mathrm{Re}\, \{\Gamma(\alpha_s(\mu)\} \right]$$

Probability of not emitting a soft wide-angle gluon etc.

Threshold-resummed differential F_2^c

For the threshold-resummed partonic cross section, using methods of Oderda, EL, Sterman for single-particle inclusive cross sections, we find to NLL accuracy ($\lambda = b_0 \alpha_s \ln N$)

$$\omega_g\left(N, s, t_1, u_1, \mu_F^2, \mu_R^2\right) = \omega_0(s, t_1, u_1) \times \exp\left[\ln N \, h_1(\lambda) + h_2\left(\lambda, \frac{m^2}{\mu_R^2}, \frac{m^2}{\mu_F^2}, s, t_1, u_1\right)\right]$$

with

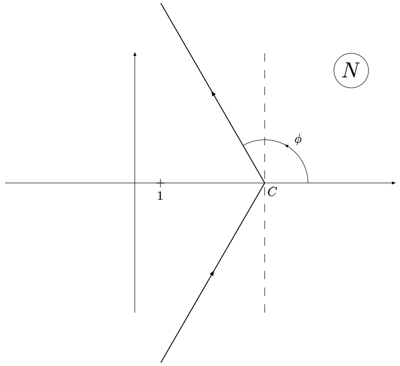
$$h_1(\lambda) = \frac{A_g^{(1)}}{2\pi b_0 \lambda}\left[2\lambda + (1-2\lambda)\ln(1-2\lambda)\right]$$

$$\begin{aligned} h_2(\lambda) = & -\frac{A_g^{(1)}}{\pi b_0}\gamma_E \ln(1-2\lambda) - \frac{\bar{B}_g^{(1)}}{2\pi b_0}\ln(1-2\lambda) - \frac{A_g^{(2)}}{2\pi^2 b_0^2}\left[2\lambda + \ln(1-2\lambda)\right] \\ & -\frac{A_g^{(1)}}{\pi b_0}\lambda \ln\left(\frac{m^2}{\mu_F^2}\right) + \frac{A_g^{(1)}}{2\pi b_0}\ln\left(\frac{m^2}{\mu_R^2}\right)\left[2\lambda + \ln(1-2\lambda)\right] \\ & -\frac{C_A}{2\pi b_0}\ln(1-2\lambda)\left(2L_u - \ln\left(\frac{s}{m^2}\right)\right) + \frac{\mathrm{Re}\Gamma_S^{(1)}}{\pi b_0}\ln(1-2\lambda)\ , \end{aligned}$$

From N space to momentum space

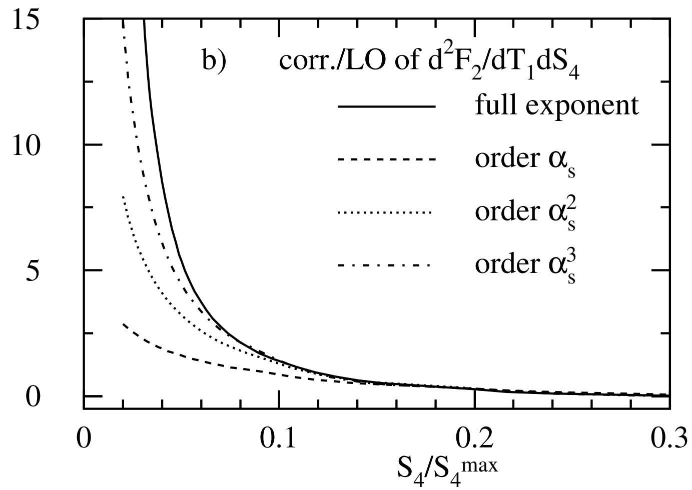
Easy to work with formulae in N space,
but for numerics go back. Recall

$$\frac{d^2 F_2^Q(S_4)}{dT_1\, dU_1} = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} e^{NS_4} \phi_g(N)\, \omega\Big(N, T_1, U_1\Big)$$



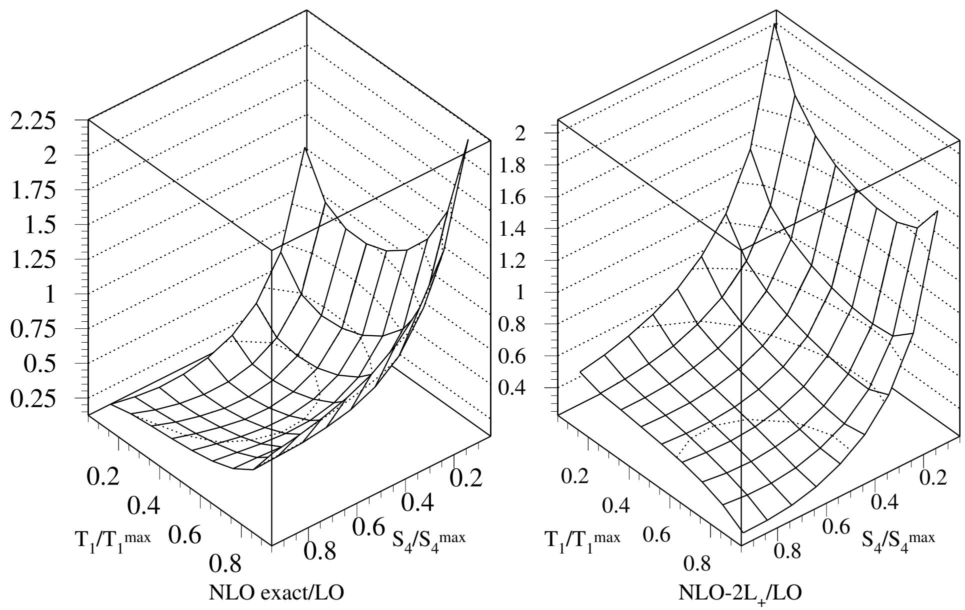
We chose for C the minimal Catani, Mangano, Nason, Trentadue contour, and toy PDF's

How well do finite order expansions converge to the resummed answer?



NLO K-functions

What about the size of the one-loop corrections for general T_1, S_4?



See large corrections at small S_4 for *all* T_1. No special behaviour at T_1 phase space edges.

Resummed curves for such plots to be completed..

Tower resummation

Since subleading logs might be important, we looked at another way of organization logs to all order: tower resummation (A. Vogt)

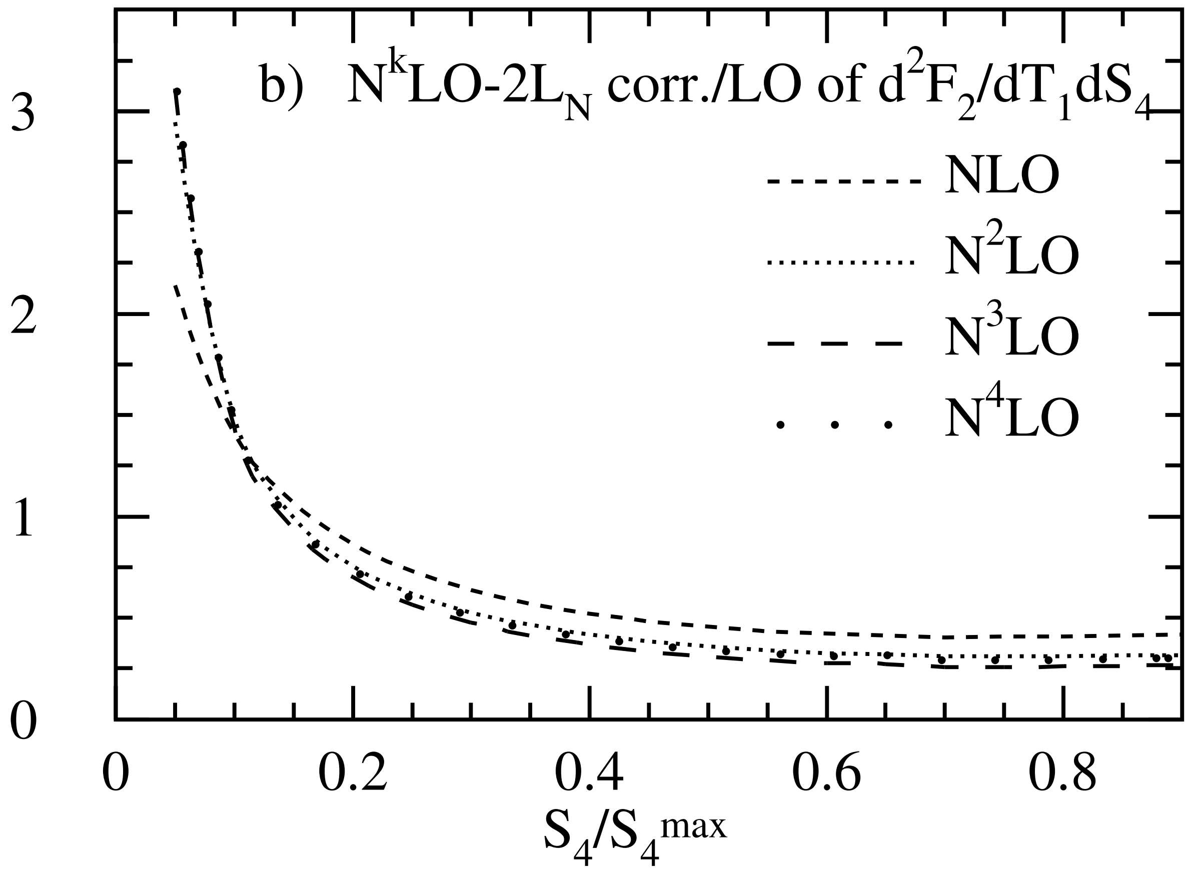
$$\omega = h_{00}(\alpha_s) \left[1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_s}{\pi} \right)^k \left(c_{k1}\, L^{2k} + c_{k2}\, L^{2k-1} + c_{k3}\, L^{2k-2} + \ldots \right) \right] .$$

Coefficients $c_{k1}, c_{k2}\, c_{k3}$ known for all k. To assess the effect of N-space or momentum space, can take appropiate L. (Landau pole singularity/ambiguity here shifted to high tower numbers.)

So far, only N space logs.

Threshold-resummed differential F_2^c

Rather rapid convergence to final answer.



Eynck, EL, Vogt

Summary

- We studied BNLO characteristics of HERA observable with possibly notable impact on
  - gluon PDF (from charm data), finite order and threshold-resummed
  - charm PDF (from bottom data), finite order and threshold-resummed
  - can be related by pure computation to bottom PDF e.g. ($\sim$ LHC)
- Found further evidence that momentum space is not always best for theoretical description.
- Using resummation quantitatively requires quantifying its uncertainties, studied these extensively
- Threshold resummation formalism now generalized to multi-differential cross sections Catani, Mangano, Nason