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LHC-HERA Workshop

“A Study of Leading–Log Gluon Amplitudes at High Energies with Non–Zero Momentum Transfer”

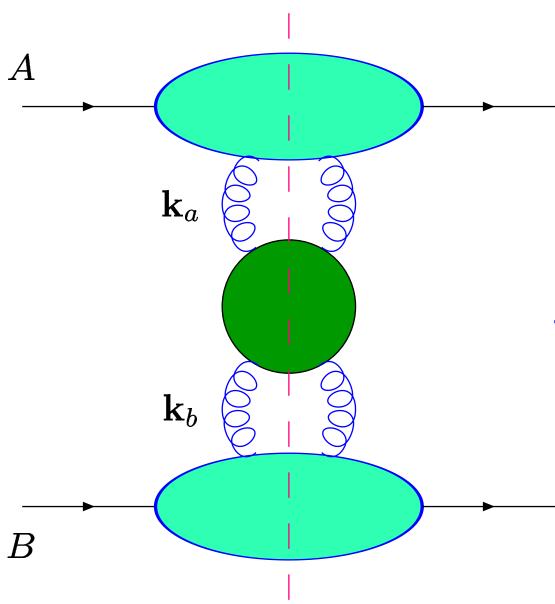
Agustín Sabio Vera*
(University of Hamburg)

In collaboration with **Jeppe R. Andersen**
(Cambridge)

*Alexander von Humboldt Fellow

LL BFKL Equation: $(\alpha_S \Delta)^n$

$$\sigma(s) = \int \frac{d^2\mathbf{k}_a}{\mathbf{k}_a^2} \int \frac{d^2\mathbf{k}_b}{\mathbf{k}_b^2} \Phi_A(\mathbf{k}_a)\, \Phi_B(\mathbf{k}_b)\, f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$$



$\phi_A(\mathbf{k}_a)$ Process-dependent

$f(\mathbf{k}_a, \mathbf{k}_b, \Delta)$ Process-independent

$\phi_B(\mathbf{k}_b)$ Process-dependent

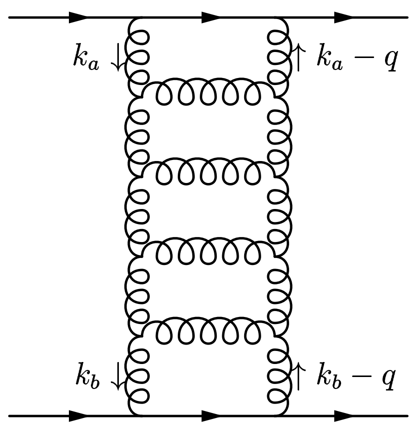


Figure 1: Contribution for quark–quark scattering with colour singlet exchange.

The diffractive scattering of two probes within BFKL can be described by the differential cross–section

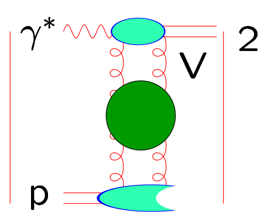
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{|A(s,t)|^2}{16\pi s^2}$$

with the scattering amplitude reading

$$\frac{|A(s,t)|}{s} = \left| \int \mathrm{d}^2\mathbf{k_1} \int \mathrm{d}^2\mathbf{k_2}\, \mathbf{\Phi}_\mathsf{A}(\mathbf{k}_1,\mathbf{q})\, \mathbf{\Phi}_\mathsf{B}(\mathbf{k}_2,\mathbf{q})\, \frac{\mathsf{f}\,(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathsf{Y})}{(\mathbf{k}_1-\mathbf{q})^2\mathbf{k}_2^2} \right|$$

$\Phi_\mathsf{A}(\mathbf{k}_1,\mathbf{q})$, $\Phi_\mathsf{B}(\mathbf{k}_2,\mathbf{q})$ process–dependent impact factors and the four–point gluon Green's function, $f(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q},\mathsf{Y})$, is universal.

Large–t Diffraction:



The Non–Forward LL BFKL Equation:

Integral equation for Mellin transform in rapidity of the four–point gluon Green's function, $f_\omega(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q})$:

$$
\begin{aligned}
\omega f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q}) \quad &= \quad \delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)+\frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{k}'\left[\frac{-\mathbf{q}^2}{(\mathbf{k}'-\mathbf{q})^2\,\mathbf{k}_a^2}f_\omega\left(\mathbf{k}',\mathbf{k}_b,\mathbf{q}\right)\right. \\
&+\frac{1}{(\mathbf{k}'-\mathbf{k}_a)^2}\left(f_\omega\left(\mathbf{k}',\mathbf{k}_b,\mathbf{q}\right)-\frac{\mathbf{k}_a^2}{\mathbf{k}'^2+(\mathbf{k}_a-\mathbf{k}')^2}f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q})\right) \\
&\left.+\frac{1}{(\mathbf{k}'-\mathbf{k}_a)^2}\left(\frac{(\mathbf{k}_a-\mathbf{q})^2\,\mathbf{k}'^2}{(\mathbf{k}'-\mathbf{q})^2\,\mathbf{k}_a^2}f_\omega\left(\mathbf{k}',\mathbf{k}_b,\mathbf{q}\right)-\frac{(\mathbf{k}_a-\mathbf{q})^2}{(\mathbf{k}'-\mathbf{q})^2+(\mathbf{k}_a-\mathbf{k}')^2}f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q})\right)\right],
\end{aligned}
$$

where $\mathbf{k_a}$ and $\mathbf{k_b}$ describe the two–dimensional transverse momenta of the exchanged gluons in the t–channel and we define $\bar{\alpha}_s \equiv \alpha_s N_c/\pi$. The driving term $\delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b)$ corresponds to a simple two gluon exchange.

It is more convenient to rearrange the terms in this expression: and, as the integration variable, to use the transverse momenta of the s–channel gluons, $\mathbf{k} = \mathbf{k}' - \mathbf{k}_a$; i.e.

$$\begin{aligned}
\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}) &= \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \\
+\frac{\bar{\alpha}_s}{2\pi} \int d^2\mathbf{k} \Bigg\{ \Bigg[\frac{1}{\mathbf{k}^2} \left(1 + \frac{(\mathbf{k}_a - \mathbf{q})^2 (\mathbf{k} + \mathbf{k}_a)^2}{(\mathbf{k} + \mathbf{k}_a - \mathbf{q})^2 \mathbf{k}_a^2} \right) &- \frac{\mathbf{q}^2}{(\mathbf{k} + \mathbf{k}_a - \mathbf{q})^2 \mathbf{k}_a^2} \Bigg] f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b, \mathbf{q}) \\
&- \frac{1}{\mathbf{k}^2} \left(\frac{\mathbf{k}_a^2}{(\mathbf{k} + \mathbf{k}_a)^2 + \mathbf{k}^2} + \frac{(\mathbf{k}_a - \mathbf{q})^2}{(\mathbf{k} + \mathbf{k}_a - \mathbf{q})^2 + \mathbf{k}^2} \right) f_\omega(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}) \Bigg\} .
\end{aligned}$$

It is convenient to use

$$\begin{aligned}
f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b, \mathbf{q}) &= f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b, \mathbf{q}) \left(\theta\left(\mathbf{k}^2 - \lambda^2\right) + \theta\left(\lambda^2 - \mathbf{k}^2\right) \right) \\
&\simeq f_\omega(\mathbf{k} + \mathbf{k}_a, \mathbf{k}_b, \mathbf{q})\, \theta\left(\mathbf{k}^2 - \lambda^2\right) + f_\omega(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q})\, \theta\left(\lambda^2 - \mathbf{k}^2\right)
\end{aligned}$$

The gluon Green's function is insensitive to the value of the slicing parameter for small values of λ.

Therefore:

$$\begin{aligned}\left(\omega-\omega_0\left(\mathbf{k}_a,\mathbf{q},\lambda\right)\right)f_\omega\left(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q}\right) &= \delta^{(2)}\left(\mathbf{k}_a-\mathbf{k}_b\right)\\ &+\int\frac{d^2\mathbf{k}}{\pi\mathbf{k}^2}\theta\left(\mathbf{k}^2-\lambda^2\right)\xi\left(\mathbf{k}_a,\mathbf{k},\mathbf{q}\right)f_\omega\left(\mathbf{k}+\mathbf{k}_a,\mathbf{k}_b,\mathbf{q}\right)\end{aligned}$$

Notation:

$$\xi\left(\mathbf{k}_a,\mathbf{k},\mathbf{q}\right) = \frac{\bar{\alpha}_s}{2}\left(1+\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2\left(\mathbf{k}+\mathbf{k}_a\right)^2-\mathbf{q}^2\mathbf{k}^2}{\left(\mathbf{k}+\mathbf{k}_a-\mathbf{q}\right)^2\mathbf{k}_a^2}\right)$$

and

$$\begin{aligned}\omega_0\left(\mathbf{k}_a,\mathbf{q},\lambda\right) &= \frac{\bar{\alpha}_s}{2\pi}\int\frac{d^2\mathbf{k}}{\mathbf{k}^2}\left[\theta\left(\lambda^2-\mathbf{k}^2\right)\left(1+\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2\left(\mathbf{k}+\mathbf{k}_a\right)^2-\mathbf{q}^2\mathbf{k}^2}{\left(\mathbf{k}+\mathbf{k}_a-\mathbf{q}\right)^2\mathbf{k}_a^2}\right)\right.\\ &- \left.\frac{\mathbf{k}_a^2}{\left(\mathbf{k}+\mathbf{k}_a\right)^2+\mathbf{k}^2}-\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2}{\left(\mathbf{k}+\mathbf{k}_a-\mathbf{q}\right)^2+\mathbf{k}^2}\right]\end{aligned}$$

The $\mathbf{q}=\mathbf{0}$ limit:

$$\omega_0\left(\mathbf{k}_a,\mathbf{0},\lambda\right) = \frac{\bar{\alpha}_s}{\pi}\int\frac{d^2\mathbf{k}}{\mathbf{k}^2}\left[\theta\left(\lambda^2-\mathbf{k}^2\right)-\frac{\mathbf{k}_a^2}{\left(\mathbf{k}+\mathbf{k}_a\right)^2+\mathbf{k}^2}\right] = -\bar{\alpha}_s\ln\frac{\mathbf{k}_a^2}{\lambda^2}$$

The non–forward trajectory can be approximated by

$$\omega_0\left(\mathbf{k}_a,\mathbf{q},\lambda\right) \approx -\frac{\bar{\alpha}_s}{2}\left(\ln\frac{\mathbf{k}_a^2}{\lambda^2}+\ln\frac{\left(\mathbf{k}_a-\mathbf{q}\right)^2}{\lambda^2}\right)$$

Solution:

$$\begin{aligned} f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q}) &= \frac{1}{\omega-\omega_0(\mathbf{k}_a,\mathbf{q},\lambda)}\Big\{\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b) \\ &\quad + \int \frac{d^2\mathbf{k}}{\pi\mathbf{k}^2}\theta\left(\mathbf{k}^2-\lambda^2\right)\xi(\mathbf{k}_a,\mathbf{k},\mathbf{q})\, f_\omega(\mathbf{k}+\mathbf{k}_a,\mathbf{k}_b,\mathbf{q})\Big\}\end{aligned}$$

$$\begin{aligned} f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q}) &= \frac{\delta^{(2)}(\mathbf{k}_a-\mathbf{k}_b)}{\omega-\omega_0(\mathbf{k}_a,\mathbf{q},\lambda)} \\ &+ \int \frac{d^2\mathbf{k}_1}{\pi\mathbf{k}_1^2}\frac{\theta\left(\mathbf{k}_1^2-\lambda^2\right)\,\xi(\mathbf{k}_a,\mathbf{k}_1,\mathbf{q})}{\omega-\omega_0(\mathbf{k}_a,\mathbf{q},\lambda)}\frac{\delta^{(2)}(\mathbf{k}_a+\mathbf{k}_1-\mathbf{k}_b)}{\omega-\omega_0(\mathbf{k}_a+\mathbf{k}_1,\mathbf{q},\lambda)} \\ &+ \int \frac{d^2\mathbf{k}_1}{\pi\mathbf{k}_1^2}\int \frac{d^2\mathbf{k}_2}{\pi\mathbf{k}_2^2}\frac{\theta\left(\mathbf{k}_1^2-\lambda^2\right)\,\xi(\mathbf{k}_a,\mathbf{k}_1,\mathbf{q})}{\omega-\omega_0(\mathbf{k}_a,\mathbf{q},\lambda)}\frac{\theta\left(\mathbf{k}_2^2-\lambda^2\right)\,\xi(\mathbf{k}_a+\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})}{\omega-\omega_0(\mathbf{k}_a+\mathbf{k}_1,\mathbf{q},\lambda)} \\ &\quad \times\frac{\delta^{(2)}(\mathbf{k}_a+\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_b)}{\omega-\omega_0(\mathbf{k}_a+\mathbf{k}_1+\mathbf{k}_2,\mathbf{q},\lambda)} \\ &\quad + \cdots \end{aligned}$$

$$f(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q},\Delta) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} d\omega\ e^{\omega\Delta} f_\omega(\mathbf{k}_a,\mathbf{k}_b,\mathbf{q})$$

where Δ is the rapidity span of the BFKL ladder.

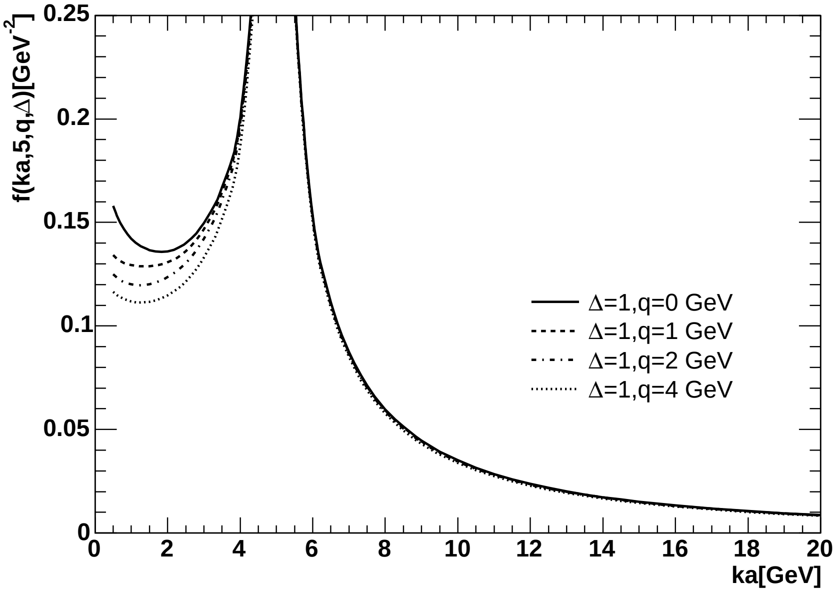
The final solution looks like:

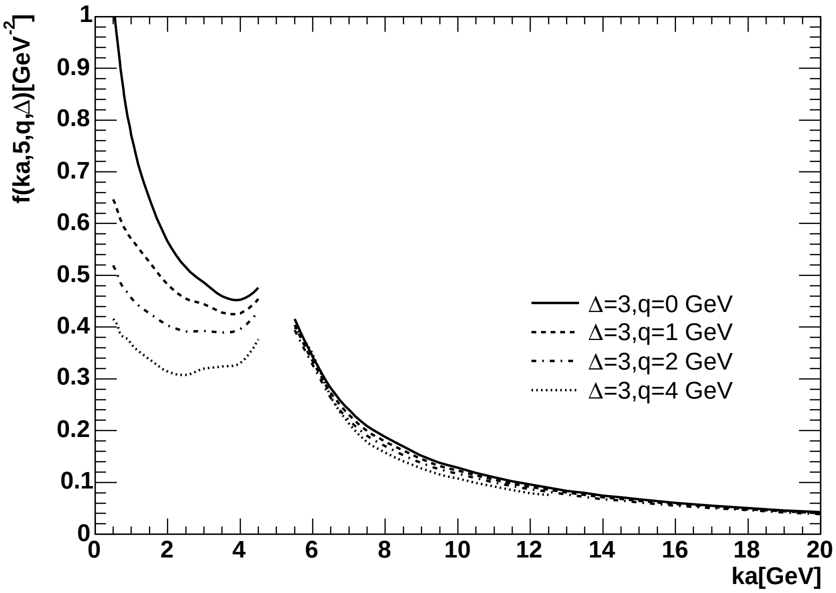
$$f(\mathbf{k}_a, \mathbf{k}_b, \mathbf{q}, \Delta) = \left(\frac{\lambda^2}{\mathbf{k}_a^2}\frac{\lambda^2}{(\mathbf{k}_a - \mathbf{q})^2}\right)^{\frac{\bar{\alpha}_s}{2}\Delta} \left\{ \delta^{(2)}(\mathbf{k}_a - \mathbf{k}_b) \right.$$

$$+ \quad \sum_{n=1}^{\infty} \prod_{i=1}^{n} \int d^2\mathbf{k}_i \frac{\theta\left(\mathbf{k}_i^2 - \lambda^2\right)}{\pi \mathbf{k}_i^2} \xi\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l, \mathbf{k}_i, \mathbf{q}\right)$$

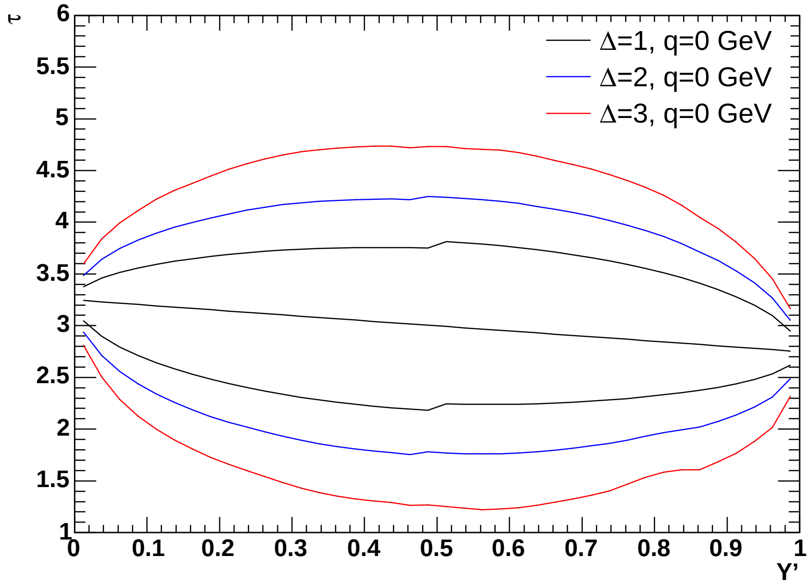
$$\left. \times \int_0^{y_{i-1}} dy_i \left(\frac{\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l\right)^2}{\left(\mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l\right)^2} \frac{\left(\mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l - \mathbf{q}\right)^2}{\left(\mathbf{k}_a + \sum_{l=1}^{i} \mathbf{k}_l - \mathbf{q}\right)^2} \right)^{\frac{\bar{\alpha}_s}{2} y_i} \delta^{(2)}\left(\sum_{l=1}^{n} \mathbf{k}_l + \mathbf{k}_a - \mathbf{k}_b\right) \right\}$$

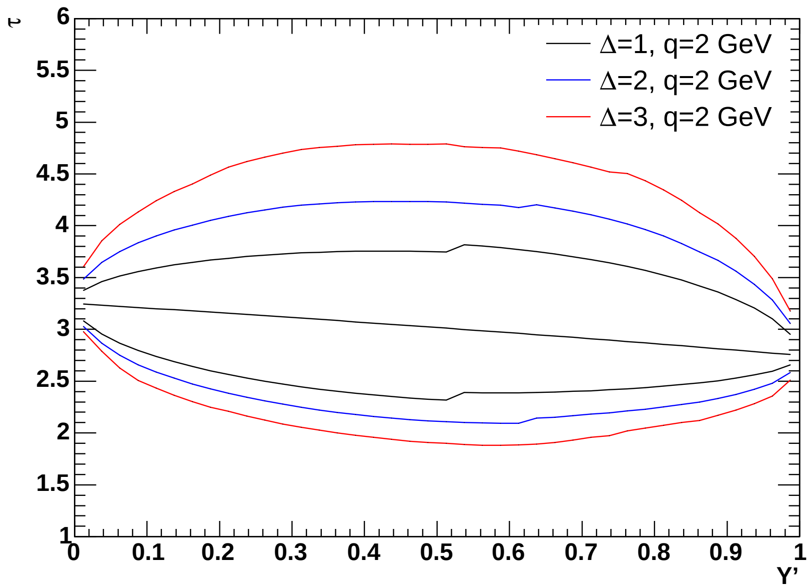
The gluon Green's function:





The diffusion picture:





Outlook:

- Method applicable at NLO, use dimensional regularisation (Andersen, SV)

- The NLO kernel almost ready (Fadin)

- With NLO impact factors full NLO cross sections are close (Bartels, Ivanov, Fadin ...)

- Convergence of the expansion for non–zero t?

- Infrared diffusion cut off by momentum transfer $\mathbf{q}$