NNLO Scheme Invariant Evolution of Unpolarized DIS Structure Functions

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- Motivation
- QCD Evolution Equations
- Scheme Invariant Evolution
- Numerical Results
- Conclusions

- The final HERA-II data, together with the world data, will allow to reduce experimental errors on α_s to $\sim 1\%$
- On the other side the theoretical error on the determination of the strong coupling constant in NLO analyses is $\Delta\alpha_s \sim 5\%$
- In order to match the claimed experimental accuracy NNLO results are necessary on the theoretical side
- Recently computed 3-loop Anomalous Dimensions were the only missing piece in order to perform a full NNLO study of DIS Structure Functions

[S. Moch, J. A. .M. Vermaseren and A. Vogt, Nucl. Phys. B688, (2004), 101; Nucl. Phys. B691, (2004), 129]

- Our aim is to perform both a Standard and a Scheme Invariant analysis of unpolarized DIS structure functions in order to:
  - Determine α_s with an accuracy of $\mathcal{O}(1\%)$
  - Extract the parton distribution functions with fully correlated errors

- Evolution Equations of DIS Structure Functions do exhibit factorization and renormalization scheme dependencies
- Renormalization scheme dependence is removed only if the perturbative series is summed to all orders
- When considering factorization scheme dependence we have two viable approaches
  - Consider process-independent scheme-dependent evolution equations for PDFs (Standard QCD analysis)
  - Consider process-dependent scheme-independent evolution equations for observables (Scheme Invariant analysis)

Standard QCD analysis

- Introduce a parametrization of the PDFs at a given reference scale
- Evolve the PDFs to the scale Q^2 via evolution equations for mass factorization
- Build structure functions as a combination of PDFs and Wilson Coefficients
- Perform a multi-parameter fit to the data to determine the PDF parameters and $\alpha_{\rm s}$

Scheme Invariant Evolution

- Extract the parametrization of observable quantities at the initial scale Q_0^2 from data
- Determine the value of the observables at the scale Q^2 using evolution equations with physical anomalous dimensions
- Perform a one-parameter fit to the data to determine $\alpha_{\rm s}$

The two analyses are complementary and performing both of them will help reduce the theoretical and conceptual errors

Our notation

$$a_s(\mu^2) \equiv \frac{\alpha_s(\mu^2)}{4\pi} \qquad \mu^2 \frac{d a_s(\mu^2)}{d\mu^2} = -\sum_{n=0}^{\infty} \beta_n a_s^{n+2}(\mu^2)$$

where, for $SU(3)$ we have

$$\beta_0 = 11 - \frac{2}{3}N_f \qquad \beta_1 = 102 - \frac{38}{3}N_f$$
$$\beta_2 = \frac{2857}{2} - \frac{5033}{18}N_f + \frac{325}{54}N_f^2$$

The expanded form for the coupling constant up to 3-loop is

$$a_s(Q^2) = \frac{1}{\beta_0 L}\left\{1 - \frac{\beta_1 \ln L}{\beta_0^2 L} + \frac{\beta_1^2 \ln^2 L - \beta_1^2 \ln L + \beta_2\beta_0 - \beta_1^2}{\beta_0^4 L^2}\right\}$$

where

$$L = \ln \frac{Q^2}{\Lambda^2_{QCD}}$$

- We work in Mellin space, where the Mellin transform of a function is defined as

$$f(N) = \int_0^1 dx x^{N-1} f(x)$$

while anomalous dimensions are related to DGLAP splitting functions through

$$\gamma_{ij}^N = -2 \int_0^1 dx x^{N-1} P_{ij}(x)$$

- Working in Mellin space is instrumental (almost necessary) for numerical implementation (for example when including Heavy Flavours)

- The coupled evolution of the singlet and gluon parton distributions can be mapped into the evolution of a pair of structure functions

$$\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t) = \begin{pmatrix} C_{A,\Sigma}^N & C_{A,g}^N \\ C_{B,\Sigma}^N & C_{B,g}^N \end{pmatrix} \begin{pmatrix} \Sigma^N \\ G^N \end{pmatrix}(t)$$

- The observables, then, satisfy the matrix evolution equation

$$\frac{d}{dt}\begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t) = -\frac{1}{4}\mathbf{K}^N \begin{pmatrix} F_A^N \\ F_B^N \end{pmatrix}(t)\,, \qquad t = -\frac{2}{\beta_0}\ln\frac{a_s(Q^2)}{a_s(Q_0^2)}$$

and the physical anomalous dimensions are

$$K_{IJ}^N = \left[-4\frac{\partial C_{I,m}^N(t)}{\partial t}\left(C^N\right)^{-1}_{m,J}(t) - \frac{\beta_0 a_s(Q^2)}{2\beta(a_s(Q^2))} C_{I,m}^N(t)\gamma_{mn}^N(t)\left(C^N\right)^{-1}_{n,J}(t)\right]$$

The physical anomalous dimensions K_{IJ}^N are factorization scheme invariants

- When considering scheme invariant evolution different pairs of structure functions can be chosen:
  - F_2, $\partial F_2/\partial t$
  - F_2, F_L
  - g_1, $\partial g_1/\partial t$ (in polarized DIS)

Leading Order:

[W. Furmanski and R. Petronzio, Z. Phys. C11, (1982), 293]

$$K_{22}^{N(0)} = 0 \qquad K_{2d}^{N(0)} = -4$$

$$K_{d2}^{N(0)} = \frac{1}{4}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) \qquad K_{dd}^{N(0)} = \gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}$$

Next-to-Leading Order:

$$
\begin{aligned}
K_{22}^{N(1)} &= K_{2d}^{N(1)} = 0 \\
K_{d2}^{N(1)} &= \frac{1}{4}\left(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)} + \gamma_{gg}^{N(1)}\gamma_{qq}^{N(0)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\right) \\
&\quad - \frac{\beta_0}{2}\left(\gamma_{qq}^{N(1)} - \frac{\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}}{\gamma_{qg}^{N(0)}}\right) \\
&\quad - \frac{2\beta_1}{2\beta_0}\left(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\right) + \frac{\beta_0}{2}C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
&\quad - \frac{\beta_0}{2}\frac{C_{2,g}^{N(1)}}{\gamma_{qg}^{N(0)}}\left[\left(\gamma_{qq}^{N(0)}\right)^2 - \gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + 2\gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)} - 2\beta_0\gamma_{qq}^{N(0)}\right] \\
K_{dd}^{N(1)} &= \gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)} + 4\beta_0 C_{2,q}^{N(1)} - 2\beta_1 - \frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\right) \\
&\quad - \frac{2\beta_0}{\gamma_{qg}^{N(0)}}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \gamma_{qg}^{N(1)}\right]
\end{aligned}
$$

Next-to-next-to-Leading Order:

$$
\begin{aligned}
K_{22}^{N(2)} &= K_{2d}^{N(2)} = 0 \\
K_{d2}^{N(2)} &= \frac{1}{4}\Bigl(\gamma_{qq}^{N(2)}\gamma_{gg}^{N(0)} + \gamma_{qq}^{N(0)}\gamma_{gg}^{N(2)} - \gamma_{qg}^{N(2)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(2)} + \gamma_{qq}^{N(1)}\gamma_{gg}^{N(1)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(1)}\Bigr) \\
&+ \frac{\beta_0}{2}\Bigl[C_{2,q}^{N(1)}\bigl(\gamma_{qq}^{N(1)} + \gamma_{gg}^{N(1)}\bigr) - \bigl(C_{2,q}^{N(1)}\bigr)^2\bigl(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\bigr) - 3C_{2,g}^{N(1)}\gamma_{gq}^{N(1)}\Bigr] \\
&- \beta_0\Bigl[\gamma_{qq}^{N(2)} + 2\gamma_{gq}^{N(0)}\bigl(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\bigr) - C_{2,q}^{N(2)}\bigl(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)}\bigr)\Bigr] \\
&+ \beta_0^2\Bigl[3\bigl(C_{2,q}^{N(1)}\bigr)^2 - 4C_{2,q}^{N(2)}\Bigr] + \frac{\beta_1}{2}\Bigl[\gamma_{qq}^{N(1)} - C_{2,q}^{N(1)}\bigl(\gamma_{qq}^{N(0)} + \gamma_{gg}^{N(0)} + 2\beta_0\bigr) + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\Bigr] \\
&- \frac{\beta_1}{2\beta_0}\Bigl(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)} + \gamma_{qq}^{N(0)}\gamma_{gg}^{N(1)} - \gamma_{qg}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(1)}\Bigr) \\
&+ \frac{3}{4}\frac{\beta_1^2}{\beta_0^2}\Bigl(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} - \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\Bigr) - \frac{\beta_2}{2\beta_0}\Bigl(\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)} + \gamma_{qg}^{N(0)}\gamma_{gq}^{N(0)}\Bigr) \\
&+ \frac{1}{\gamma_{qg}^{N(0)}}\Bigl\{\frac{\beta_1}{2}\gamma_{qq}^{N(0)}\Bigl[C_{2,g}^{N(1)}\bigl(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\bigr) - \gamma_{gg}^{N(0)}\Bigr] + 2\beta_0^3 C_{2,q}^{N(1)}C_{2,g}^{N(1)} \\
&+ \beta_0^2\Bigl[4\gamma_{qq}^{N(0)}\bigl(C_{2,g}^{N(2)} - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\bigr) - C_{2,g}^{N(1)}C_{2,q}^{N(1)}\bigl(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\bigr) + C_{2,q}^{N(1)}\gamma_{qg}^{N(1)} \\
&+ C_{2,g}^{N(1)}\gamma_{qq}^{N(1)} + \bigl(C_{2,g}^{N(1)}\bigr)^2\gamma_{gq}^{N(0)}\Bigr] \\
&+ \beta_0\Bigl[C_{2,g}^{N(1)}C_{2,q}^{N(1)}\gamma_{qq}^{N(0)}\bigl(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\bigr) + \gamma_{qq}^{N(0)}\bigl(C_{2,g}^{N(1)}\gamma_{gg}^{N(1)} + C_{2,g}^{N(2)}\gamma_{gg}^{N(0)}\bigr)\Bigr]
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{\beta_0}{2}\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)}\gamma_{gg}^{N(0)}+\gamma_{gq}^{N(0)}\gamma_{qg}^{N(1)}-\frac{3}{2\gamma_{qq}^{N(0)}\gamma_{qq}^{N(1)}}\right)+\gamma_{qq}^{N(1)}\gamma_{gq}^{N(1)}\right.\\
&\left.\left.+\left(C_{2,g}^{N(1)}\right)^2\left(\gamma_{gg}^{N(0)}\gamma_{gq}^{N(0)}-\frac{3}{2}\gamma_{gq}^{N(0)}\gamma_{qq}^{N(0)}\right)\right]\right\}\\
&+\frac{2\beta_0}{\left(\gamma_{qg}^{N(0)}\right)^2}\left\{-\beta_0^2\left(C_{2,g}^{N(1)}\right)^2\gamma_{qq}^{N(0)}+\beta_0\left[-C_{2,g}^{N(1)}\gamma_{qq}^{N(0)}\gamma_{qg}^{N(1)}\right.\right.\\
&\left.+\left(C_{2,g}^{N(1)}\right)^2\gamma_{qq}^{N(0)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}+\frac{\gamma_{qq}^{N(0)}\gamma_{gg}^{N(0)}}{2}\right)\right]\\
&\left.-\frac{1}{2}\left[\left(C_{2,g}^{N(1)}\right)^2\gamma_{qq}^{N(0)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)^2-\gamma_{qq}^{N(0)}\left(\gamma_{qg}^{N(1)}\right)^2\right]+C_{2,g}^{N(1)}\gamma_{qg}^{N(1)}\gamma_{qq}^{N(0)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\right\}
\end{aligned}
$$

$$
\begin{aligned}
K_{dd}^{N(2)} =\; &\gamma_{qq}^{N(2)}+\gamma_{gg}^{N(2)}-4\beta_0\left[\left(C_{2,q}^{N(1)}\right)^2-2C_{2,q}^{N(2)}\right]-4\beta_2+\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\left(\gamma_{qq}^{N(0)}+\gamma_{gg}^{N(0)}\right)\\
&-\frac{\beta_1}{\beta_0}\left(\gamma_{qq}^{N(1)}+\gamma_{gg}^{N(1)}-2\beta_1\right)+\frac{4\beta_0}{\gamma_{qg}^{N(0)}}\left\{4\beta_0\left(C_{2,g}^{N(2)}-C_{2,q}^{N(1)}C_{2,g}^{N(1)}\right)+\gamma_{qg}^{N(2)}\right.\\
&\left.+\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)}-C_{2,g}^{N(2)}\right)-C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)}-\gamma_{gg}^{N(1)}-2\beta_1\right)-\left(C_{2,g}^{N(1)}\right)^2\gamma_{gq}^{N(0)}\right\}\\
&+\frac{2\beta_0}{\left(\gamma_{qg}^{N(0)}\right)^2}\left\{-4\beta_0^2\left(C_{2,g}^{N(1)}\right)^2-4\beta_0 C_{2,g}^{N(1)}\left[\gamma_{qg}^{N(1)}-C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\right]\right.\\
&\left.-\left(\gamma_{qg}^{N(1)}\right)^2+2C_{2,g}^{N(1)}\gamma_{qg}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)-\left[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\right]^2\right\}
\end{aligned}
$$

Instead of F_L we consider

$$\widetilde{F}_L(Q^2) = \frac{F_L(Q^2)}{a_s(Q^2)C_{L,g}^{N(1)}}$$

which is also factorizations scheme independent due to the fact that the first order Wilson coefficients $C_{L,q}^{N(1)}$ and $C_{L,g}^{N(1)}$ are scheme invariants.

Leading Order:

[S. Catani, Z. Phys. C75, (1997), 665]

$$K_{22}^{N(0)} = \gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)} \qquad K_{2L}^{N(0)} = \gamma_{qg}^{N(0)}$$

$$K_{L2}^{N(0)} = \gamma_{gq}^{N(0)} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 \gamma_{qg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)$$

$$K_{LL}^{N(0)} = \gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}$$

Next-to-Leading Order:

[J. Blümlein, V. Ravindran and W. L. van Neerven, Nucl. Phys. B586, (2000),349]

$$K_{22}^{N(1)} = \gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)\right] + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}$$
$$- \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right]\gamma_{qg}^{N(0)} + 2\beta_0\left(C_{2,q}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\right)$$

$$K_{2L}^{N(1)} = \gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} + 2\beta_0) + \left(C_{2,q}^{N(1)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)} - \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right)\gamma_{qg}^{N(0)}$$

$$K_{LL}^{N(1)} = \gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{gg}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)}\right) - C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{gg}^{N(0)}$$
$$- \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}C_{2,g}^{N(1)}\gamma_{qq}^{N(0)} + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)}\right]\gamma_{qg}^{N(0)} + 2\beta_0\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}$$

$$
\begin{aligned}
K_{L2}^{N(1)} = {} & \gamma_{gq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qq}^{N(0)}\right) - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{gg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{gg}^{N(0)}\right) \\
& - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 \left(\gamma_{qg}^{N(1)} - \frac{\beta_1}{\beta_0}\gamma_{qg}^{N(0)}\right) + \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)}\right]\gamma_{qq}^{N(0)} \\
& - \left[\left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^3 C_{2,g}^{N(1)} + 2\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} - \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,q}^{N(1)}\right]\gamma_{qg}^{N(0)} \\
& + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,g}^{N(1)} - C_{2,q}^{N(1)} + \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right)\gamma_{gq}^{N(0)} \\
& - \left[\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} + \left(\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2 C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{2,q}^{N(1)}\right]\gamma_{gg}^{N(0)} + 2\beta_0\left(\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
K_{22}^{N(2)} = {} & \gamma_{qq}^{N(2)} + C_{2,g}^{N(1)} \gamma_{gq}^{N(1)} + \gamma_{gq}^{N(0)} \left(C_{2,g}^{N(2)} - C_{2,g}^{N(1)} C_{2,q}^{N(1)} \right) + 2\beta_0 \left[2C_{2,q}^{N(2)} - \left(C_{2,q}^{N(1)} \right)^2 - \frac{C_{L,q}^{N(2)} C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}} \right] \\
& + \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} \left[C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right) - \gamma_{qg}^{N(1)} \right] - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \Bigg[\gamma_{qg}^{N(2)} - C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)} \gamma_{gq}^{N(0)} \right) \\
& + \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 4\beta_0 \right) \left(C_{2,g}^{N(1)} C_{2,q}^{N(1)} - C_{2,g}^{N(2)} \right) + \gamma_{qg}^{N(0)} \left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)} C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}} \right) \Bigg] \\
& + \frac{\left(C_{L,q}^{N(1)} \right)^2}{\left(C_{L,g}^{N(1)} \right)^2} \left[\left(C_{2,q}^{N(1)} \right)^2 \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0 \right) - C_{2,g}^{N(1)} \gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)} \left(C_{2,g}^{N(1)} C_{2,q}^{N(1)} - C_{2,g}^{N(2)} \right) \right] \\
& + \frac{C_{L,g}^{N(2)}}{\left(C_{L,g}^{N(1)} \right)^2} \left[C_{L,q}^{N(1)} \gamma_{qg}^{N(0)} + C_{L,q}^{N(2)} \gamma_{qg}^{N(0)} - C_{2,g}^{N(1)} C_{L,q}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0 \right) \right] - 2 \frac{C_{2,g}^{N(1)} C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)} \right)^2} \gamma_{qg}^{N(0)} \\
& - \frac{\beta_1}{\beta_0} \Bigg\{ \gamma_{qg}^{N(1)} + \gamma_{qg}^{N(0)} \left(C_{2,g}^{N(1)} - \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}} \right) + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \left[C_{2,g}^{N(1)} \left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} \right) - \gamma_{qg}^{N(1)} \right] \\
& + \frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)} \right)^2} \gamma_{qg}^{N(0)} \left(C_{L,g}^{N(2)} - C_{L,q}^{N(1)} C_{2,g}^{N(1)} \right) \Bigg\} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) \left(\gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \gamma_{qg}^{N(0)} \right)
\end{aligned}
$$

$$
\begin{aligned}
K_{2L}^{N(2)} = {} & \gamma_{qg}^{N(2)} + C_{2,q}^{N(1)}\gamma_{qg}^{N(1)} + C_{2,q}^{N(2)}\gamma_{qg}^{N(0)} - 2\beta_0 C_{2,q}^{N(1)} C_{2,g}^{N(1)} - C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) \\
& - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(2)}\gamma_{gq}^{N(0)}\right) + \gamma_{qg}^{N(0)}\left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right) \\
& + \frac{\beta_1}{\beta_0}\left[\frac{\gamma_{qg}^{N(0)}}{C_{L,g}^{N(1)}}\left(C_{L,g}^{N(2)} - C_{2,g}^{N(1)} C_{L,q}^{N(1)}\right) + C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - C_{2,q}^{N(1)}\gamma_{qg}^{N(0)} - \gamma_{qg}^{N(1)}\right] \\
& + \frac{\gamma_{qg}^{N(0)}}{\left(C_{L,g}^{N(1)}\right)^2}\left(C_{L,g}^{N(2)} - C_{2,g}^{N(1)} C_{L,q}^{N(1)}\right)^2 + \frac{1}{C_{L,g}^{N(1)}}\Bigg[\left(C_{2,g}^{N(1)} C_{L,q}^{N(2)} - C_{2,q}^{N(1)} C_{L,g}^{N(2)} + C_{2,g}^{N(2)} C_{L,q}^{N(1)}\right) \\
& - C_{2,g}^{N(1)}\left(C_{2,g}^{N(1)} C_{L,q}^{N(1)} - C_{L,g}^{N(2)}\right)\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} - 2\beta_0\right) - \frac{\gamma_{qg}^{N(1)}}{C_{2,g}^{N(1)}}\Bigg] \\
K_{L2}^{N(2)} = {} & \gamma_{gq}^{N(2)} - C_{2,q}^{N(2)}\gamma_{gq}^{N(0)} + C_{2,q}^{N(1)}\left(C_{2,q}^{N(1)}\gamma_{gq}^{N(0)} - \gamma_{gq}^{N(1)}\right) + \frac{\beta_1}{\beta_0}\Bigg\{\gamma_{gq}^{N(1)} + \left(\frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}} - C_{2,q}^{N(1)}\right)\gamma_{gq}^{N(0)} \\
& + \frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left[\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} - C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right] \\
& - \frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\Bigg[2C_{L,q}^{N(2)}\gamma_{qg}^{N(0)} + C_{L,q}^{N(1)}\gamma_{qg}^{N(1)} - C_{L,q}^{N(1)} C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - C_{L,q}^{N(1)} C_{2,q}^{N(1)}\gamma_{qg}^{N(0)} \\
& - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} C_{L,g}^{N(2)}\gamma_{qg}^{N(0)}\Bigg]\Bigg\} + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0}\right)\left[\gamma_{gq}^{N(0)} + \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)} - \gamma_{qq}^{N(0)} - \frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)\right]
\end{aligned}
$$

$$+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}} \Big\{ \gamma_{qq}^{N(2)} - \gamma_{gg}^{N(2)} - C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} - \gamma_{gq}^{N(1)}\right) + \left(C_{2,g}^{N(2)} - 2C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right)\gamma_{gq}^{N(0)}$$

$$+ \left[\left(C_{2,q}^{N(1)}\right)^2 - C_{2,q}^{N(2)}\right]\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)\Big\} + \frac{C_{L,g}^{N(2)}}{C_{L,g}^{N(1)}}\left(\gamma_{gq}^{N(1)} - C_{2,q}^{N(1)}\gamma_{gq}^{N(0)}\right)$$

$$+\frac{C_{L,q}^{N(2)}}{C_{L,g}^{N(1)}}\left[\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)} - C_{2,q}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)} + 2\beta_0\right)\right] - 2\frac{\left(C_{L,q}^{N(2)}\right)^2}{\left(C_{L,g}^{N(1)}\right)^2}\gamma_{qg}^{N(0)}$$

$$+2\beta_0\frac{C_{L,g}^{N(2)}}{\left(C_{L,g}^{N(1)}\right)^2}\left(C_{L,q}^{N(1)}C_{2,q}^{N(1)} - C_{L,q}^{N(2)}\right) + 2\frac{C_{L,q}^{N(2)}C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\Big[C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{qq}^{N(0)} + \beta_0\right)$$

$$+C_{2,q}^{N(1)}\gamma_{qg}^{N(0)} - \gamma_{qg}^{N(1)}\Big] + \frac{\left(C_{L,q}^{N(1)}\right)^2}{\left(C_{L,g}^{N(1)}\right)^2}\Bigg\{ - \gamma_{qg}^{N(2)} + C_{2,q}^{N(2)}\gamma_{qg}^{N(0)} + C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right)$$

$$+C_{2,q}^{N(1)}\left(\gamma_{qg}^{N(1)} - C_{2,q}^{N(1)}\gamma_{qg}^{N(0)}\right) + C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)} - \gamma_{gg}^{N(1)} + C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}\right)$$

$$-C_{2,q}^{N(1)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)} - \gamma_{gg}^{N(0)}\right) - \left(\frac{C_{L,g}^{N(2)} - C_{L,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2\gamma_{qg}^{N(0)}\Bigg\}$$

$$+\frac{1}{\left(C_{L,g}^{N(1)}\right)^3}\Big\{\left(C_{L,q}^{N(1)}\right)^3\Big[\Big(C_{2,q}^{N(1)}C_{2,g}^{N(1)} - C_{2,g}^{N(2)} + \frac{\beta_1}{\beta_0}C_{2,g}^{N(1)}\Big)\gamma_{qg}^{N(0)} - C_{2,g}^{N(1)}\gamma_{qg}^{N(1)}$$

$$\left(C_{2,g}^{N(1)}\right)^2\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\Bigg]+\left(C_{L,q}^{N(1)}\right)^2\Bigg[C_{L,g}^{N(2)}\gamma_{qg}^{N(1)}-\left(C_{2,q}^{N(1)}C_{L,g}^{N(2)}+3C_{L,q}^{N(2)}C_{2,g}^{N(1)}\right)\gamma_{qg}^{N(0)}$$

$$-C_{L,g}^{N(2)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}+2\beta_0\right)\Bigg]+C_{L,q}^{N(1)}C_{L,g}^{N(2)}\left(\beta_0C_{L,g}^{N(2)}+C_{L,q}^{N(2)}\gamma_{qg}^{N(0)}\right)\Bigg\}$$

$$K_{LL}^{N(2)} = \gamma_{gg}^{N(2)}-C_{2,g}^{N(1)}\gamma_{gq}^{N(1)}+\left(C_{2,g}^{N(1)}C_{2,q}^{N(1)}-C_{2,g}^{N(2)}\right)\gamma_{gq}^{N(0)}+\left(\frac{\beta_1^2}{\beta_0^2}-\frac{\beta_2}{\beta_0}\right)\left(\gamma_{gg}^{N(0)}+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}\right)$$

$$+\frac{\beta_1}{\beta_0}\Bigg[C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}-\gamma_{gg}^{N(1)}-\frac{C_{2,q}^{N(2)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(0)}-\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\gamma_{qg}^{N(1)}+\frac{C_{2,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)$$

$$-\frac{C_{L,q}^{N(1)}}{\left(C_{L,g}^{N(1)}\right)^2}\left(C_{L,q}^{N(1)}C_{2,g}^{N(1)}-C_{2,g}^{N(2)}+C_{L,g}^{N(2)}\right)\gamma_{qg}^{N(0)}\Bigg]+\frac{C_{L,q}^{N(1)}}{C_{L,g}^{N(1)}}\left(\frac{C_{L,g}^{N(2)}-C_{L,q}^{N(1)}C_{2,g}^{N(1)}}{C_{L,g}^{N(1)}}\right)^2\gamma_{qg}^{N(0)}$$

$$+\frac{1}{C_{L,g}^{N(1)}}\Big[C_{L,q}^{N(1)}\gamma_{qg}^{N(2)}+C_{L,q}^{N(2)}\gamma_{qg}^{N(1)}-C_{L,q}^{N(1)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(1)}-\gamma_{gg}^{N(1)}+C_{2,g}^{N(1)}\gamma_{gq}^{N(0)}+C_{2,q}^{N(1)}\gamma_{qq}^{N(0)}\right)$$

$$-C_{L,q}^{N(2)}C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}-2\beta_0\right)-C_{L,q}^{N(1)}C_{2,g}^{N(2)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)\Big]$$

$$\frac{1}{\left(C_{L,g}^{N(1)}\right)^2}\Bigg\{\left(C_{L,q}^{N(1)}\right)^2\Bigg[\left(C_{2,g}^{N(2)}-C_{2,g}^{N(1)}C_{2,q}^{N(1)}\right)\gamma_{qg}^{N(0)}-C_{2,g}^{N(1)}\left(C_{2,g}^{N(1)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)-\gamma_{qg}^{N(1)}\right)\Bigg]$$

$$+2\beta_0C_{L,g}^{N(2)}\left(C_{2,g}^{N(1)}C_{L,q}^{N(1)}-C_{L,g}^{N(2)}\right)+C_{L,q}^{N(1)}\Big[2C_{2,g}^{N(1)}C_{L,q}^{N(2)}\gamma_{qg}^{N(0)}+C_{2,g}^{N(1)}C_{L,g}^{N(2)}\left(\gamma_{qq}^{N(0)}-\gamma_{gg}^{N(0)}\right)$$

$$-C_{L,g}^{N(2)}\left(C_{L,q}^{N(2)}\gamma_{qg}^{N(0)}+C_{L,q}^{N(1)}\gamma_{qg}^{N(1)}\right)\Big]\Bigg\}$$

Solution of the Evolution Equation: U-matrix formalism

[J. Blümlein and A. Vogt, Phys. Rev. D58, (1998), 014020]

- We write the solution as a perturbation around the LO solution

$$\mathbf{F}_{LO}(N,a_s)=\left(\frac{a_s}{a_0}\right)^{-\mathbf{K}_0/2\beta_0}\mathbf{F}(N,a_0)\equiv\mathbf{L}(N,a_s,a_0)\mathbf{F}(N,a_0)$$

with

$$\mathbf{F}(N,a_s)\equiv\begin{pmatrix}F_A^N\\ F_B^N\end{pmatrix}(t)$$

- The expansion reads

$$\begin{aligned}\mathbf{F}(N,a_s) &= \mathbf{U}(N,a_s)\mathbf{L}(N,a_s,a_0)\mathbf{U}^{-\mathbf{1}}(\mathbf{N},\mathbf{a_s})\mathbf{F}(N,a_0)\\ &= \left[1+\sum_{k=1}^{\infty}a_s^k\mathbf{U}_k(N)\right]\mathbf{L}(N,a_s,a_0)\left[1+\sum_{k=1}^{\infty}a_0^k\mathbf{U}_k(N)\right]^{-1}\mathbf{F}(N,a_0)\end{aligned}$$

- Up to 3-loops the solution is

$$\begin{aligned}\mathbf{F}(N,a_s)=&\left[\mathbf{L}+a_s\mathbf{U}_1\mathbf{L}-a_0\mathbf{L}\mathbf{U}_1\right.\\ &\left.+a_s^2\mathbf{U}_2\mathbf{L}-a_sa_0\mathbf{U}_1\mathbf{L}\mathbf{U}_1+a_0^2\mathbf{L}\left(\mathbf{U}_1^2-\mathbf{U}_2\right)\right]\mathbf{F}(N,a_0)\end{aligned}$$

- We write the LO physical anomalous dimension $\mathbf{K}^{(0)}$ as

$$\mathbf{K}^{(0)} = \lambda_- \mathbf{e}^- + \lambda_+ \mathbf{e}^+ \qquad \mathbf{e}^{\pm} = \frac{\pm 1}{\lambda_+ - \lambda_-}\left(\mathbf{K}^0 - \lambda_{\mp} \mathbb{1}\right)$$

and

$$\lambda_{\pm} = \frac{1}{2}\left[\mathbf{K}^{(0)}_{11} + \mathbf{K}^{(0)}_{22} \pm \sqrt{\left(\mathbf{K}^{(0)}_{11} - \mathbf{K}^{(0)}_{22}\right)^2 + 4\mathbf{K}^{(0)}_{12}\mathbf{K}^{(0)}_{21}}\right]$$

- The U matrices are expressed in terms of the physical anomalous dimensions as

$$\mathbf{U}_j = -\frac{1}{j}\left[\mathbf{e}^- \widetilde{\mathbf{K}}^{(j)} \mathbf{e}^- + \mathbf{e}^+ \widetilde{\mathbf{K}}^{(j)} \mathbf{e}^+\right] + \frac{\mathbf{e}^+ \widetilde{\mathbf{K}}^{(j)} \mathbf{e}^-}{\lambda_- - \lambda_+ - j} + \frac{\mathbf{e}^+ \widetilde{\mathbf{K}}^{(j)} \mathbf{e}^-}{\lambda_- - \lambda_+ - j}$$

where

$$\widetilde{\mathbf{K}}^{(1)} = \mathbf{K}^{(1)}, \qquad \widetilde{\mathbf{K}}^{(j)} = \mathbf{K}^{(j)} \sum_{i=1}^{j-1} \mathbf{K}^{(j-i)} \mathbf{U}_i \quad \text{if} \quad j \neq 1$$

- The explicit form of the solution at NNLO is the following

$$
\begin{aligned}
\mathbf{F}(N,a_s) = \Bigg\{ e^{-\lambda_-\,t/4} \Bigg[& \mathbf{e}^- + (a_s - a_0)\mathbf{K}^{(1)}_{--} + \left(a_s \frac{e^{-\lambda_+\,t/4}}{e^{-\lambda_-\,t/4}} - a_0 \right) \frac{\mathbf{K}^{(1)}_{-+}}{r_+ - r_- + 1} \\
& + \left(a_s^2 - a_0^2\right) \frac{\mathbf{K}^{(2)}_{--}}{2} + \left(a_s^2 \frac{e^{-\lambda_+\,t/4}}{e^{-\lambda_-\,t/4}} - a_0^2 \right) \frac{\mathbf{K}^{(2)}_{-+}}{r_+ - r_- + 2} \\
& - a_s a_0 \left(\mathbf{K}^{(1)}_{---} + \frac{\mathbf{K}^{(1)}_{--+}}{r_+ - r_- + 1} + \frac{\mathbf{K}^{(1)}_{+--}}{r_- - r_+ + 1} \right. \\
& \left. + \frac{\mathbf{K}^{(1)}_{+-+}}{(r_- - r_+ + 1)(r_+ - r_- + 1)} \right) \Bigg] + (- \leftrightarrow +) \Bigg\} \mathbf{F}(N,a_0)
\end{aligned}
$$

where

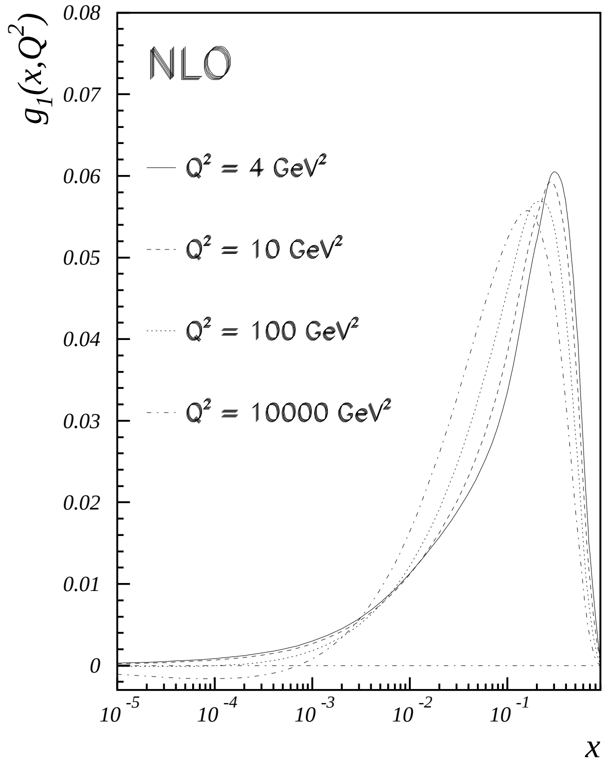
$$
\mathbf{K}^{(1)}_{ij} = \mathbf{e}^i \widetilde{\mathbf{K}}^{(1)} \mathbf{e}^j \qquad \mathbf{K}^{(1)}_{ijk} = \mathbf{e}^i \widetilde{\mathbf{K}}^{(1)} \mathbf{e}^j \widetilde{\mathbf{K}}^{(1)} \mathbf{e}^k \qquad i,j,k = \pm\,.
$$

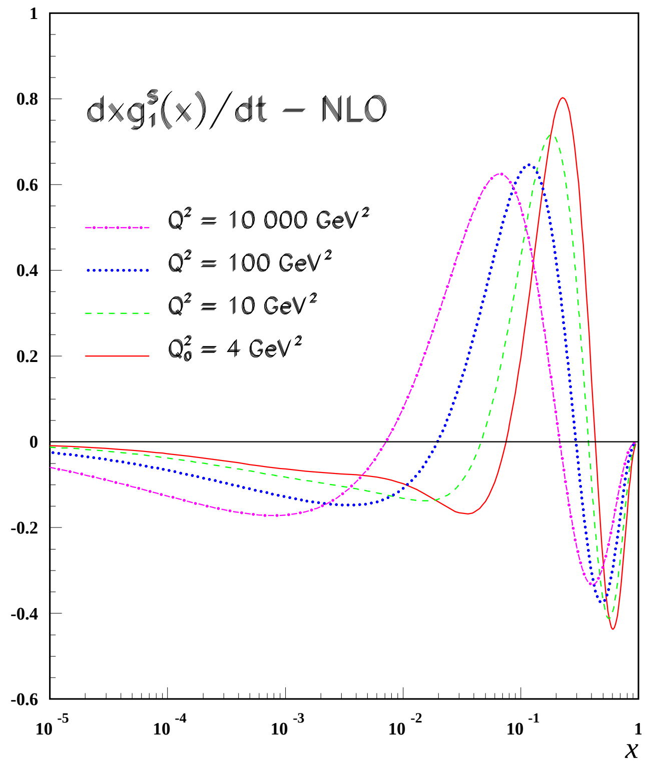
- Heavy flavour contribution to DIS structure function in the kinematic regime of HERA is known to be sizable (up to $20 - 40\%$ for $F_2^{e.m.}$, depending on the actual event kinematics)
- Any analysis of DIS structure functions aiming to $\sim 1\%$ accuracy needs to take into account heavy quark effects on F_2 and F_L
- A parametrization of heavy flavour Wilson coefficients in Mellin space has recently been derived in a form which allows direct incorporation in evolution codes.

[S. I. Alekhin and J. Blümlein, Phys. Lett. B594, (2004), 299]

[J. Blümlein, H. Böttcher, Nucl. Phys. B636, (2002), 225]

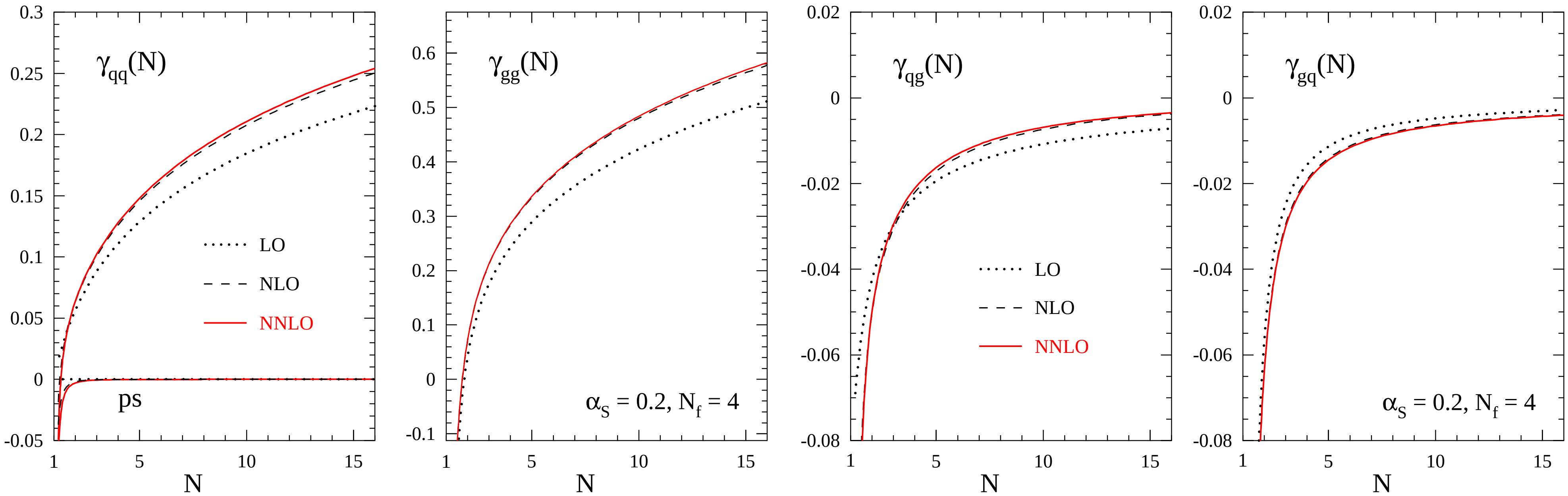
- A combined Standard-Scheme Invariant Analysis has been performed up to NLO in polarized DIS, considering the structure functions g_1 and $\partial g_1/\partial t$
- Both analyses yield values for $\alpha_s(M_Z^2)$ in accord with the world average
- A difference in $\Lambda_{QCD}^{(4)} = 12\mathrm{MeV}$ is found between the values obtained in the two analyses
- Results obtained considering 3 massless flavours



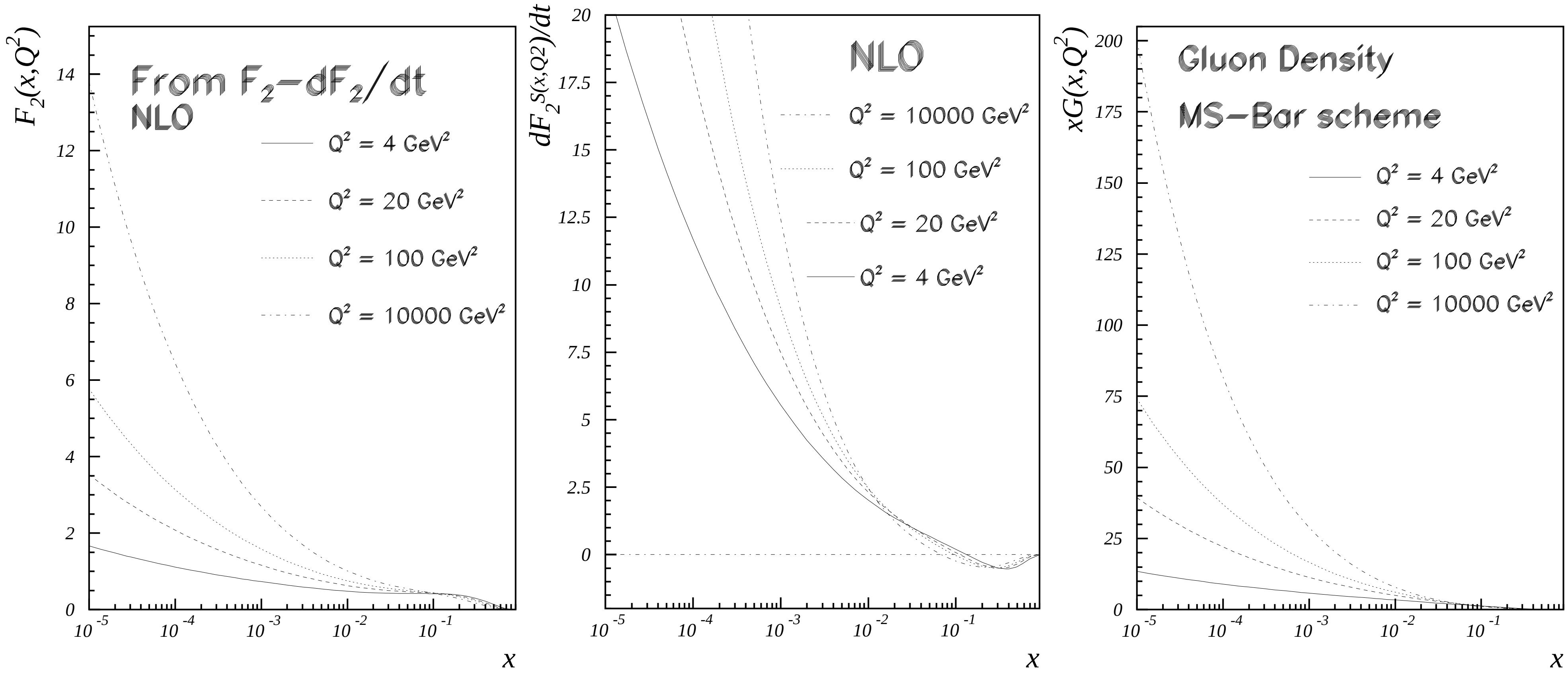


- A complete numerical implementation of the NNLO analysis is almost finished
- The results on 3-loop anomalous dimensions show a good convergence of the perturbative series, with small differences going from NLO to NNLO
- Due to these results (and the results of the NS analysis) we expect slight modifications also in the Singlet analysis of the structure functions.

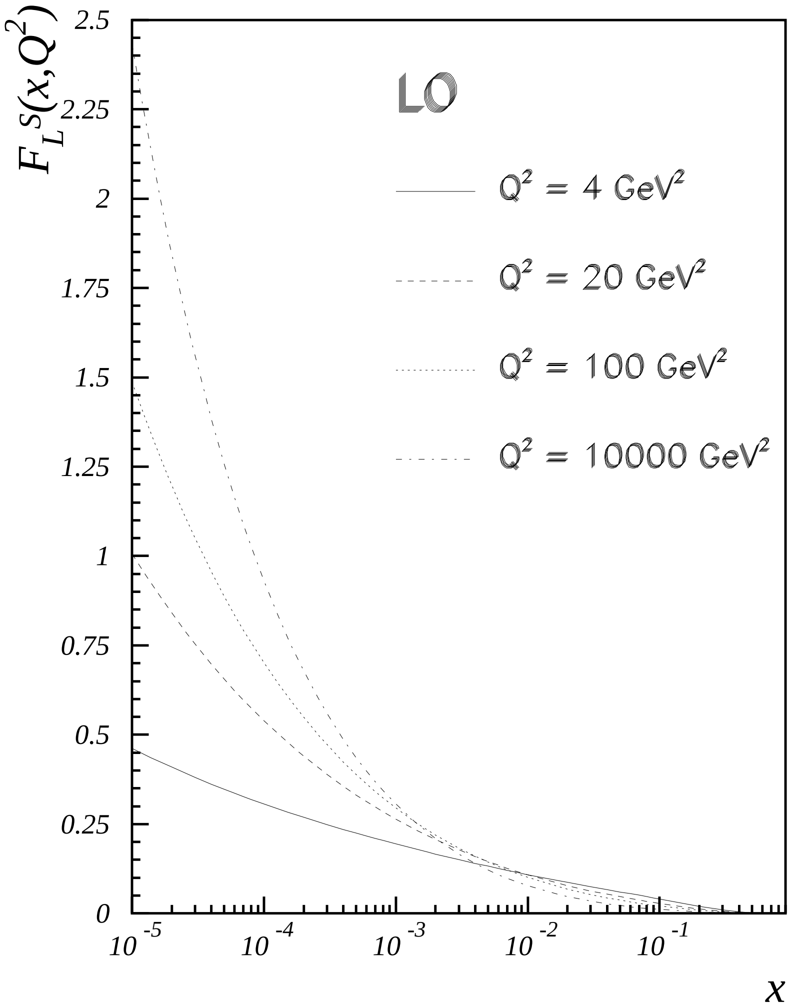
[S. Moch, J. A. .M. Vermaseren and A. Vogt, Nucl. Phys. B688, (2004), 101; Nucl. Phys. B691, (2004), 129]

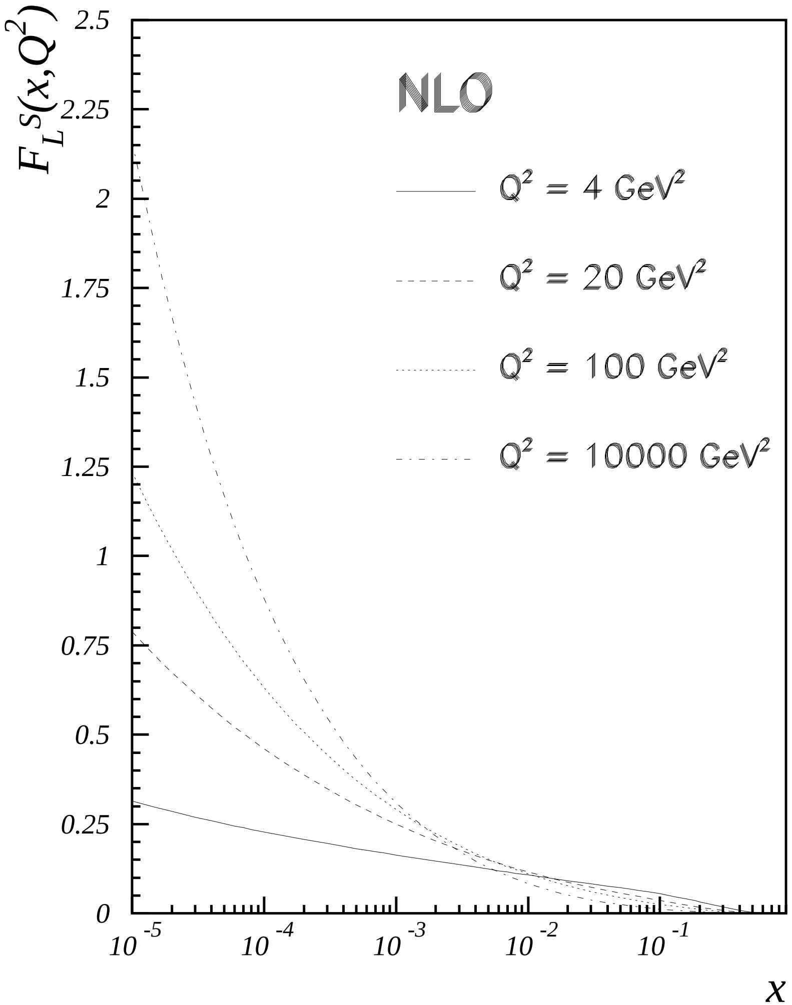


- F_2 and $\partial F_2/\partial t$ obtained from NLO scheme invariant evolution with 4 massless flavours
- Parton Distribution Functions can be extracted in any factorization scheme ($e.g.$ the gluon PDF in the $\overline{MS}$ scheme)



- The longitudinal structure function F_L in LO and NLO as obtained from scheme invariant evolution of F_2, F_L





- Upcoming measurements of DIS structure functions will allow to reduce experimental errors on α_{s} to the level of 1%
- On the theoretical side NNLO analysis are required in order to match such an accuracy
- Combining standard QCD analysis and fits based on scheme invariant evolution could provide a method to reduce theoretical and conceptual errors in the determination of α_{s}
- We aim to perform a combined analysis up to NNLO of DIS structure functions to extract α_{s} and a set of parton distribution functions with fully correlated errors
- Numerical implementation of NLO evolution for massless quarks is completed
- Complete NNLO analysis and implementation of Heavy Flavour contribution soon to come ...