Non-Markovian (constrained) Monte Carlo Algorithm for QCD evolution

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S. Jadach and M. Skrzypek

stanislaw.jadach@ifj.edu.pl, maciej.skrzypek@ifj.edu.pl

HNINP-PAS (IFJ-PAN), Cracow, Poland

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The problem and motivation

Basic facts:

- Markovian MC implementing the QCD/QED evolution equations is basic ingredient in all parton shower type MCs
- Unconstrained forward Markovian MC, with evolution kernels from perturbative QCD/QED, can only be used for FSR (inefficient for ISR).
- For the ISR cascade the elegant Backward Markovian MC algorithm of Sjostrand (Phys.Lett. 157B, 1985) is a widely adopted remedy.
- Backward Markovian MC does not solve the QCD evolution eqs. It merely exploits their solutions coming from the external non-MC methods

The problem:

- Is it possible to invent an efficient MC algorithm, non-Markovian, solving internally the evolution eqs. by its own?

Motivation:

- More freedom in the modeling the ISR parton shower,
- Easier MC modeling of the unintegrated parton distributions $D_k(p_T, x)$
- MC modeling of the CCFM class of the QCD calculations/models.

Vocabulary

- $\boxed{\text{Markovian MC algorithm}}$

  The algorithm in which the number of emission (determining the dimension of the dimension of the integral, phase space), is generated as the last variable

- $\boxed{\text{non-Markovian MC algorithm}}$

  The algorithm in which the number of emission (the dimension of the integral), is generated as one of the first variables.

- $\boxed{\text{Constrained MC algorithm = CMC}}$

  The integration domain restricted to a less-dimensional hyperspace by means of inserting the $\delta(F(x_1, ..., x_n))$ function.
  (Energy-momentum Conserving $\delta^{(4)}(P - \sum p_i)$ is a well known example.)
  CMC algorithm can generate efficiently points in this subspace.
  The distribution on the hyperspace is usually much more complicated (to generate).

- $\boxed{\text{Evolution equation leading to Markovian process}}$

  $\partial_t N_I(t) = \sum_L P_{IK}(t) N_K(t)$, where $P_{II}(t) \equiv -\sum_{X \neq I} P_{XI}(t)$,
  I, K can be discrete, continuous or mixture of both.
  QCD case: $\sum_K \to \sum\limits_{k=Q,G} \int_0^1 dx/x$ and $P_{K_2 K_1} \to (x_2/x_1) P_{k_2,k_1}(x_2/x_1)$

Pure bremsstrahlung from $k = G, q, \bar{q}$ line

Iterative solution of the QCD evolution equations,
for evolution $t_0 \to t$, where $t = \ln Q$ is the evolution time:

$$x\mathcal{D}_{kk}(t,t_0;x) = e^{-\Phi_k(t,t_0)} \bigg\{ \delta_{x=1} + \\ + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{t_0}^{t} dt_i \int_0^1 dz_i\; \mathcal{P}^{\Theta}_{kk}(t_i,z_i)\; \delta_{x=\prod_{i=1}^{n} z_i} \bigg\},$$

Notation:

- $\mathcal{P}_{kk}(t,z) \equiv \frac{\alpha(t)}{\pi} z P_{kk}(t) = \mathcal{P}^{\delta}(t)\delta_{z=1} + \mathcal{P}^{\Theta}_{kk}(t,z)$,
  $\mathcal{P}^{\Theta}_{kk}(t,z) = \mathcal{P}_{kk}(t,z)\theta_{1-z>\varepsilon}$
  where $P_{ik}(z)$ are QCD DGLAP kernels, see next slide.
- Sudakov formfactor: $\Phi_k(t,t_0) = \int_{t_0}^{t} dt'\; \mathcal{P}^{\delta}_{kk}(t')$.
- $\theta_{x>0} = 1$ for $x > y$ and $= 0$ otherwise;
- $\delta_{x=y} \equiv \delta(x-y)$.
- IR cut $\varepsilon << 1$, does not depend on t (this is OK for DGLAP).

QCD LL kernels

Table of the elements in the LL kernels ($T_f = n_f T_R$), $Q = q + \bar{q}$

IK	$A^{(0)}_{KK}$	$B^{(0)}_{KK}$	$C^{(0)}_{IK}$	$D^{(0)}_{IK}(z)$	$\hat{D}_{IK}(z)$	$\int dz D^{(0)}_{IK}(z)$
GG	$\frac{11}{6}C_A - \frac{2}{3}T_f$	$2C_A$	$2C_A$	$2C_A(-2+z-z^2)$	0	$-\frac{11}{3}C_A$
QG	$-$	$-$	0	$2T_f(z^2+(1-z)^2)$	$2T_f$	$\frac{4}{3}T_f$
QQ	$\frac{3}{2}C_F$	$2C_F$	0	$C_F(-1-z)$	0	$-\frac{3}{2}C_F$
GQ	$-$	$-$	$2C_F$	$C_F(-2+z)$	0	$-\frac{3}{2}C_F$

$$P_{ik}(z) = \delta(1-z)\delta_{ik}A_{kk} + \frac{1}{(1-z)_+}\delta_{ik}B_{kk} + \frac{1}{z}C_{ik} + D_{ik}(z).$$

For the purpose of the MC generation temporary simplifications:

$$zP_{kk}(z) \to z\hat{P}_{kk}(z) = zB_{kk}\left(\frac{1}{1-z} + \frac{1}{z}\right) = B_{kk}\frac{1}{1-z},$$

$$\mathcal{P}^{\Theta}_{kk}(t,z) \to \hat{\mathcal{P}}^{\Theta}_{kk}(t,z) = \alpha(t)\; z\hat{P}^{\Theta}_{kk}(z) = \frac{2B_{kk}}{\beta_0(t-t_\Lambda)}\;\frac{\theta_{1-z>\varepsilon}}{1-z}$$

$$\mathcal{P}^{\delta}_{kk}(t) = \frac{\alpha_s(t)}{\pi}\left\{B_{kk}\ln\frac{1}{\varepsilon} - A_{kk}\right\}$$

Mapping of variables etc.

The t dependence of $\alpha(t)$ compensated by means of mapping $t_i \to \tau_i = \ln(t_i - \ln \Lambda_0)$ and we also introduce new energy variable $y_i \equiv \ln(1 - z_i)$:

$$x\mathcal{D}_{kk}(\tau, \tau_0; x) = e^{-\Phi_k(\tau,\tau_0)} \Bigg\{ \delta_{x=1} +$$

$$+ x^{\omega_k} \sum_{n=1}^{\infty} \frac{1}{n!} b_k^n \prod_{i=1}^{n} \int\limits_{\ln(1-\varepsilon)}^{\ln(1-x)} dy_i \; \delta_{x=\prod_{i=1}^{n} z_i(y_i)} \int\limits_{\tau_0}^{\tau} d\tau_i \; w_P \Bigg\},$$

NOTATION:

- Sudakov formfactor: $\Phi_k(\tau, \tau_0) = (\tau - \tau_0) \left(b_k \ln \frac{1}{\varepsilon} - a_k \right)$
- MC compensating weight $w_P = x^{-\omega_k} \prod_{j=1}^{n} \frac{\hat{P}_{kk}^{\Theta}(z_j)}{P_{kk}^{\Theta}(z_j)}$
- where dummy x^{ω_k} introduced to optimize final MC weight distribution
- constants: $b_k \equiv \frac{2}{\beta_0} B_{kk}, \quad a_k \equiv \frac{2}{\beta_0} A_{kk}.$

The energy CONSTRAINT is our target

The constraint is: $x = \prod_{i=1}^{n} z_i(y_i) = F(y_1, y_2, \ldots, y_n)$.

Conveniently rewritten as $\boxed{\ln\frac{1}{x} = \sum_{j=0}^{n} f(y_j)}$, $f(y_j) = -\ln\left(1 - \exp\left(y_i\right)\right) = -\ln z_j$.

It also determines upper integration limit: $y_i \in (y_{\min}, y_{\max}) = \big(\ln\varepsilon, \ln(1-x)\big)$.
Ordering energy variables y_i, defining $y_0 \equiv 0$, yields:

$$x\mathcal{D}_{kk}(\tau,\tau_0;x) = e^{-\Phi_k(\tau,\tau_0)}\Big\{\delta_{x=1} +$$
$$+\, x^{\omega_k - 1}\sum_{n=1}^{\infty} b_k^n \prod_{i=1}^{n}\int_{y_{\min}}^{y_{\max}} dy_i\, \theta_{y_i > y_{i-1}}\, \delta\left(\ln\frac{1}{x} - \sum_j f(y_j)\right)\int_{\tau_0}^{\tau} d\tau_i\, w_P\Big\}.$$

- Function $f(y_i)$ is very steeply (exponentially) rising, hence the constraint $x = \prod_{i=1}^{n} z_i(y_i)$ is "saturated" by a single z_j, while other ones $z_i \simeq 1$.
- In other words, $y_j \simeq y_{\max} = \ln(1-x)$, and other ones $y_i, \;\; i \neq j$ move freely within the $(y_{\min}, y_{\max})$.
- Due to ordering, $y_n \simeq y_{\max}$ effectively takes responsibility for satisfying the constraint.

1. How do we get rid (satisfy) the energy constraint?

STEP 1:

Perform the following simple linear transformation:

$$\boxed{y_i = y'_i - Y},$$

where Y is "adjusted" such that $y'_n = y_n + Y = y_{\max}$.

The introduction of Y variables is countered by the δ-function:

$$x\mathcal{D}_{kk}(t,t_0;x) = e^{-\Phi_k(t,t_0)}\Bigg\{\delta_{x=1} +$$

$$+ x^{\omega_k - 1}\sum_{n=1}^{\infty} b_k^n \int dY \prod_{i=1}^{n} \int\limits_{y_{\min}}^{y_{\max}} dy_i\ \theta_{y_i > y_{i-1}}\,\delta(y_n + Y - y_{\max})$$

$$\times\,\delta\left(\ln\frac{1}{x} - \sum_j f(y_j)\right)\int\limits_{\tau_0}^{\tau} d\tau_i\ w_P\Bigg\}.$$

2. How do we satisfy the energy constraint?

STEP 2:

Change variables $y_i \to y'_i$. Jacobian is equal one!

$$x\mathcal{D}_{kk}(t,t_0;x) = e^{-\Phi_k(t,t_0)}\Big\{\delta_{x=1} + x^{\omega_k - 1}\sum_{n=1}^{\infty} b_k^n \int dY$$

$$\times \prod_{i=1}^{n}\int_{y_{\min}}^{y_{\max}} dy'_i \; \theta_{y'_i > y'_{i-1}} \delta(y'_n - y_{\max})\delta\left(\ln\frac{1}{x} - \sum_j f(y'_j - Y)\right)\int_{\tau_0}^{\tau} d\tau_i \; w_P\Big\},$$

IMPORTANT!

- We are able to preserve the same integration limits $y'_i \in (y_{\min}, y_{\max})$ in spite of $\theta_{y_1 > y_{\min}} = \theta_{y'_1 > Y + y_{\min}}$. Luckily we shall get $Y \geq 0$ in the next step!
- To be consistent we redefine $w_P \to w_P \times \theta_{y'_1 > Y + y_{\min}}$.
- Notation consistent by adding $y'_0 \equiv y_{\min}$.
- We have got $y'_n = y_{\max}$ as we wanted!!!

3. How do we satisfy the energy constraint?

STEP 3: Eliminate the constraint $\delta(x - F(y'_j))$ by means of the Y-integration:

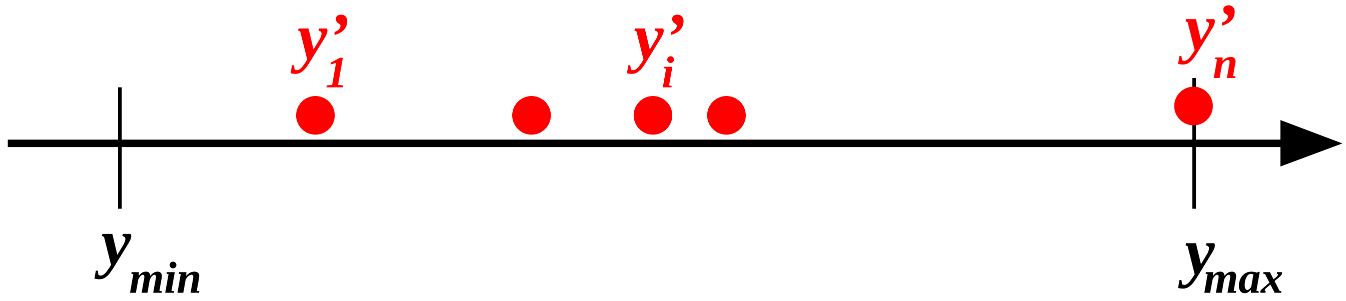
$$x\mathcal{D}_{kk}(t, t_0; x) = e^{-\Phi_k(t,t_0)} \Big\{ \delta_{x=1} +$$

$$+ x^{\omega_k - 1} \sum_{n=1}^{\infty} b_k^n \prod_{i=1}^{n} \int_{y_{\min}}^{y_{\max}} dy'_i \; \theta_{y'_i > y'_{i-1}} \delta(y'_n - y_{\max})$$

$$\times \frac{1}{|\partial_Y \ln F(y'_j - Y)|_{Y=Y_0}} \int_{\tau_0}^{\tau} d\tau_i \; w_P \Big\},$$

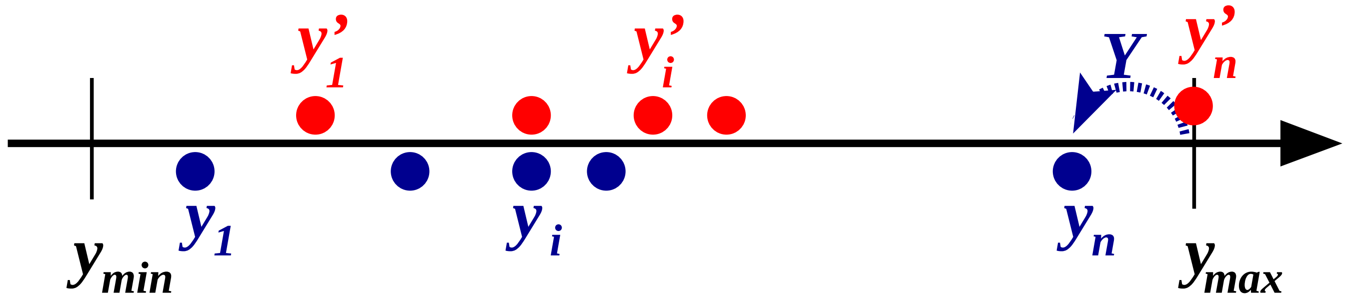
- $1/|\partial_Y \ln F|$ enters the MC weight. Does it destroy the weight??!!
- $Y_0 = Y_0(x, y'_1, ..., y'_n)$ is the solution of the transcendent equation $x = F(y'_j - Y)$.
- Check whether $y_1 = y'_1 - Y_0(x, y'_1, ..., y'_n) \geq y_{\min}$. About 1/3 evens trashed.
- Effectively $\delta(x - F(y'_1, ..., y'_n))$ is traded for $\delta(y'_n - y_{max})$.
- The parallel shift $\{\Sigma : y' \to y\}$ maps y' from the $(n-1)$-dim. simplex S_{n-1} defined by $y_n = y_{max}$, into target hyperspace R_{n-1} defined by $x = F(y_j)$.
- In fact $R_{n-1} \in \Sigma(S_{n-1})$, thanks to $Y_0(x, y'_1, ..., y'_n) > 0$!!!

Linear shift: $y'_i \to y_i = y'_i - Y$



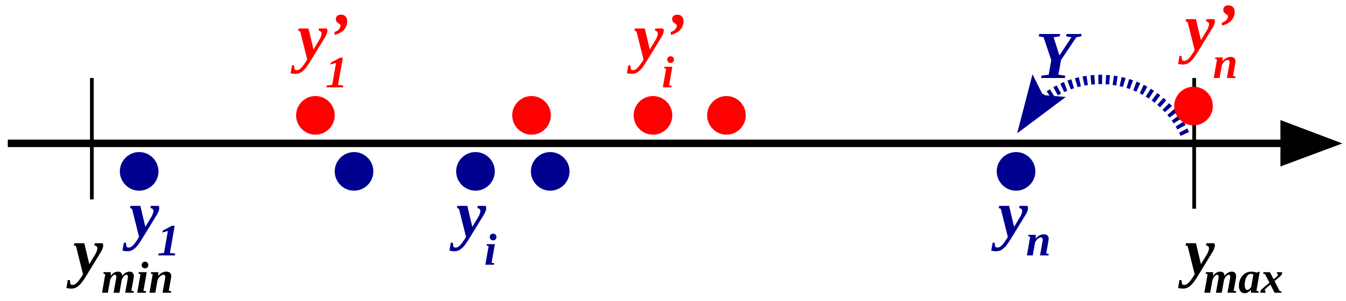
- Begin with y'_i such that one of them $y_n \equiv y_{\rm max}$

Linear shift: $y'_i \to y_i = y'_i - Y$



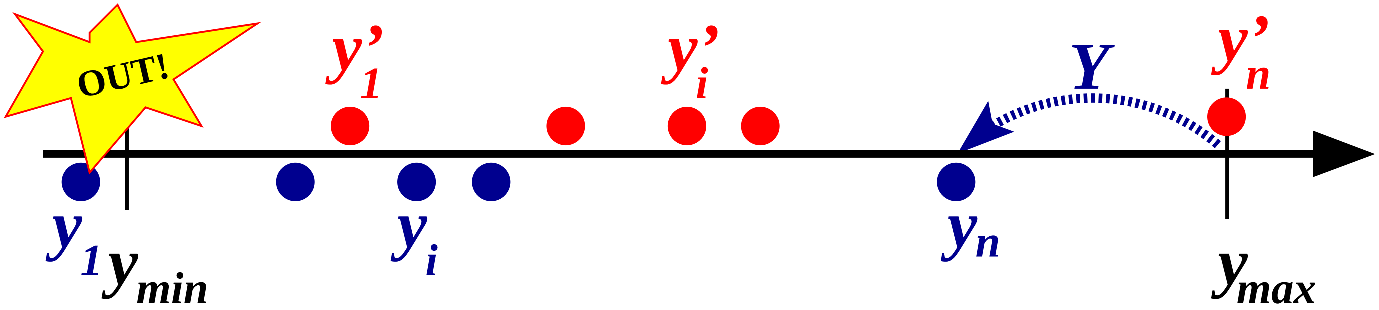
- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i

Linear shift: $y'_i \to y_i = y'_i - Y(y'_1, y'_2, ..., y'_n)$



- Begin with y'_i such that one of them $y_n \equiv y_{\max}$
- Shift $y'_i \to y_i$ by Y, where Y solves constraint condition $\prod z_i = x$
- Y is therefore complicated function of all y'_i
- Sometimes the smallest y'_i is shifted OUT of the phase space, below IR the limit $y_{\min}$. Such an event gets MC weight $w = 0$

Master formula for the bremsstrahlung Monte Carlo

$$
x\mathcal{D}_{kk}(\tau,\tau_0;x) = e^{(\tau-\tau_0)a_k} \sum_{n=0}^{\infty} \Big\{ e^{b_k \mathcal{R}(\varepsilon)} \delta_{n=0}\delta_{x=1} + \delta_{n>0}\theta_{1-x>\varepsilon} e^{b_k \mathcal{R}(1-x)} \frac{b_k x^{\omega_k -1}}{x g(x)}
$$
$$
\times P_n\big(b_k[\mathcal{R}(1-x)-\mathcal{R}(\varepsilon)]\big) \prod_{i=1}^{n} \int_0^1 dr_i \; \frac{\delta(1-\max r_j)}{n} \int_0^1 ds_i \; w^{\#} \Big\}
$$

NOTATION:

- Mapping $\tau_i = \tau_0 + s_i(\tau-\tau_0)$
- Mapping $z_i = 1 - e^{y_i} = 1 - \exp\big(y_{\min} + r_i(y_{\max} - y_{\min}) - Y\big)$
- Poisson distribution: $P_n(\lambda) = e^{-\lambda}\lambda^n/n!, \quad \lambda = < n >$.
- $\mathcal{R}(x) \equiv (\tau - \tau_0)\ln(x)$ (implicitly depends on $\tau - \tau_0$).
- MC weight: $\boxed{w^{\#} = w_P \; \frac{x g(x)}{|\partial_Y \ln F(Y_0)|} \; \theta_{y'_1 - Y_0 > y_{\min}}}$,
- where $g(x) = |\partial_y \ln z(y)|_{z=x} = \frac{1-x}{x}$ is to stabilize the MC weight.
- Ordering of y'_i is here relaxed (to get explicit $1/(n-1)!$ for Poisson).

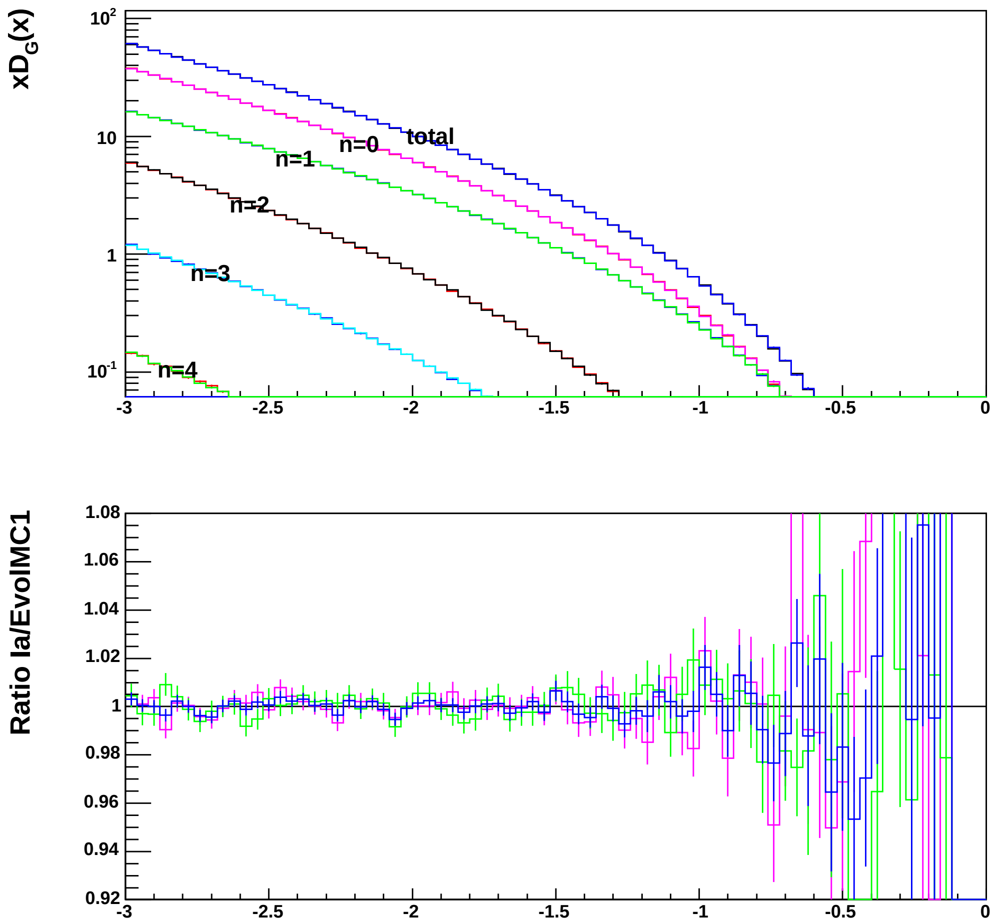
Gluon bremsstrahlung MC algorithm overview:

- The out-most integration variable is total x, the same as in hard process $H(x)$.
- Neglecting temporarily MC weight $w^{\#}$ we can sum/integrate analytically the entire series of integrals in the master eq.: $\sigma_k = \int_{\epsilon_1}^{1} dx\ H(x)\ D_{kk}(t, t_0; x) =$

$$= \int_{\epsilon_1}^{1} \frac{dx}{x}\ H(x) \int_{0}^{\exp(b_k \mathcal{R}(1-x))} dR\ Z(R)^{\omega_k - 2} e^{(\tau-\tau_0)a_k}\ \frac{x}{Z(R)} D_k\left(\frac{x}{Z(R)}, t_0\right)$$

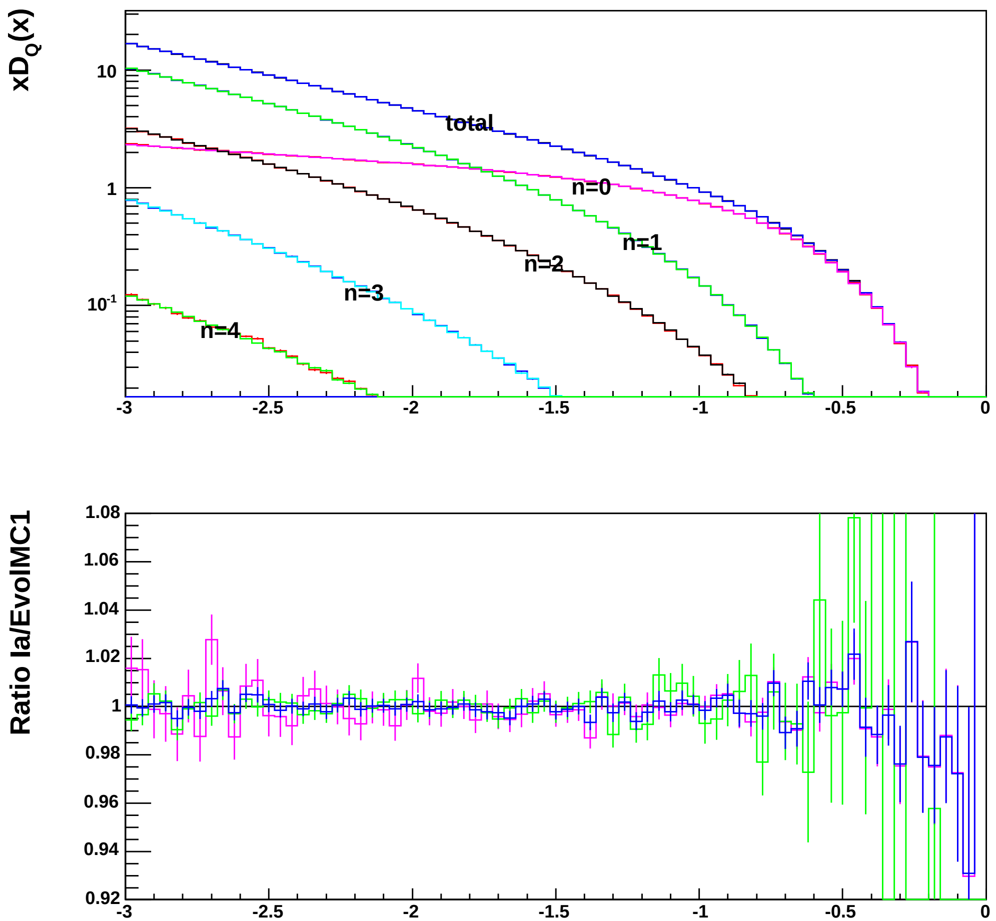
- Mapping: $R(Z) = e^{b_k \mathcal{R}(1-Z)} = (1-Z)^{b_k(\tau-\tau_0)}$
- Generation of R (and of k) is done by `Foam`, general purpose MC tool.
- Knowing $Z(R)$, if $Z > 1-\varepsilon$ the emission multiplicity n is generated according to Poisson P_{n-1} (Non-Markovian!!!), otherwise $n = 0$ and $Z = 1$.
- Variables $s_i, i = 1, 2, ..., n$ are generated and mapped into $\tau_i(s_i)$ and $t_i(\tau_i)$. They are ordered.
- Unordered variables $r_i \in (0, 1)$ are generated, such that one of them is equal 1 (rescaling). Mapped into $y_i'(r_i)$.
- The solution $Y = Y_0$ of the transcendent equation $\ln F(y_j' - Y) - \ln x = 0$ is found numerically (NB. derivative $\partial_Y \ln F$ for MC weight obtained as byproduct).
- With Y_0 at hand, variables $z_i(y_i(y_i'))$ are evaluated.

Test of Gluon bremsstrahlung Constrained MC



Histograms $n = 0$ represents pure gluon bremsstrahlung out of gluon line.
Starting distribution is gluon in proton at $Q = 1$GeV.
Plotted distribution is at 1TeV.
Compared are results from unconstrained Markovian MC (EvolFMC) and the new non-Markovian constrained MC (EvolCMC).
They agree to within statistical error $\sim 0.25\%$ (100M events)!

Test of Gluon bremsstrahlung Constrained MC



Histograms $n = 0$ represents pure gluon bremsstrahlung out of quark line.
Starting distribution is gluon in proton at $Q = 1$GeV.
Plotted distribution is at 1TeV.
Compared are results from unconstrained Markovian MC (EvolFMC) and new the non-Markovian constrained MC (EvolCMC).
They agree to within statistical error $\sim 0.25\%$ (210M events)!

Hierarchic reorganization of the emission chain (cascade)

Beyond pure bremsstrahlung, the full DGLAP non-Markovian MC requires two-level organization of the emission chain:
(S) Flavor transmutation super-level $G \to Q \to G \to Q \to G \to \dots$
(B) Bremsstrahlung sub-level, any No. of gluon emissions ($Q \to Q$, $G \to G$).

Starting point is the usual iterative solution ($k \equiv k_n$) of the QCD evolution equations:

$$xD_k(\tau, x) = e^{-(\tau-\tau_0)R_k} xD_k(\tau_0, x) +$$
$$+ \sum_{n=1}^{\infty} \sum_{k_{n-1}\dots k_1 k_0} \left[\prod_{j=1}^{n} \int_{\tau_0}^{\tau} d\tau_j \; \theta_{\tau_j > \tau_{j-1}} \right] \int_0^1 dx_0 \left[\prod_{i=1}^{n} \int_0^1 dz_i \right]$$
$$\times\, e^{-(\tau-\tau_n)R_k} \left[\prod_{i=1}^{n} \mathcal{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; e^{-(\tau_i-\tau_{i-1})R_{k_{i-1}}} \right] x_0 D_{k_0}(\tau_0, x_0) \delta_{x = x_0 \prod_{i=1}^{n} z_i},$$

Notation:

- Kernels: $\mathcal{P}^{\Theta}_{k_i k_{i-1}}(z_i) = \frac{\alpha(t_0)}{\pi} \; z_i P_{k_i k_{i-1}}(z_i) \theta_{1-z_i > \varepsilon}$
- Transition rates: $R_k = \sum_j \int_0^{1-\epsilon} dz \; \mathcal{P}^{\Theta}_{jk}(z) = \sum_j R_{jk}$

Hierarchic reorganization of the emission chain (cascade)

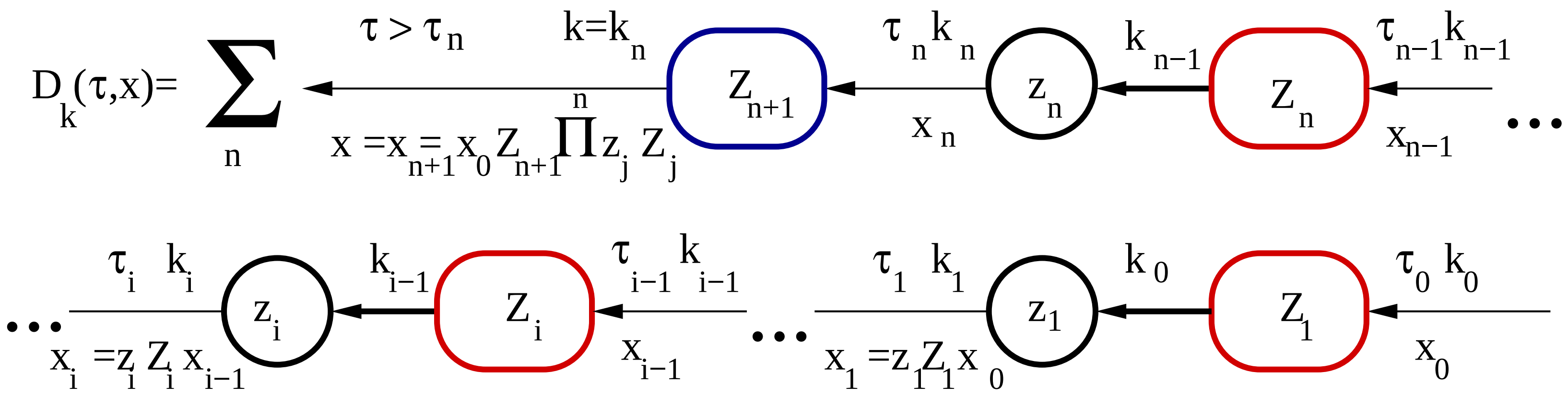
The key point is to isolate segments of pure gluon bremsstrahlung with the transformation of the summation order (indexing) which looks schematically as follows:

$$\sum_{n=0}^{\infty}\sum_{k_{n-1}\dots,k_1 k_0} X_{k_n k_{n-1}\dots k_1 k_0} = \sum_{n=0}^{\infty}\sum_{\substack{k_{n-1}\dots,k_1 k_0\\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}}$$

$$\sum_{j_n,j_{n-1}\dots j_0=1}^{\infty} X_{k_n^{(j_n)}\dots k_n^{(2)}k_n^{(1)}k_{n-1}^{(j_{n-1})}\dots k_{n-1}^{(2)}k_{n-1}^{(1)}\dots\dots k_1^{(j_1)}\dots k_1^{(2)}k_1^{(1)}k_0^{(j_0)}\dots k_0^{(2)}k_0^{(1)}}$$

- In the above we have $k_r^{(j_r)} = \dots = k_r^{(2)} = k_r^{(1)}$ and the purpose of the upper index in this context is simply to show that the same index k is repeated j_r times.
- However, variables $z_r^{(m)}$ and $\tau_r^{(m)}, \quad r = 1,2,\dots,n, \quad m = 1,2,\dots,j_r$ are truly independent and the upper index truly differentiates them.
- The aim is now to show that one can factorize out the pure bremstrahlung functions $\mathcal{D}_{kk}(\tau,x|\tau_0,x_0)$ and identify the remaining functions and integrations.
- I omitt the details of the formal combinatoric proof of the above transformation.
- Alternative derivations: directly from evolution eqs. or using functional methods.

Two-level, hierarchic organization of the emission chain



- Black circle is $G \to Q$ or $Q \to G$ flavor transmutation
- Red oval is pure bremsstrahlung segment
- Blue oval is the last bremsstrahlung segment, just before the hard process.
- x_0 and k_0 are starting values at the proton, at the low energy scale τ_0, $Q_0 \sim 1$GeV.

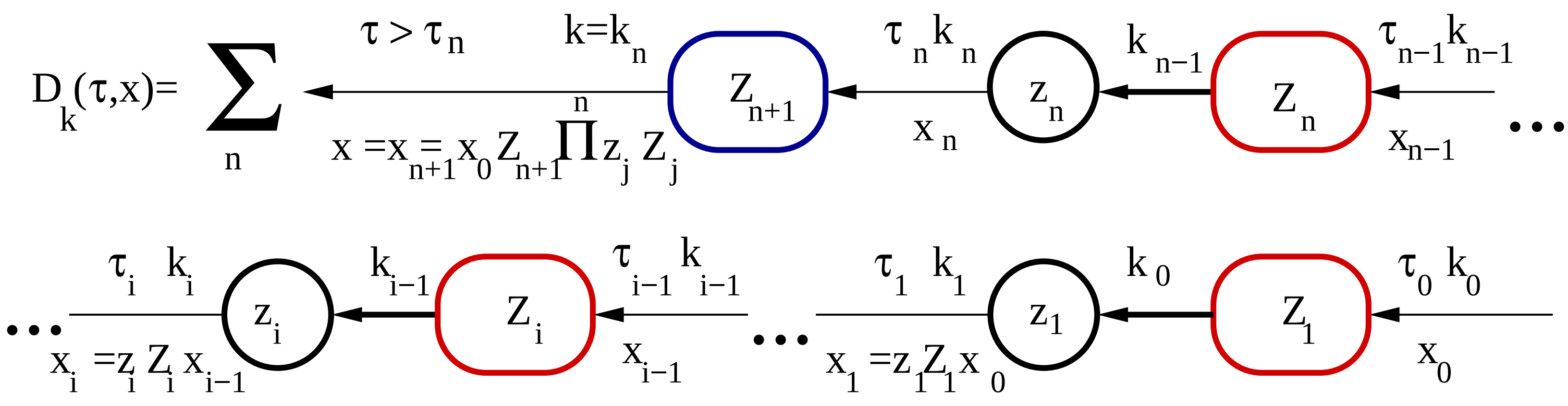
Two-level hierarchic: $\mathcal{D}_{kk}$ are also multi-integrals!

$$D_k(\tau,x) = \int dZ\; dx_0\; \mathcal{D}_{kk}(\tau,Z|\tau_0)\; D_k(\tau_0,x_0)\delta_{x=Zx_0} +$$

$$+\sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0 \\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}} \int_0^1 dZ_{n+1} \left[\prod_{j=1}^{n} \int_{\tau_0}^{\tau} d\tau_j\; \theta_{\tau_j>\tau_{j-1}} \int_0^1 dz_j \int_0^1 dZ_j\right] \int_0^1 dx_0$$

$$\times\, \mathcal{D}_{kk}(\tau,Z_{n+1}|\tau_n) \left[\prod_{i=1}^{n} \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; \mathcal{D}_{k_{i-1}k_{i-1}}(\tau_i,Z_i|\tau_{i-1})\right]$$

$$\times\, D_{k_0}(\tau_0,x_0)\delta\left(x - x_0 Z_{n+1}\prod_{i=1}^{n} z_i Z_i\right), \qquad k\equiv k_n.$$



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$$\times\; \mathcal{D}_{kk}(\tau, Z_{n+1}|\tau_n) \left[\prod_{i=1}^{n} \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i)\; \mathcal{D}_{k_{i-1}k_{i-1}}(\tau_i, Z_i|\tau_{i-1})\right]$$

$$\times\; D_{k_0}(\tau_0,x_0)\delta\Big(x - x_0 Z_{n+1}\prod_{i=1}^{n} z_i Z_i\Big), \qquad k\equiv k_n,$$

$$\mathcal{D}_{kk}(\tau,Z|\tau_0) = \frac{e^{\Phi_k(\tau,\tau_0)}}{Z}\left\{\delta_{Z=1} + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int_{\tau_0}^{\tau} d\tau_i\; \theta_{\tau_i>\tau_{i-1}}\int_0^1 dz_i\; z_i \mathbf{P}^{\Theta}_{kk}(z_i)\delta_{Z=\prod_{i=1}^{n} z_i}\right\}$$

NOTATION:

- Pure bremss. Sudakov formfactor $\Phi_k(\tau,\tau_0) = (\tau-\tau_0)(a_k + b_k \ln\varepsilon)$
- Kernel $\times$ coupling const: $\mathbf{P}^{\Theta}_{k_1 k_2}(z) = \frac{2}{\beta_0} P_{k_1 k_2}(z)\theta_{1-z>\varepsilon}$
- Other: $b_k \equiv \frac{2}{\beta_0} B_{kk}, \quad a_k \equiv \frac{2}{\beta_0} A_{kk},$

CMC =Non-Markovian constrained MC, for full DGLAP

- Neglecting temporarily $w^{\#}$ inside the segments $\mathcal{D}_{kk}$, gluon bremsstrahlung sub-level, we can itegrate/sum analytically over all variables of the sub-level
- The overall (energy) x-constraint δ-function is eliminated using $\int dx_0$
- We are left with the $3n+1$-dim. integrals (n= No. of flavor changes) of the flavor-changing super-level, the INTEGRAND FOR Foam is the following:

$$
\begin{aligned}
D_k(\tau,x) = {} & x^{-1}\int_x^1 dZ \int\limits_0^{R(x)} dR_1\; Z(R_1)^{\omega_k-2} e^{a_k(\tau-\tau_0)}\; x_0 D_k(\tau_0,x_0) + \\
& + x^{-1}\sum_{n=1}^{\infty} \sum_{\substack{k_{n-1}\dots,k_1 k_0\\ k_n\neq k_{n-1}\neq\dots\neq k_1\neq k_0}} \prod_{j=1}^{n}\int_{\tau_0}^{\tau} d\tau_j\; \theta_{\tau_j>\tau_{j-1}} \int\limits_0^{R(x)} dR_{n+1}\; Z(R_{n+1})^{\omega_k-2} e^{a_k(\tau-\tau_n)} \\
& \times \left[\prod_{i=1}^{n}\int\limits_{x_{i+1}}^{1} dz_i\; \mathbf{P}^{\Theta}_{k_i k_{i-1}}(z_i) \int\limits_0^{R(x_{i+1}/z_i)} dR_i\; Z(R_i)^{\omega_{k_{i-1}}-2} e^{a_{k_{i-1}}(\tau_i-\tau_{i-1})}\right] \\
& \times x_0 D_{k_0}(\tau_0,x_0),
\end{aligned}
$$

$$
R(Z_i) = (1-Z_i)^{b_{k_{i-1}}(\tau_i-\tau_{i-1})}, \quad Z(R_i) = 1-\exp\left((b_{k_{i-1}}(\tau_i-\tau_{i-1}))^{-1}\ln R_i\right),
$$

CMC algorithm of type I, full DGLAP

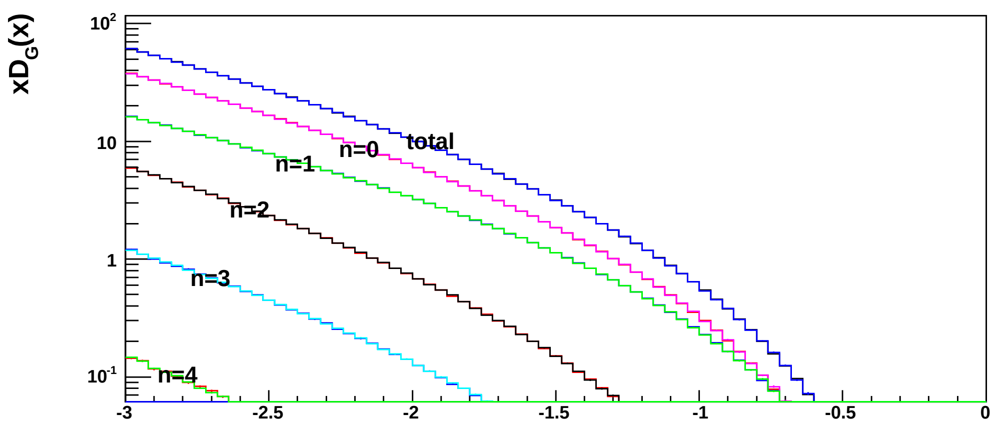
CMC algorithm description

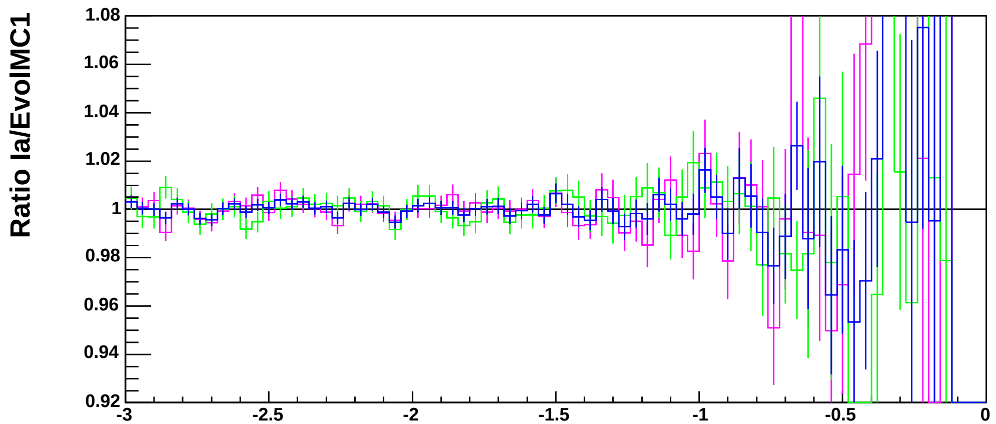
- Generate super-level variables n, k_i, τ_i Z_i and z_i using `Foam` general purpose MC tool.
- Limiting no. of flavor transition ($G \to Q$ and $Q \to G$) to $n = 0, 1, 2, 3$ is enough, for the $\sim 0.2\%$ precision.
- For each pure gluon bremsstralung segment defined by Z_i and (τ_i, τ_{i-1}), $i = 1, 2, ..., n+1$, gluon emission variable $(z_j^{(i)}, \tau_j^{(i)},\ j = 1, 2, ..., n^{(i)}$, are generated using previously described dedicated CMC.
- Weight$= 1$ events available!

Numeric tests

- In the next slides we show numerical results from such a non-Markovian CMC `EvolCMC` for “evolution” ranging from $Q = 1$GeV to $Q = 1$TeV, $x > 10^{-3}$,
- They are compare them with the results of the Markovian uncontrained evolution of our own `EvolFMC`
- `EvolFMC` was previously x-checked with `QCDnum16` and `ACHEB`
- The greement of Nonmarkovian `EvolCMC` and Markovian `EvolFMC` is excelent, $\sim 0.25\%$.

Test of non-Markovian Constrained MC, DGLAP case





$n = 0$: $G \to G$

$n = 1$: $Q \to G$ and any no. of gluon emissions out of Q and G,

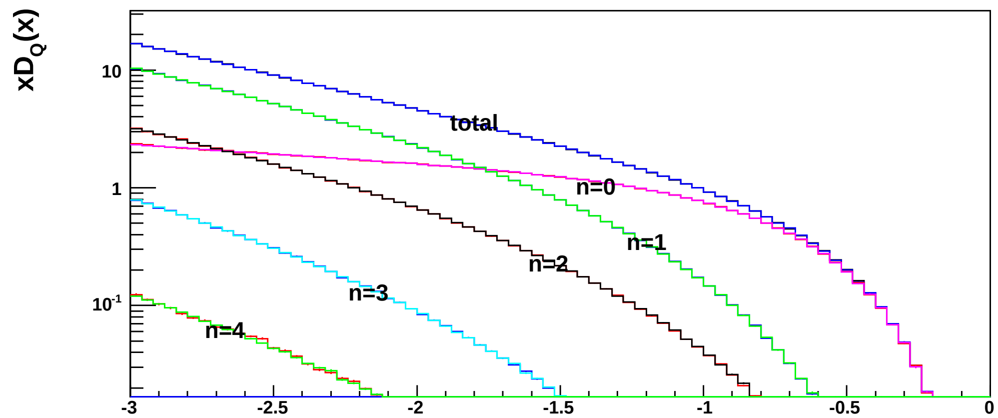
$n = 2$: $G \to Q \to G$, etc.

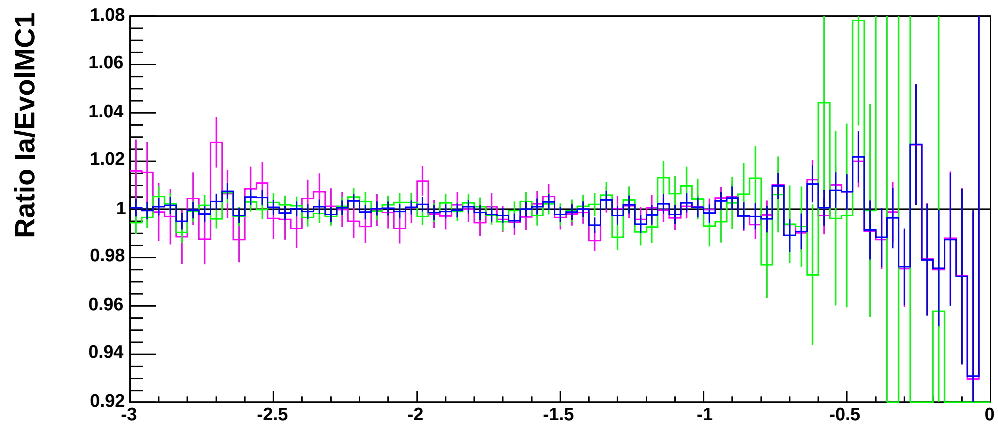
$n = 3$: $Q \to G \to Q \to G$, etc.

$n = 4$: $G \to Q \to G \to Q \to G$, etc. “Total” is the sum of $n = 0, 1, 2, 3, 4$.

Evolution from proton at $Q = 1$GeV up to 1TeV. New non-Markovian CMC (EvolCMC) agrees with unconstrained Markovian MC (EvolFMC) to within $\sim 0.25\%$! (100M)

Test of non-Markovian Constrained MC, DGLAP case





$n = 0$: $Q \to Q$
$n = 1$: $G \to Q$ and any no. of gluon emissions out of Q and G,
$n = 2$: $G \to Q \to G \to Q$, etc.
$n = 3$: $G \to Q \to G \to Q$, etc.
$n = 4$: $Q \to G \to Q \to G \to Q$, etc. “Total” is the sum of $n = 0, 1, 2, 3, 4$.
Evolution from proton at $Q = 1$GeV up to 1TeV. New non-Markovian CMC (EvolCMC) agrees with unconstrained Markovian MC (EvolFMC) to within $\sim 0.25\%$! (210M)

Other recent works on CMC algorithms and PLANS

- Alternative non-Markovian CMC algorithm class II exists, see contribution to Loops and Legs, Zinnowitz April 2004.
  So far implemented for pure bremsstrahlung only.
  Higher dimensionality of the Foam integrand in the full DGLAP case:-(
- Algorithm CMC class I (of this presentation) is already implemented/tested for the z-dependent $\alpha_S((1-z)Q)$.
  This is relevant for parton showers and modeling the CCFM evolution.
- The unintegrated parton distributions $D_k(k_T, x)$ are already calculated from the one-loop type CCFM model (Placzek& Golec) in the Markovian EvolFMC framework. This will be ported to the non-Markovian EvolCMC.
- $\overline{MS}$ NLL corrections are already implemented in the Markovian EvolFMC (W.Placzek and K.Golec) and will be ported to the non-Markovian CMC soon.
- Generally our aim are MC models/programs for unintegrated PDFs for W and Z production at LHC based on CCFM-type evolution, but keeping compatibility with the DGLAP as close as possible.
- It will take a few months to have first complete MC. (Next summer?)
- This is, of course, very close to CASCADE approach.
- Fitting $F_2(Q, x)$ of DIS with our CMCs at some point? Yes.

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