Differential distributions at NNLO in QCD

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Hera-LHC

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Outline

- Introduction
  - Why NNLO?
  - Why differential?
- Method I: Inclusive and semi-inclusive phase-space integrations
- W, Z boson rapidity distributions (C.A, Dixon, Melnikov, Petriello)
- Method II: Arbitrarily differential phase-space integrations
- Differential distributions for Higgs boson production via gluon fusion (C.A., Melnikov, Petriello)

Why NNLO?

- Precision measurements for processes with large cross-sections and clean experimental signals
  - reduced dependence on arbitrary scales
  - include a bigger+more realistic variety of kinematic configurations
- Trustworthy separation of perturbative (structure functions) from non-perturbative (pdf's) physics.
- Cross-sections with slowly convergent perturbative expansion

Why differential?

- Computing at NNLO is a challenge *(discover and automate very sophisticated methods)*
- Very good record for total cross-sections+decay rates $e^+e^- \to$ *hadrons, DIS, Drell-Yan, Higgs boson production, etc*
- Total cross-sections are idealized- unrealistic observables:
  - ignore detector+other acceptances
  - not possible to attach shower+hadronization
  - The physics output from studying differential distributions is usually superior

Technical challenges

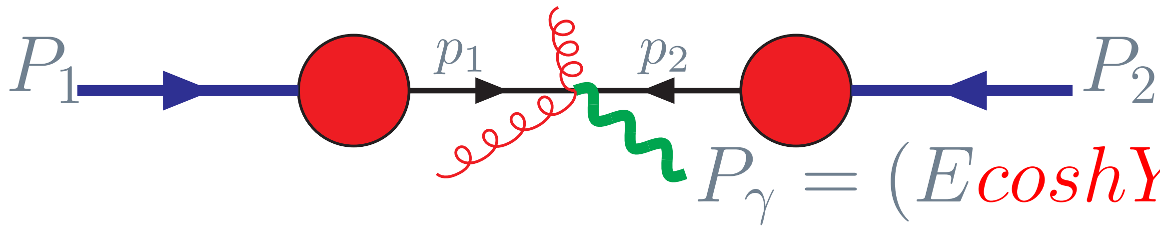
- Total cross-sections: Integrations over the phase-space are very similar to loop integrations

$$\delta(p^2 - m^2) \to \frac{1}{p^2 - m^2 + i0} - \frac{1}{p^2 - m^2 - i0}$$

  - Use loop-methods *(very well developed in the last few years)*
  - Infrared singularities pop easily out by doing “loop-integrations”.
- Phase-space integrals for differential distributions require a very different treatment
  - Infrared singularities must be extracted before the integrations
  - Evaluate the finite integrals numerically in order to permit the computation of many different observables
- It is technically a big challenge to move from inclusive cross-sections to differential distributions.

Electroweak gauge boson rapidity distr.

with Lance Dixon, Kirill Melnikov and Frank Petriello



- First differential distribution at NNLO in QCD.
- By doing a "total cross-section" calculation

$$\frac{d\sigma}{dY} \sim \int d\,(\text{Phase-Space})\,(\text{Matrix-Elements})\,\delta\left(u - \frac{2p_1 \cdot P_\gamma}{2p_2 \cdot P_\gamma}\right), \quad u = \frac{x_1 e^{2Y}}{x_2}$$

$$\delta\left(u - \frac{2p_1 \cdot P_\gamma}{2p_2 \cdot P_\gamma}\right) \to \frac{p_2 \cdot P_\gamma}{P_\gamma \cdot [p_1 - u p_2] + i0} - c.c$$

- Applicable to a multitude of "semi-inclusive" quantities

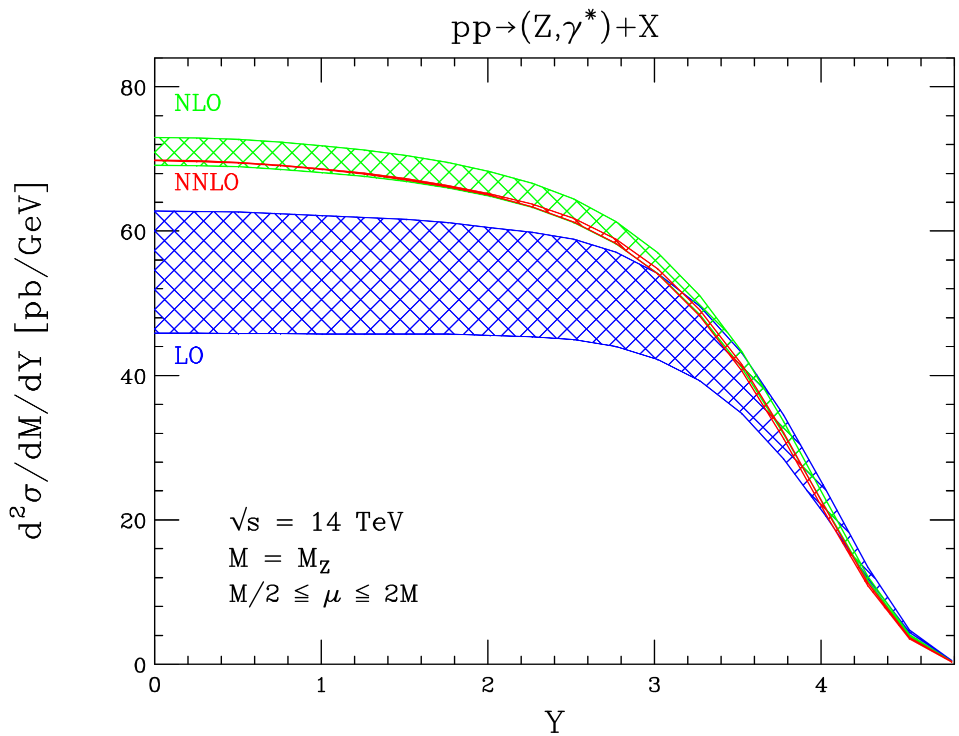
Physics information of $d\sigma/dY$

- High precision measurements at fixed-target collisions (E866), the Tevetron, and the LHC.
- Standard caddle: extract pdf's and partonic luminosities

$$\frac{d\sigma}{dY} = [\text{quark density}]_1 \left(\frac{M}{E_{cm}} e^{Y}\right) \times [\text{quark density}]_2 \left(\frac{M}{E_{cm}} e^{-Y}\right) + \mathcal{O}\left(\alpha_s\right)$$

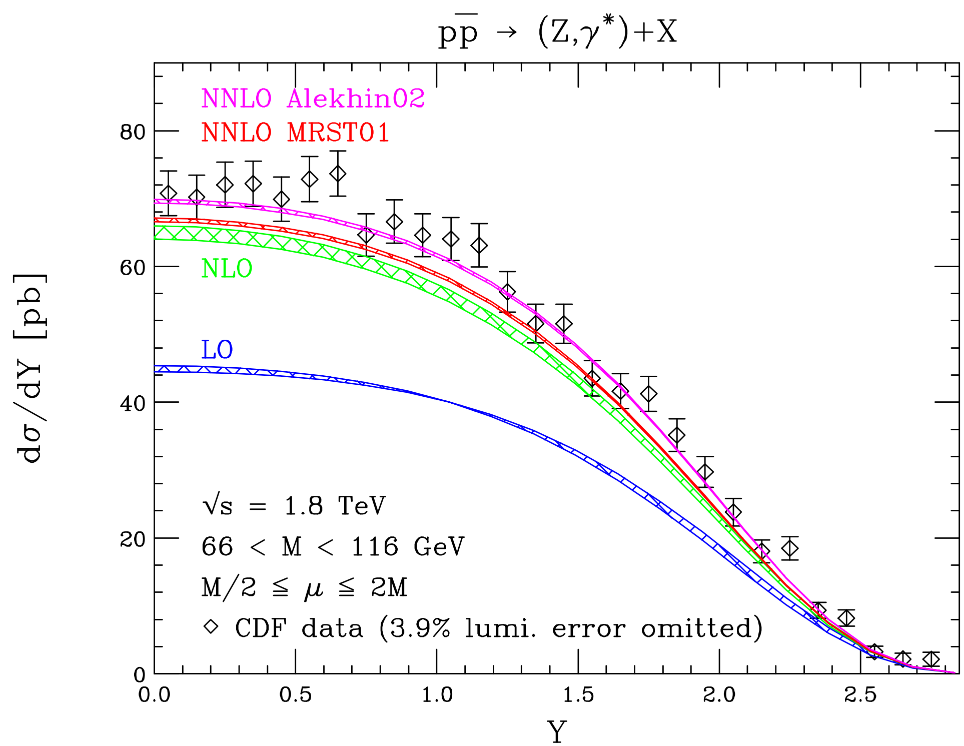
- Precision electroweak measurements at hadron colliders
  - weak mixing angle from forward-backward asymmetry
  - W-mass measurements (sensitive to pdf's)
- Determination of new gauge boson couplings to quarks, ...

On-shell Z boson at the LHC



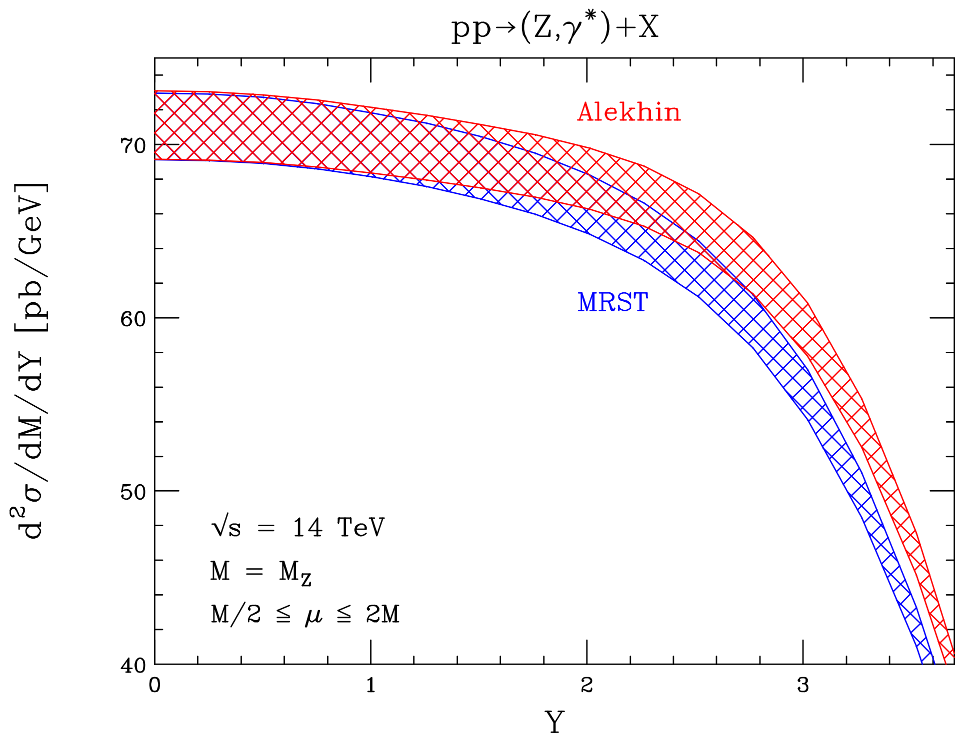
- small NNLO scale uncertainty: $(30\% - 25\%)(LO) \to (6\%)(NLO) \to 0.1\%(Y = 0) - 1\%(Y \leq 3) - 3\%(Y \simeq 4)(NNLO)$
- shape stabilizes at NNLO

On-shell Z boson at the Tevatron



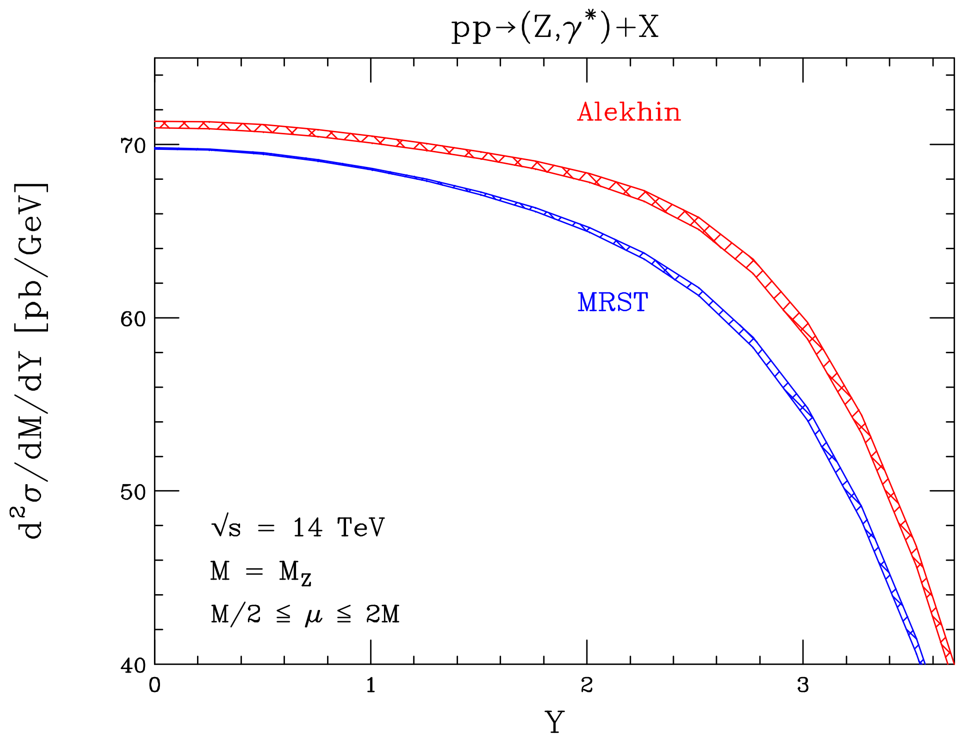
- small (2%) scale variation at LO - non-monotonic behaviour
- proper NLO variation $(2\% - 5\%)$, $\leq 1\%$ at NNLO
- $\frac{\delta\sigma_{NLO}}{\sigma_{LO}} \simeq 20\% - 45\%$, $\frac{\delta\sigma_{NNLO}}{\sigma_{LO}} \simeq 3\% - 4\%$

MRST versus Alekhin



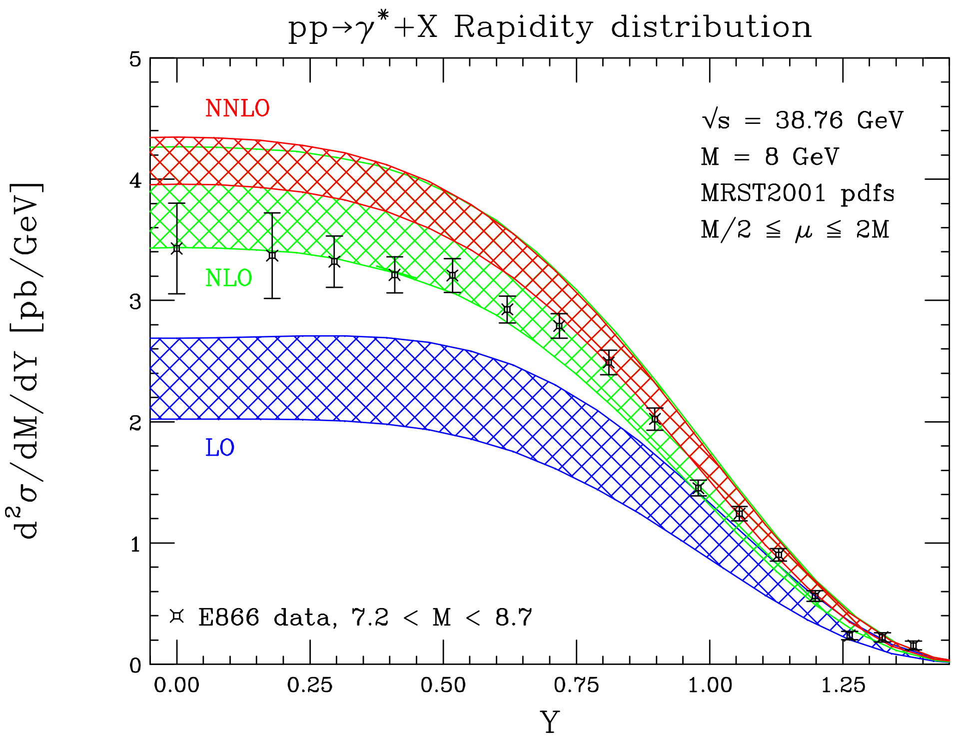
- Alekhin set: DIS data only - NNLO consistent
- MRST set: Global analysis
- Indistinguishable at NLO

MRST versus Alekhin



- Alekhin set: DIS data only - NNLO consistent
- MRST set: Global analysis
- Indistinguishable at NLO
- NNLO can resolve the discrepancies

Low energy DY production (E866)



- NNLO distribution sharper in central rapidity regions.
- Data lower than NNLO $\rightarrow$ smaller $\bar{q}$ densities

Fixed order partonic Monte-Carlos

with Kirill Melnikov and Frank Petriello

- A cross-section is:

$$\sigma = \sum_n \int d\left(\text{Phase-Space}_n\right) \left(\text{Matrix-Elements}\right)_n$$
$$\times Observable\left(\text{PhaseSpace vars}\right)$$

  - Obs, an arbitrarily complicated function to describe the experimentally measured configurations of the phase-space $\rightarrow$ NUMERICAL INTEGRATION
  - Divergent Matrix-Elements $\rightarrow D = 4 - 2\epsilon$

- TASK: Expose $1/\epsilon$ poles of individual terms; cancel them against each other; calculate the finite remainder numerically (Monte-Carlo integration).

Lessons from NLO

- Amplitudes factorize in terms of UNIVERSAL terms (dipoles) in singular limits (soft, collinear).
- Use factorization properties to construct finite integrands

$$\sigma = \int dPS_{n+1} \left[(\text{Matrix Elements })_{n+1} - (\text{Born })_n \; Dipole \right] + \int dPS_n \left[\int Dipole \; (\text{Born}) + (\text{ Matrix-elements})_n \right]$$

- A lot of effort is going into formulating a dipole-like approach at NNLO: (Gehrmann, Gehrmann de-Rider, Glover, Heinrich, Kilgore, Kosower, Weinzierl)
- First partial results for $e^+e^- \to 3$jets at NNLO, (Gehrmann, Gehrmann de-Rider, Glover)

Computing without factorization

- A dipole approach at NNLO is aesthetically appealing
- When completed, it will make use of a deeper understanding of quantum field theory $\longrightarrow$ physicist friendly approach.

However:

- “Dipoles” are not simple to find or integrate
- Mathematical ambition: We must be able to compute the required phase-space integrals anyway
- How about a computer-friendly approach?

The method of expansions in plus-distributions

- Change variables

$$\sigma = \int_0^1 d\lambda_1 d\lambda_2 \ldots \left| \frac{\partial(\text{Phase-Space})}{\partial(\lambda_1, \lambda_2, \ldots)} \right| (\text{Matrix Elements})\ (\text{Observ.})$$

- Expand in ϵ, using plus distributions, the combination:

$$\mathcal{I} = (\text{Jacobian}) \times (\text{Matrix-Elements})$$

- Compute terms in the expansion numerically for the $Obs.$ at hand.

$$\int_0^1 d\lambda \left[\frac{1}{\lambda}\right]_+ \times Obs. = \int_0^1 d\lambda \frac{Obs.(\lambda) - Obs(0)}{\lambda}$$

Expanding singular terms in ϵ

- For factorized singularities:

$$\mathcal{I} = \lambda_1^{-1+\epsilon} f(\lambda_1, \lambda_2, \ldots)$$

- substitute

$$\lambda^{-1+\epsilon} = \frac{\delta(\lambda)}{\epsilon} + \left[\frac{1}{\lambda}\right]_+ + \epsilon \left[\frac{\ln \lambda}{\lambda}\right]_+ + \frac{\epsilon^2}{2!}\left[\frac{\ln^2 \lambda}{\lambda}\right]_+ + \ldots$$

- At NNLO we find more complicated singularities:

  overlapping (Binoth, Henrich; Hepp; Denner, Roth), pseudothresholds (C.A, Melnikov, Petriello)

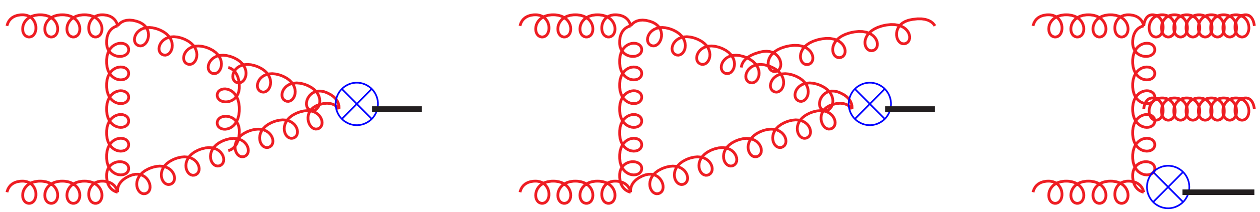
- They factorize by splitting the integration region:

$$\int_0^1 dx dy (x+y)^{-2+\epsilon} = \int_0^1 dx \int_0^x dy (x+y)^{-2+\epsilon} + \int_0^1 dy \int_0^y dx (x+y)^{-2+\epsilon}$$

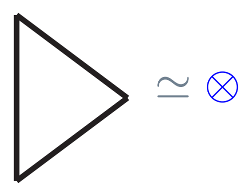
Applications

Partonic NNLO Monte-Carlos for:

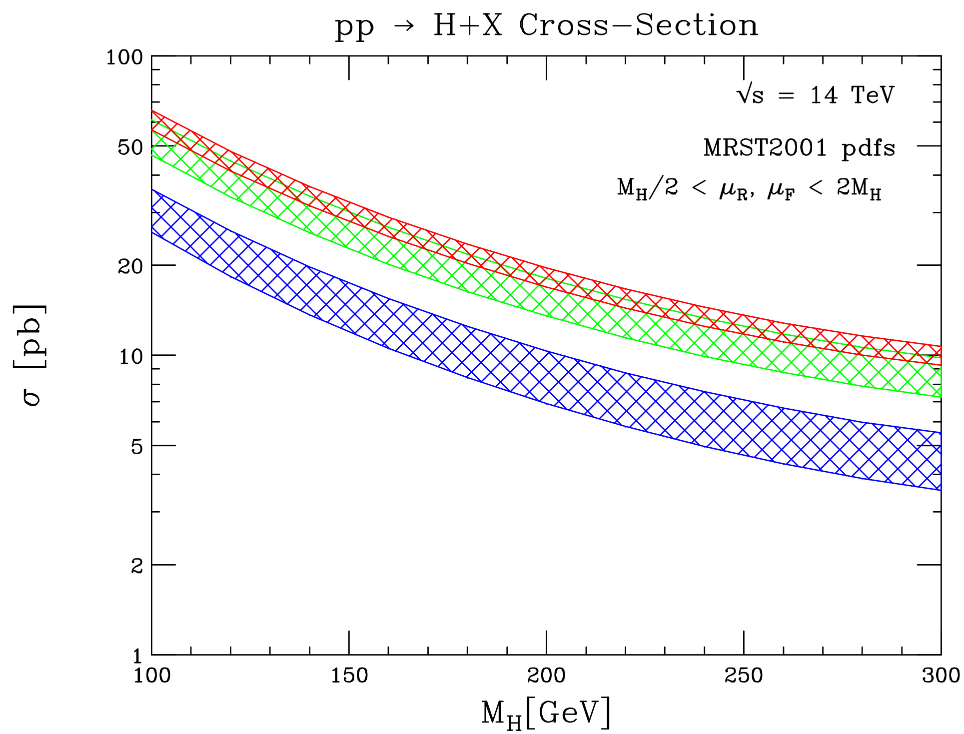
- $e^+e^- \rightarrow 2$jets, *hep-ph/0311311, hep-ph/0402280*
- Higgs boson production via gluon fusion, *hep-ph/0409088*



- in the limit $m_h << 2m_{top}$:

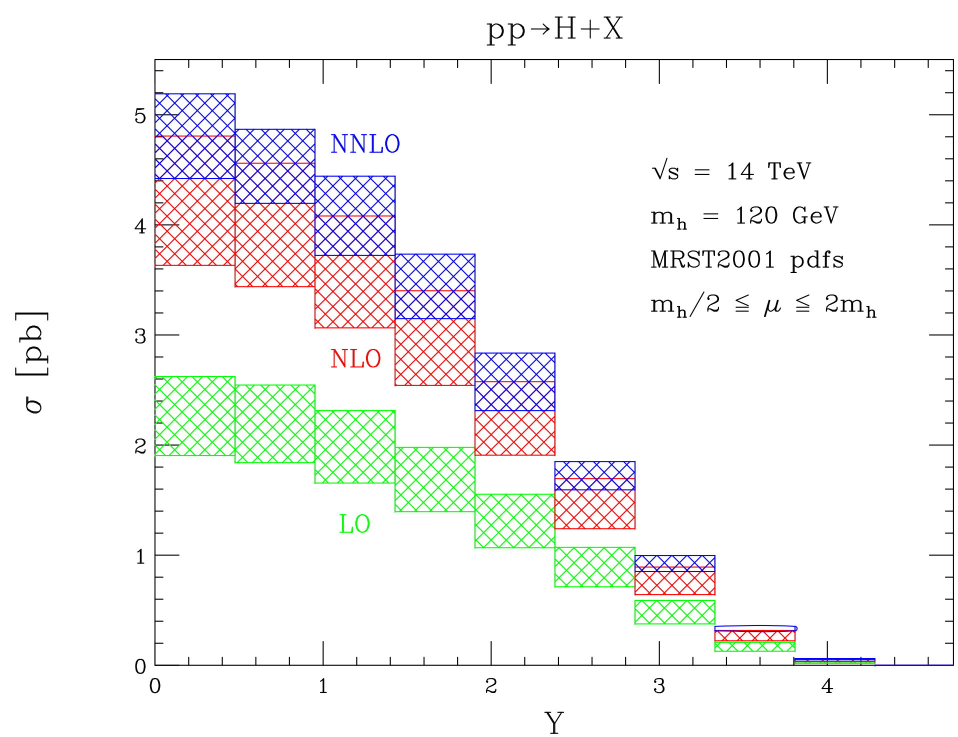


Total Cross-section



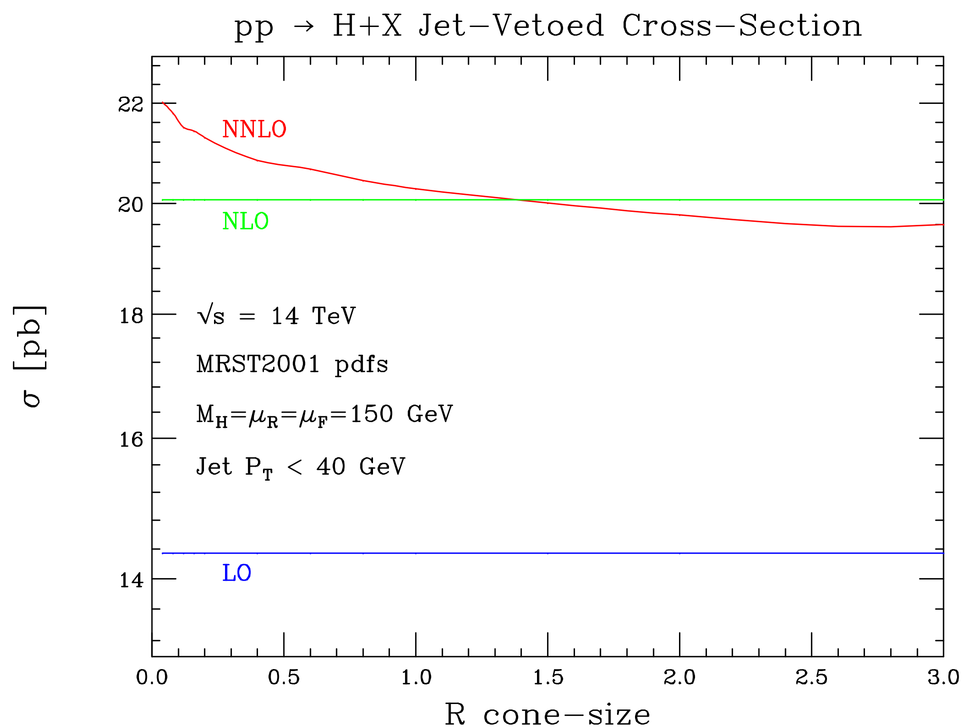
- Reproduce the total cross-section, *Harlander, Kilgore; C.A., Melnikov; Ravindran, Smith, van Neerven*.
- Remember the large NLO (70%) and NNLO (30%) corrections, and large scale variation ($\sim 15 - 20\%$ at NNLO)
- MC evaluation: 30min for the Tevatron and 90min for the LHC on a 2.4GHz desktop.
- Relatively smooth numerical integration

Higgs rapidity distribution



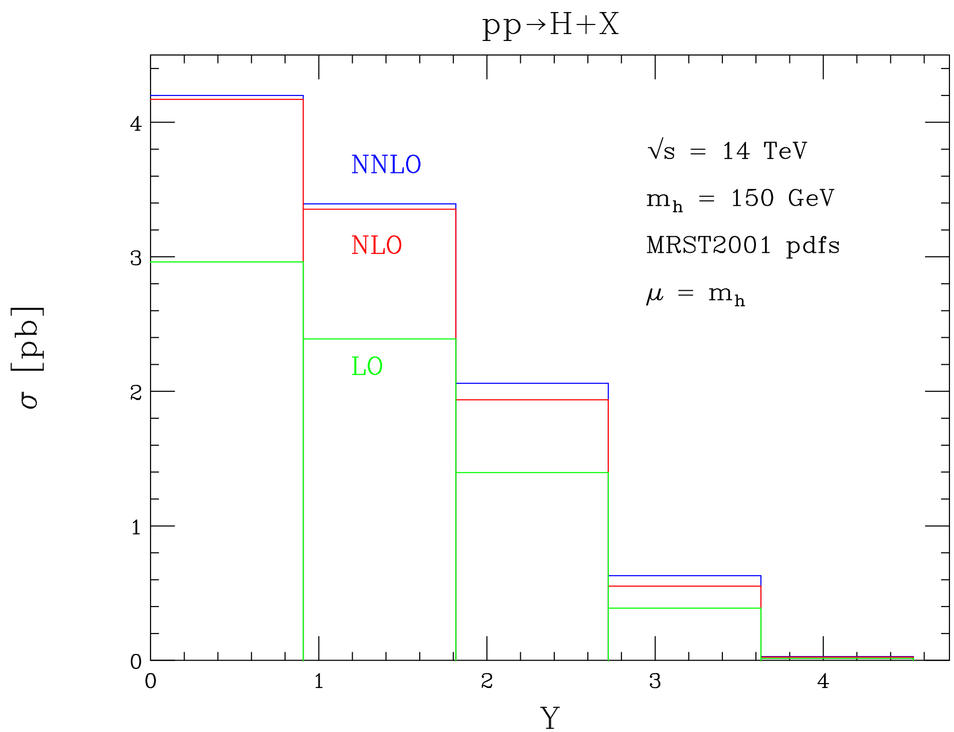
- bin-integrated rapidity distribution (MC statistical error 1%)
- Similar scale variations to the total cross-section; large K-factors.
- Small rapidity dependence of the K-factors

Veto on high-P_T jets



- We veto on events with jets $p_t^{jet} > 40GeV$
- $R_{ij} = \sqrt{\Delta\phi_{ij}^2 + \Delta\eta_{ij}^2}$: Two partons form a jet if $R_{ij} < R$ *(e.g. Catani et al., hep-ph/011164)*
- LO and NLO insensitive to the clustering algorithm. NNLO R-variation is 13%

Rapidity distribution with a jet-veto



- Cut affects more severely the NNLO than the NLO cross-section.
- NLO: $< P_t^H >\sim 38\,\mathrm{GeV}$
- NNLO: $< P_t^H >\sim 45\,\mathrm{GeV}$

Work in progress

- We are studying distributions for the decay of the Higgs into photons (taking faithfully into account isolation and other experimental cuts).
- Other final states (W^+W^-, ZZ) in the list “to do”
- Public code and more results in a forthcoming publication
- Fully differential NNLO MC for Drell-Yan lepton-pairs, pseudoscalar Higgs, etc, require straightforward insertions of the appropriate matrix-elements into our code (plus distribution implementation)

Conclusions-Outlook

- We now have general methods for NNLO
  - inclusive differential distributions
  - arbitrarily differential distributions
- Drell-Yan and Higgs boson production are the first applications
- Very important input for high precision studies of basic observables at the LHC
- Cleaner extraction of pdfs - precise LHC luminometer
- New physics searches: Confident comparisons with precision electroweak data at hadron colliders
- MC@NNLO?