Testing the NonCommutative Standard Model at Colliders

Thorsten Ohl
— Würzburg University —
⟨ohl@physik.uni-wuerzburg.de⟩

in collaboration with: Jürgen Reuter (PC), Ana Alboteanu & Reinhold Rückl (HC)

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Quantum mechanics: measurements of position and momentum complementary

$$\Delta x_i \cdot \Delta p_j \geqslant \frac{\hbar}{2}\delta_{ij}$$

i. e. the corresponding operators do not commute

$$[x_i, p_j] = x_i p_j - p_j x_i = i\hbar\delta_{ij}$$

Currently no experimental evidence for complementarity of position measurements:

$$[x_\mu, x_\nu] \stackrel{?}{=} 0$$

However

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu} = i\frac{C_{\mu\nu}}{\Lambda_{\text{NC}}{}^2}$$

possible, as long as the characteristic energy scale Λ_{NC} large enough, i. e. the characteristic length scale

$$l_{\text{NC}} = \frac{1}{\Lambda_{\text{NC}}}$$

small enough compared to the characteristic scales of present experiments.

- Fundamental length scale
  - x_μ-continuum $\Rightarrow$ lattice of eigenvalues of the operators $\hat{x}_\mu$ [Snyder, Wess]
  - smooth cut-off for $E > \Lambda_{NC}$
  - $\because$ internal and space-time symmetries no longer commuting
  - $\therefore$ richer symmetry structures
- String theory
  - NCQFT is low energy limit of certain string theories [Seiberg/Witten]
  - more than 1600 citations since the August of 1999 ...
  - ☹ no prediction for the value of Λ_{NC}

☺ Why not? *Schön ist, Mutter Natur, deiner Erfindung Pracht* and everything

- not excluded by experiment,
- mathematically consistent and elegant, as well as
- observable at the next generation of experiments

should be studied: *Ist ein großer Gedanke, Ist des Schweißes der Edlen wert!*

special case and useful approximation: $\theta^{\mu\nu}$ constant 4×4-matrix:

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu} = i\frac{1}{\Lambda_{NC}^2}C^{\mu\nu} = i\frac{1}{\Lambda_{NC}^2}\begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -B^3 & B^2 \\ E^2 & B^3 & 0 & -B^1 \\ E^3 & -B^2 & B^1 & 0 \end{pmatrix}$$

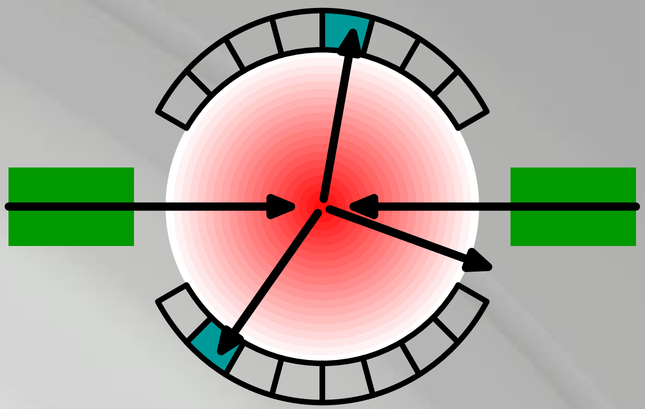
NB: "electric" and "magnetic" components $\vec{E}$ (i. e. θ^{0i}) und $\vec{B}$ (i. e. θ^{ij}) play very different rôles (theoretically as well as phenomenologically)

"Fundamentalistic approach":

- construct observables as functions of the operators $\hat{x}_\mu$
- develop scattering theory on noncommutative spaces

☹ too complicated (for me)

Collider prepares an initial state $|\text{in}\rangle$, the interaction S under study transforms it and a detector measures the overlap of the resulting state with a final state $|\text{out}\rangle$.



$\therefore$ particle physics experiments study spacial coordinates x_μ oder $\hat{x}_\mu$ not directly, but functions of these coordinates instead: states and fields

$\therefore$ results of measurements codified in effective lagrangians as products of functions:

$$\mathcal{L}_{\text{eff.}}(x) = \cdots + g_2 \bar{\psi}(x)\gamma_\mu(1-\gamma_5)\psi'(x)W^\mu(x)$$
$$+ g_3 \sum_{a,b,c} f_{abc} \frac{\partial A^a_\nu}{\partial x_\mu}(x) A^{b,\mu}(x) A^{c,\nu}(x) + \cdots$$

- simpler, but equivalent realization of NCQFT: replace pointwise product of functions of noncommuting variables

$$(fg)(\hat{x}) = f(\hat{x})g(\hat{x})$$

by Moyal-Weyl $*$-products of functions of commuting variables:

$$(f*g)(x) = f(x)e^{\frac{i}{2}\overleftarrow{\partial^\mu}\theta_{\mu\nu}\overrightarrow{\partial^\nu}}g(x) = f(x)g(x) + \frac{i}{2}\theta_{\mu\nu}\frac{\partial f(x)}{\partial x_\mu}\frac{\partial g(x)}{\partial x_\nu} + \mathcal{O}(\theta^2)$$

Then

$$(x_\mu * x_\nu)(x) = x_\mu x_\nu + \frac{i}{2}\theta_{\mu\nu}$$

und in particular

$$[x_\mu \overset{*}{,} x_\nu](x) = (x_\mu * x_\nu)(x) - (x_\nu * x_\mu)(x) = i\theta_{\mu\nu}$$

NB: higher orders in $\theta_{\mu\nu}$ required to make the $*$-product associative:

$$(f*g)*h = f*(g*h)$$

Gauge principle: the workhorse of theoretical particle physics for ≈35 years

$$\text{matter fields}: \psi \to \psi' = e^{ig\chi}\psi$$

$$\text{gauge fields}: A_\mu \to A'_\mu = e^{ig\chi} A_\mu e^{-ig\chi} + \frac{i}{g} e^{ig\chi} \left(\partial_\mu e^{-ig\chi}\right)$$

with covariant derivative and field strength

$$D_\mu = \partial_\mu - igA_\mu \to D'_\mu = e^{ig\chi} D_\mu e^{-ig\chi}$$

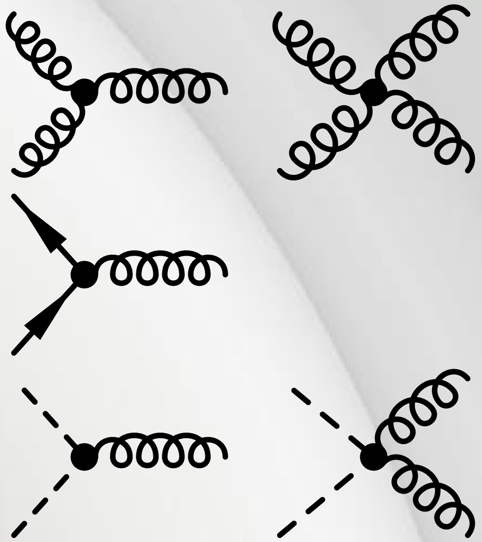
$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \to F'_{\mu\nu} = e^{ig\chi} F_{\mu\nu} e^{-ig\chi}$$

finite set of building blocks for interactions

$$\mathcal{L} = -\frac{1}{8}\operatorname{tr}\left(F_{\mu\nu}F^{\mu\nu}\right)$$

$$+\bar\psi(\not{D} - m)\psi$$

$$+(D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|)$$



apparent noncommutative generalization:

$$\psi \to \psi' = e^{ig\chi *}\psi = \psi + ig\chi*\psi + \frac{(ig)^2}{2!}\chi*\chi*\psi + \mathcal{O}(\chi^3)$$

$$A_\mu \to A'_\mu = e^{ig\chi *}A_\mu e^{-ig\chi *} + \frac{i}{g}e^{ig\chi *}\left(\partial_\mu e^{-ig\chi *}\right)$$

$$= A_\mu + ig[\chi \overset{*}{,} A_\mu] + \frac{(ig)^2}{2!}[\chi \overset{*}{,} [\chi \overset{*}{,} A_\mu]] + \partial_\mu\chi + ig[\chi \overset{*}{,} \partial_\mu\chi] + \mathcal{O}(\chi^3)$$

wipes out the differences between abelian and non-abelian gauge theories:

$\therefore$ $A'_\mu \neq A_\mu + \partial_\mu\chi$ even if $[\chi, A_\mu] = 0$, because $[\chi \overset{*}{,} A_\mu] \neq 0$

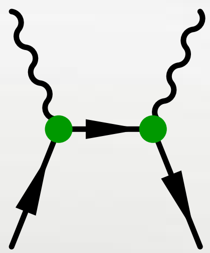
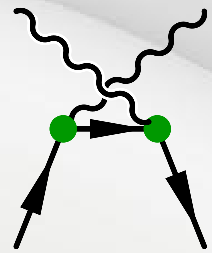
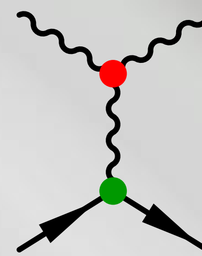
$\therefore$ $F_{\mu\nu} \neq \partial_\mu A_\nu - \partial_\nu A_\mu$ even if $[A_\mu, A_\nu] = 0$, because $[A_\mu \overset{*}{,} A_\nu] \neq 0$

☺ gold plated signature:

self couplings of neutral gauge bosons γ and Z allowed on tree-level!

$\therefore$ commutative gauge theories: form and strength of the couplings among gauge bosons determined completely by couplings of gauge bosons to matter!

- a single coupling for each non-abelian gauge theory
- also in noncommutative generalizations of QED:

$g_M^2 \cdot$  $+ g_M^2 \cdot$  $+ g_M g_{TGC} \cdot$  $\stackrel{!}{=} 0 \Rightarrow \boxed{g_M = g_{TGC}}$

- incompatible with the hypercharge quantum numbers in the standard model:

$$Y(L_e, e_R, \nu_{e,R}, L_{u,d}, u_R, d_R) = (-1, -2, 0, 1/3, 4/3, -2/3)$$

- furthermore: $SU(N)$ can not be realised, since only $U(N)$ closes:

$$[A_\mu \stackrel{*}{,} A_\nu]_- = [A_\mu^a T^a \stackrel{*}{,} A_\nu^b T^b]_- = \frac{1}{2}[A_\mu^a \stackrel{*}{,} A_\nu^b]_+ [T^a, T^b]_- + \frac{1}{2}[A_\mu^a \stackrel{*}{,} A_\nu^b]_- [T^a, T^b]_+$$

- solution: spontaneous symmetry breaking $U(N) \to SU(N) \times U(1)$ und hypercharges from mixing [Sheikh-Jabbari et al., 2000]

Express noncommutative entities as functions of commutative entities

$$\hat{A}_\mu(x) = \hat{A}_\mu(A_{\nu_1}(x), \partial_{\nu_1}A_{\nu_2}(x), \partial_{\nu_1}\partial_{\nu_2}A_{\nu_3}(x), \ldots, \theta)$$
$$\hat{\chi}(x) = \hat{\chi}(\chi(x), \partial_{\nu_1}\chi(x), \ldots, A_{\nu_1}(x), \partial_{\nu_1}A_{\nu_2}(x), \ldots, \theta)$$
$$\hat{\psi}(x) = \hat{\psi}(\psi(x), \partial_{\nu_1}\psi(x), \ldots, A_{\nu_1}(x), \partial_{\nu_1}A_{\nu_2}(x), \ldots, \theta)$$

realize noncommutative gauge transformations through commutative gauge transformations:

$$\hat{A}(A,\theta) \to \hat{A}'(A,\theta) = e^{ig\hat{\chi}*}\hat{A}_\mu(A,\theta)e^{-ig\hat{\chi}*} + \frac{i}{g}e^{ig\hat{\chi}*}\left(\partial_\mu e^{-ig\hat{\chi}*}\right) \stackrel{!}{=} \hat{A}(A',\theta)$$

$$\hat{\psi}(\psi,A,\theta) \to \hat{\psi}'(\psi,A,\theta) = e^{ig\hat{\chi}*}\hat{\psi} \stackrel{!}{=} \hat{\psi}(\psi',A',\theta)$$

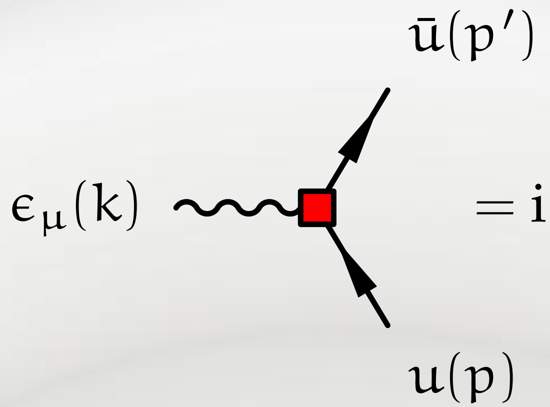
Solution (not unique) as power series in θ:

$$\hat{A}_\mu(x) = A_\mu(x) + \frac{1}{4}\theta^{\rho\sigma}\left[A_\sigma(x), \partial_\rho A_\mu(x) + F_{\rho\mu}(x)\right]_+ \quad + \mathcal{O}((\theta^{\mu\nu})^2)$$
$$\hat{\psi}(x) = \psi(x) + \frac{1}{2}\theta^{\rho\sigma}A_\sigma(x)\partial_\rho\psi(x) + \frac{i}{8}\theta^{\rho\sigma}\left[A_\rho(x), A_\sigma(x)\right]_-\psi(x) \quad + \mathcal{O}((\theta^{\mu\nu})^2)$$
$$\hat{\chi}(x) = \chi(x) + \frac{1}{4}\theta^{\rho\sigma}\left[A_\sigma(x), \partial_\rho\chi(x)\right]_+ \quad + \mathcal{O}((\theta^{\mu\nu})^2)$$

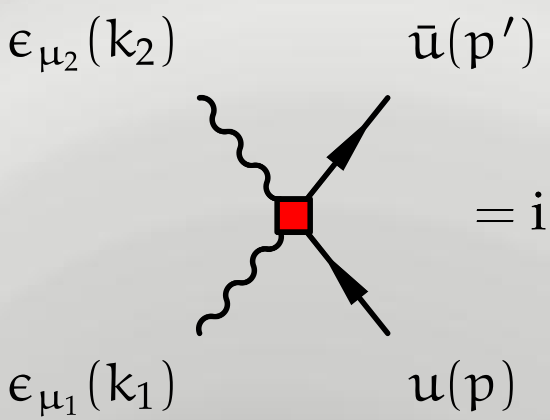
New interactions among gauge and matter fields from the expansion

$$g\bar{\hat{\psi}}(x)\hat{\not{A}}(x)\hat{\psi}(x) = g\bar{\psi}(x)\not{A}(x)\psi(x) + \mathcal{O}(\theta^{\mu\nu})$$

i. e.



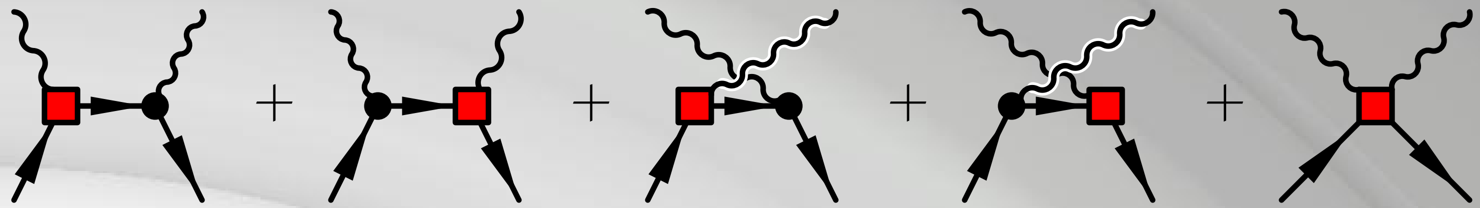
$$= ig \cdot \frac{i}{2}\left[(k\theta)_\mu \not{p} + (\theta p)_\mu \not{k} - (k\theta p)\gamma_\mu\right]$$



$$= ig^2 \cdot \frac{i}{2}\left[(\theta(k_1 - k_2))_{\mu_1}\gamma_{\mu_2} - (\theta(k_1 - k_2))_{\mu_2}\gamma_{\mu_1} - \theta_{\mu_1\mu_2}(\not{k}_1 - \not{k}_2)\right]$$

(all momenta outgoing)

☺ Ward identity already satisfied by



three gauge boson vertices not required!

☹ no prediction for three gauge vertices

- TODO:
  - ∵ Seiberg-Witten maps are not constructed from commutators alone
  - ☹ leaves Lie algebra and enters enveloping associative algebra: in general infinite dimensional!
  - ☺ *Wo aber Gefahr ist, wächst / Das Rettende auch.*: Seiberg-Witten maps not unique: freedom sufficient for eliminating the unwanted degrees of freedom

In the enveloping algebra, the trace

$$S_{\text{gauge}} = -\frac{1}{8}\int d^4x \; \mathrm{tr}\left(\frac{1}{G^2} F_{\mu\nu} * F^{\mu\nu}\right)$$

depends on the representation ($1/G^2$ commutes with all of $SU(3)_C \times SU(2)_L \times U(1)_Y$)

- ∴ coupling constant for three gauge boson vertices not unique

e. g. trace in der sum of all representations appearing in the standard model:

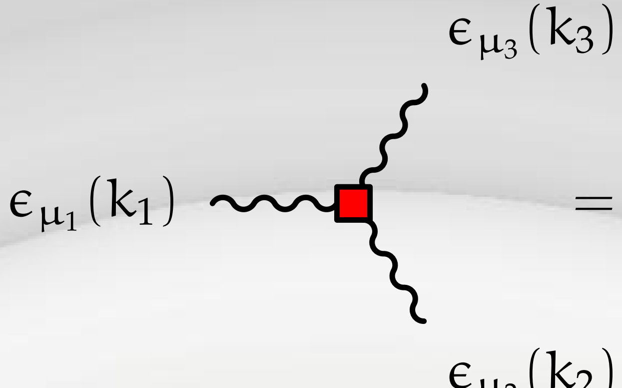
- ∴ constraints for the eigenvalues $1/g_i^2$ of $1/G^2$ in the representations $(i = 1, 2, \ldots, 6)$

  1. sum rules from matching to standard model

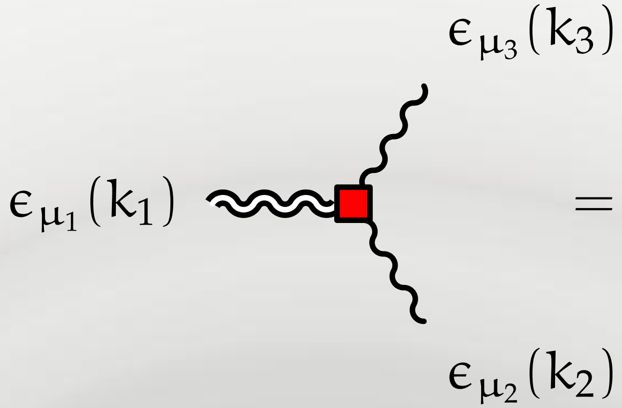
$$\frac{1}{g_s^2} = \frac{1}{g_3^2} + \frac{1}{g_4^2} + \frac{2}{g_5^2}, \; \frac{1}{g^2} = \frac{1}{g_2^2} + \frac{3}{g_5^2} + \frac{2}{g_6^2}, \; \frac{1}{g'^2} = \cdots$$

  2. positivity

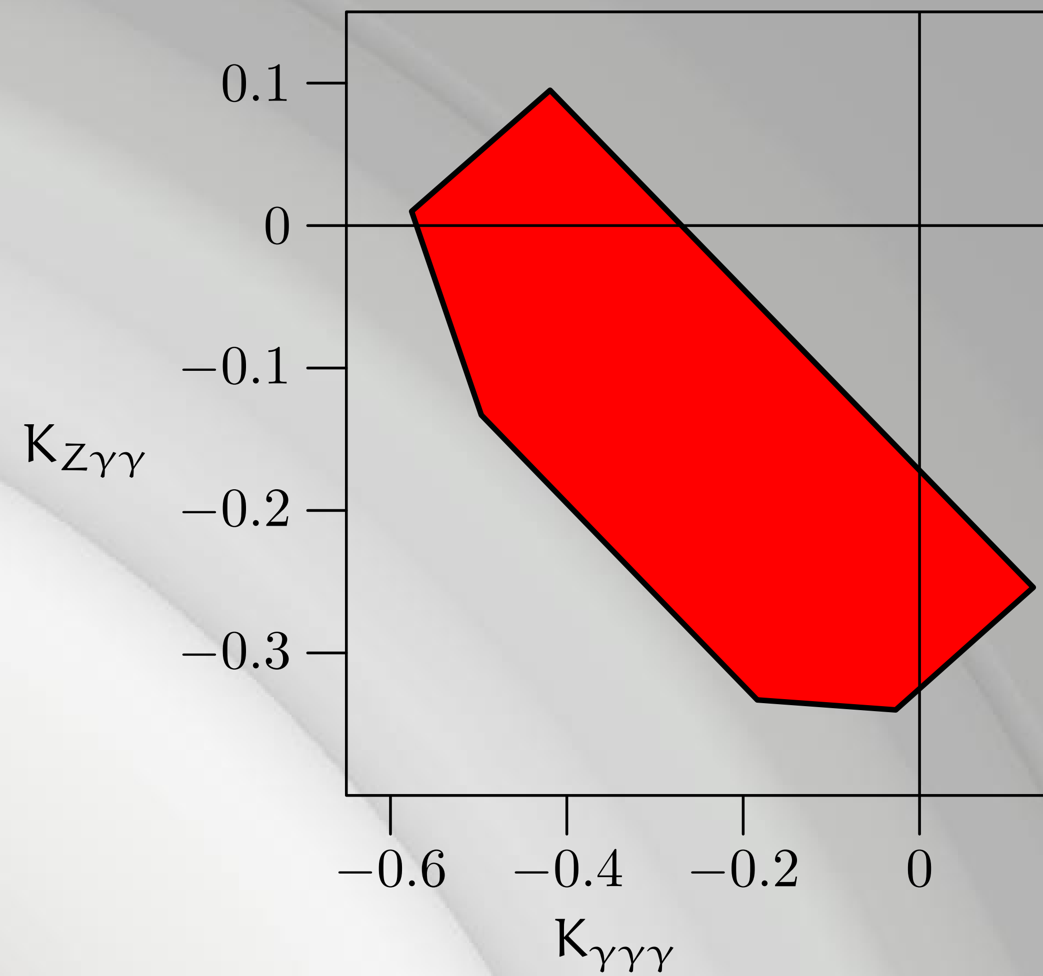
$$\frac{1}{g_i^2} \geqslant 0$$

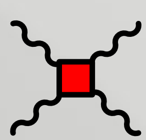
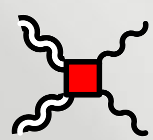


$= iK_{\gamma\gamma\gamma} \cdot \ldots$



$= iK_{Z\gamma\gamma} \cdot \ldots$



NB: quartic vertices (e. g.  , ) are $\mathcal{O}((\theta^{\mu\nu})^2)$!

$$|A|^2 = |A^{\text{SM}}|^2 + (A^{\text{SM}})^* A_1^{\text{NC}} + (A_1^{\text{NC}})^* A^{\text{SM}} + |A_1^{\text{NC}}|^2 + (A^{\text{SM}})^* A_2^{\text{NC}} + (A_2^{\text{NC}})^* A^{\text{SM}} + \mathcal{O}((\theta^{\mu\nu})^3)$$

☹ $\mathcal{O}((\theta^{\mu\nu})^2)$ Lagrangian of the NCSM not yet known

$\therefore$ study $\mathcal{O}(\theta^{\mu\nu})$-interference

- imaginary contributions to A:

  1. width of unstable particles

$$\frac{1}{p^2 - m_Z^2 + i m_Z \Gamma_Z}$$

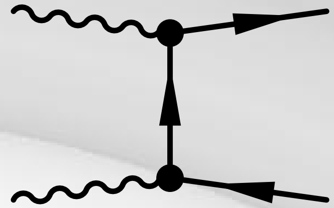
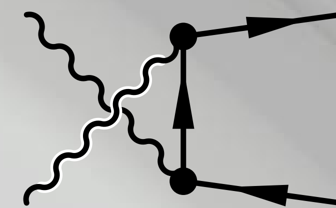
  2. $\mathrm{tr}(\cdots\gamma_5\cdots) \to i\epsilon_{\mu\nu\rho\sigma}\cdots$ requires more than 4 independent four vectors:

  $\therefore$ $2 \to 3$-processes or polarization

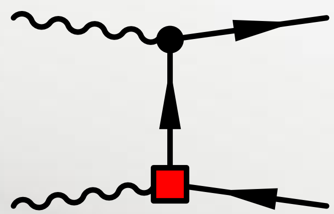
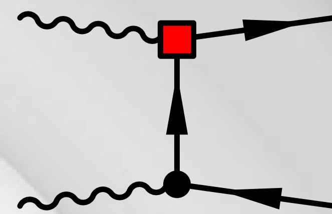
☺ $\gamma\gamma \to f\bar{f}$ requires polarization anyway:

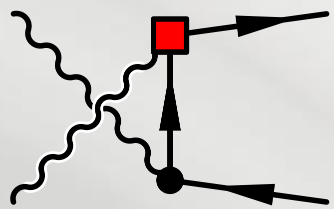
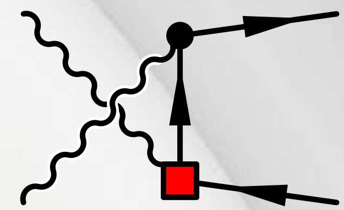
beam spectra from Compton backscattering peaking at high energies

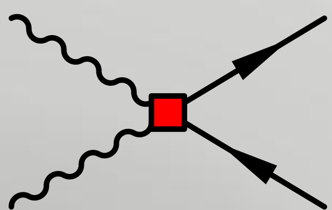
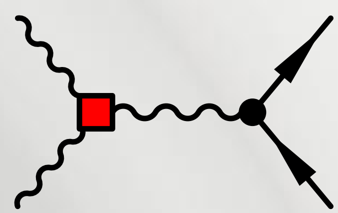
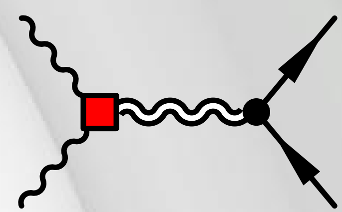
In the standard model one diagram in the t- and u-channel for $\gamma(k_1)\gamma(k_2) \to f(p_1)\bar{f}(p_2)$:

$A_t^{\text{SM}} =$  , $A_u^{\text{SM}} =$  .

NCSM:

$A_{t,1}^{\text{NC}} =$  , $A_{t,2}^{\text{NC}} =$  ,

$A_{u,1}^{\text{NC}} =$  , $A_{u,2}^{\text{NC}} =$  .

$A_c^{\text{NC}} =$  , $A_{s,\gamma}^{\text{NC}} =$  , $A_{s,Z}^{\text{NC}} =$  .

[ΘΩ, Reuter, arXiv:hep-ph/0406098, Phys. Rev. D]

Representation of the noncommutativity θ as a rank two Weyl-van der Waerden-spinor ϕ_{AB}

$$\theta_{A\dot{A},B\dot{B}} = \theta^{\mu\nu}\bar{\sigma}_{\mu,A\dot{A}}\bar{\sigma}_{\nu,B\dot{B}} = \phi_{AB}\epsilon_{\dot{A}\dot{B}} + \bar{\phi}_{\dot{A}\dot{B}}\epsilon_{AB}$$

with $(\phi_{AB})^* = \bar{\phi}_{\dot{A}\dot{B}}$ and

$$\phi_{11} = -E_- - iB_-,\ \phi_{12} = E_3 + iB_3 = \phi_{21},\ \phi_{22} = E_+ + iB_+$$

with $E_\pm = E^1 \pm iE^2$, $B_\pm = B^1 \pm iB^2$.

All contractions can be expressed as spinor products

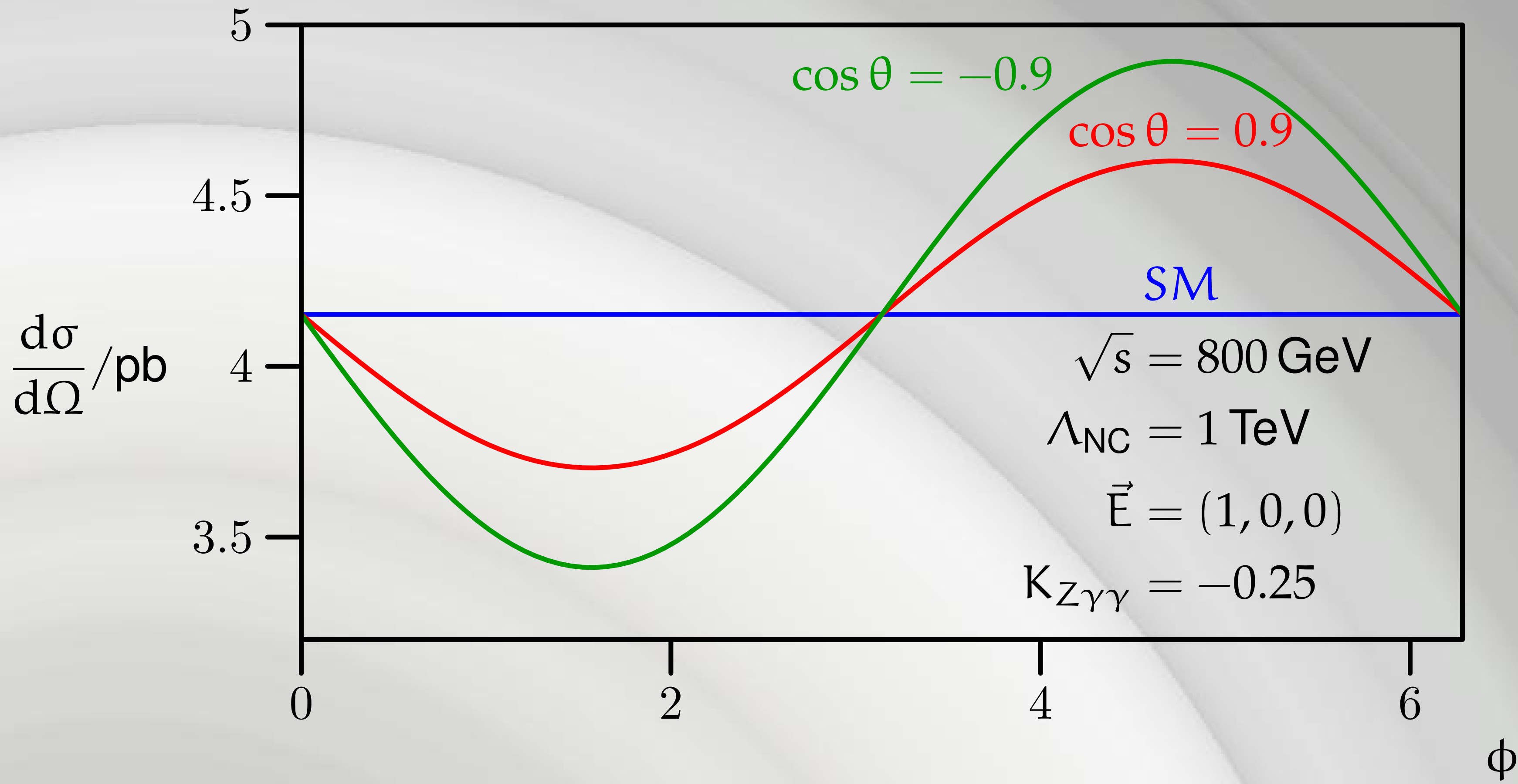
$$(V_1\theta V_2) = \frac{1}{2}\text{Re}\left[\langle v_1 v_2\rangle^* \langle v_1 \phi v_2\rangle\right]$$

with $\langle p\phi q\rangle = \phi_{11}p_2q_2 + \phi_{22}p_1q_1 - \phi_{12}(p_1q_2 + p_2q_1)$.

$$A_{u,1}^{(+,-)} = \frac{-e^2Q_f^2}{\sqrt{2}u}\frac{\langle k_1p_2\rangle\langle p_1k_2\rangle^*}{\langle p_2k_1\rangle^*}\left[(\epsilon_2\theta p_1)\langle k_2p_1\rangle\langle p_1p_2\rangle^* + \sqrt{2}(k_2\theta p_1)\langle k_2p_2\rangle^*\right]$$

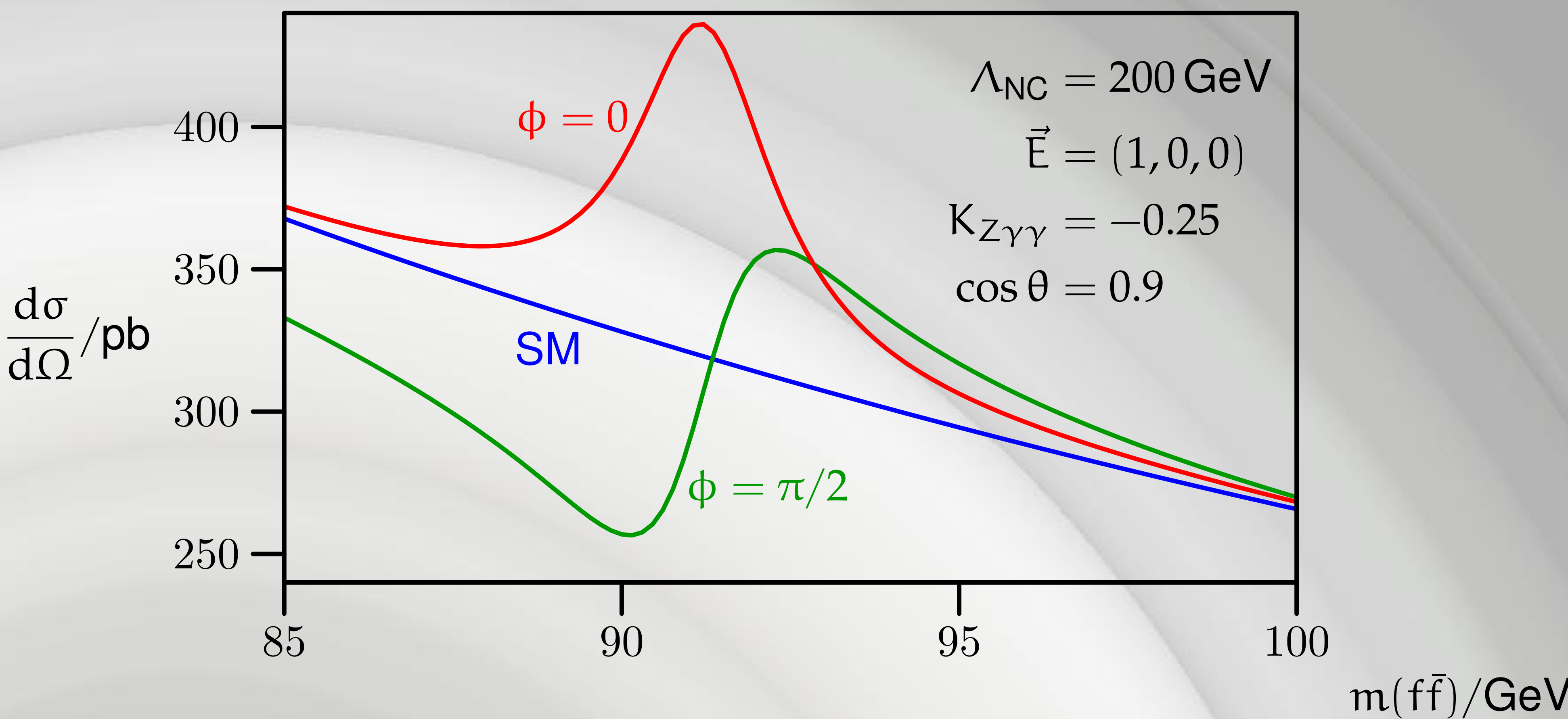
$$A_{u,1}^{(-,+)} = 0, \qquad \text{etc.}$$

Polarized differential cross section depends on the azimuthal angle ϕ:

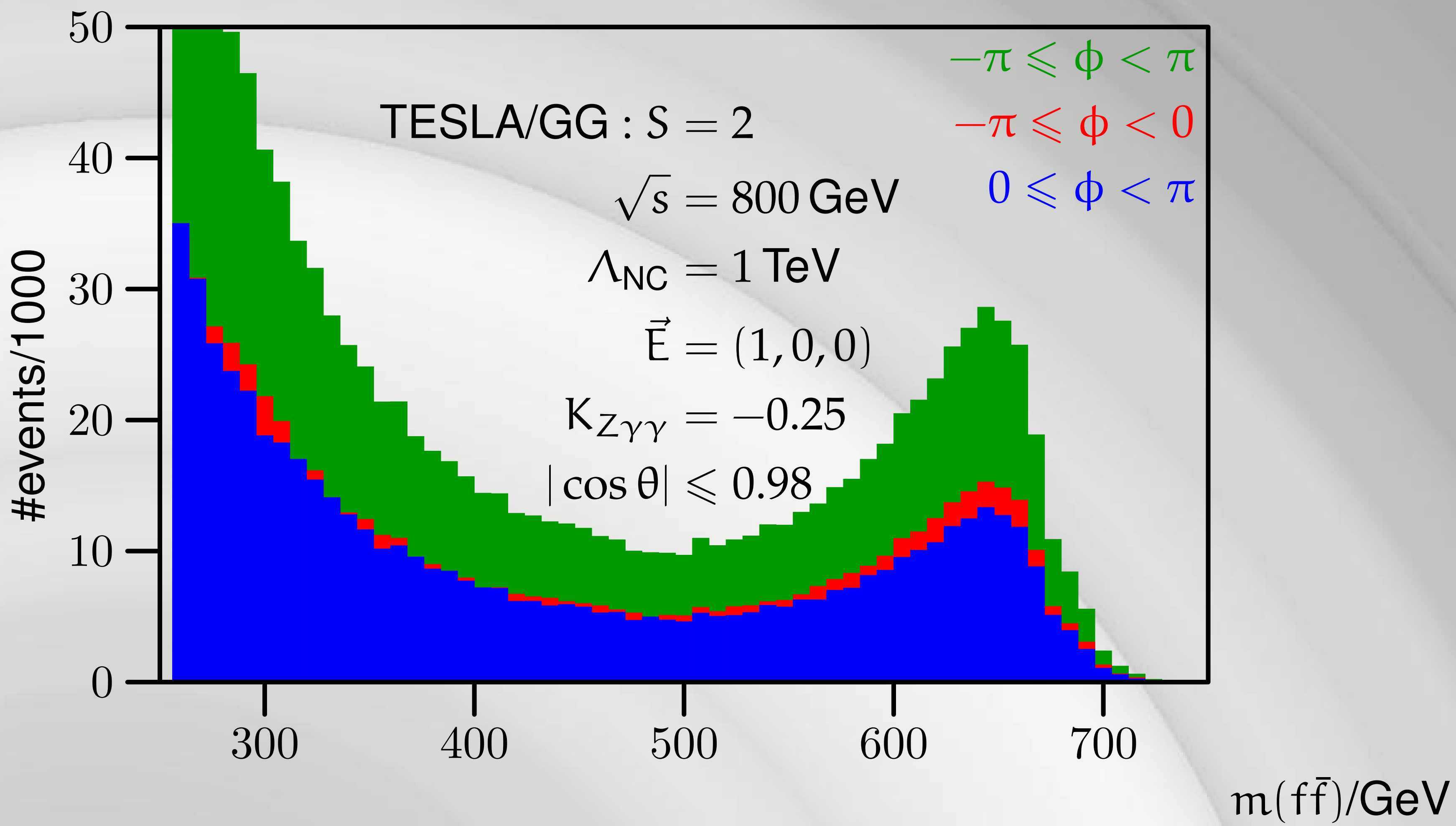


$\theta^{\mu\nu}$ fixes two directions $\vec{E}$ und $\vec{B}$ $\Longrightarrow$ rotational invariance lost!

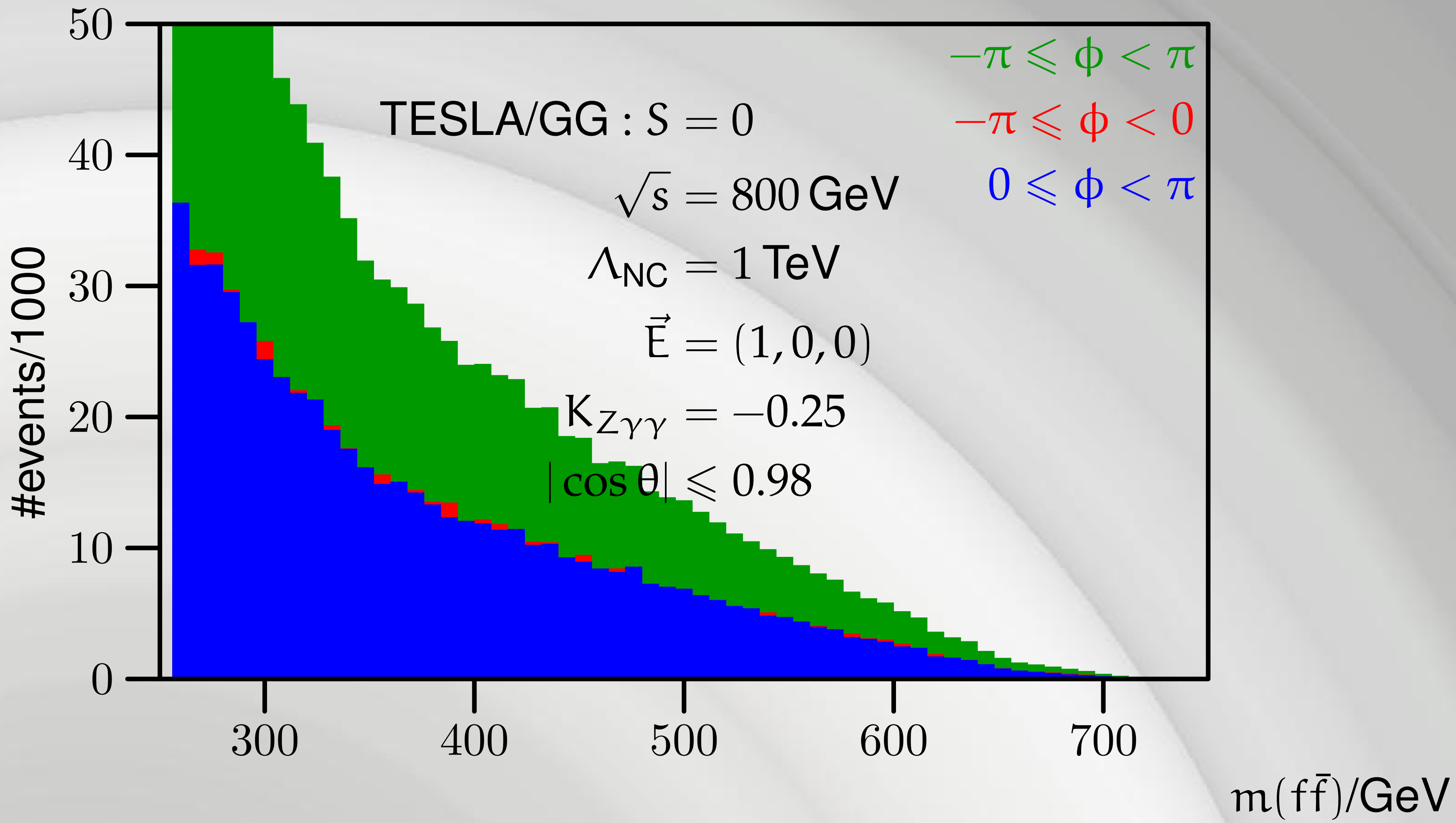
Z-boson in the s-channel as interference



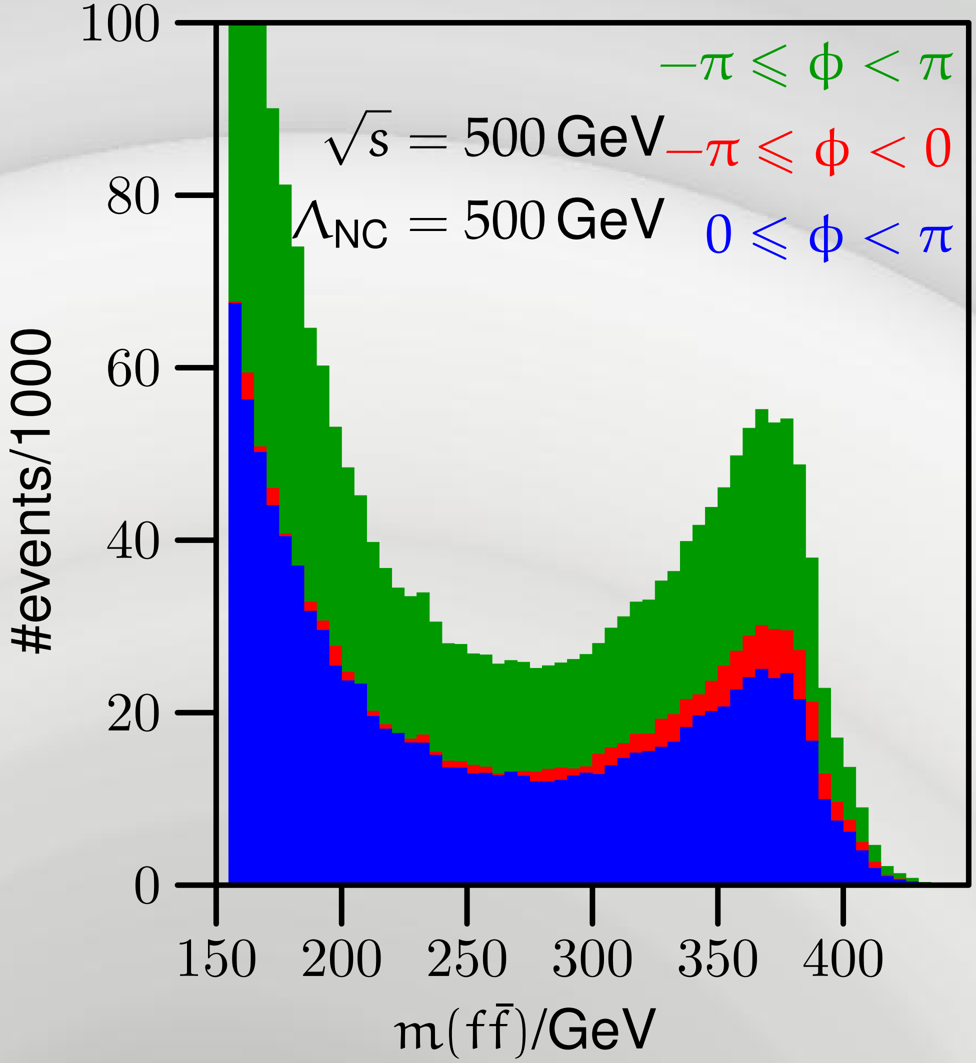
Number of events in the semispheres $\phi < 0$ and $\phi > 0$ for $\sqrt{s} = 800$ GeV

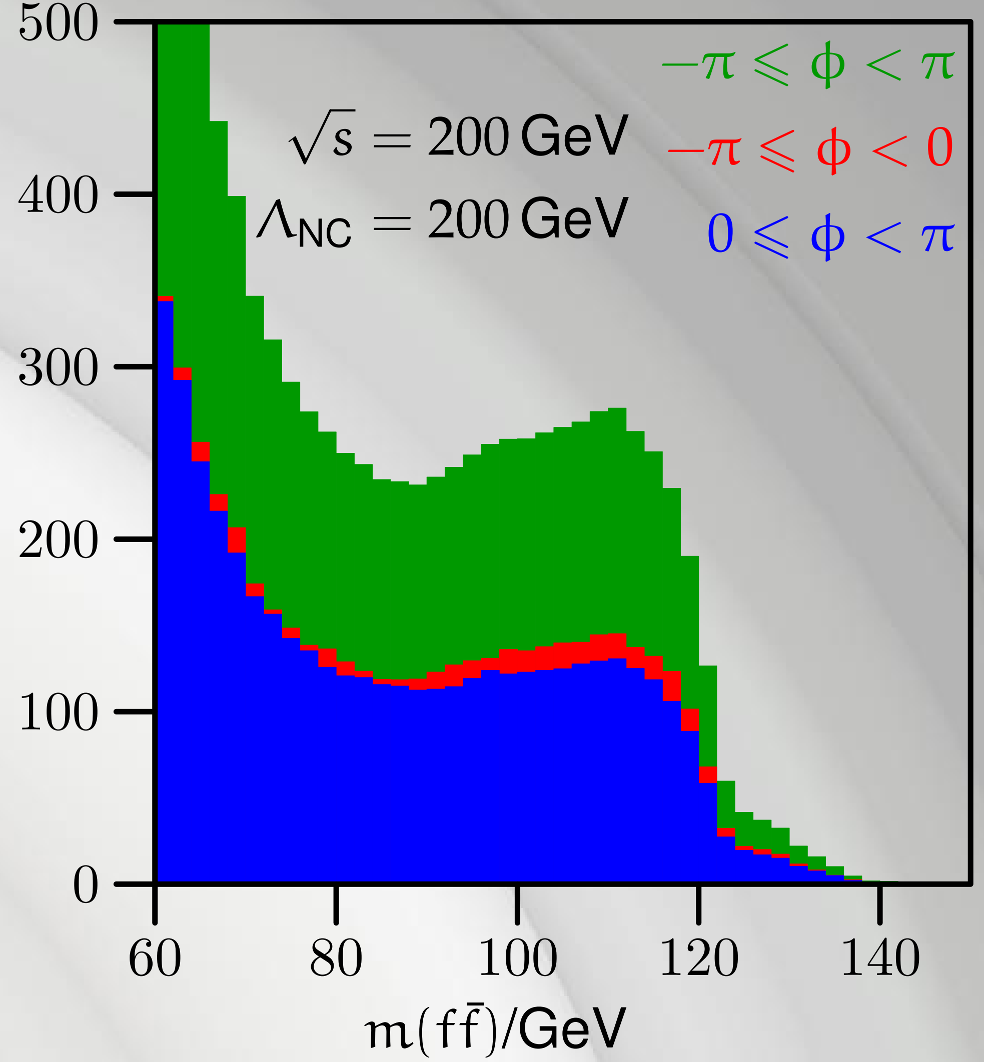


No signal in the Higgs-friendly $S = 0$ mode:

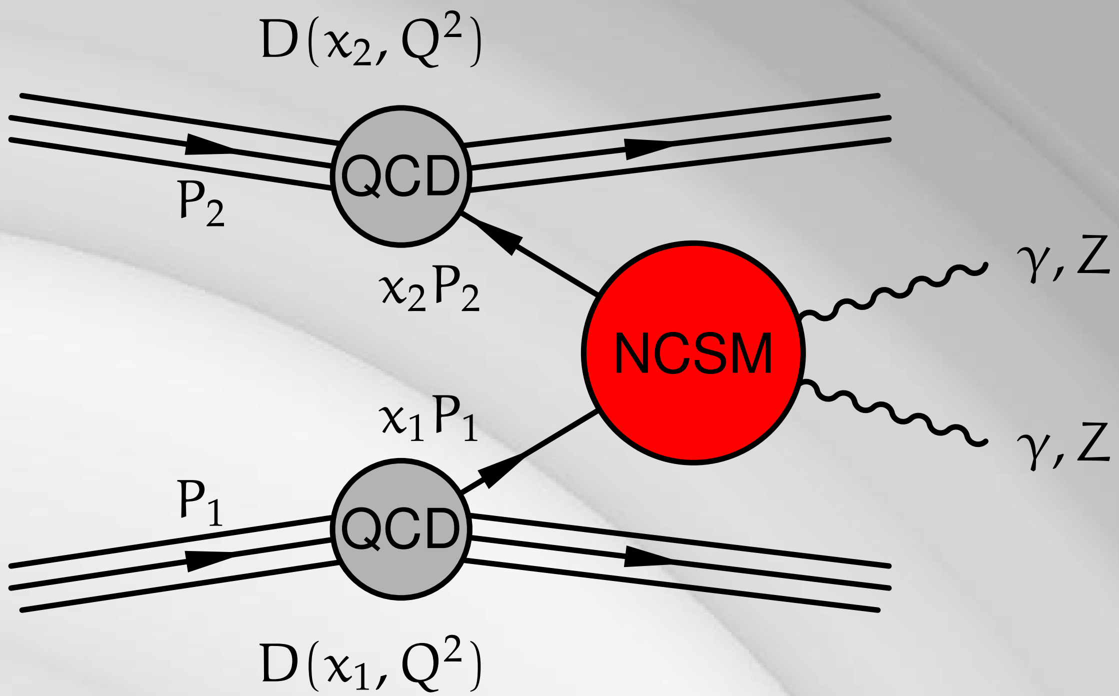


lower energies yield lower physics reach:





Much sooner: LHC (and Tevatron)



[plots by Ana Alboteanu]

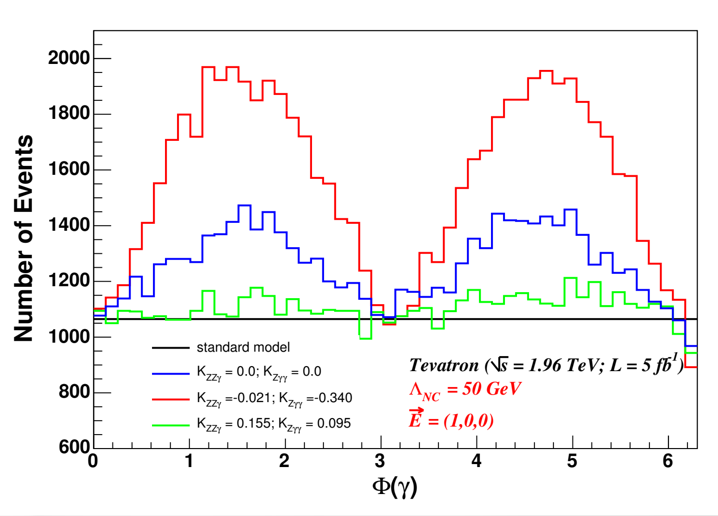
- cross section from manual calculation
- event generation using WHIZARD (omni-purpose event generator, [Kilian])

$$\sqrt{s} = 1.96\,\text{TeV} \qquad \int L = 5\,\text{fb}^{-1}$$

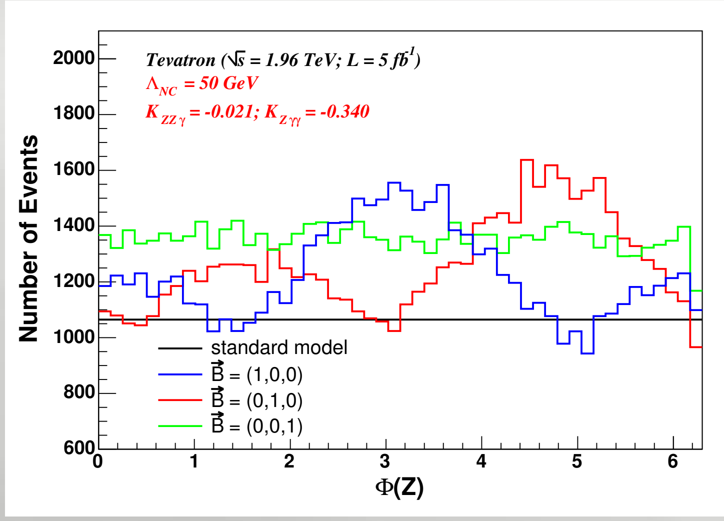
$$5^\circ \leqslant \theta_{Z,\gamma} \geqslant 175^\circ \qquad p_T \geqslant 10\,\text{GeV}$$

$$\Lambda_{NC} = 50\,\text{GeV}$$

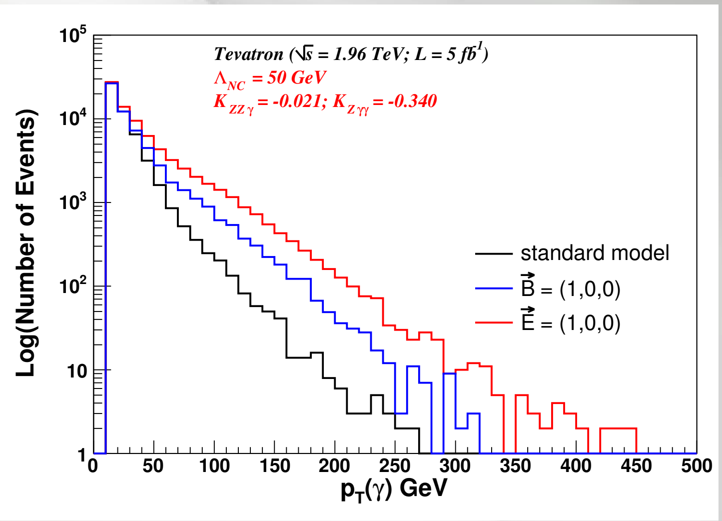
Gauge boson couplings $Z\gamma\gamma$ and $ZZ\gamma$:



Spatial orientation of $\vec{B}$:



Nice high-p_T signature:

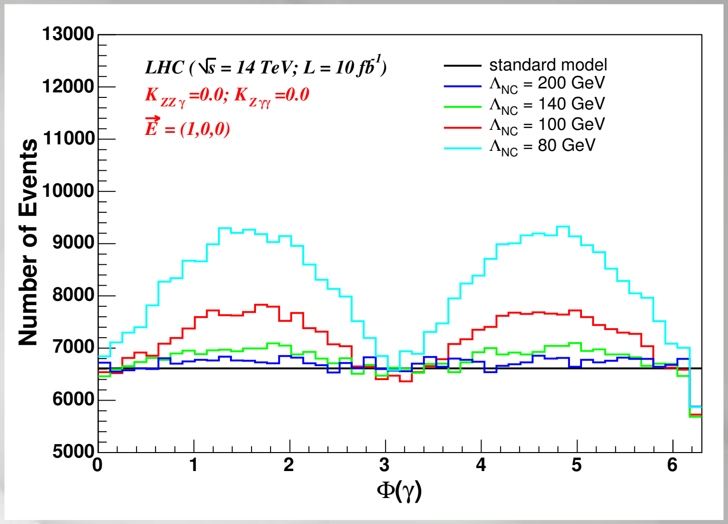


Sensitivity to Λ_{NC}:

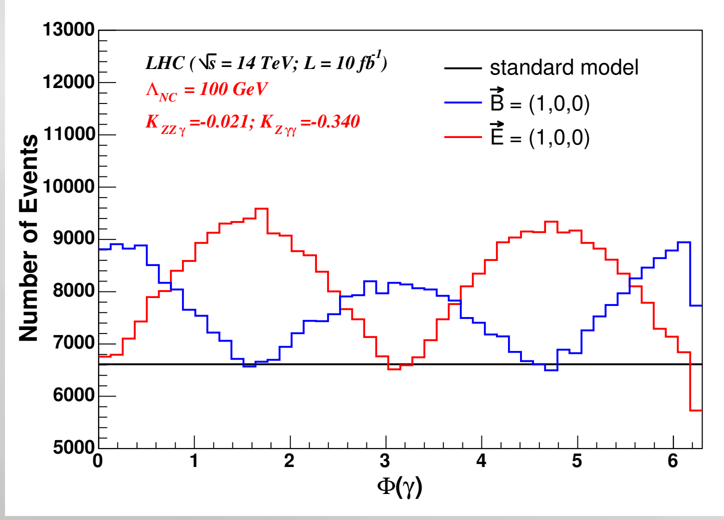
$$\sqrt{s} = 14\,\mathrm{TeV} \qquad \int L = 10\,\mathrm{fb}^{-1}$$

$$5^\circ \leqslant \theta_{Z,\gamma} \geqslant 175^\circ \qquad p_T \geqslant 10\,\mathrm{GeV}$$

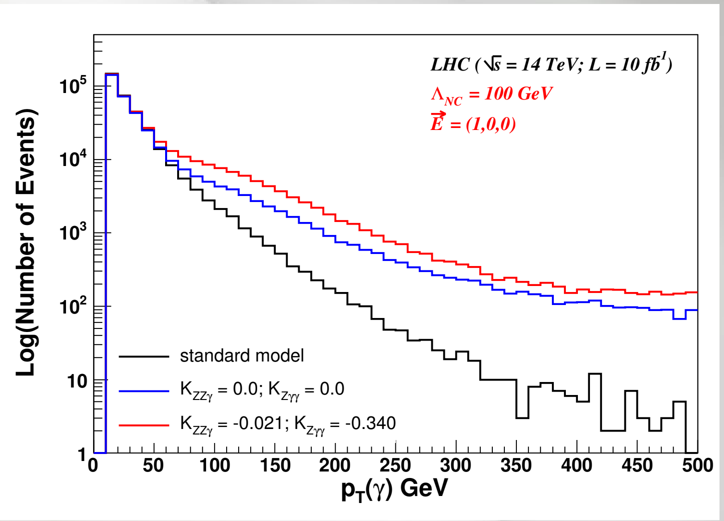
$$\Lambda_{NC} = 100\,\mathrm{GeV}$$



$\vec{B}$ vs. $\vec{E}$:

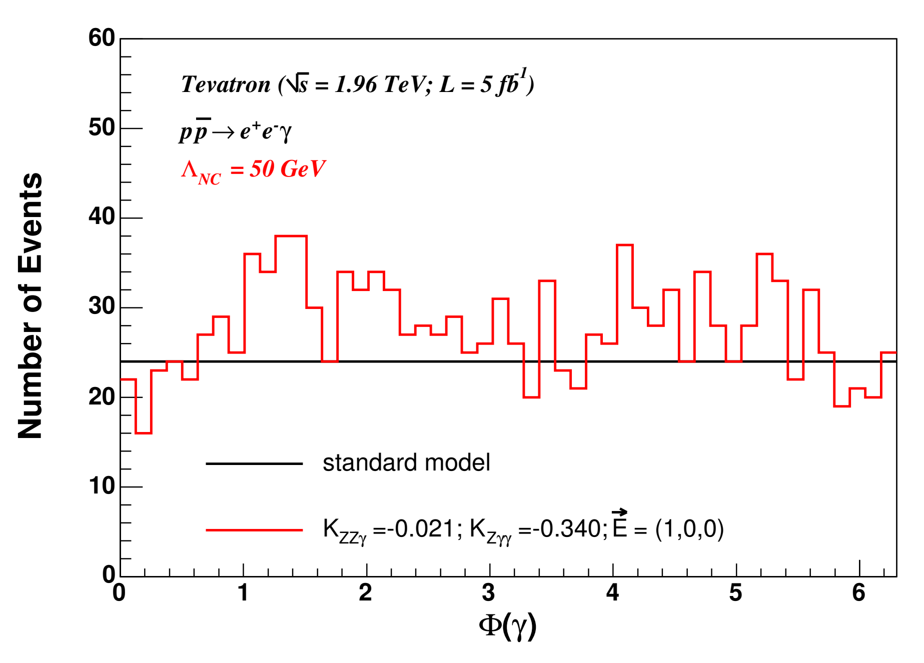


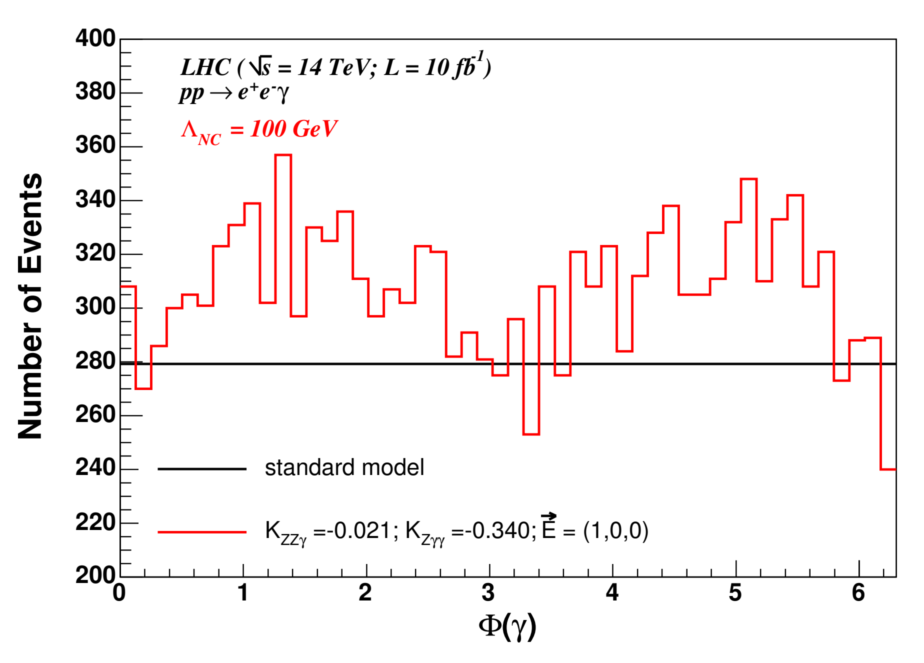
High-p_T signature:



$$\sqrt{s} = 1.96\,\text{TeV} \qquad \int L = 5\,\text{fb}^{-1}$$
$$p_T(\gamma) \geqslant 10\,\text{GeV} \qquad p_T(e^+, e^-) \geqslant 25\,\text{GeV}$$
$$|\eta| \leqslant 2.5 \qquad \Lambda_{NC} = 50\,\text{GeV}$$

$$\sqrt{s} = 14\,\text{TeV} \qquad \int L = 10\,\text{fb}^{-1}$$
$$p_T(\gamma) \geqslant 10\,\text{GeV} \qquad p_T(e^+, e^-) \geqslant 25\,\text{GeV}$$
$$|\eta| \leqslant 2.5 \qquad \Lambda_{NC} = 100\,\text{GeV}$$





- NCSM-Lagrangian at $\mathcal{O}((\theta^{\mu\nu})^2)$!
- new processes
  - e^+e^- mode of the linear collider
  - $\gamma\gamma \to \gamma\gamma$
- 3-particle final states at LHC
  - complete calculation (O'Mega)
  - background
- $\gamma\nu\bar{\nu}$-interactions on cosmological scales