NLO pQCD prediction for exclusive two-photon annihilation into pseudoscalar meson pair

$$\gamma\gamma \to M^+M^- \ (M = \pi,\, K)$$

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Motivation:

- **exclusive processes** - challenging tests of QCD
- **photon induced reactions** – clean test of pQCD
- **NLO** – LO do not have much predictive power

Formalism:

- **1980 -** Brodsky and Lepage, Efremov and Radyushkin, Duncan and Mueller

Situation:

- **LO** – many exclusive processes
- **NLO** – 3 processes

Complete list of NLO calculations:

- **pion electromagnetic form factor ($\pi^{+(-)} + \gamma^* \to \pi^{+(-)}$)**

  Field at al. (1981); Dittes and Radyushkin (1981); Sarmadi(1982); Radyushkin and Khalmuradov (1985); Braaten and Tse (1987);

  complete NLO prediction: Melić, Nižić and Passek (1999,2000)

- **pion transition form factor ($\gamma^*\gamma \to \pi^0$)**

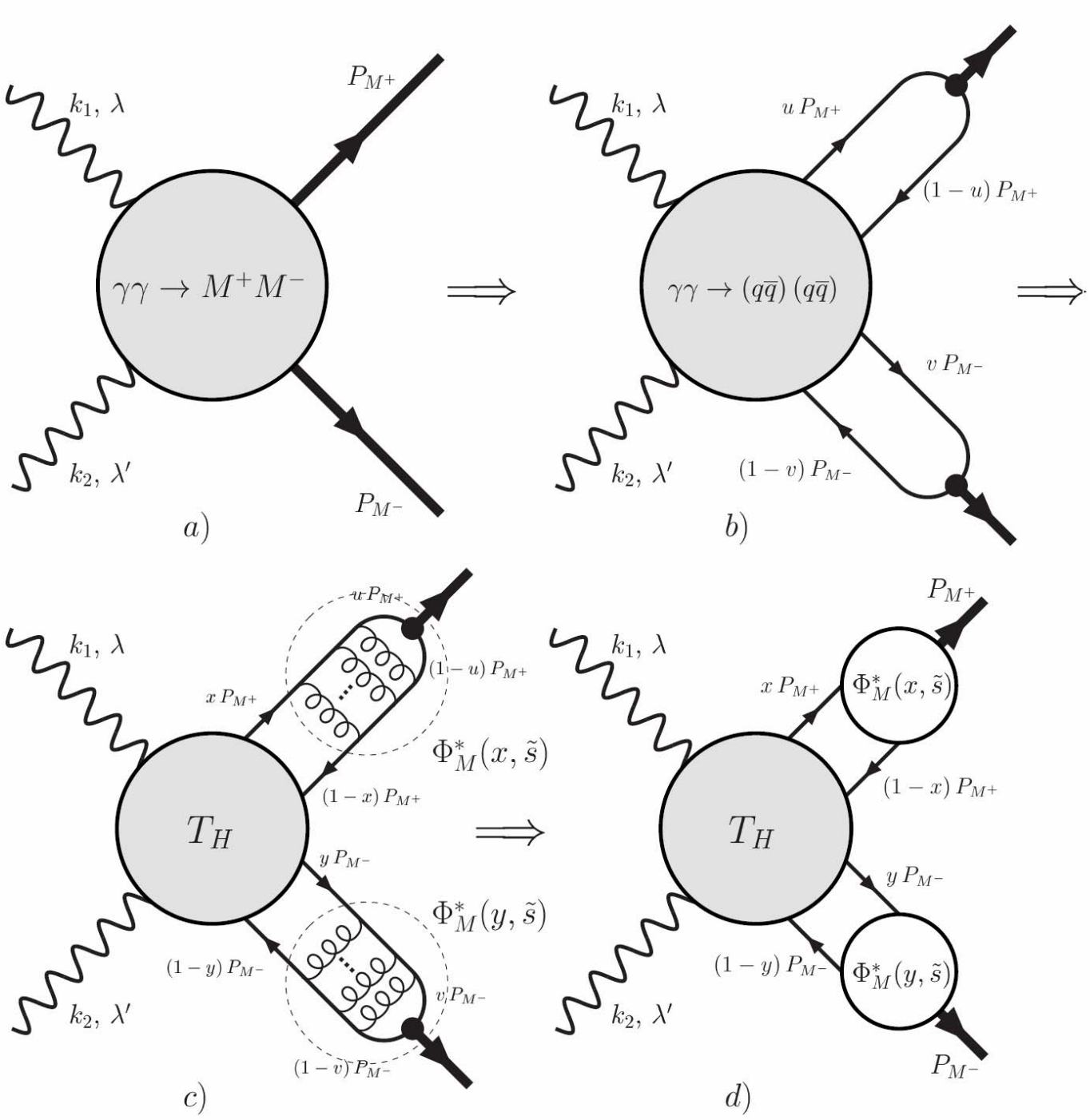
  del Aguila and Chase (1981); Brateen (1983); Kadantseva et al. (1986);

  BLM scale (NNLO): Melić, Nižić and Passek (2001)

- **two photon annihilation into meson pair ($\gamma\,\gamma \to M^+\,M^-$)**

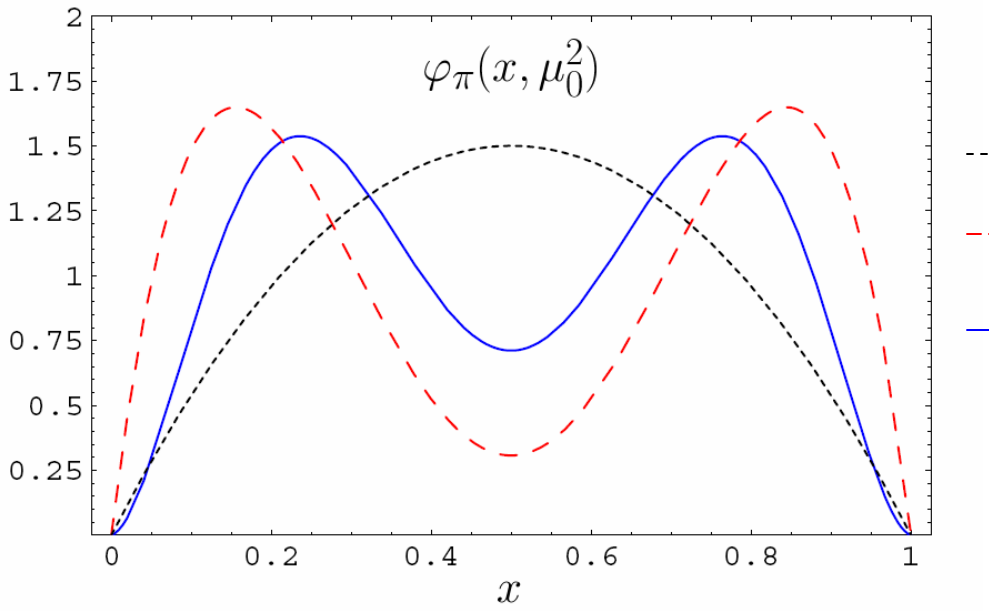
  Nižić (1987) – simplified meson distribution amplitude: $\Phi_M \sim \delta(x - 1/2)$

Formalism


$k_1,\ \lambda$
$k_2,\ \lambda'$
P_{M^+}
P_{M^-}
$\gamma\gamma \to M^+M^-$
$a)$
$u\,P_{M^+}$
$(1-u)\,P_{M^+}$
$v\,P_{M^-}$
$(1-v)\,P_{M^-}$
$\gamma\gamma \to (q\bar{q})\,(q\bar{q})$
$b)$
$x\,P_{M^+}$
$(1-x)\,P_{M^+}$
$y\,P_{M^-}$
$(1-y)\,P_{M^-}$
$\Phi^*_M(x,\tilde{s})$
$\Phi^*_M(y,\tilde{s})$
T_H
$c)$
$d)$

$$\mathcal{M}(\lambda\lambda';s,\theta_{c.m.}) = \int_0^1 dx \int_0^1 dy\ \Phi_M^*(x,\tilde{s})\,\Phi_M^*(y,\tilde{s})\,T_H(\lambda\lambda';s,\theta_{c.m.};x,y,\tilde{s})$$

Distribution amplitude:

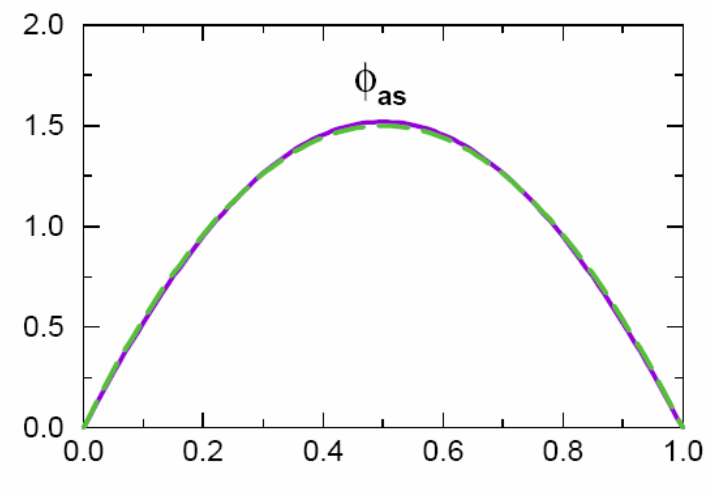


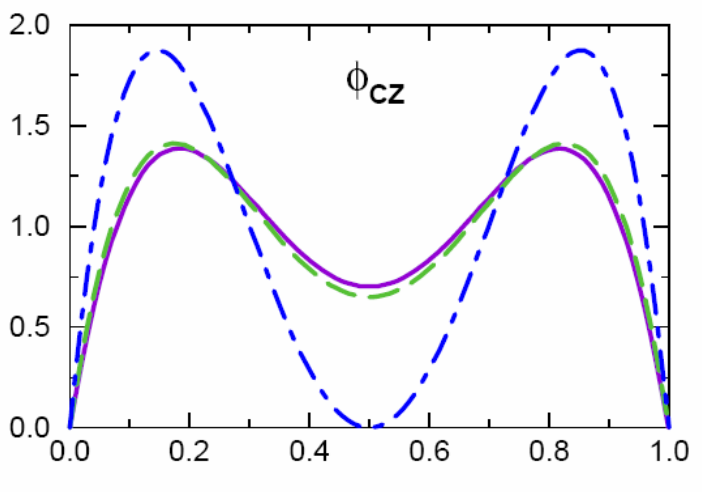
Profiles of pion DAs normalized at $\mu_0^2 = 1$ GeV^2

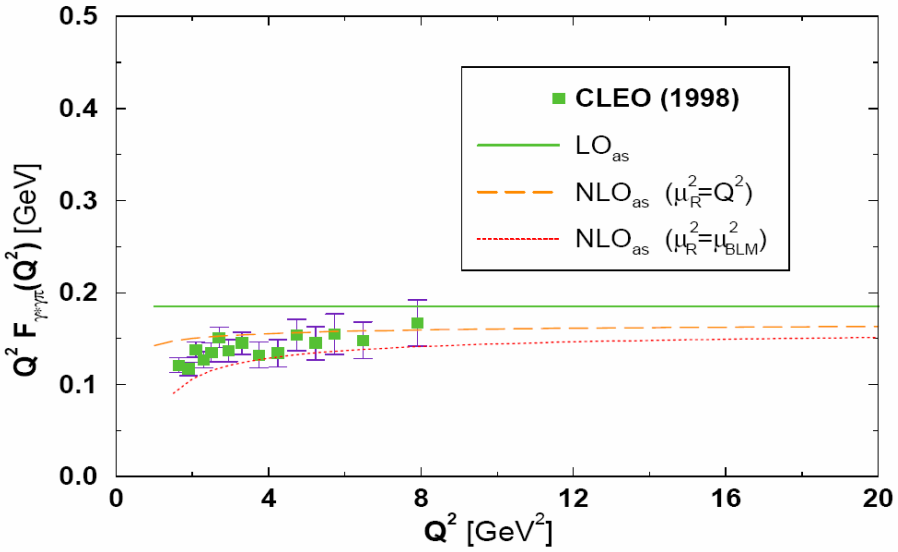
Asymptotic (as)

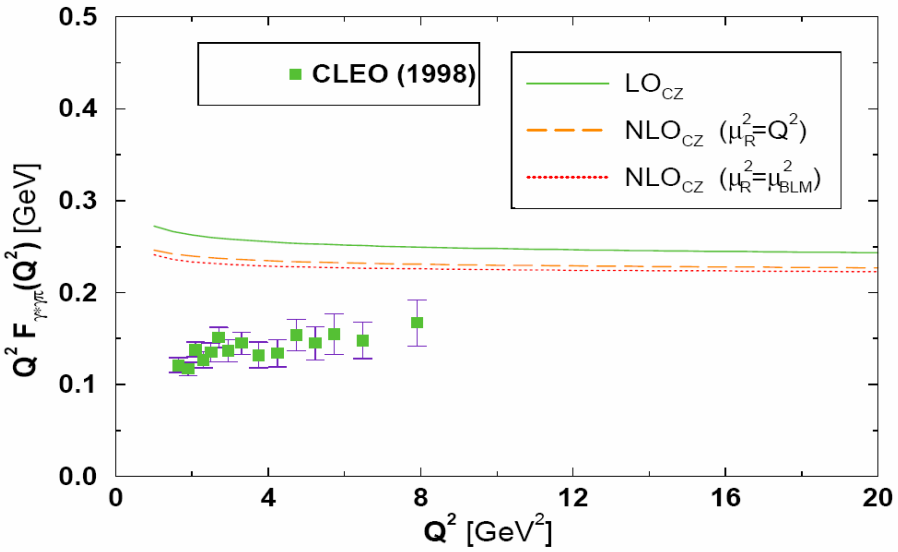
Chernyak-Zhitnitsky (CZ)

Bakulev, Mikhailov, Stefanis (BMS)

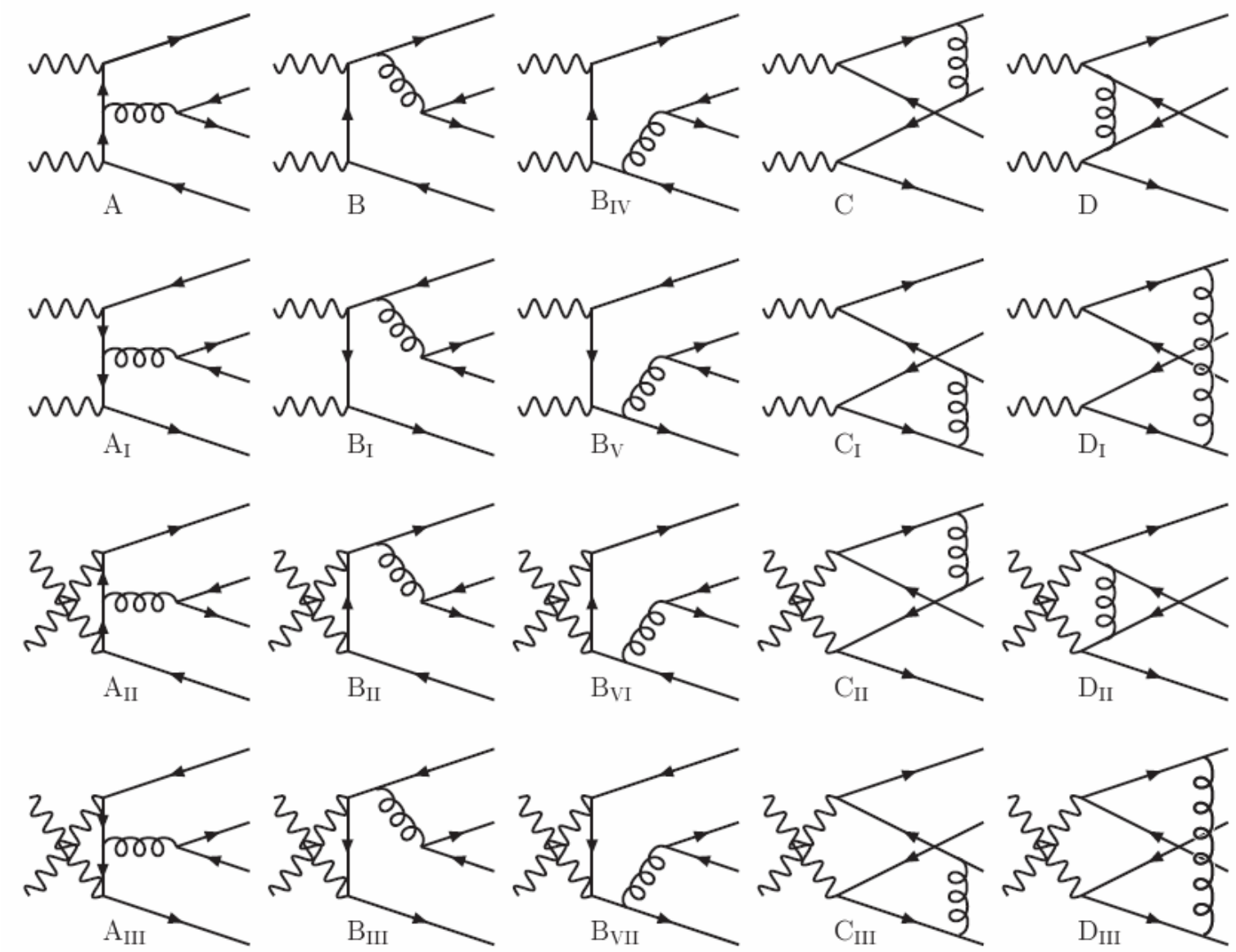

ϕ_{as}
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0


ϕ_{CZ}
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0


CLEO (1998)
LO_{as}
NLO_{as} ($\mu_R^2=Q^2$)
NLO_{as} ($\mu_R^2=\mu_{BLM}^2$)
$Q^2 F_{\gamma^*\gamma\pi}(Q^2)$ [GeV]
Q^2 [GeV^2]
0.5
0.4
0.3
0.2
0.1
0.0
0
4
8
12
16
20


CLEO (1998)
LO_{CZ}
NLO_{CZ} ($\mu_R^2=Q^2$)
NLO_{CZ} ($\mu_R^2=\mu_{BLM}^2$)
$Q^2 F_{\gamma^*\gamma\pi}(Q^2)$ [GeV]
Q^2 [GeV^2]
0.5
0.4
0.3
0.2
0.1
0.0
0
4
8
12
16
20

LO:


A
B
B_{IV}
C
D
A_I
B_I
B_V
C_I
D_I
A_{II}
B_{II}
B_{VI}
C_{II}
D_{II}
A_{III}
B_{III}
B_{VII}
C_{III}
D_{III}

$$\mathcal{M}^{(0)}(\lambda\lambda';s,Z)=\int_0^1 dx\int_0^1 dy\;\Phi_M^*(x,\tilde{s})\,\Phi_M^*(y,\tilde{s})\,T_H^{(0)}(\lambda\lambda';s,Z;x,y,\tilde{s}),$$

$$\left.\begin{matrix}T_H^{(0)}(++;s,Z;x,y,\tilde{s})\\ T_H^{(0)}(--;s,Z;x,y,\tilde{s})\end{matrix}\right\}=(4\pi)^2\alpha_e\alpha_S(s_R)C_F\frac{8(e_1-e_2)^2}{s(1-Z^2)}\frac{xy+\bar{x}\bar{y}}{xy\bar{x}\bar{y}},$$

$$\left.\begin{matrix}T_H^{(0)}(+-;s,Z;x,y,\tilde{s})\\ T_H^{(0)}(-+;s,Z;x,y,\tilde{s})\end{matrix}\right\}=(4\pi)^2\alpha_e\alpha_S(s_R)C_F\frac{8}{s\,xy\bar{x}\bar{y}}\left[(1-xy-\bar{x}\bar{y})\frac{(e_1-e_2)^2}{(1-Z^2)}\right.$$
$$\left.+e_1e_2\frac{(xy+\bar{x}\bar{y})(x\bar{y}+x\bar{y})}{(xy+\bar{x}\bar{y})^2-Z^2(xy-\bar{x}\bar{y})^2}+(e_1^2-e_2^2)\frac{(x-y)}{2}\right]$$

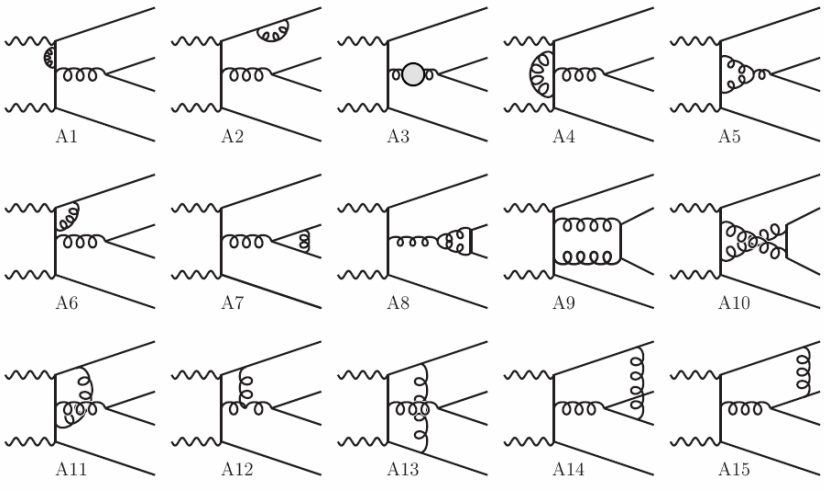
$$Z=\cos\theta_{c.m.},\quad \bar{x}=1-x,\quad \bar{y}=1-y,$$

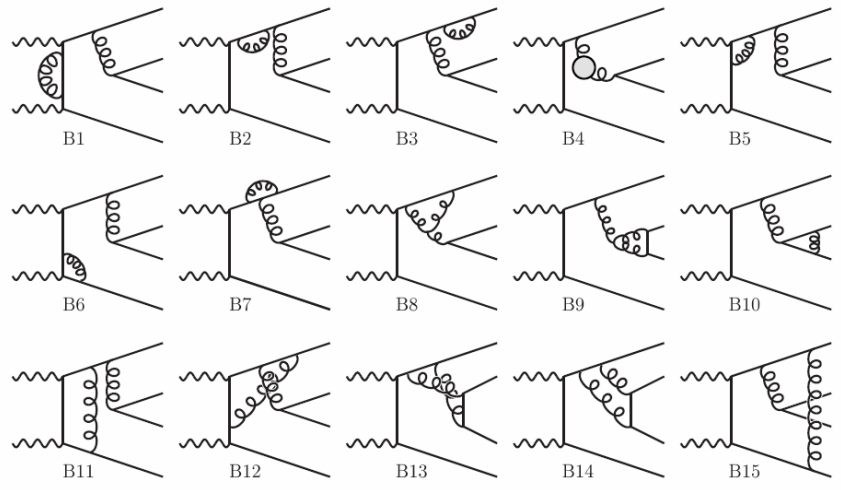
Brodsky and Lepage (Phys.Rev.D **24**, 1808 (1981))

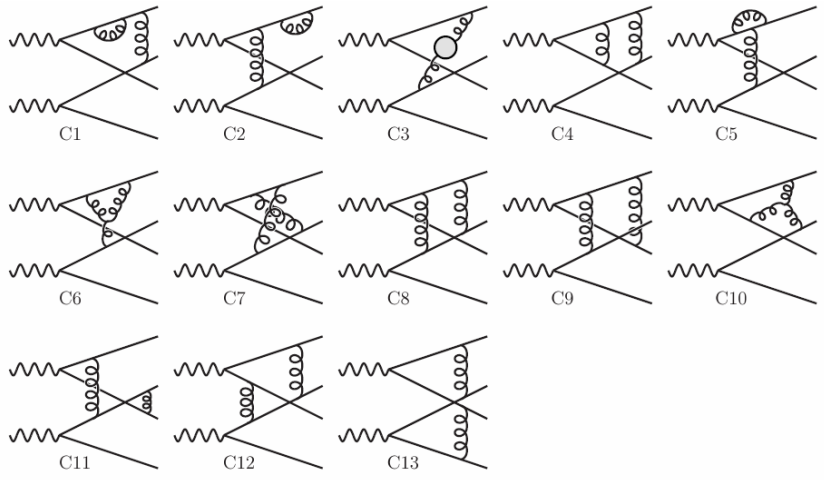
Essential parts of the hard scattering amplitudes are accidentally proportional to the pion form factor

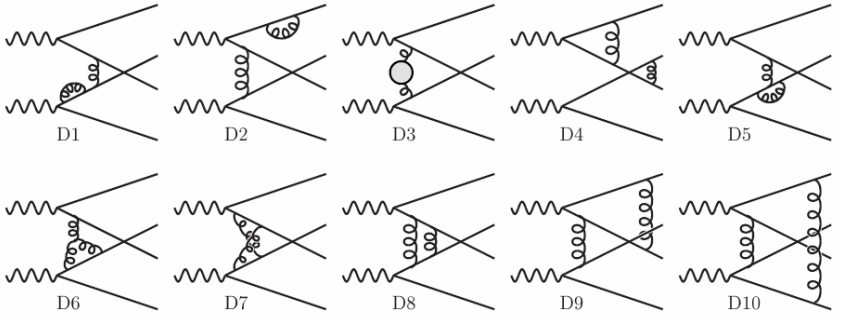
$$\frac{d\,\sigma}{d\,Z}(\gamma\gamma\to M^+M^-)\sim\frac{4|F_M(s)|^2}{1-Z^4}\frac{d\,\sigma}{d\,Z}(\gamma\gamma\to\mu^+\mu^-)$$

$$F_\pi(s)\sim 0.4\,GeV^2/s$$

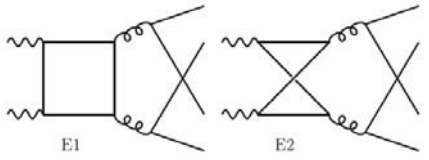








NLO = 422 diagrams (they can be generated from 58 basic diagrams)



… a few years later:

$$\sigma\left(|\cos\theta_{c.m.}|<0.6\right)=f_M^4\frac{1.035}{s^3}\alpha_S^2(s_R)\left\{1+\frac{\alpha_S(s_R)}{\pi}\left[-3.828+\frac{\beta_0}{2}\left(3.563+\ln\left(\frac{s_R}{s}\right)\right)\right]\right\}$$

Physical scales:

- BLM – mean virtuality of gluon propagator $s_{R,BLM}\simeq s/35$
- α_V scheme – QCD coupling is defined from the heavy quark potential. Renormalization scale is by definition the momentum transfer caused by the gluon. $s_{R,V}\simeq s/7$

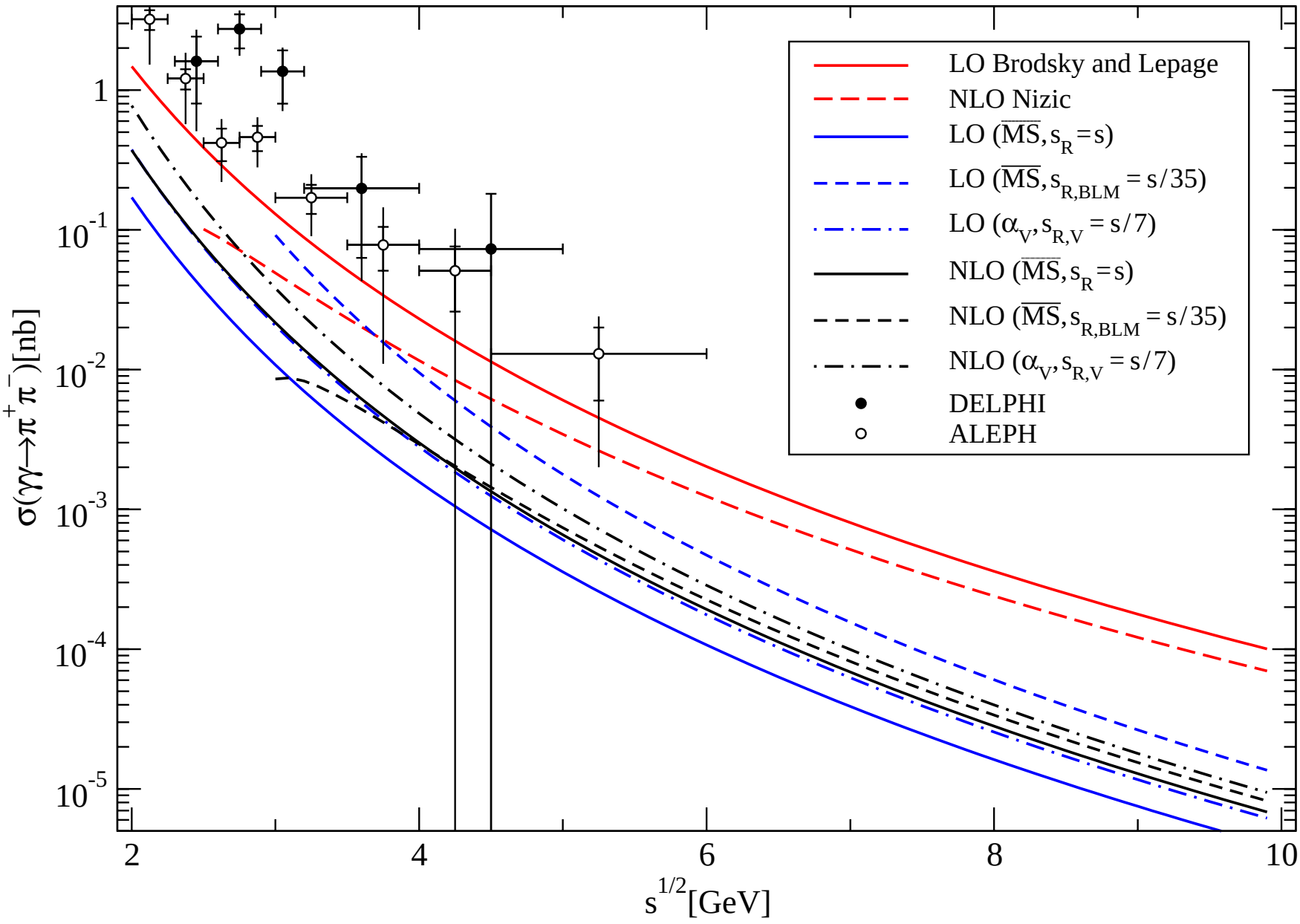
	$\overline{MS}$, $s_R=s$		$\overline{MS}$, $s_{R,BLM}=s/35$		α_V, $s_{R,V}=s/7$	
$s\,(GeV^2)$	$\alpha_S(s_R)$	NLO/LO	$\alpha_S(s_{R,BLM})$	NLO/LO	$\alpha_S(s_{R,V})$	NLO/LO
4	0.303	1.178	1.33	-1.607	0.45	1.066
10	0.253	0.982	0.71	-0.858	0.347	0.823
100	0.178	0.693	0.327	-0.395	0.221	0.523
1000	0.138	0.536	0.212	-0.257	0.162	0.384

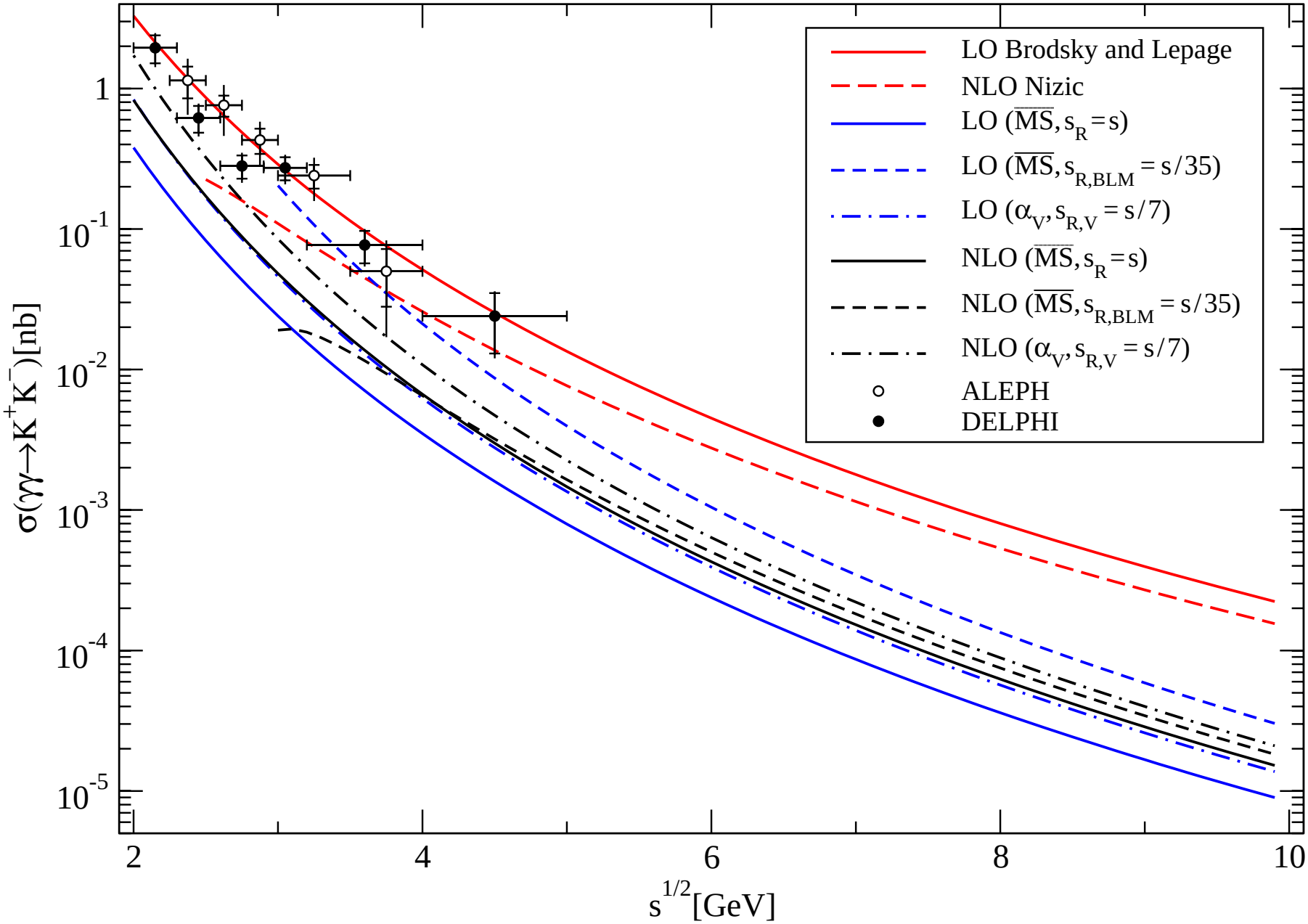
Measurements:

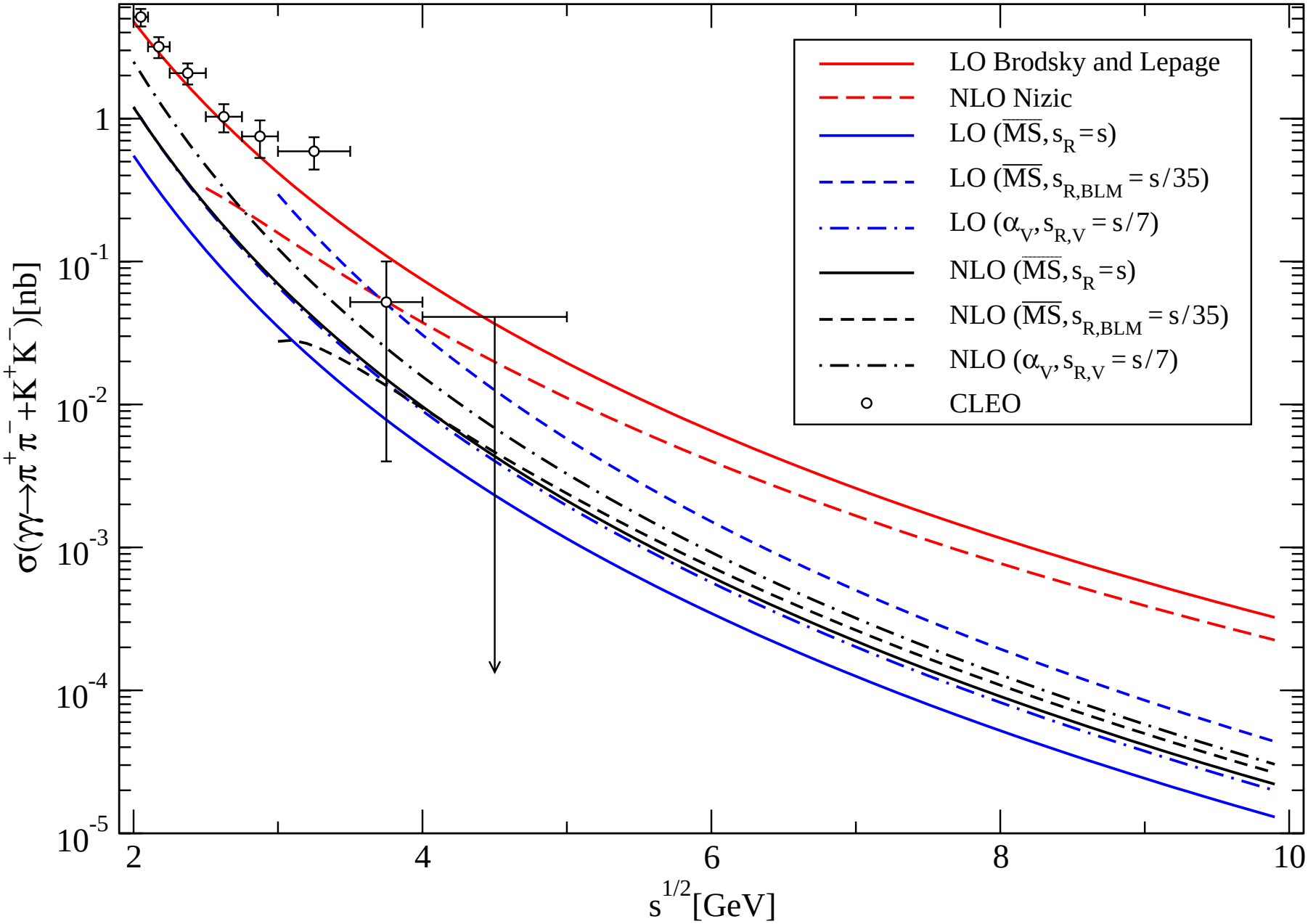
J. Dominick *et al.* [CLEO Collaboration], "Two photon production of charged pion and kaon pairs," Phys. Rev. D **50** (1994) 3027 [arXiv:hep-ph/9403379].

A. Heister *et al.* [ALEPH Collaboration], "Exclusive production of pion and kaon meson pairs in two photon collisions at LEP," Phys. Lett. B **569** (2003) 140.

K. Grzelak, [DELPHI Collaboration], "Exclusive production of charged kaon and pion pairs in photon photon collisions at LEP-2," *Prepared for International Conference on the Structure and Interactions of the Photon and 14th International Workshop on Photon-Photon Collisions (Photon 2001), Ascona, Switzerland, 2-7 Sep 2001*


LO Brodsky and Lepage
NLO Nizic
LO ($\overline{MS}, s_R = s$)
LO ($\overline{MS}, s_{R,BLM} = s/35$)
LO ($\alpha_V, s_{R,V} = s/7$)
NLO ($\overline{MS}, s_R = s$)
NLO ($\overline{MS}, s_{R,BLM} = s/35$)
NLO ($\alpha_V, s_{R,V} = s/7$)
DELPHI
ALEPH
$\sigma(\gamma\gamma \to \pi^+\pi^-)$[nb]
$s^{1/2}$[GeV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2
4
6
8
10


LO Brodsky and Lepage
NLO Nizic
LO ($\overline{MS}$, $s_R = s$)
LO ($\overline{MS}$, $s_{R,BLM} = s/35$)
LO (α_V, $s_{R,V} = s/7$)
NLO ($\overline{MS}$, $s_R = s$)
NLO ($\overline{MS}$, $s_{R,BLM} = s/35$)
NLO (α_V, $s_{R,V} = s/7$)
ALEPH
DELPHI
$\sigma(\gamma\gamma \to K^+K^-)$[nb]
$s^{1/2}$[GeV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2
4
6
8
10


LO Brodsky and Lepage
NLO Nizic
LO ($\overline{MS}, s_R = s$)
LO ($\overline{MS}, s_{R,BLM} = s/35$)
LO ($\alpha_V, s_{R,V} = s/7$)
NLO ($\overline{MS}, s_R = s$)
NLO ($\overline{MS}, s_{R,BLM} = s/35$)
NLO ($\alpha_V, s_{R,V} = s/7$)
CLEO
$\sigma(\gamma\gamma \to \pi^+\pi^-+K^+K^-)$[nb]
$s^{1/2}$[GeV]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
2
4
6
8
10

Conclusion:

- hard scattering picture is way below the experiment
- experiment is way below the hard scattering picture