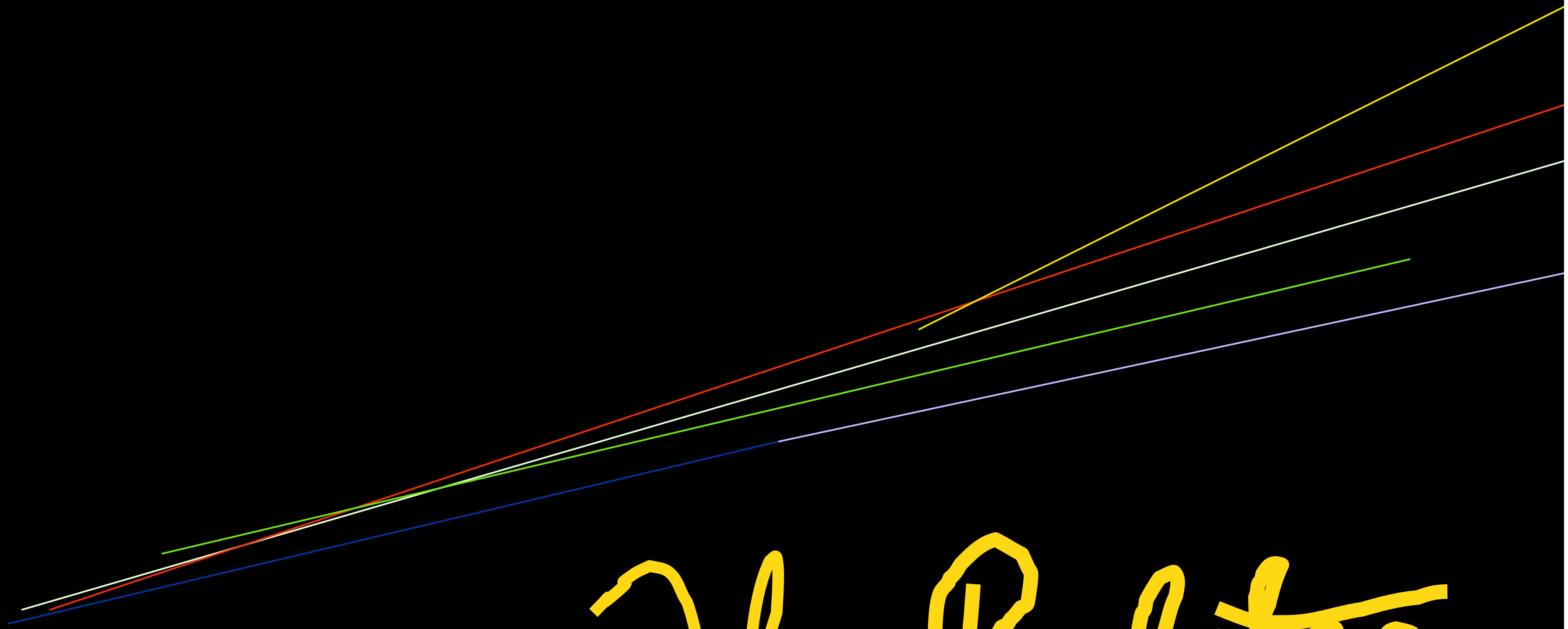
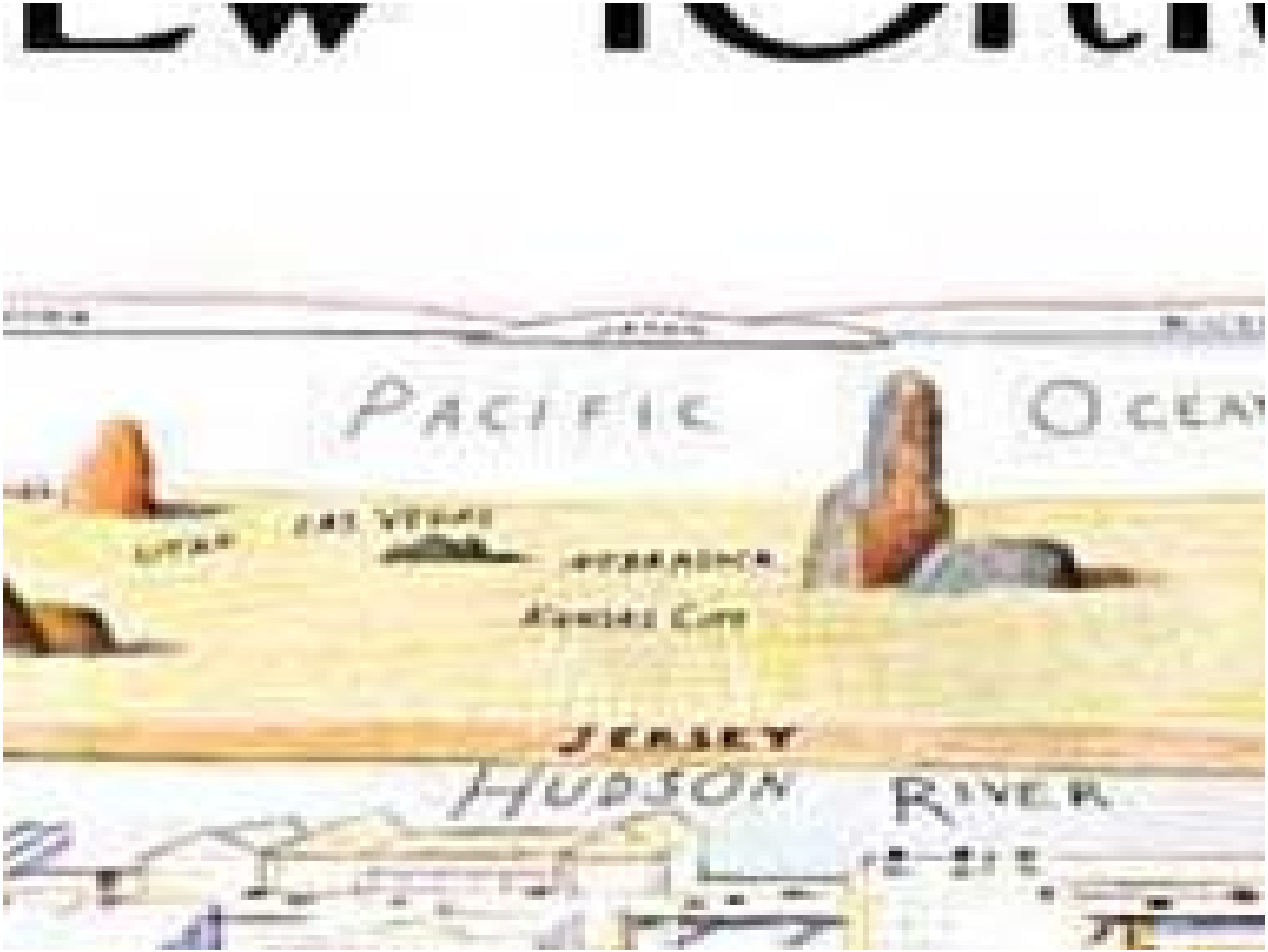
disentangling forward physics



John Ralston


PACIFIC
JERSEY
HUDSON
RIVER

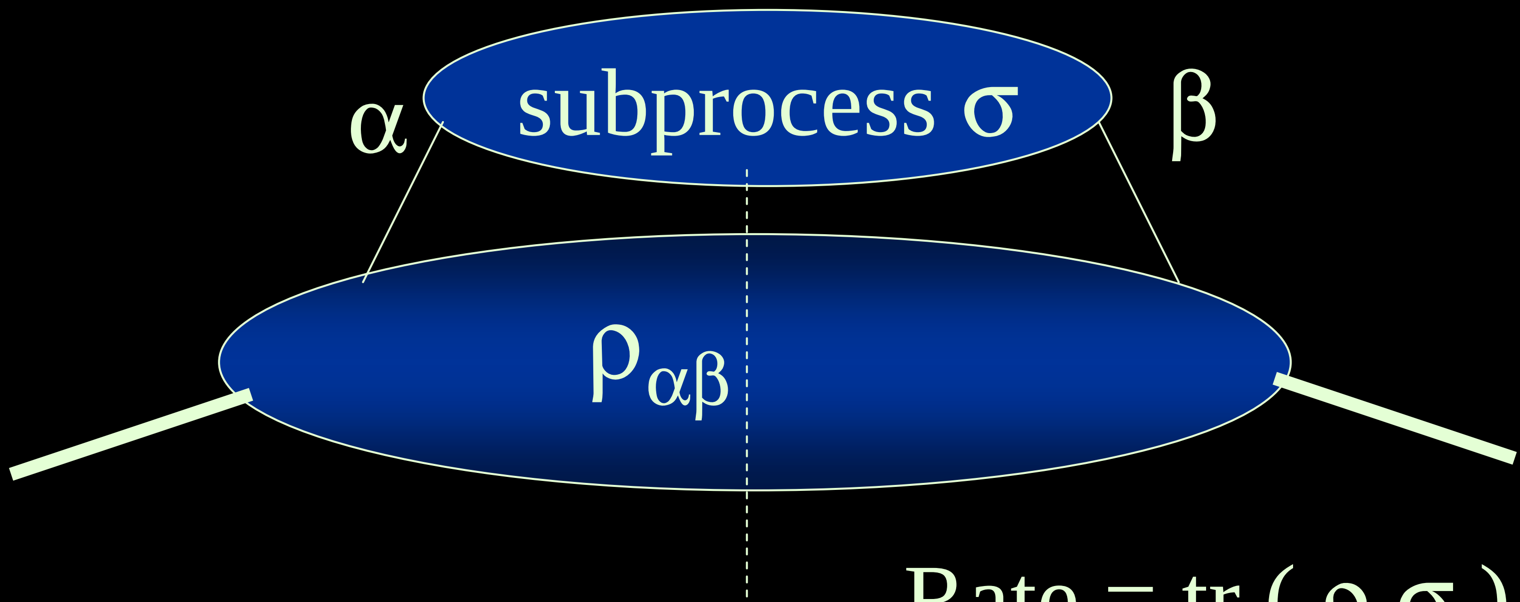
RHIC-spin: thanks Steve Heppelmann, 1990

The meeting will discuss the opportunies to make decisive measurements in the forward region at RHIC and LHC

What is Observable?

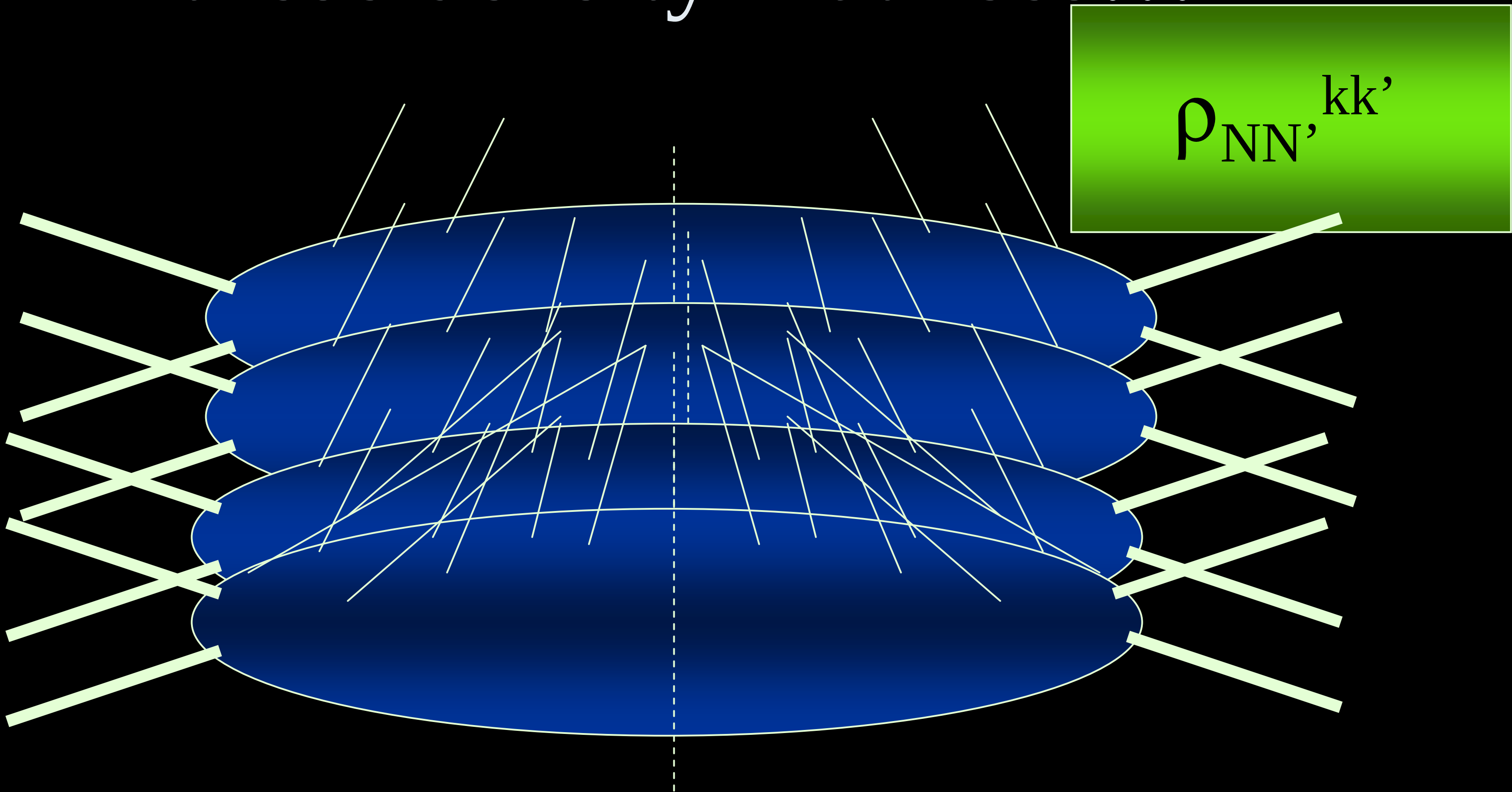
Hard scattering measures this

density matrix



Rate = tr (ρ σ)

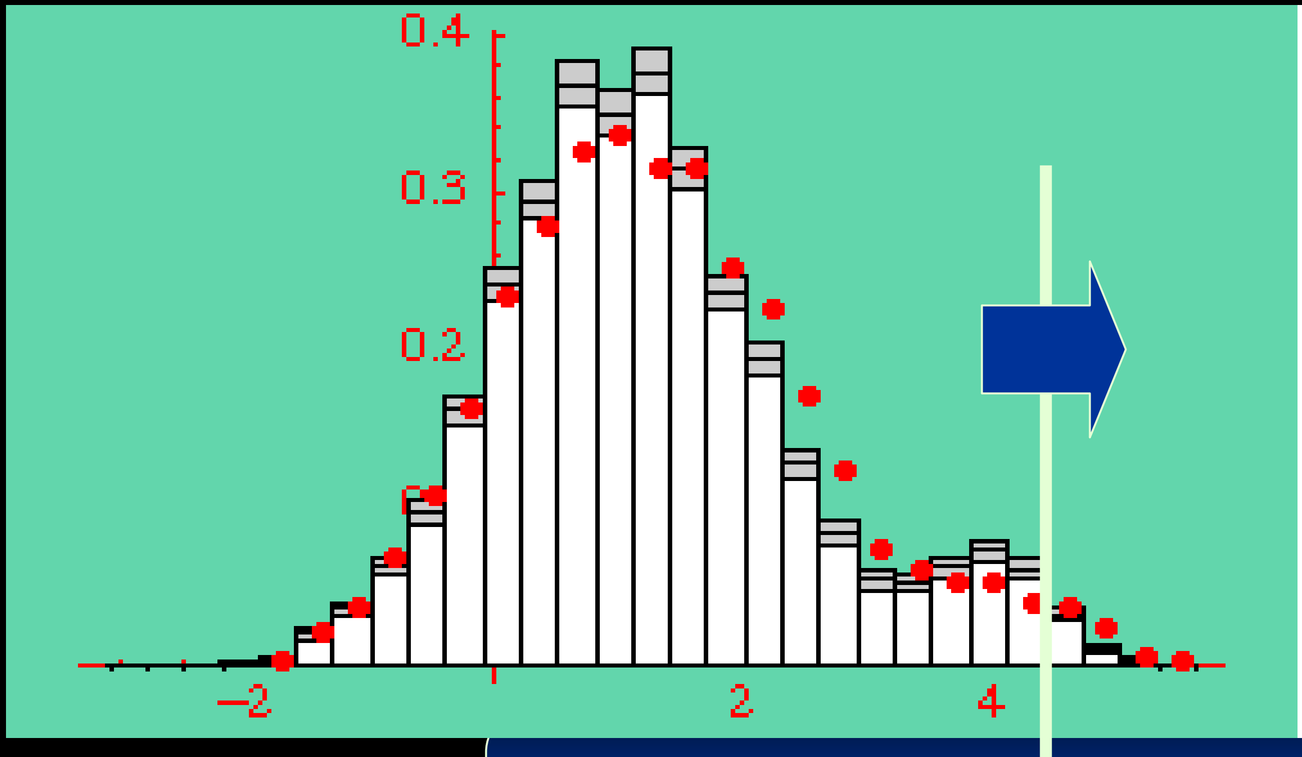
Forward scattering measures these density matrices…



No classical distributions can do the same job

No classical distributions can do the same job

Meanwhile experimenters cut on classical probability…as in 1904



- Cuts improve “signal/noise”

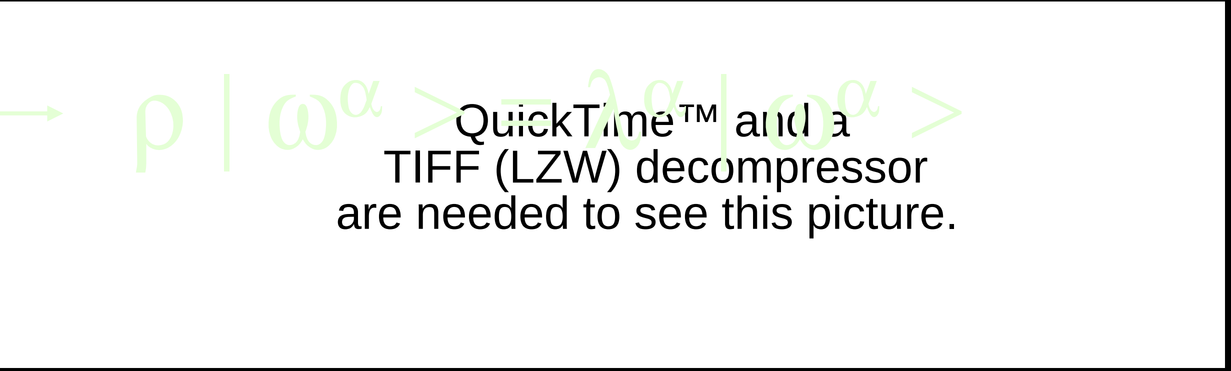
Theory, which suggests interesting signals, ought to play an equally profound role in directing how to make the cuts.

It is always more efficient to cut probability on eigenstates

- Seek special states $|\,\omega >$ so that

$$P(\omega) = \frac{<\omega|\,\rho\,|\,\omega>}{<\omega\,|\,\omega>} \longrightarrow \textit{max}$$

$$\longrightarrow \rho\,|\,\omega^{\alpha}> = \lambda^{\alpha}\,|\,\omega^{\alpha}>$$



Constructing the density matrix…a la roulette

- Basis $1=|k_1>$, $2=|k_1, k_2>$… $N=|k_1, k_2, k_3, \ldots k_N>$
- In practice, bins of rapidity y_k and number N
- Event J is a vector D_{kN}^J

0	2	0	1	…	…	…	3	0

y=0.5 y=1 y=1.5 ,,,

$< 0 | 0 >= 1;$

$< 1 | 0 >= 0$

$< 1 | 1 >= 1$

$< 1 | 2 >= 0\ldots$

Gentlemen and Ladies,
Consider Organization by Product Spaces

- |k>|N> means each k thing has N things
- N_{max} x k_{max} things encoded; lossless
- An exponentially efficient encoding
- Collect same “objects” $|\alpha>$ and “sample history s_α^J ”

$$D_{kN}^J = \Sigma_\alpha \; |\alpha> |s_\alpha^J>$$

How to divide spaces

	0	2	0	1	…	…	…	3	0
|1>	0	0	0	1	…	…	…	0	0
+									
|2 >	0	1	0	0	…	…	…	0	0
+									
|3>	0	0	0	0	…	…	…	1	0

Normalize the sampling history of each subspace

(0 1 0 1 … … … 1 1)

…m of these

$= m^{1/2}$ (0 $1/m^{1/2}$ 0 $1/m^{1/2}$ … … … $1/m^{1/2}$ $1/m^{1/2}$)

…thus the braces are normalized

John's main formula

- Joint density matrix

$$\rho_{NN'}^{kk'} = \Sigma_J \; D_{kN}^{J} D_{k'N'}^{J}$$

here D_{kN}^{J} is produced by *division* of the data *so as to represent the data record on a space of sums of products with consistent normalization conventions.*

Division is the inverse of a direct product

how this kind of probability transforms

Let $|D\rangle$ be a data record divided by singular value decomposition into mutually exclusive categories $|\alpha\rangle$ and sample history $|s_\alpha\rangle$:

$$|D\rangle = \sum_\alpha \sqrt{P_\alpha}\, |\alpha\rangle\, |s_\alpha\rangle\,. \tag{1}$$

In a different basis $\left|\tilde{\beta}\right\rangle$ use $|\alpha\rangle = \sum_\beta \left|\tilde{\beta}\right\rangle \left\langle \tilde{\beta} \mid \alpha \right\rangle = U_{\beta\alpha}\, |\alpha\rangle$ Provided the sample history is traced out, the record is statistically equivalent to one that actually measured $\tilde{\beta}$ objects.

But in that event there would be a transformed sample $|\tilde{s}_\beta\rangle$ and transformed probability $\tilde{P}_\beta$ such that

$$|D\rangle = \sum_\beta \sqrt{\tilde{P}_\beta}\left|\tilde{\beta}\right\rangle |\tilde{s}_\beta\rangle\,;$$

$$\sqrt{\tilde{P}_\beta}\, |\tilde{s}_\beta\rangle = \sum_\alpha U_{\alpha\beta}\sqrt{P_\alpha}\, |s_\alpha\rangle\,. \tag{2}$$

Given $|s_\beta\rangle$ normalized, we square to find

$$\tilde{P}_\beta = \sum_\alpha U_{\alpha\beta}\sqrt{P_\alpha} U^\dagger_{\alpha\beta}. \tag{3}$$

This is just the diagonal expectation of the density matrix, $\rho = \sum_\alpha P_\alpha\, |\alpha\rangle\, \langle\alpha|$, namely

$$\tilde{P}_\beta = \left\langle \tilde{\beta} \right| \rho \left| \tilde{\beta} \right\rangle.$$

: the probability rule of quantum theory.

definitions

- data record: categories, numbers, other orthogonal attributes plus sample history
- division: inverse of products, converts data record into products of parent vectors
- truncation: tracing out unwanted degrees of freedom, such as data history
- density matrix: a quadratic form made of the data record, the truncation of parent density matrices

… by construction

- Joint density matrix

$$\rho_{NN'}^{kk'}$$

- Truncated density matrix

$$\rho^{kk'} = tr_N(\rho_{NN'}^{kk'})$$

application: noise rejection cuts

- Almost all data is almost all noise
- Construct density matrix, optimal fit to noise (Karhunen-Loeve) ; *diagonalize*

events J, traced out

$$\Sigma_J^{10,000} \; D_k^{\,J} \; D_{k'}^{\,J} \rightarrow \Sigma_\alpha^{k\text{-max}} \; \lambda^\alpha \; e_k^{\,\alpha} \; e_{k'}^{\,\alpha}$$

space kept

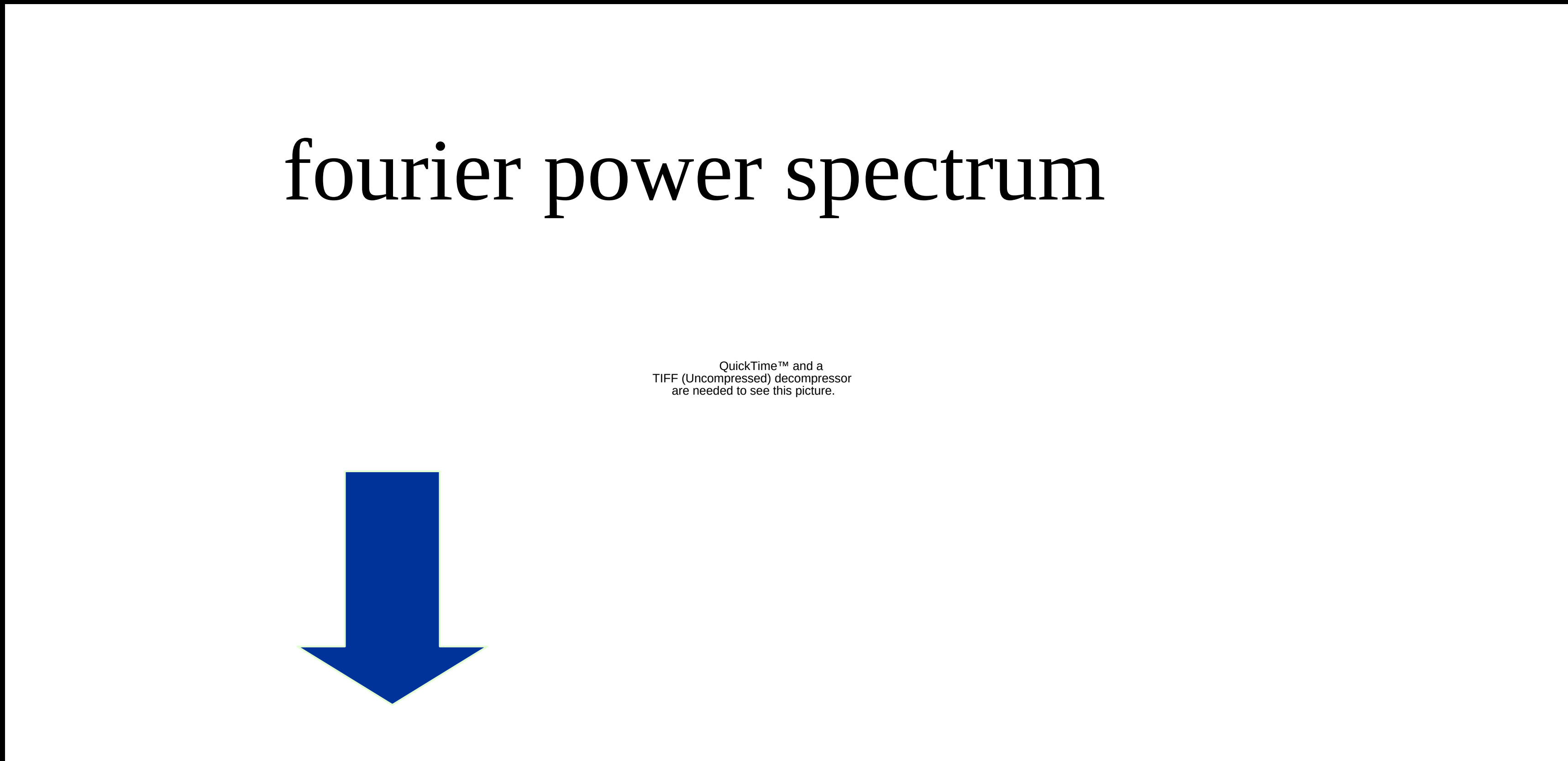
Efficiency : Let J=1…10, 000,
versus α =1…12 : *get it?*

Embed these 50 points in noise, local signal/noise=1/5, global signal/noise=1/40

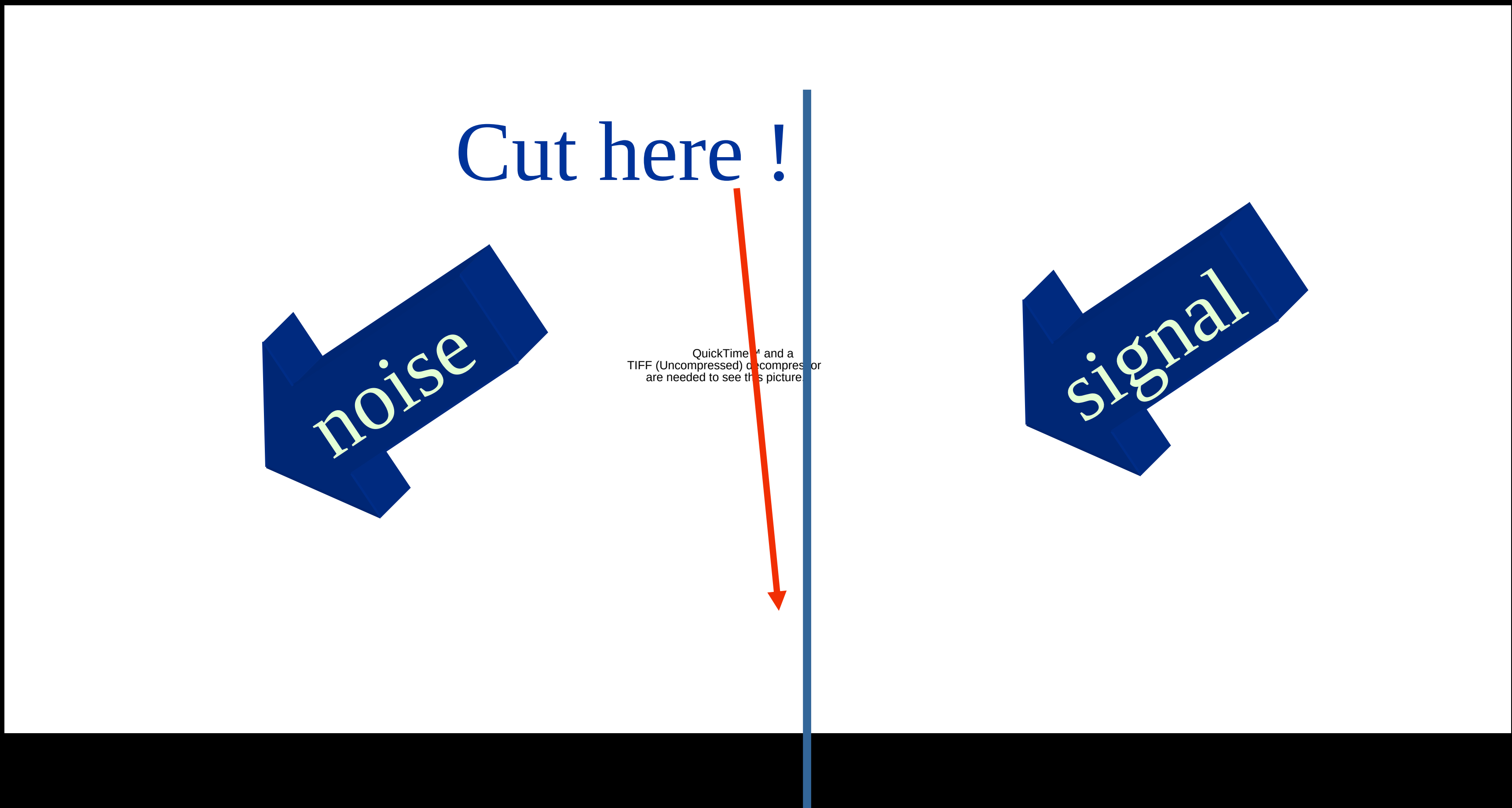
Optimal filter =1 - |noise><noise|

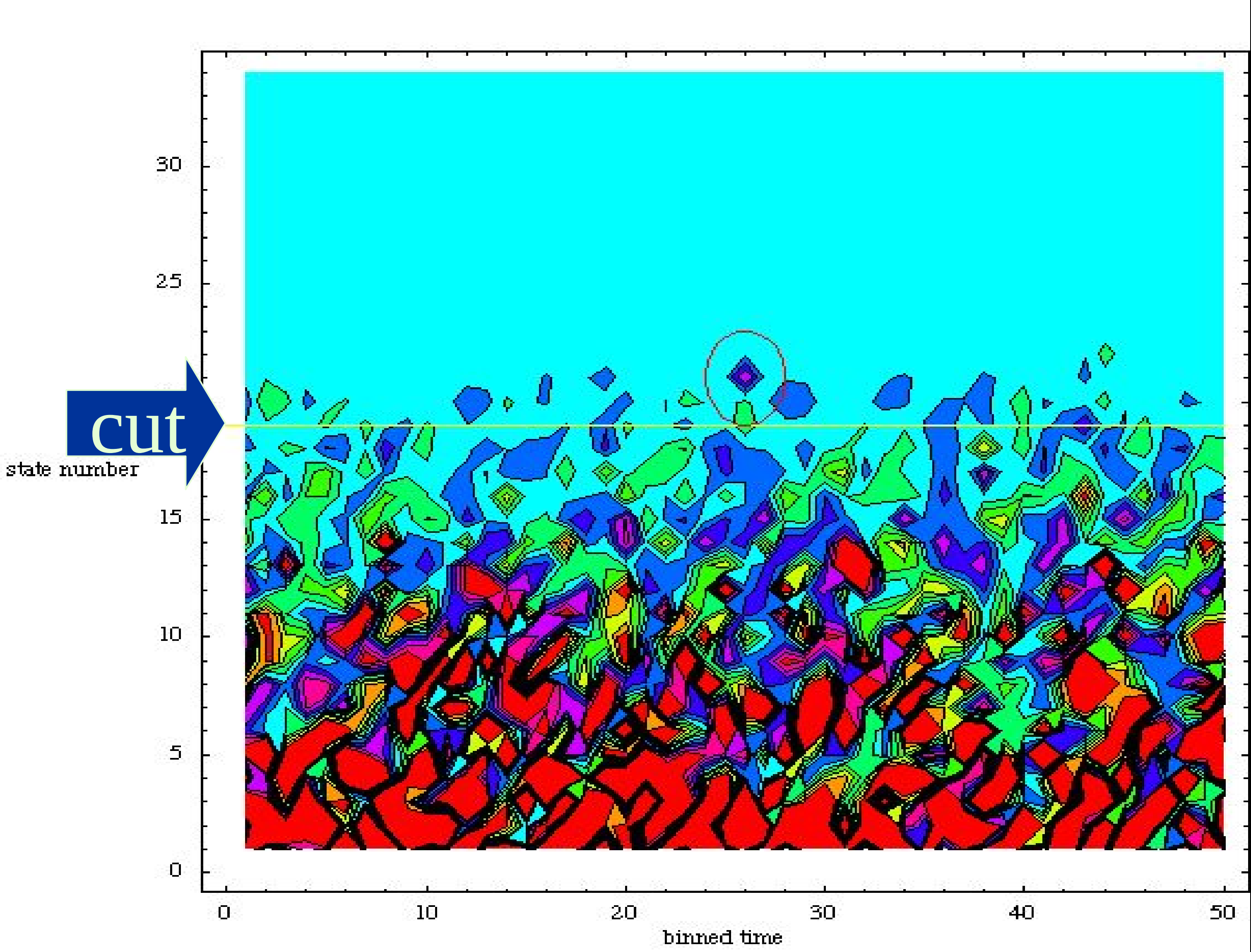
- A fairly stupid pattern of correlation, but it happens to not be random

Look at the Fourier power, a common basis

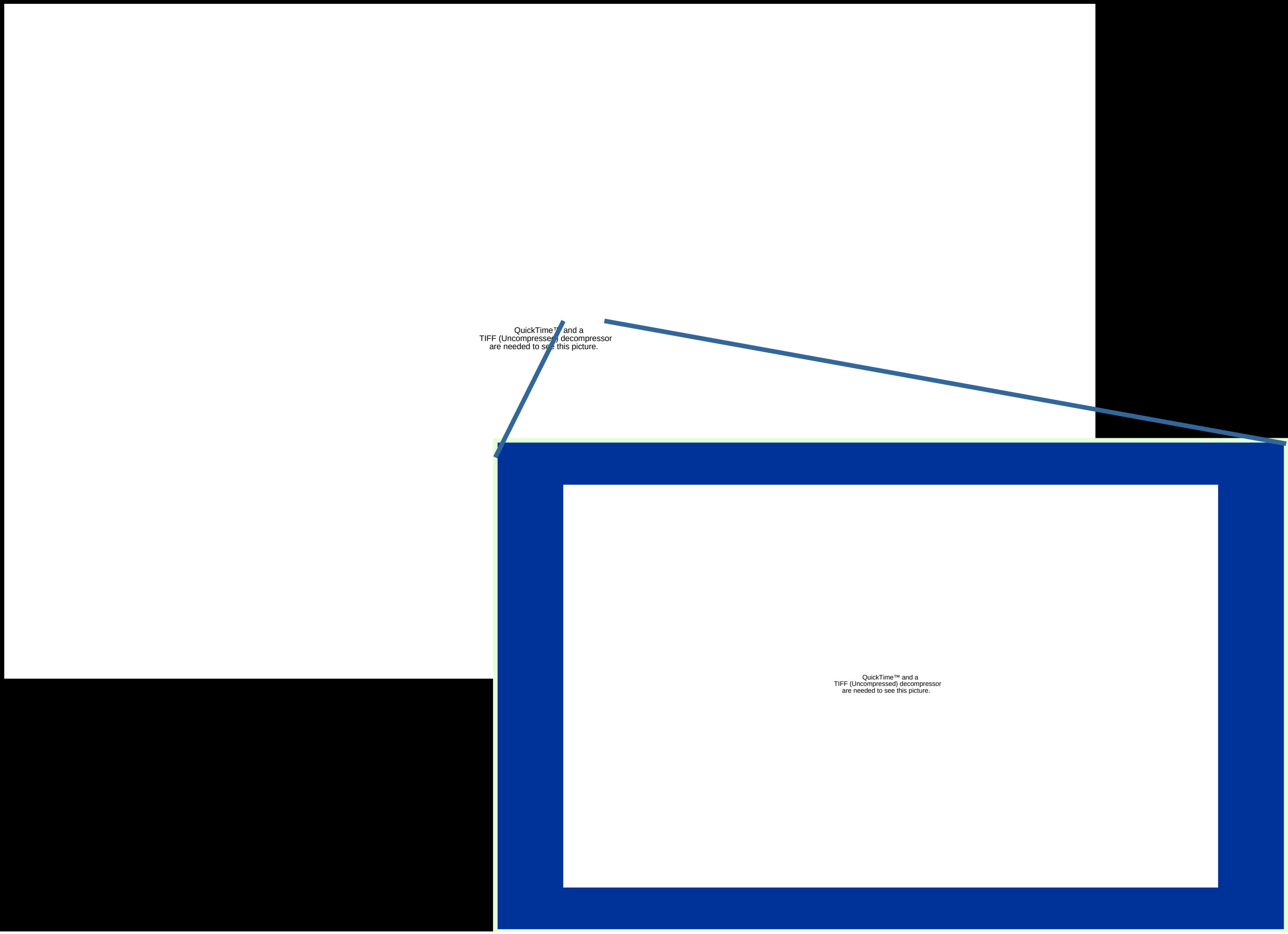


Noise spectrum *in density matrix basis* is most compact noise segregation




cut
state number
30
25
15
10
5
0
0
10
20
30
40
50
binned time

filtered projection of the data



application: FNAL T926

- April '04 Radio-Cherenkov test beam with 100 GeV protons, 500/bucket, 8 buckets, 53MHz, 3 cm antennas, coincidence + phototubes, 40 dB Gain, sampled at 1.6 x 10^{-10} s, GHz bandwidth, 20 mV rms

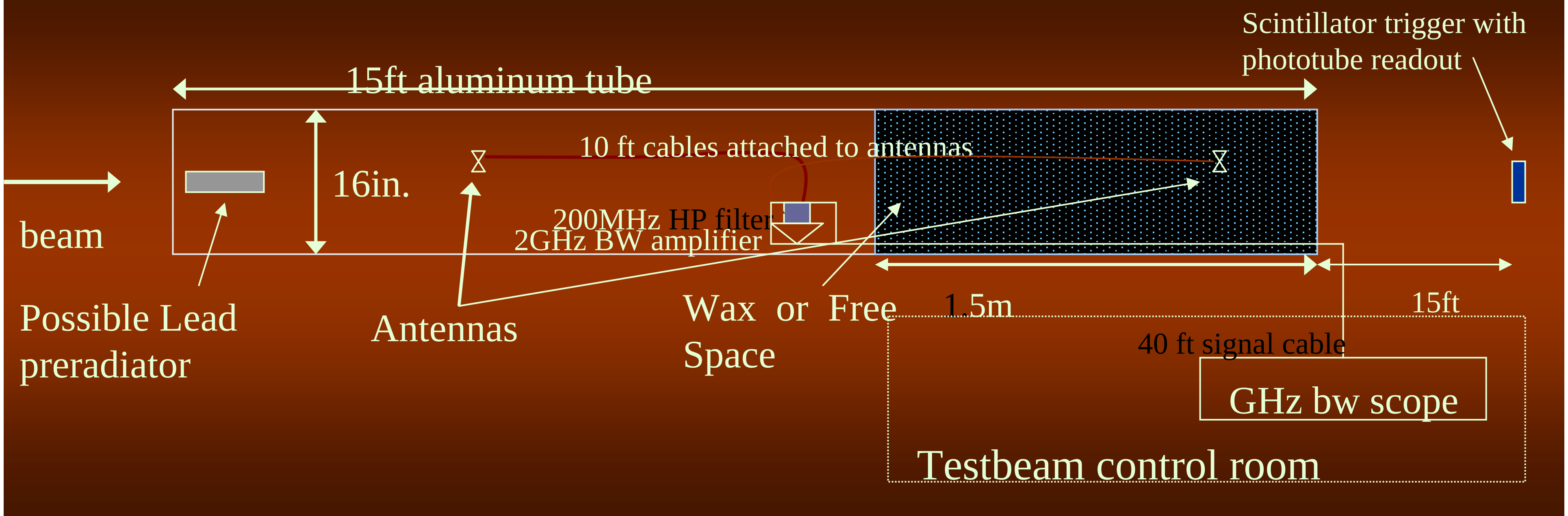
• A. Bean et al;

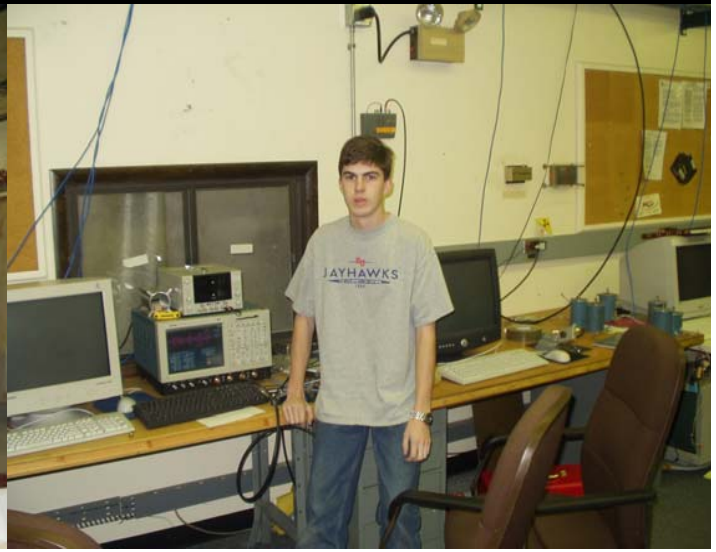
• MTEST fnal E. Ramberg

•S/N <<1 *Noise dominated*.

•Take average 400 runs -> *gain 20 in S/N*…not enough

Testbeam setup at MTest



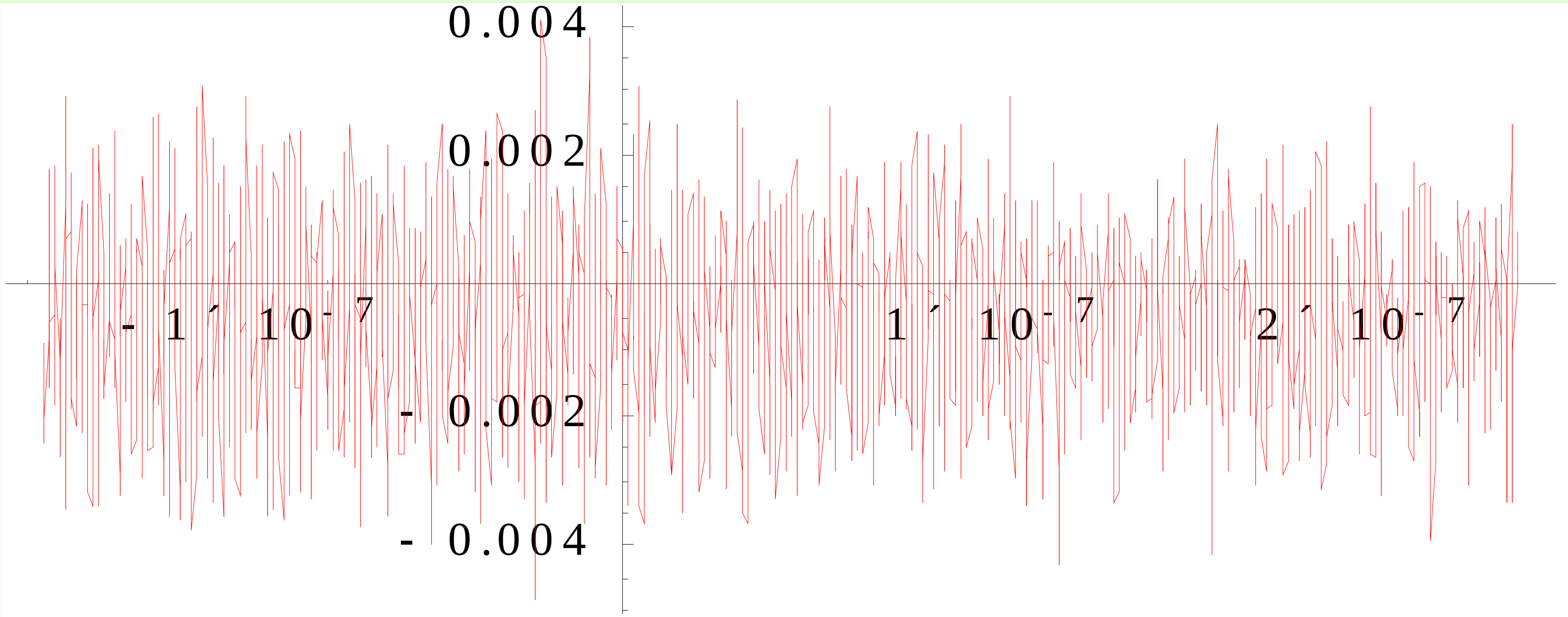


a noisy average signal…

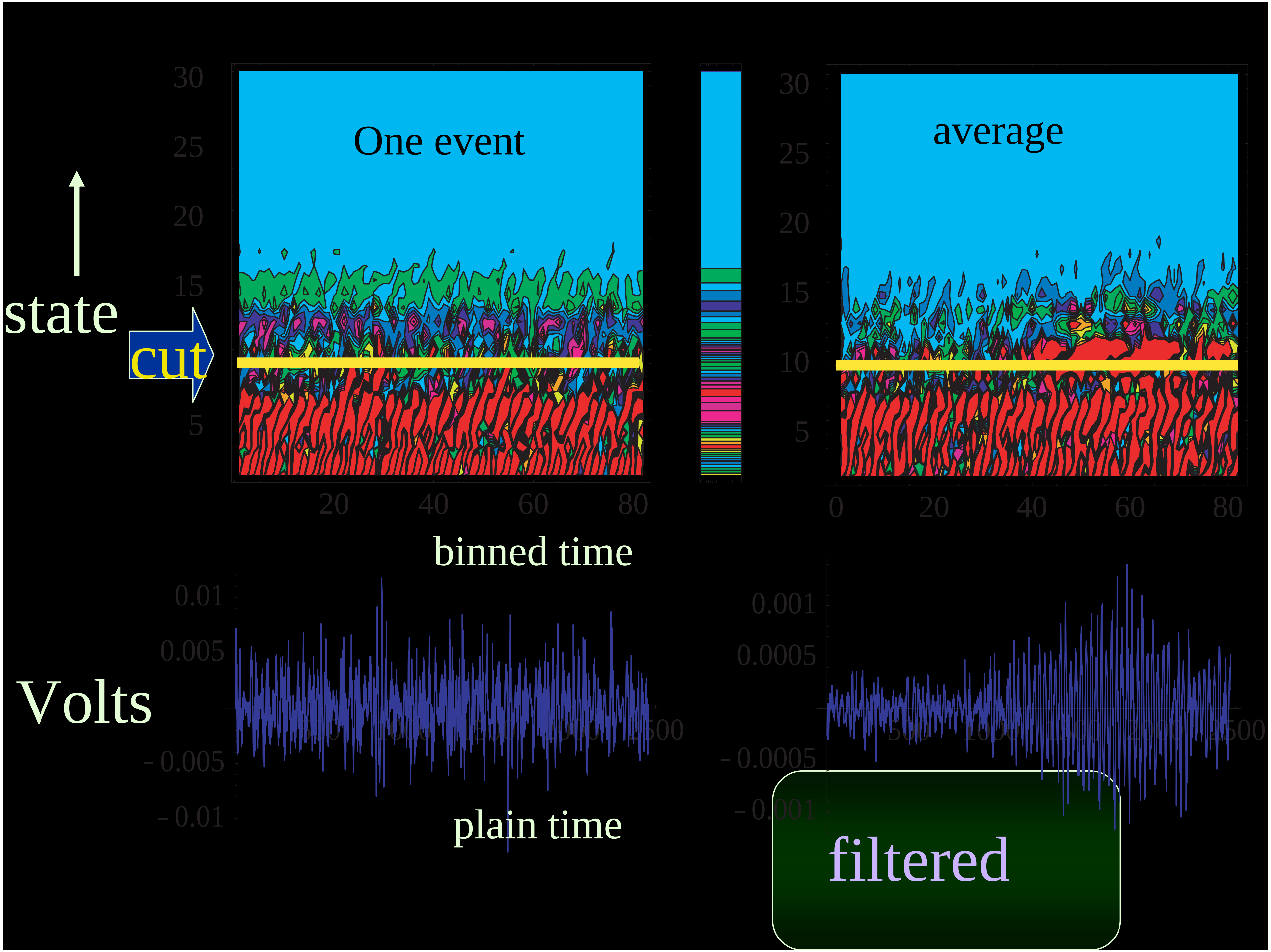
One event   rms noise O(30mV)

Average ~ 400 events   → reduce uncorrelated noise

rms noise O(1.5mV)



NEED SOMETHING MORE TO DIG OUT SIGNAL!



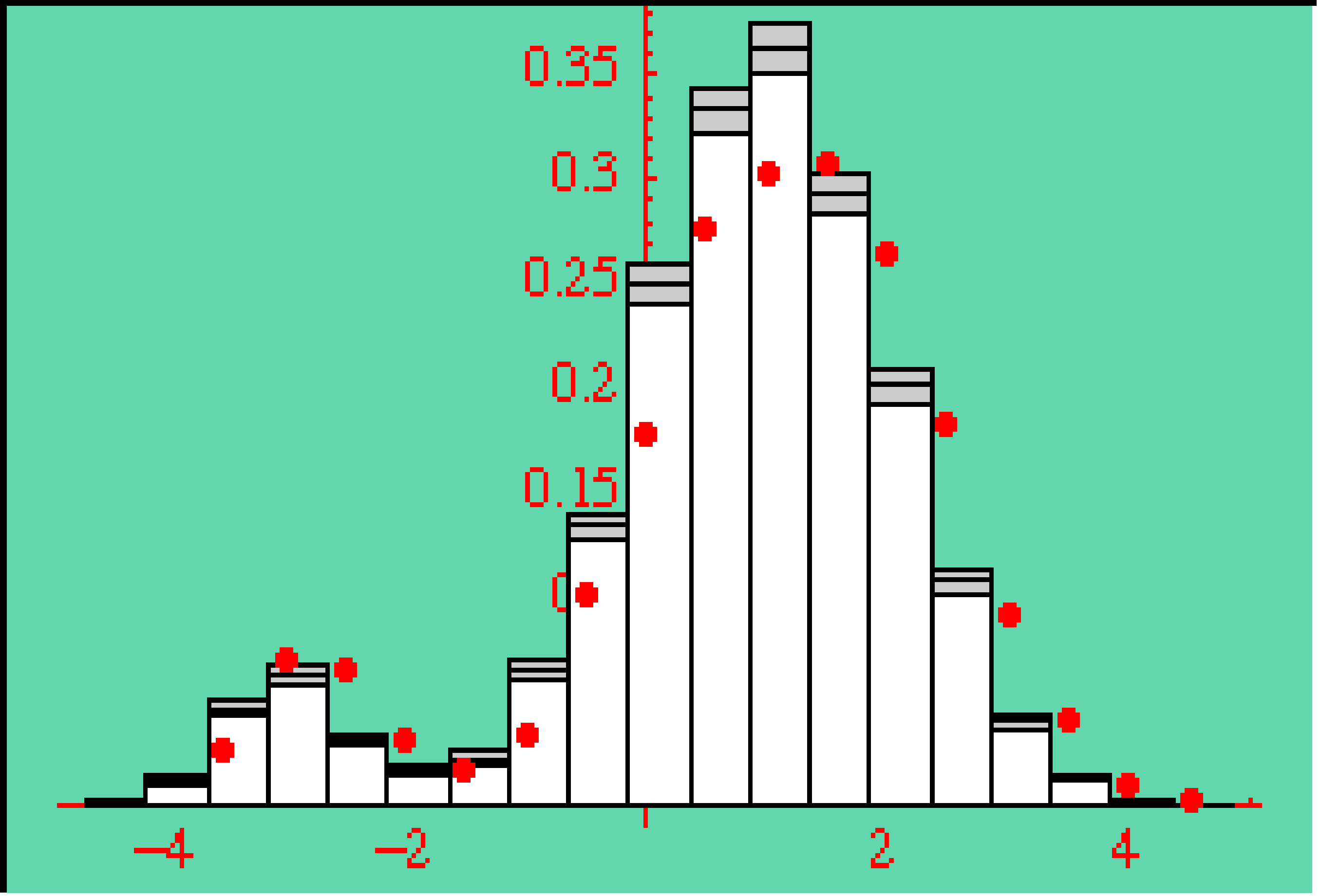
state
cut
One event
average
binned time
Volts
plain time
filtered

Back to forward multiparticle data…

data

density-matrix

histogram

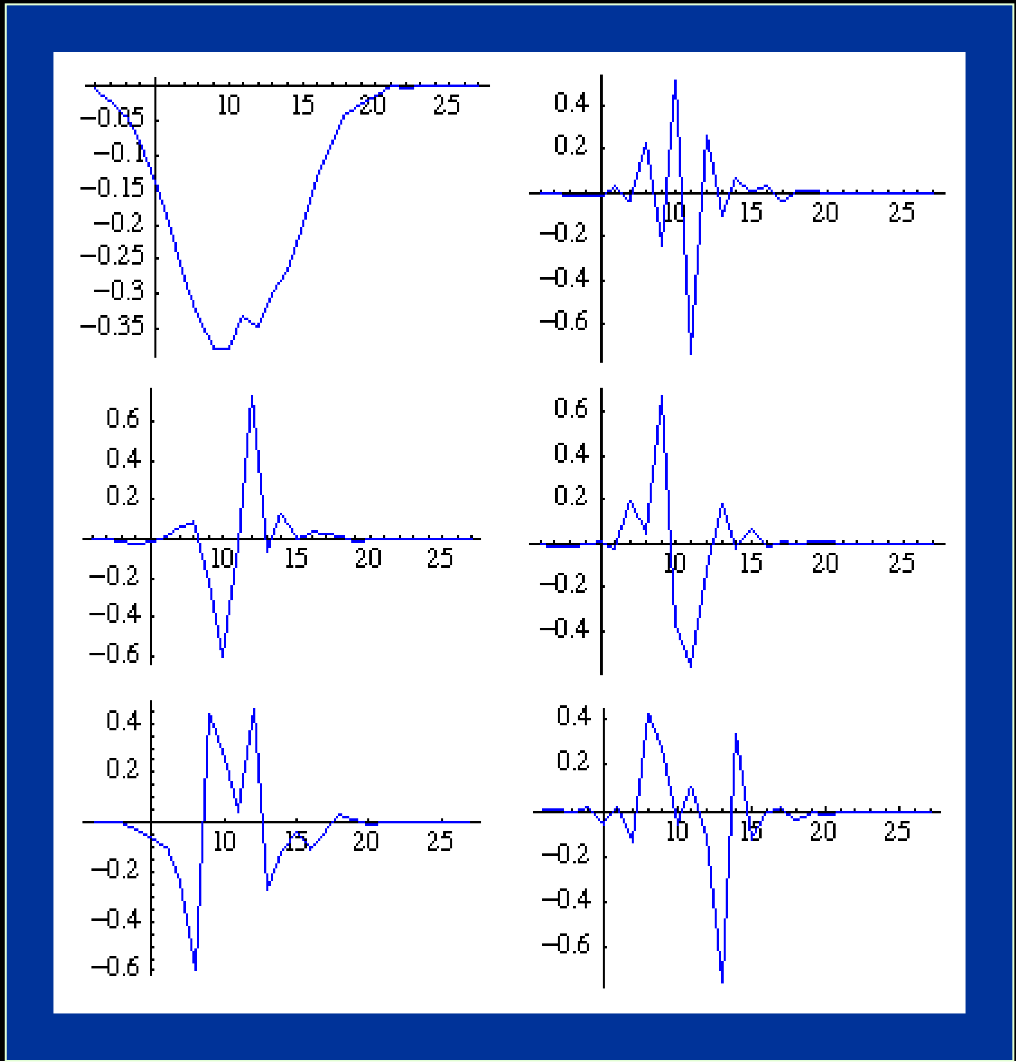


conventional and unconventional forms of probability

The first few eigenvectors of ρ, called $\psi_\alpha(k)$

- *INVARIANT, INDEPENDENT*

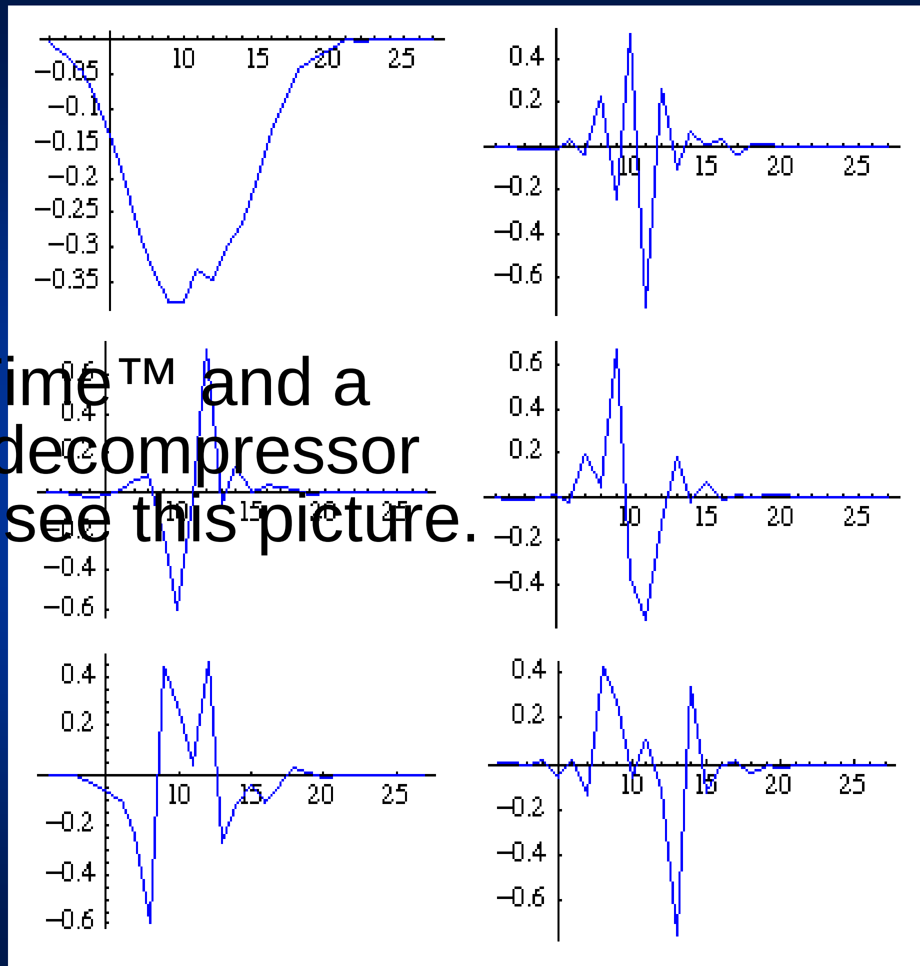
Probabilities = $|\ \psi_\alpha\ (k)\ |^2$



here are the weighted eigen-distributions

- Encodings of spatial correlations in the quantum system

And spin, of course.



Theorists might do OK on the symmetries

Fiddle empirically to find U such that

$$\rho\,' = U \rho U^{*T} = \rho$$

- Ψου ηαϖε α σψμμετρψ
- You have a symmetry

Summary overview

Classical probability inadequate;
Partons are simple density matrices;
Partons inadequate. (thanks Rudy Hwa)

More general density matrices
a proper framework;
Must be experimentally driven !

Exists by construction.

No
models
today:
your data
will drive
the
discovery