BRAHMS Meeting on Forward Physics at RHIC and LHC
The University of Kansas, Lawrence, October 22nd 2004

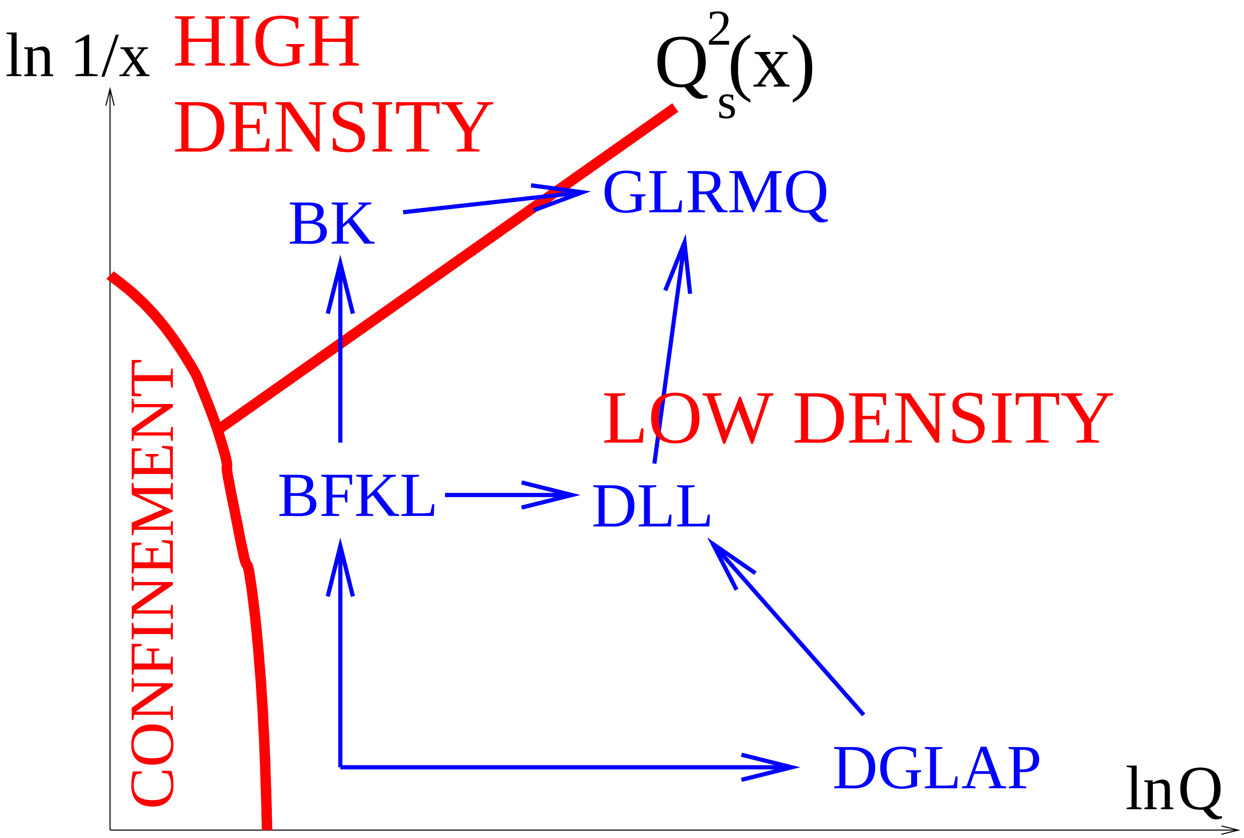
Geometric scaling: Phenomenology vs. results from BK

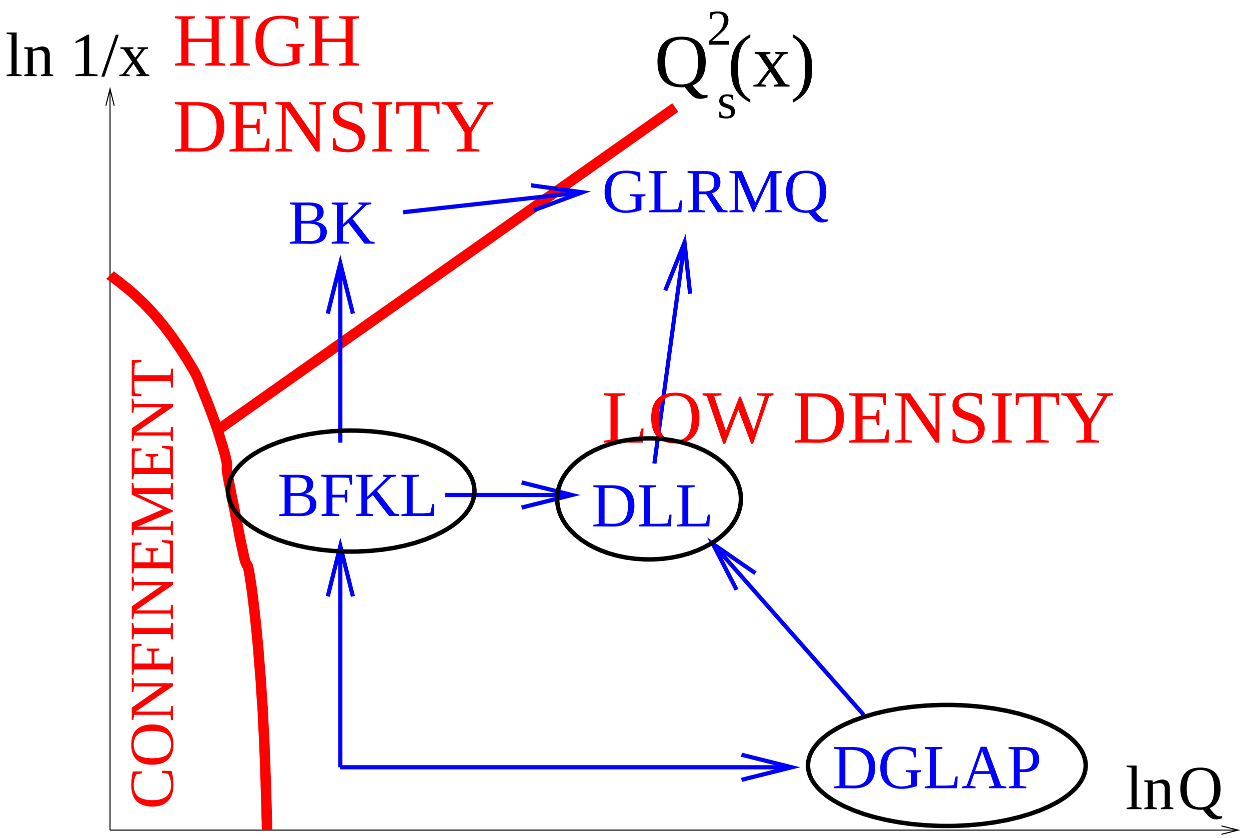
Néstor Armesto

Department of Physics, Theory Division, CERN

1. Introduction.
2. Phenomenological analysis of geometric scaling (with C. A. Salgado and U. A. Wiedemann, hep-ph/0407018).
3. Features from the BK equation (with J. L. Albacete, J. G. Milhano, C. A. Salgado and U. A. Wiedemann, hep-ph/0408216).
4. Conclusions.

The aim: regimes of QCD





- Linear evolution equations: Dokshitzer-Gribov-Lipatov-Altarelli-Parisi in $\ln Q^2$, Balistky-Fadin-Kuraev-Lipatov in $\ln{(1/x)}$.

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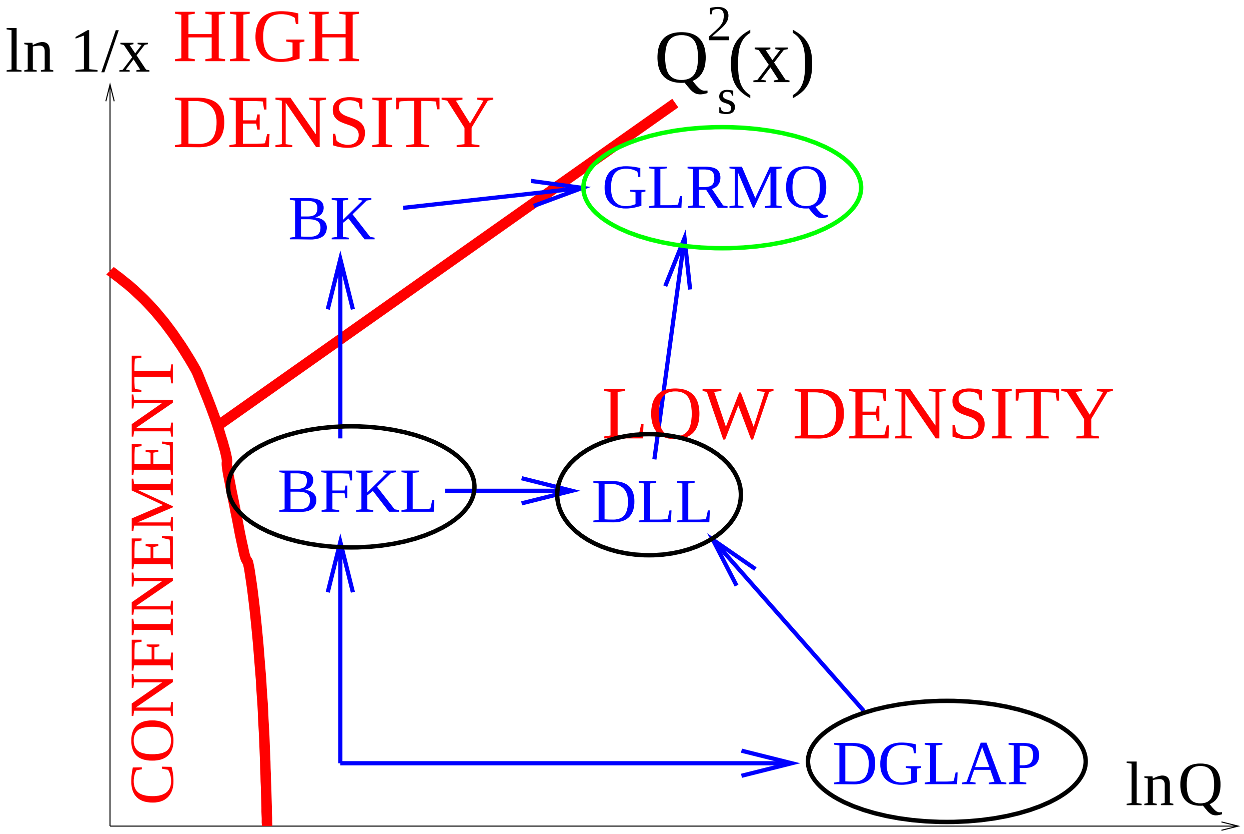

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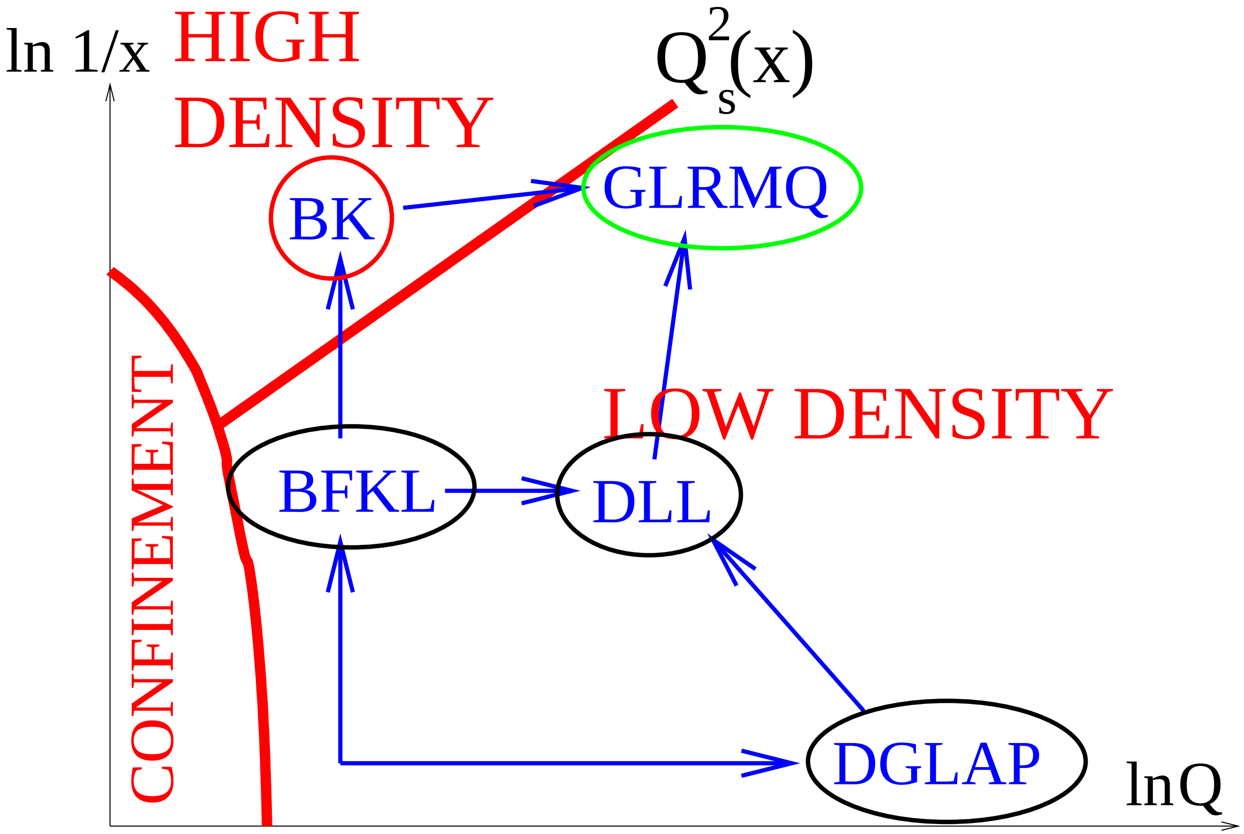

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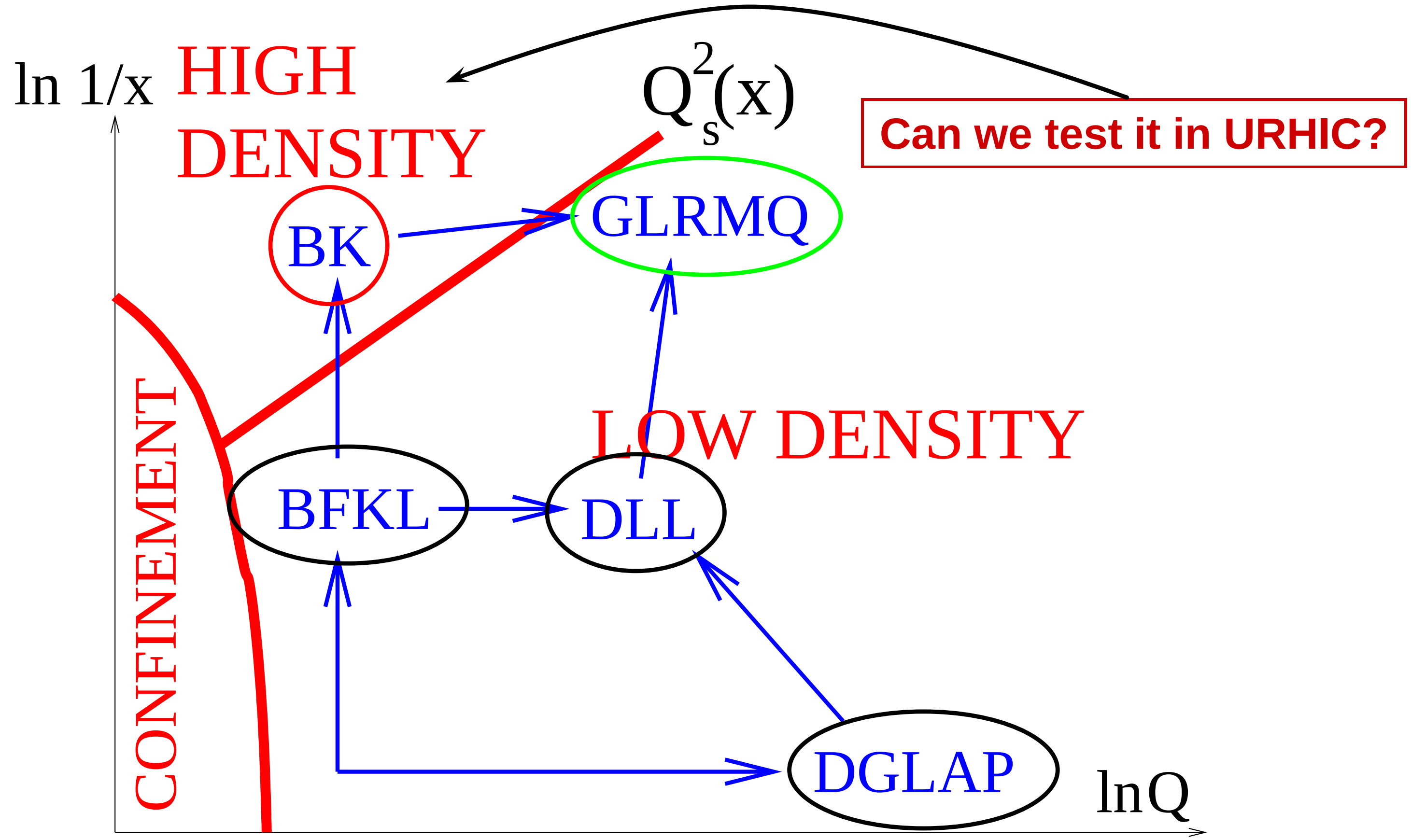

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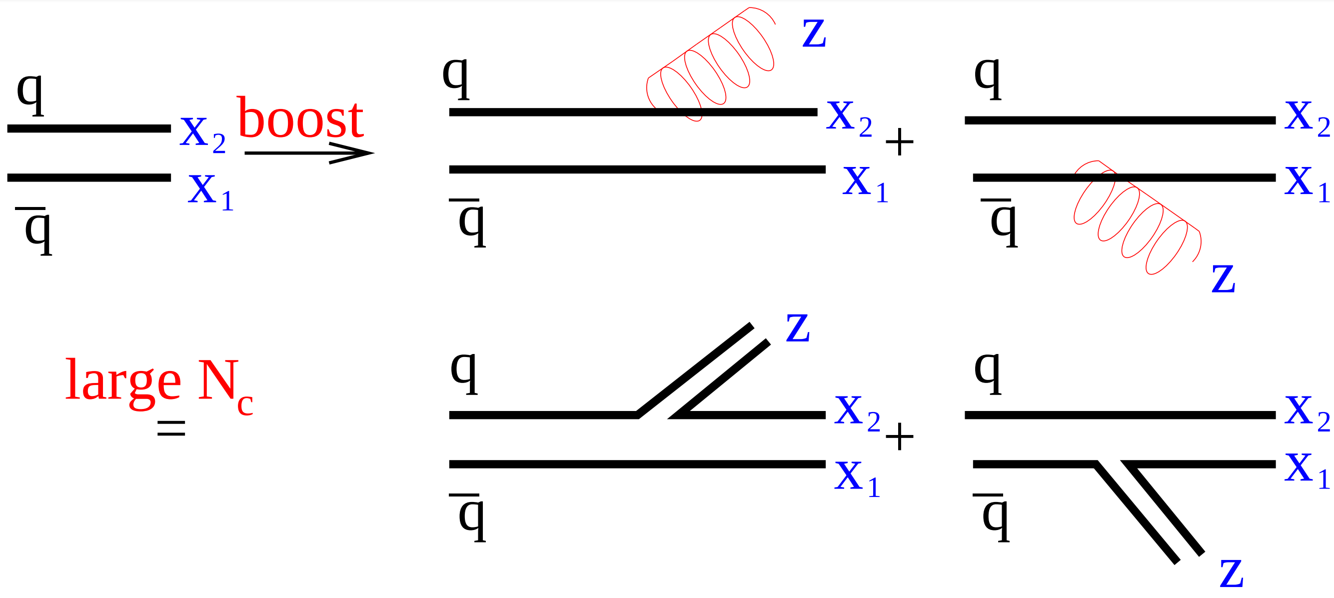
Our tool: BK equation

BK equation:



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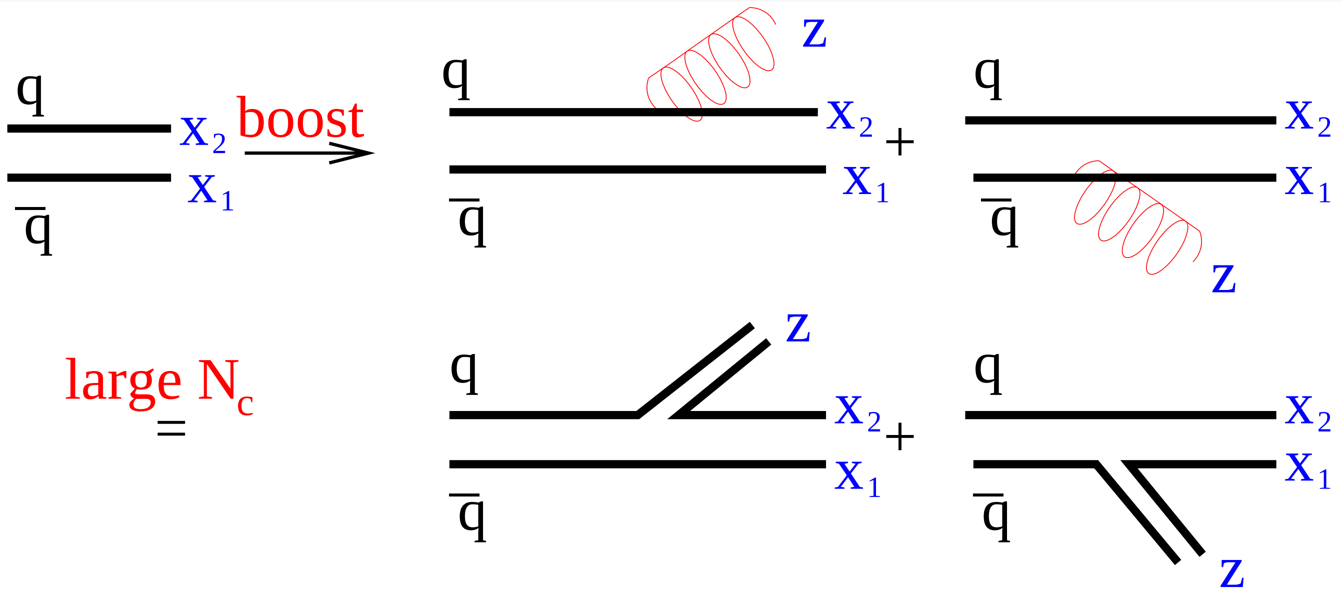

$$Y = \ln(x_0/x), \quad \bar{\alpha}_s = \alpha_s N_c/\pi, \quad y = \bar{\alpha}_s Y, \quad r = x_1 - x_2, \quad b = (x_1 + x_2)/2;$$

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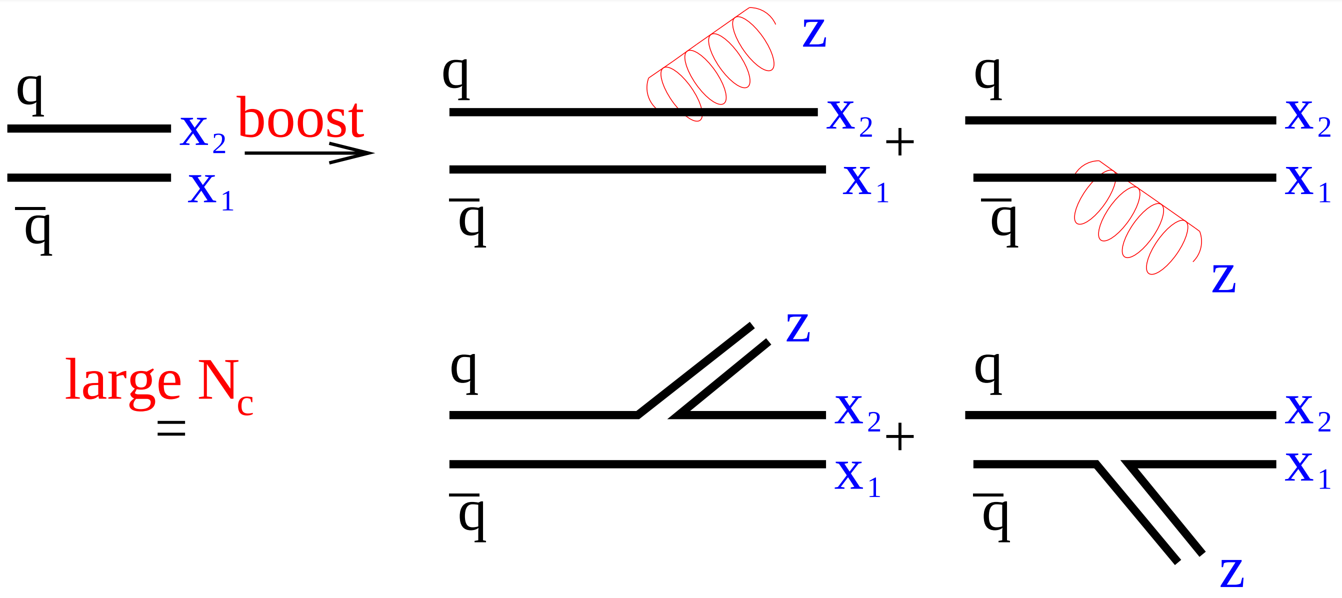

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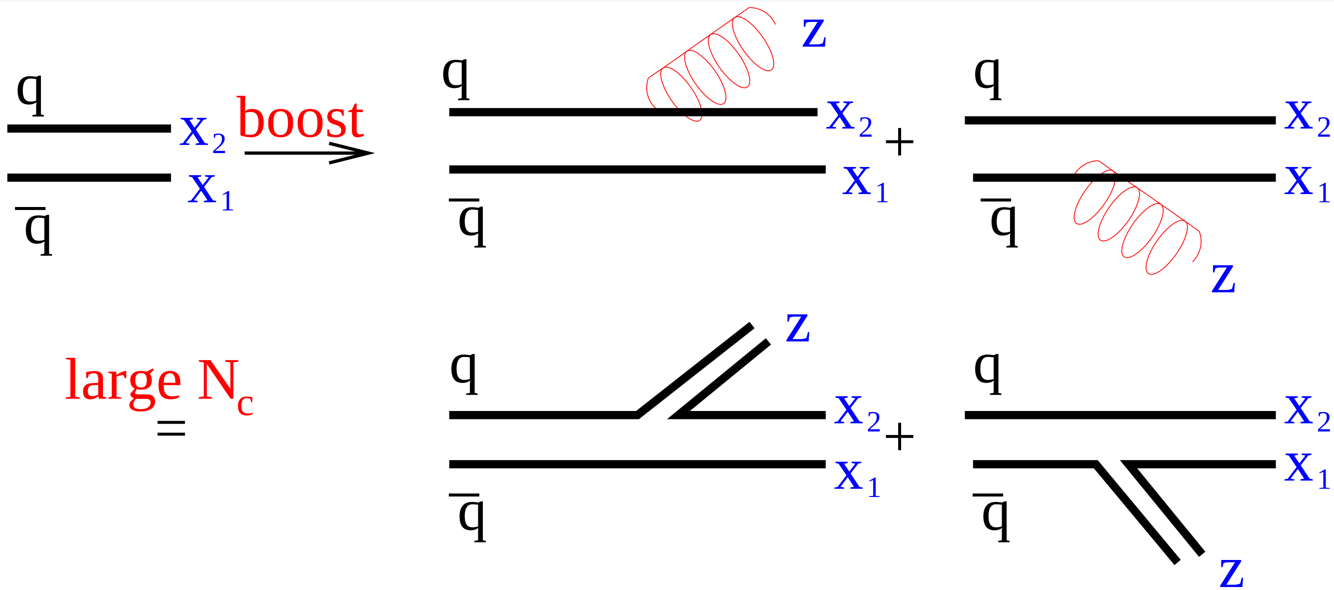

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$$\times[N(x_1,z)+N(z,x_2)-N(x_1,x_2)$$

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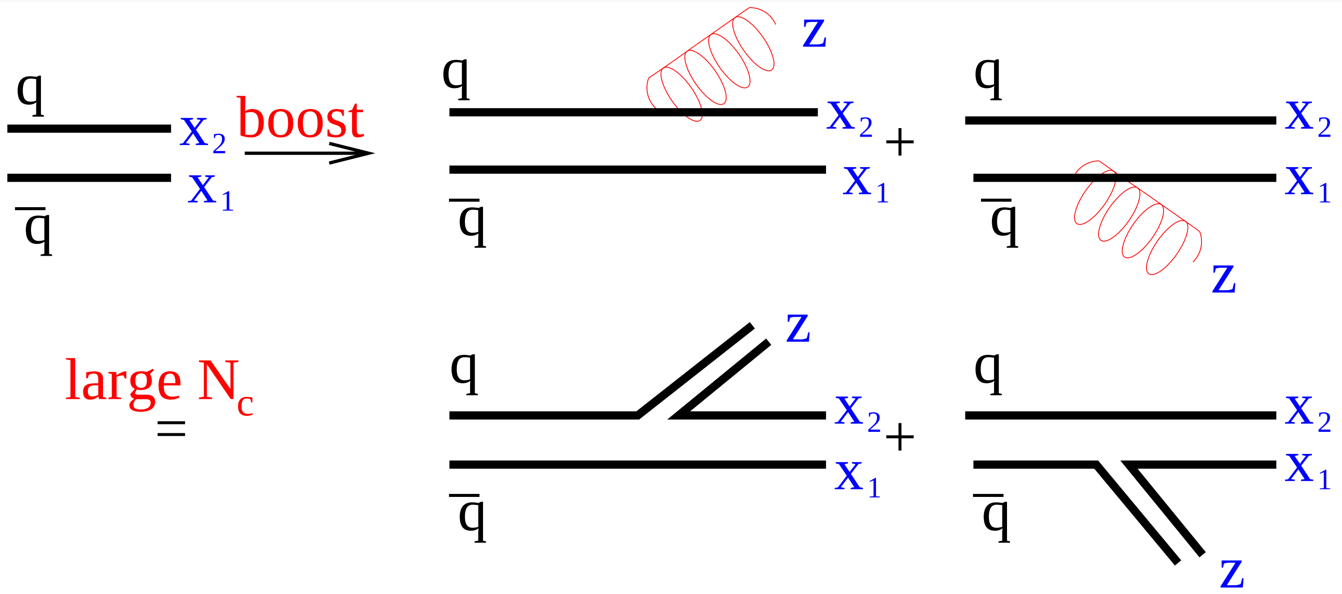

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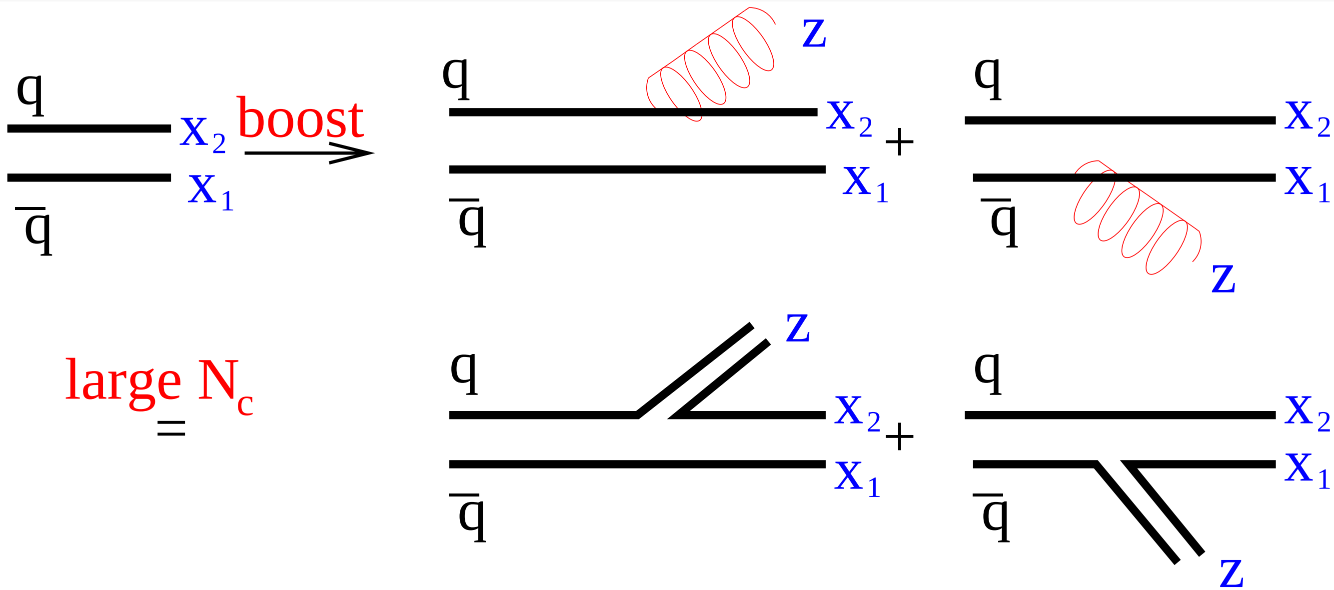
BK equation:



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$$V_F(x_1, \ldots 0) = \mathcal{P}\, e^{ig\int dz_- T^a A^+_a(x_1, z_-)},$$

α_s fixed (LL); make it running, try modifications of the kernel to mimic NLL effects.

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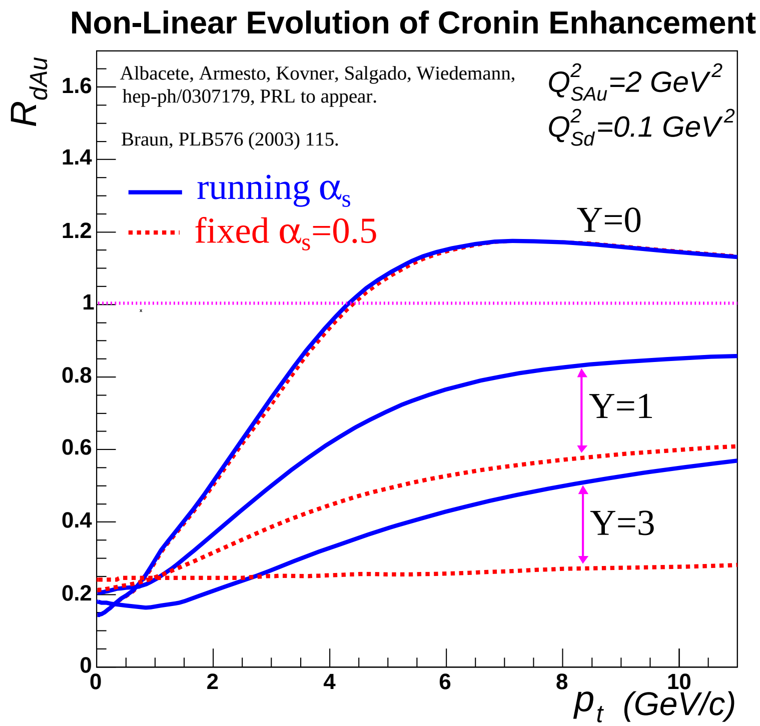
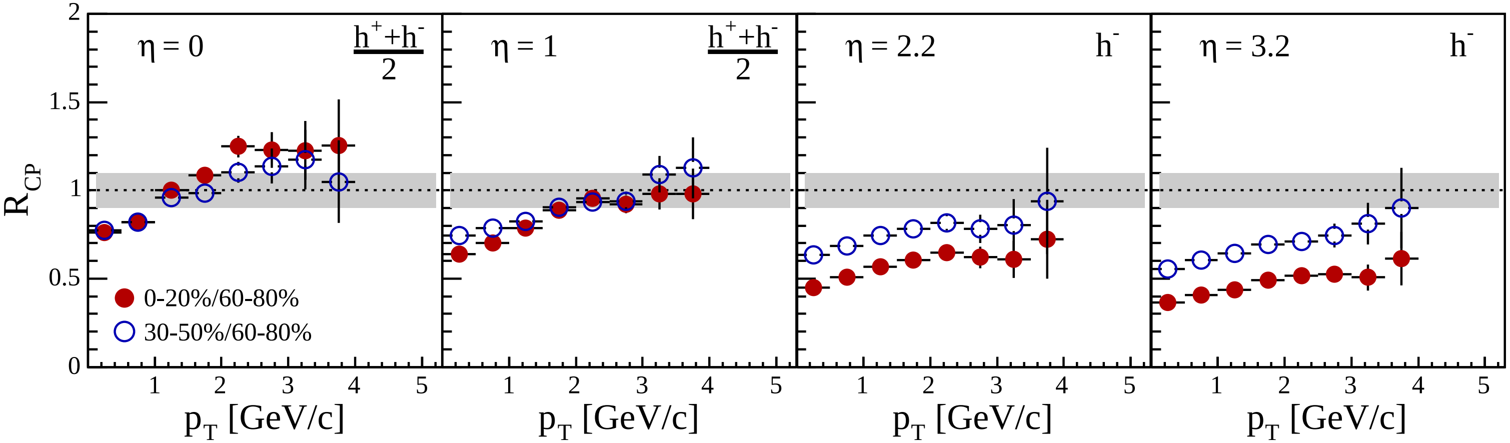
A signal: BK evolution of Cronin

- Evolution erases the Cronin effect present in the initial condition (AAKSW, PRL92(04)082001; BKW, PRD68(03)054009; KKT, PRD68(03)094013; KLM, PLB561(03)93) $\Longrightarrow$ disappearance at forward rapidities (BRAHMS Coll., nucl-ex/0403005).





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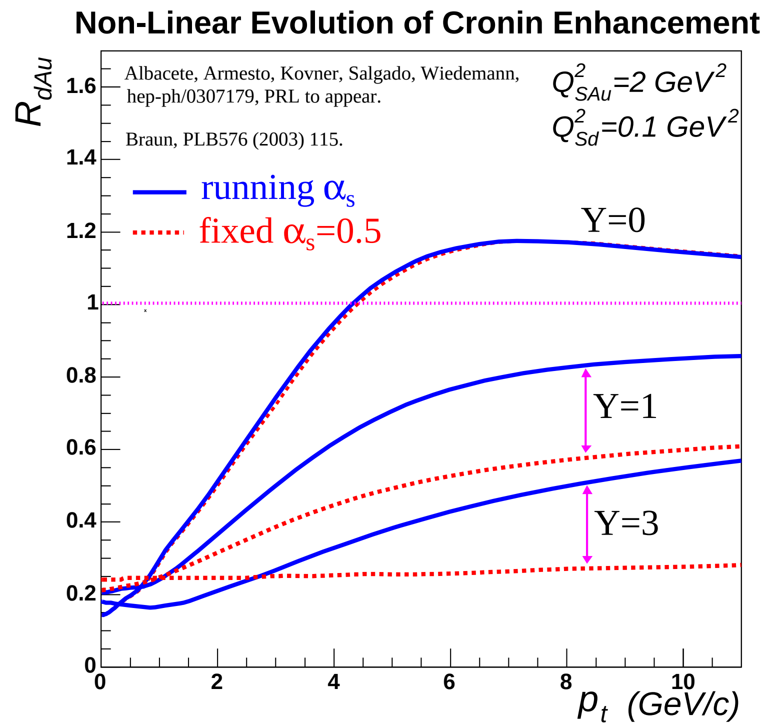

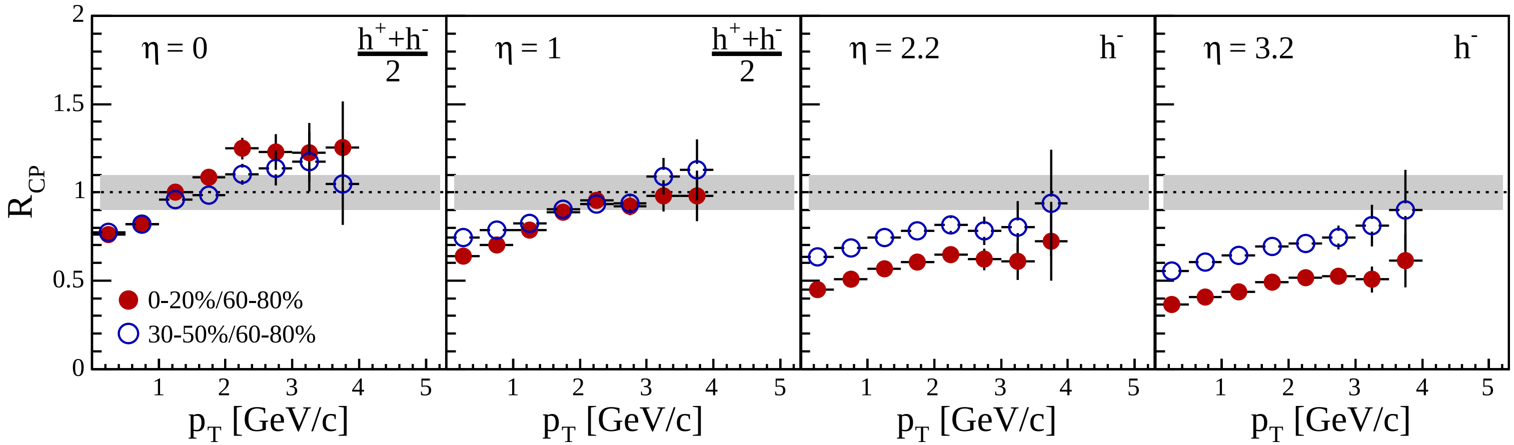

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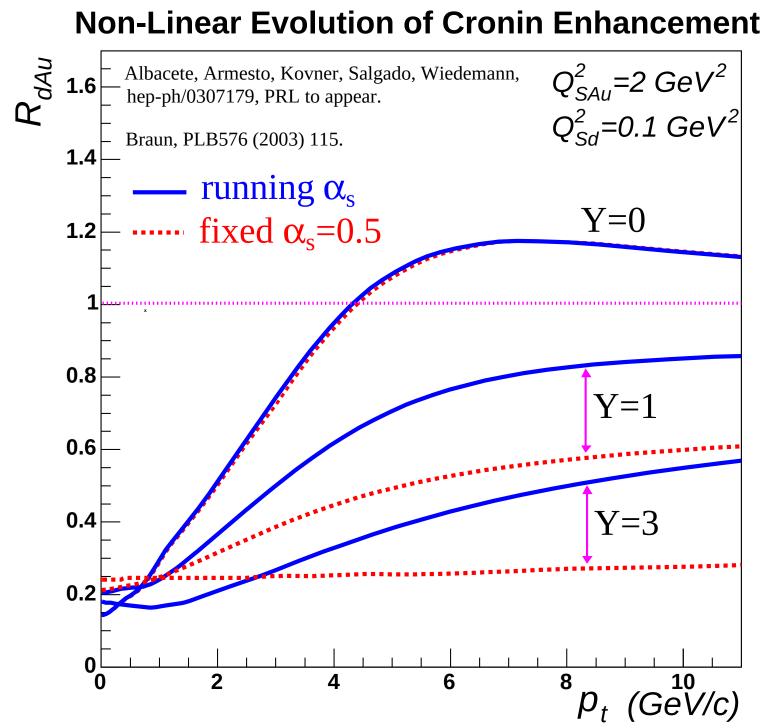

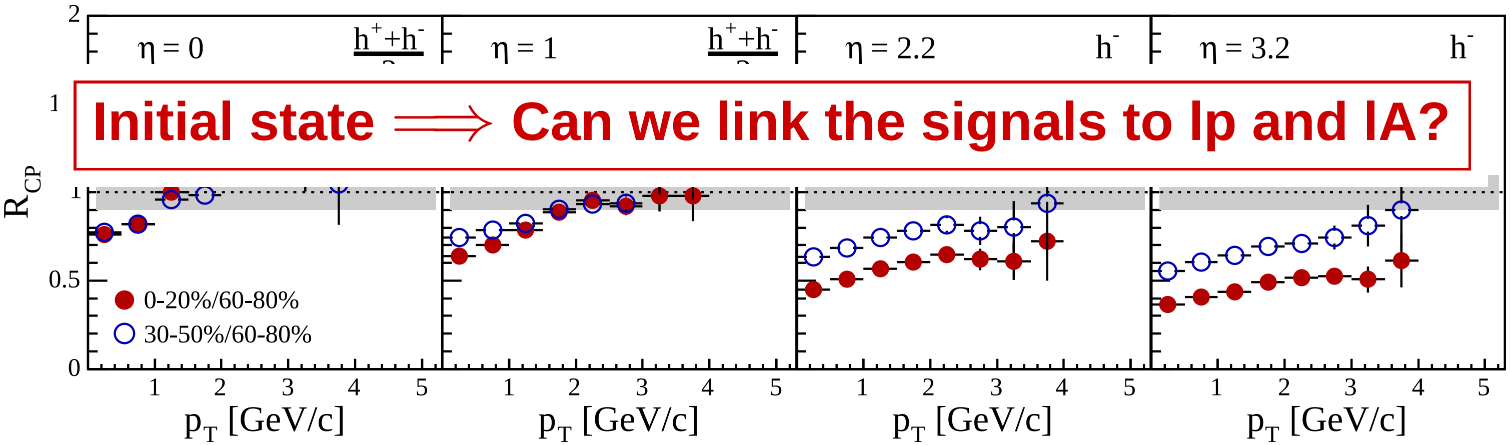

Initial state $\Longrightarrow$ Can we link the signals to lp and lA?

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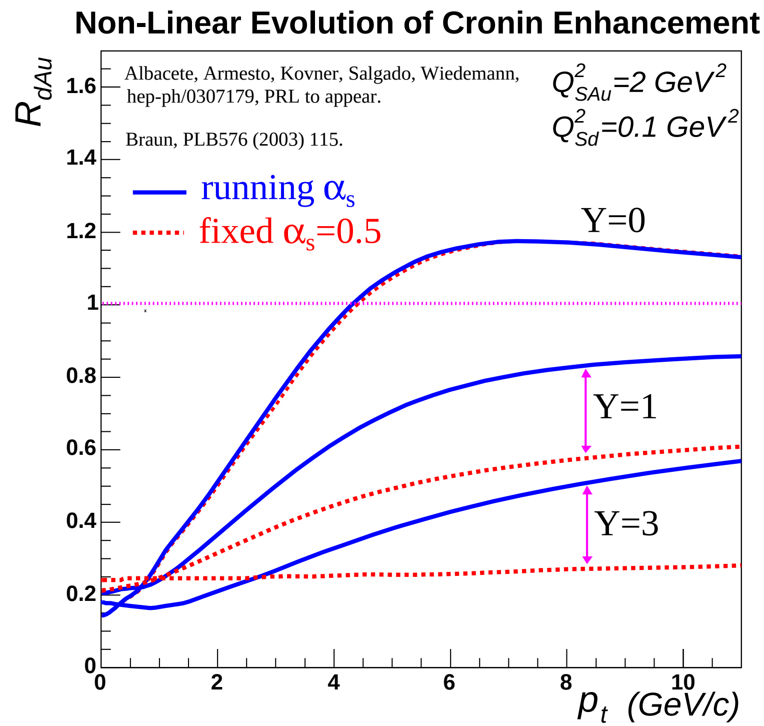

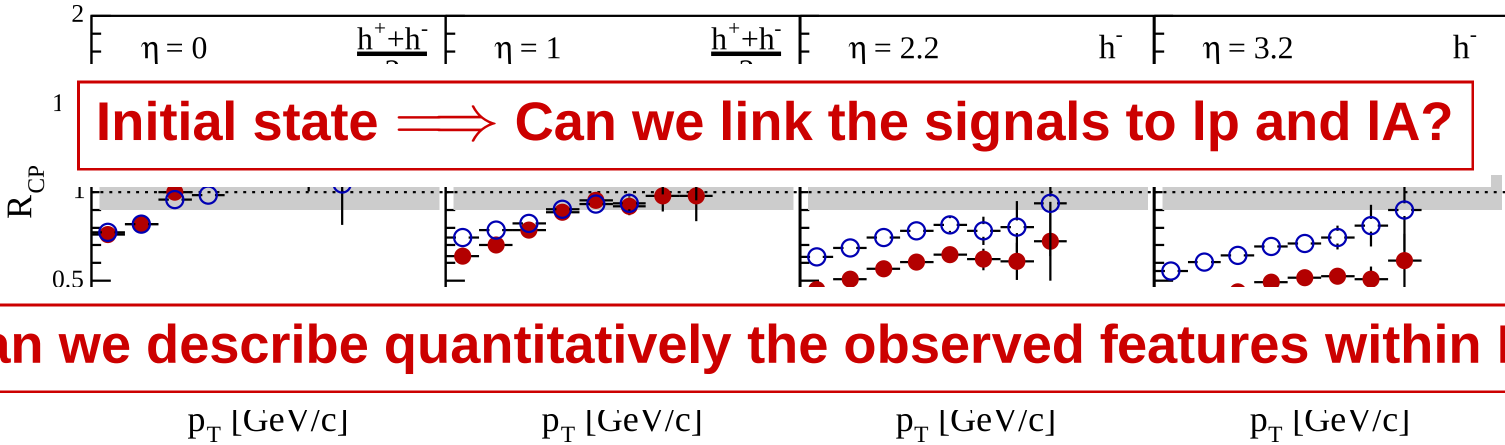

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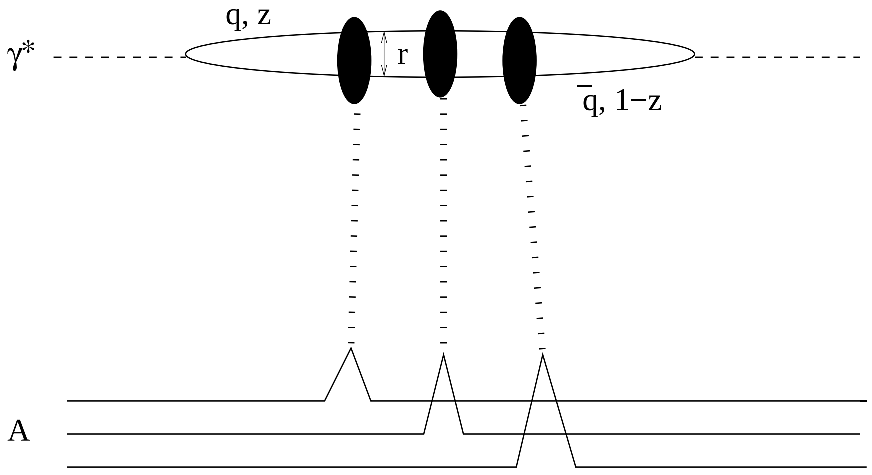
Can we describe quantitatively the observed features within BK?

2. Phenomenological analysis of geometric scaling

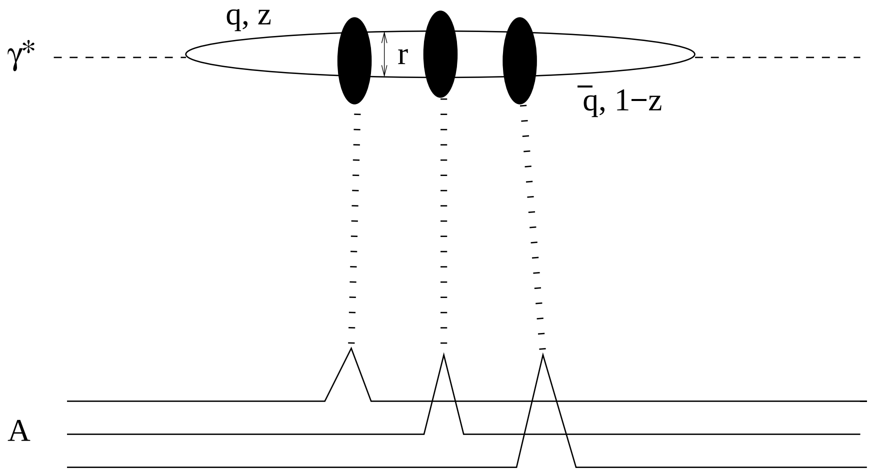
(with C. A. Salgado and U. A. Wiedemann, hep-ph/0407018)

- Scaling in lp and lA.
- Multiplicities in AA.
- Ratios at forward rapidities in pA.

Scaling in lp and lA (I)

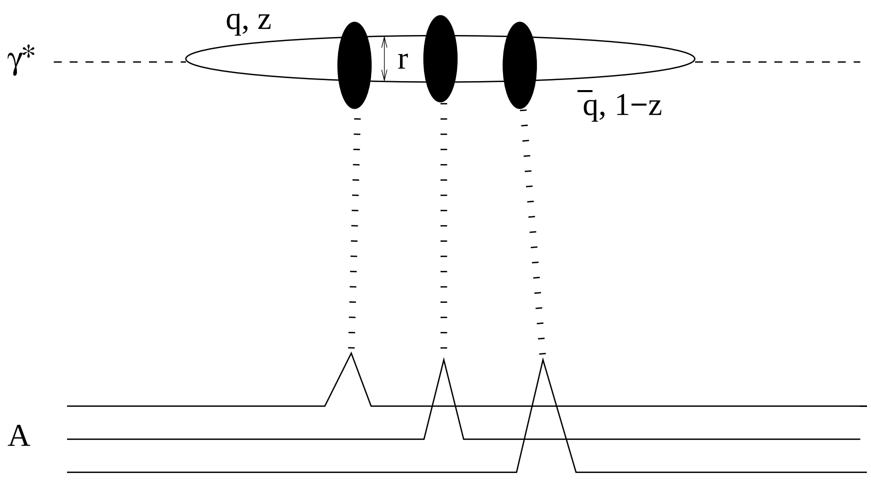


- Geometric scaling in lp collisions for $x \lesssim 0.01$ (SGBK, PRL86(01)596) strongly suggests saturation.



- Geometric scaling in lp collisions for $x \lesssim 0.01$ (SGBK, PRL86(01)596) strongly suggests saturation.
- It is best discussed within the dipole model.

$$\sigma^{\gamma^* h}_{T,L}(x,Q^2) = \int d\mathbf{r} \int_0^1 dz |\Psi^{\gamma^*}_{T,L}(Q^2,\mathbf{r},z)|^2 \underbrace{2\int d\mathbf{b}\, N_h(\mathbf{r},x;\mathbf{b})}_{\sigma^h_{\rm dip}(\mathbf{r},x)} .$$



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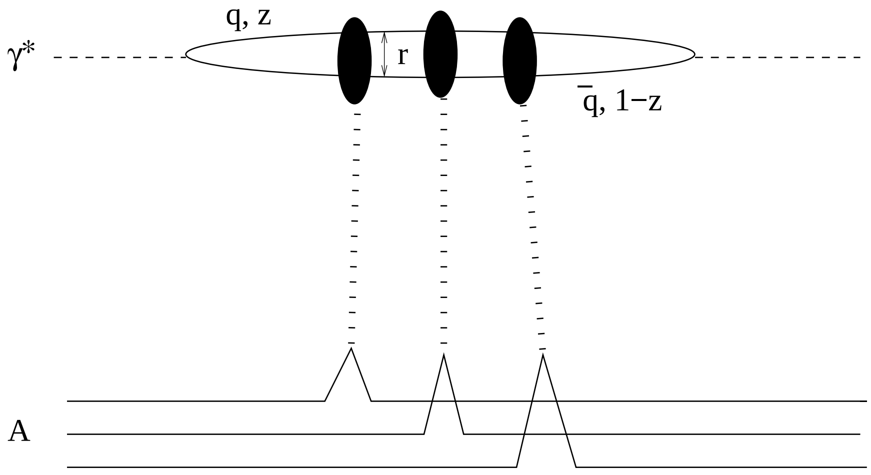
$$\text{Scaling} \Longrightarrow N_h(\mathbf{r},x;\mathbf{b}) \equiv N(rQ_{\rm s,p}(x,\mathbf{b}));\ \ \bar{\mathbf{b}} = \mathbf{b}/\sqrt{\pi R_h^2} \Longrightarrow$$



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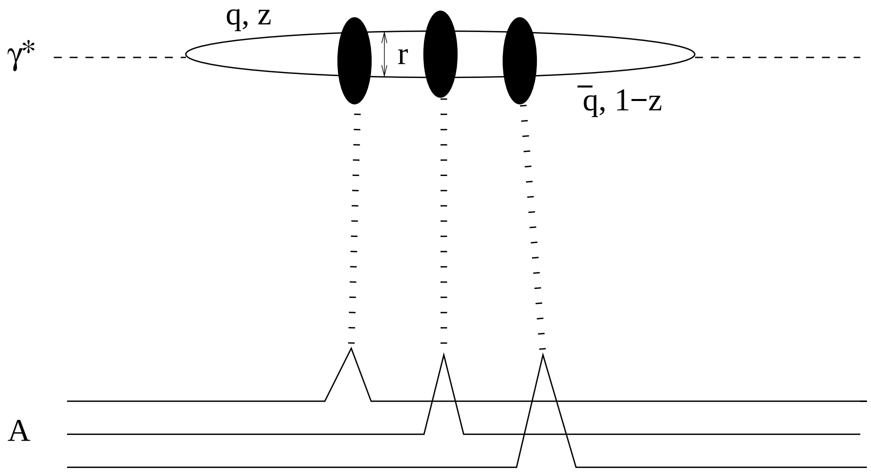
Scaling in lp and lA (I)



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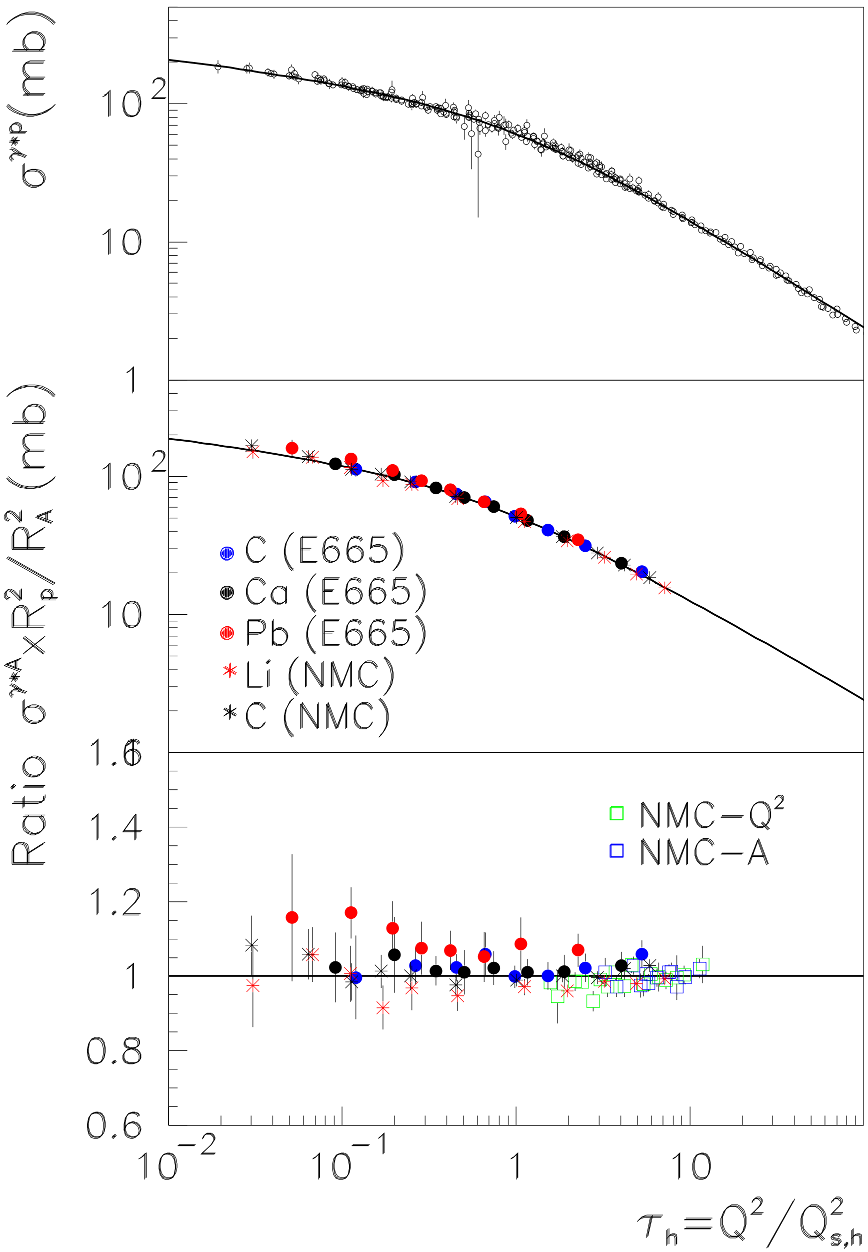
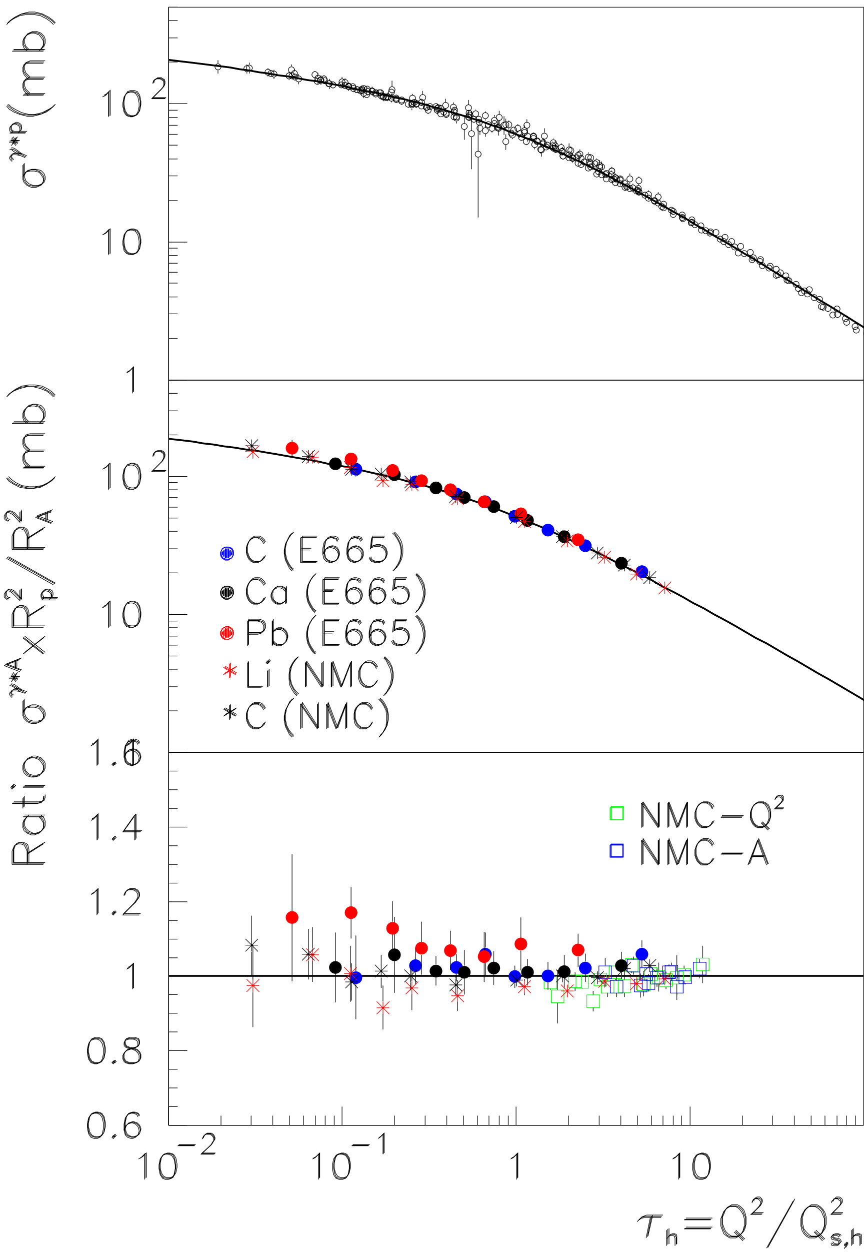
$$|\Psi_{T,L}^{\gamma^*}|^2 \overset{m_f=0}{\equiv} Q^2 f(\mathbf{r}Q,z) \Longrightarrow \sigma_{T,L}^{\gamma^* h} \equiv g\left(\tau_h = \frac{Q^2}{Q_{\rm s,h}^2(x) \equiv \langle Q_{\rm s,h}^2(x,\bar{\mathbf{b}})\rangle_{\bar{\mathbf{b}}}}\right) .$$

Scaling in lp and lA (II)



Scaling in lp and lA (II)

- $Q^2_{\rm s,p} = (\bar{x}/x_0)^{-\lambda}$ in GeV^2, $x_0 = 3.04 \cdot 10^{-4}$, $\lambda = 0.288$, $\bar{x} = x\left(\frac{Q^2+4m_f^2}{Q^2}\right)$, $m_f = 0.14$ GeV (GBW, PRD59(99)014017) $(x < 0.01)$.



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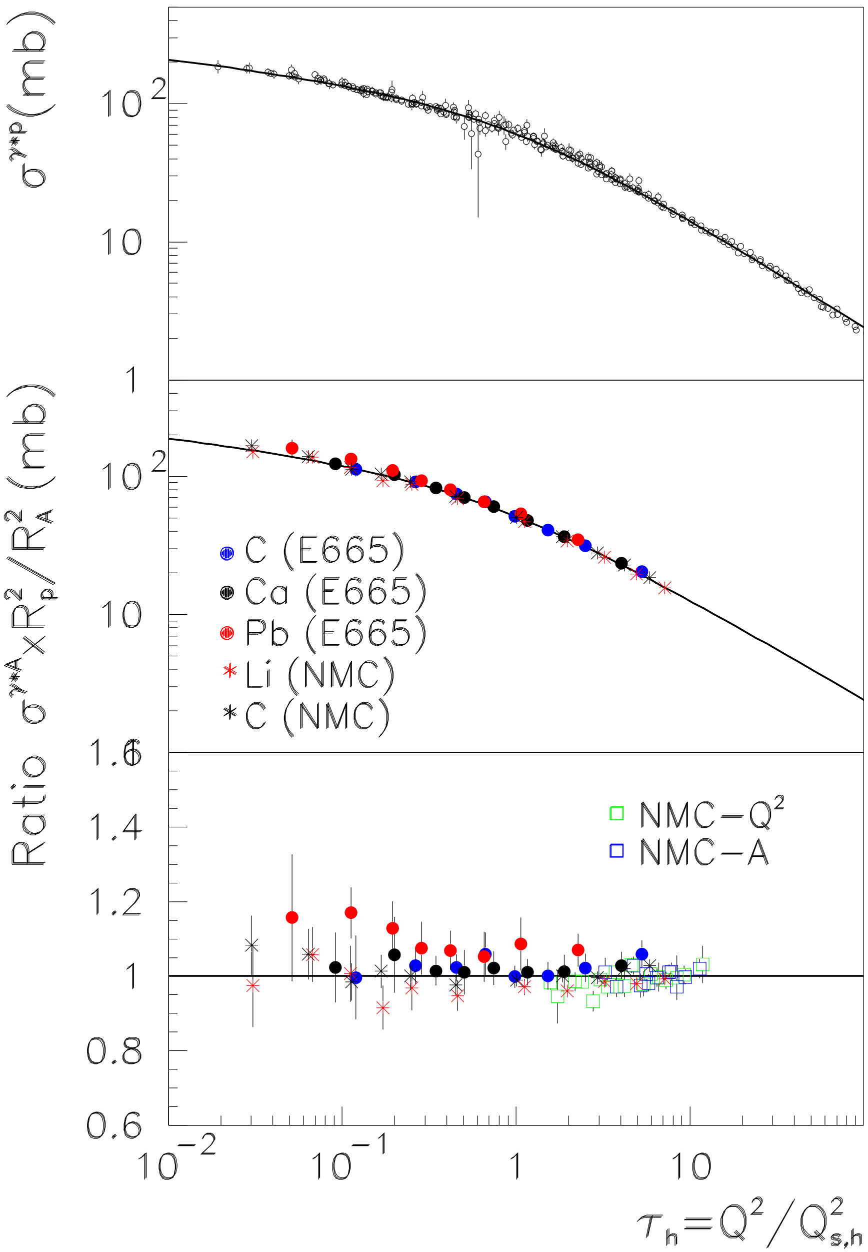
- $\sigma^{\gamma^* p}(x, Q^2) \equiv \Phi(\tau_p)$ parameterized in lp.



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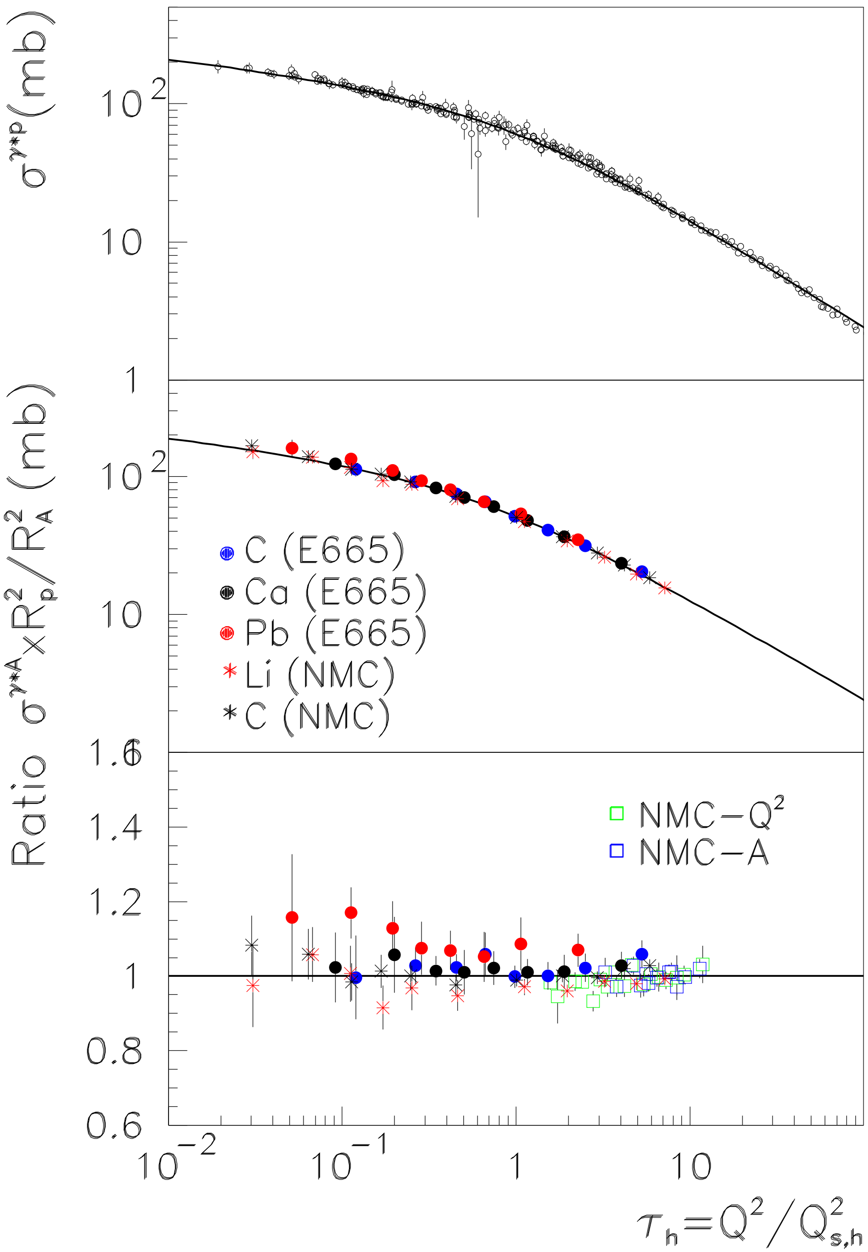
as suggested by rescaling.



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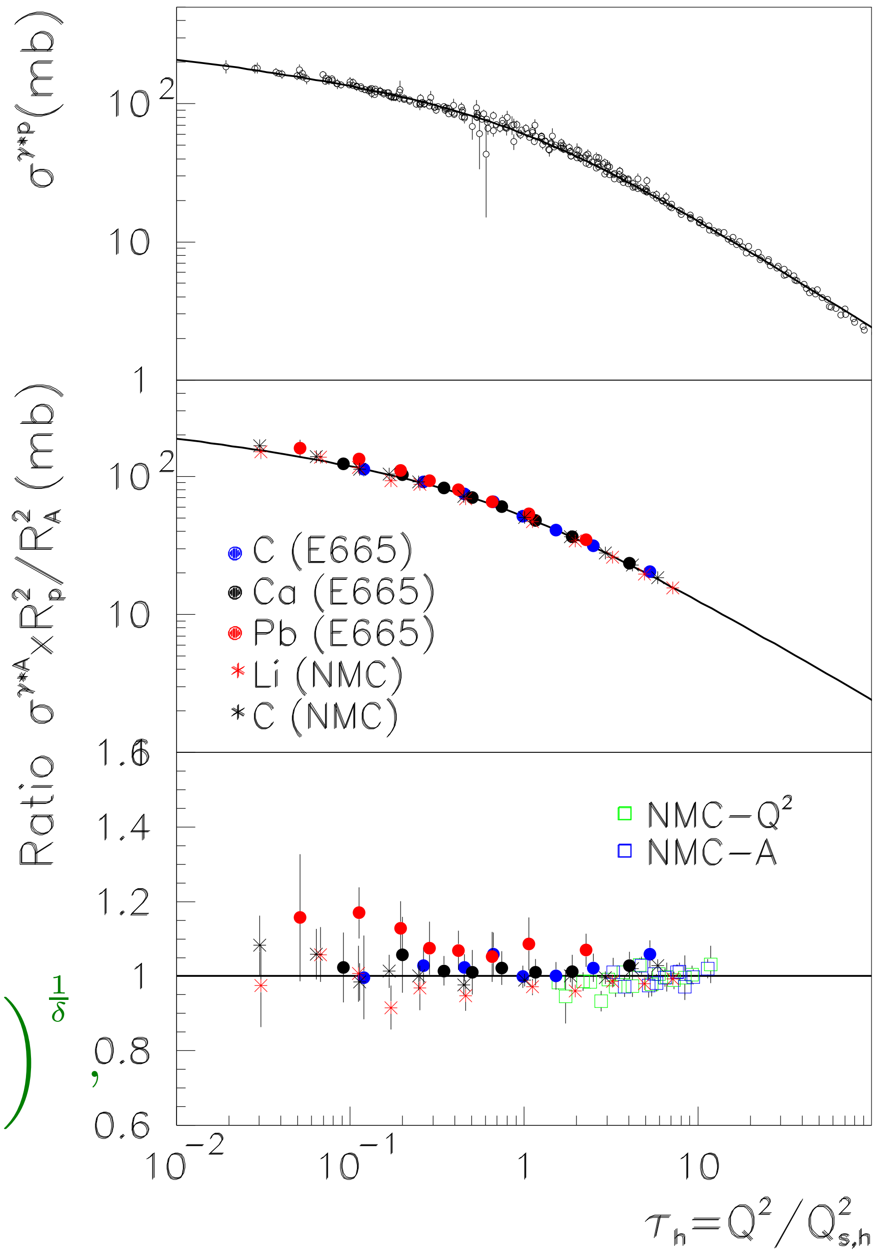
- Ansatz:

$$\frac{Q^2_{\rm s,A}}{Q^2_{\rm s,p}} = \left(\frac{A\pi R_p^2}{\pi R_A^2}\right)^{\frac{1}{\delta}} \Rightarrow \frac{\tau_A}{\tau_p} = \left(\frac{\pi R_A^2}{A\pi R_p^2}\right)^{\frac{1}{\delta}},$$

$\delta = 0.79 \pm 0.02$ $(x < 0.02)$.



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- $\sigma^{\gamma^* p}(x,Q^2) \equiv \Phi(\tau_p)$ parameterized in lp.

$$\delta = 0.79 \to 1 \Longrightarrow \chi^2/\mathrm{dof} = 0.95 \to 2.35.$$

- $$\frac{\sigma^{\gamma^* A}(\tau_A)}{\pi R_A^2} = \frac{\sigma^{\gamma^* p}(\tau_A)}{\pi R_p^2}$$

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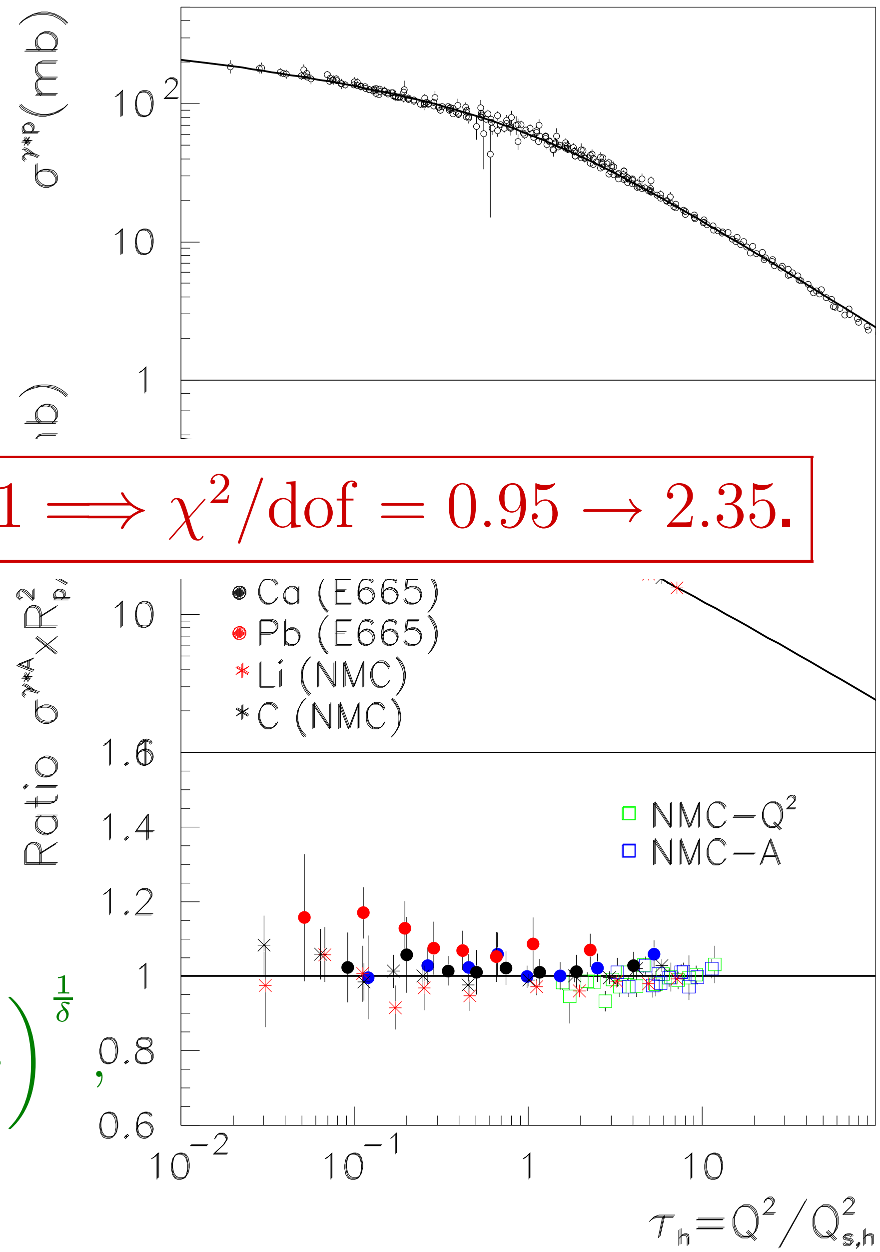

Multiplicities in AA

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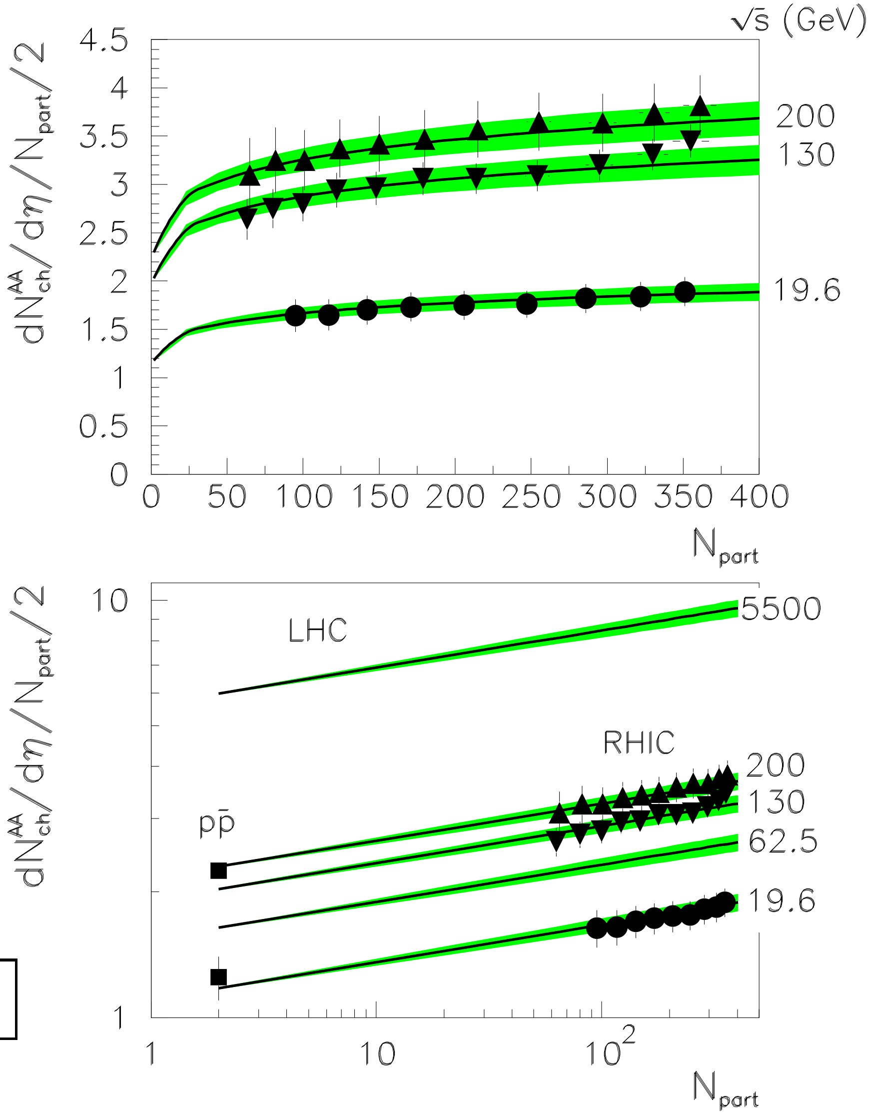

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Ratios at forward rapidities in pA

Néstor Armesto

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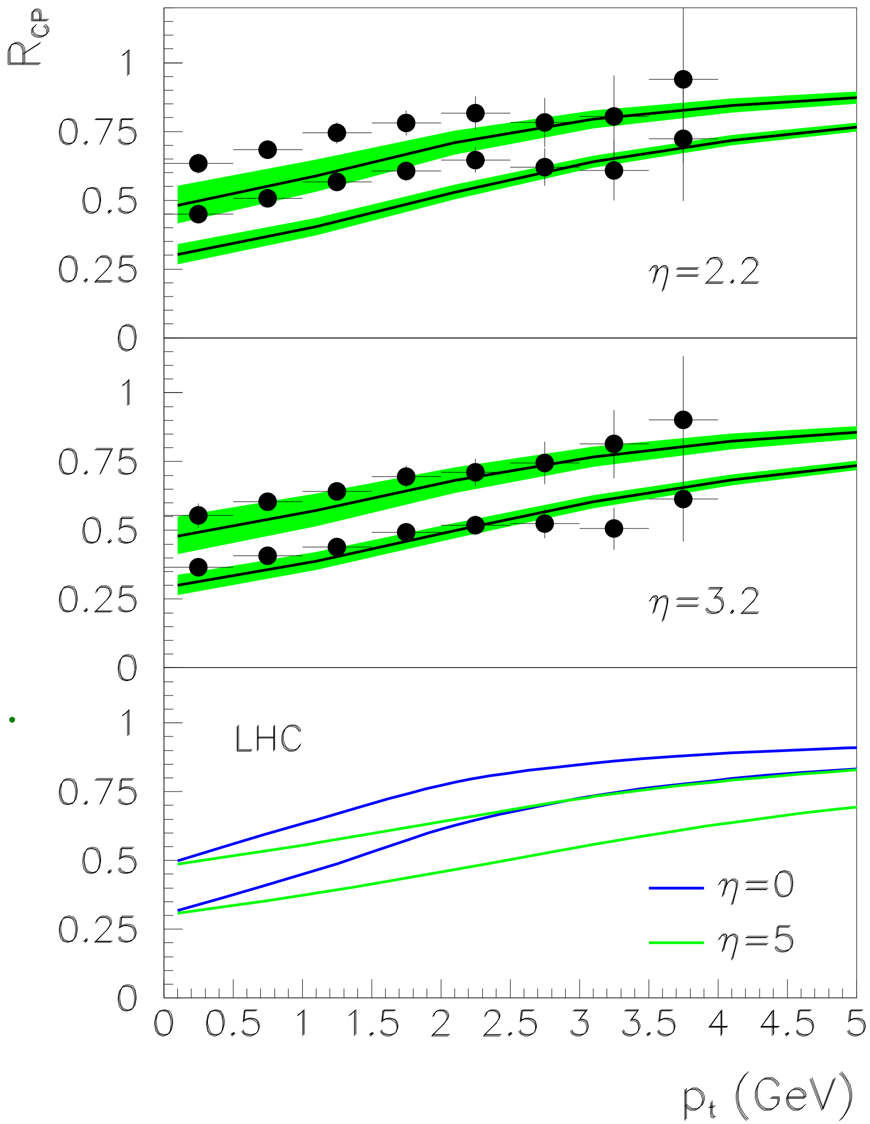
Data: BRAHMS, nucl-ex/0403005.



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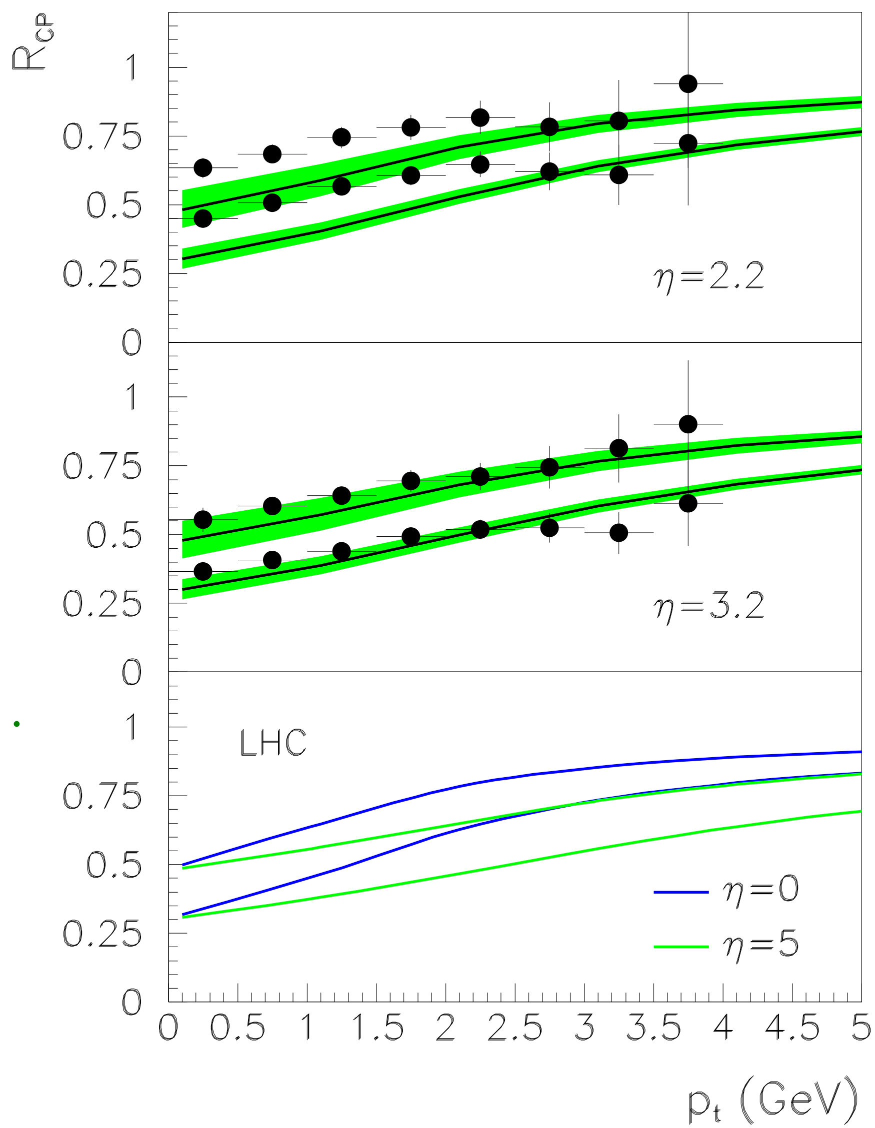
- Qualitative agreement (# adjusted parameters=0), Y- and centrality dependences in the right direction. Caveat: no Cronin.

Data: BRAHMS, nucl-ex/0403005.



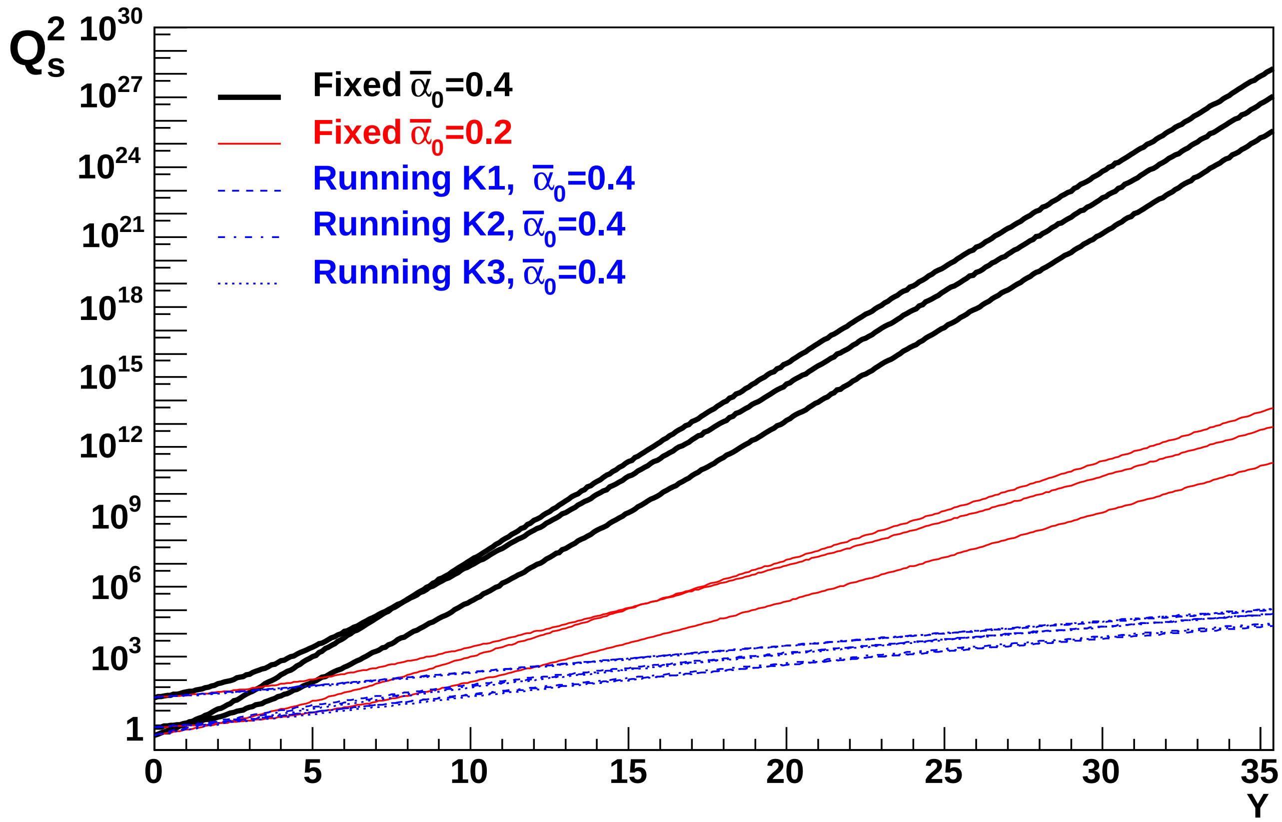
3. Features from the BK equation

(with J. L. Albacete, J. G. Milhano, C. A. Salgado and U. A. Wiedemann, hep-ph/0408216)

- Scaling, $N(Y,r) \equiv N(\tau = rQ_s(Y))$ for $Y \gg 1$ (backup).
- Small r behavior (backup).
- Rapidity dependence of Q_s.
- Nuclear size dependence of Q_s.

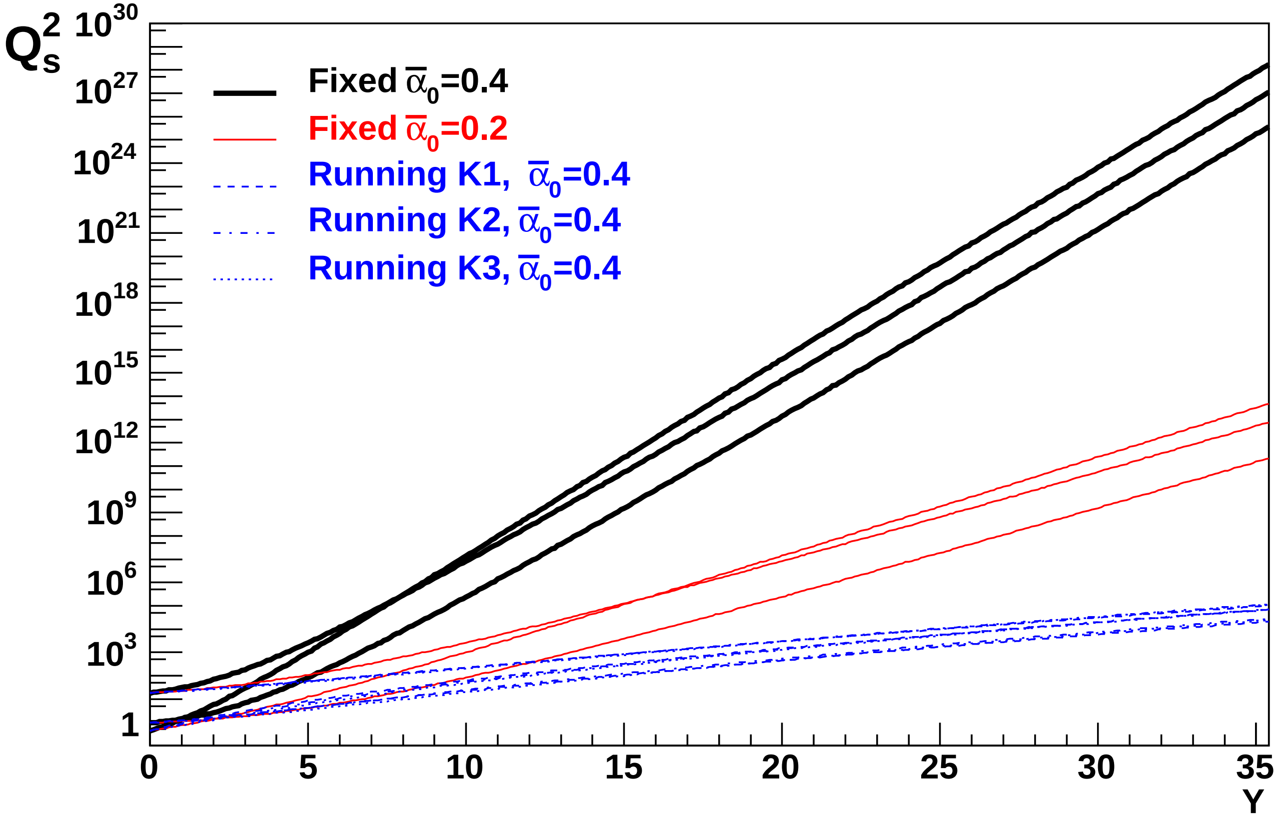
Note: results not yet with b-dependence. We have examined values of Y and A from small to huge.

Aim: study the transition to asymptotics.



Q_s^2
10^{30}
10^{27}
10^{24}
10^{21}
10^{18}
10^{15}
10^{12}
10^{9}
10^{6}
10^{3}
1
0
5
10
15
20
25
30
35
Y
Fixed $\bar{\alpha}_0$=0.4
Fixed $\bar{\alpha}_0$=0.2
Running K1, $\bar{\alpha}_0$=0.4
Running K2, $\bar{\alpha}_0$=0.4
Running K3, $\bar{\alpha}_0$=0.4

Rapidity dependence of Q_s



- $N(r = Q_s^{-1}(Y)) = 1/2$. For fixed α_s, $Q_s^2(Y) = Q_s^2(Y=0)\ \exp\left[d\bar{\alpha}_s Y\right]$; $d \simeq 4.57$ (expected $d = 4.88$ (IIM, NPA708(02)327)).

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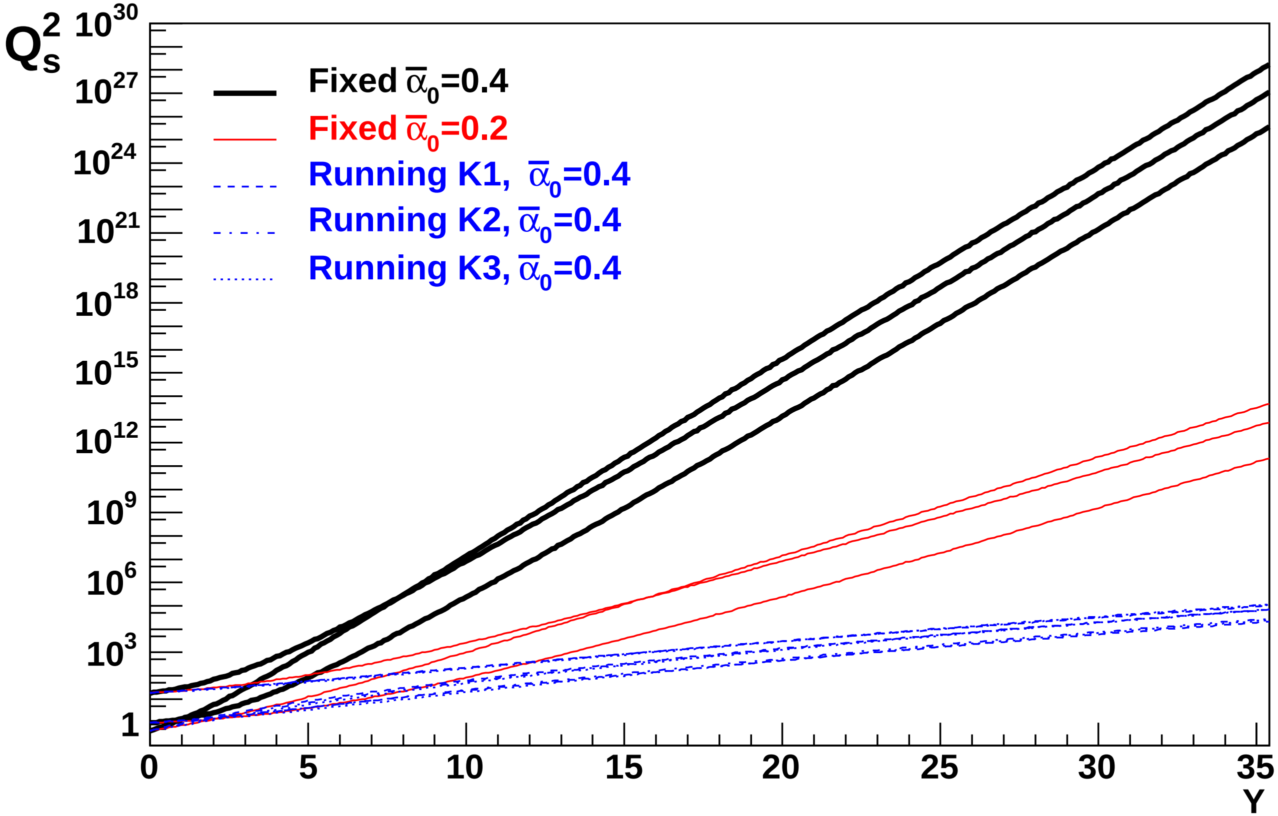

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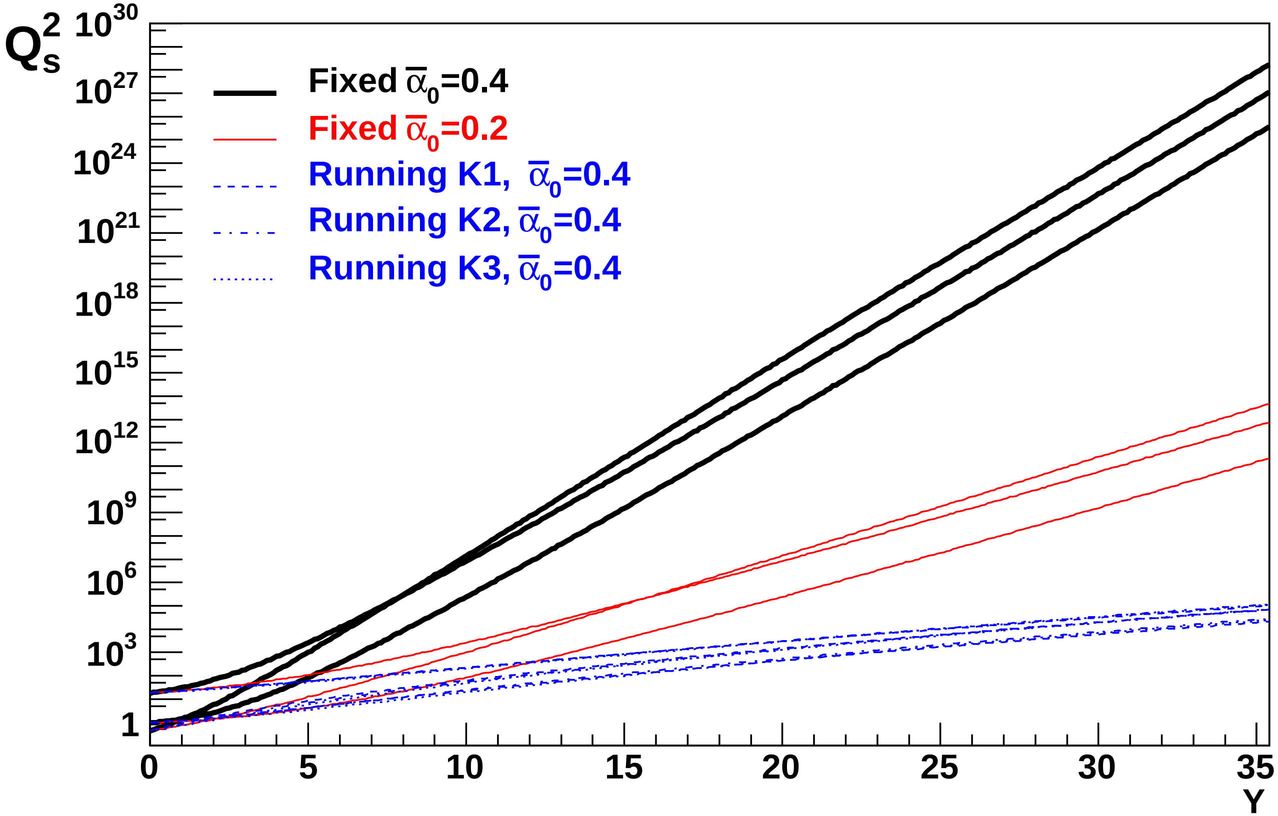

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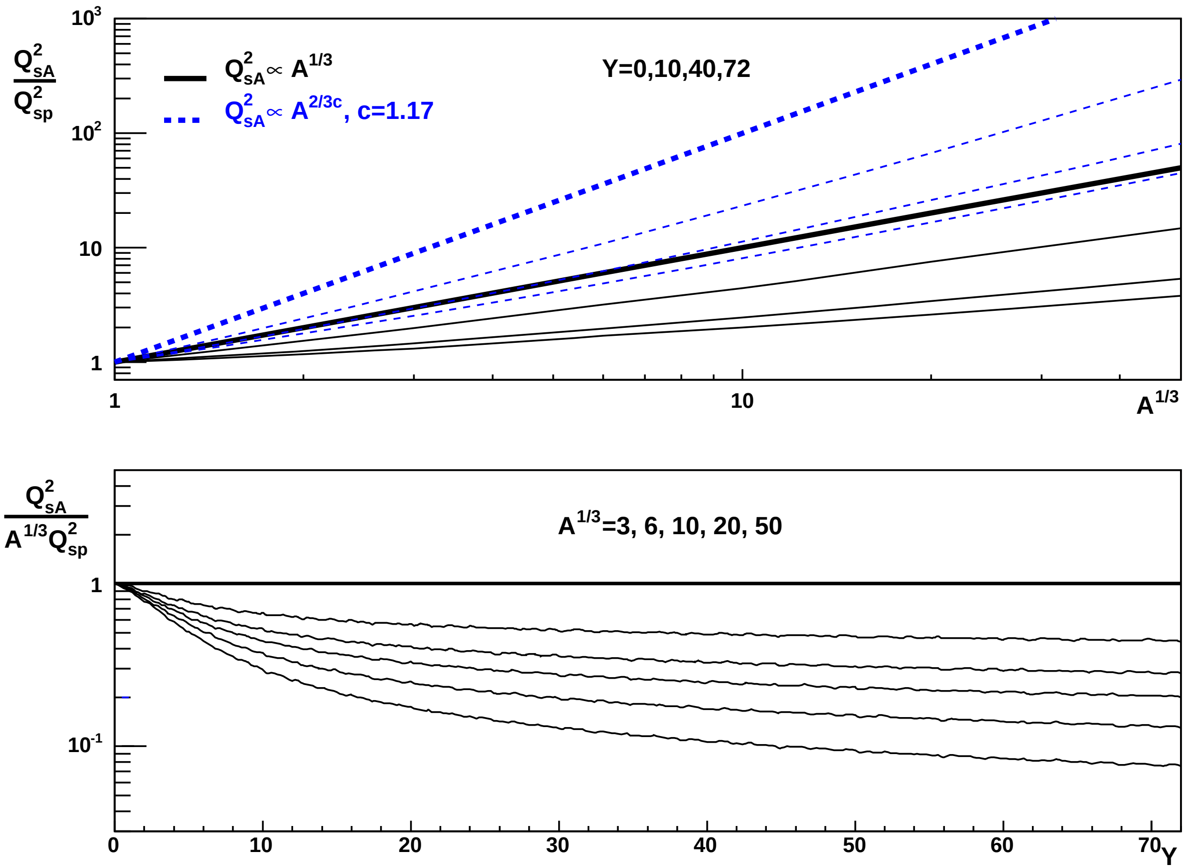
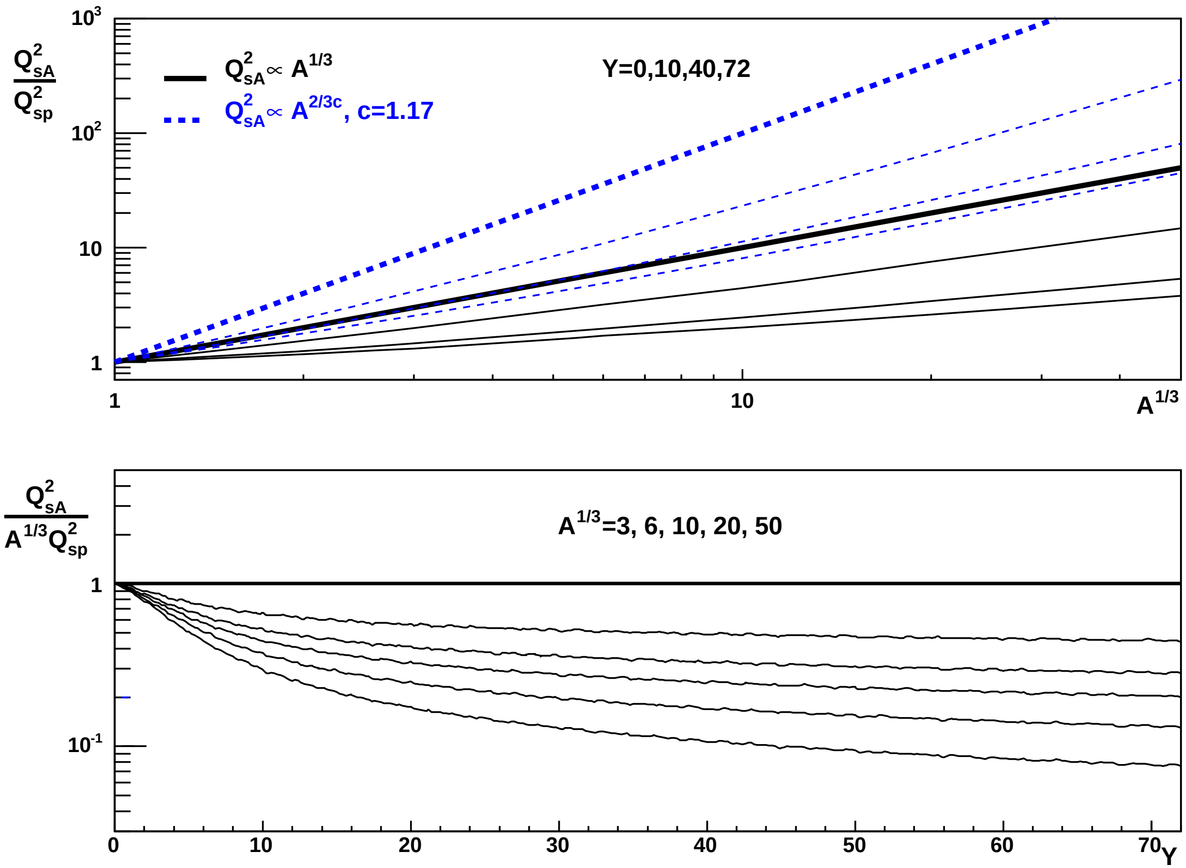
- Linear fit for running $\alpha_s \Longrightarrow d\bar{\alpha}_s \simeq 0.28$ for $Y \simeq 10$ (LGLM, NPA696(01)851; T, NPB648(03)293 // KMRS, hep-ph/0406135; KS, hep-ph/0408117; CLSV, hep-ph/0408333).



Q^2_{sA}/Q^2_{sp}
$Q^2_{sA} \propto A^{1/3}$
$Q^2_{sA} \propto A^{2/3c}$, c=1.17
Y=0,10,40,72
10^3
10^2
10
1
1
10
$A^{1/3}$
$Q^2_{sA}/(A^{1/3}Q^2_{sp})$
$A^{1/3}$=3, 6, 10, 20, 50
1
10^{-1}
0
10
20
30
40
50
60
70
Y



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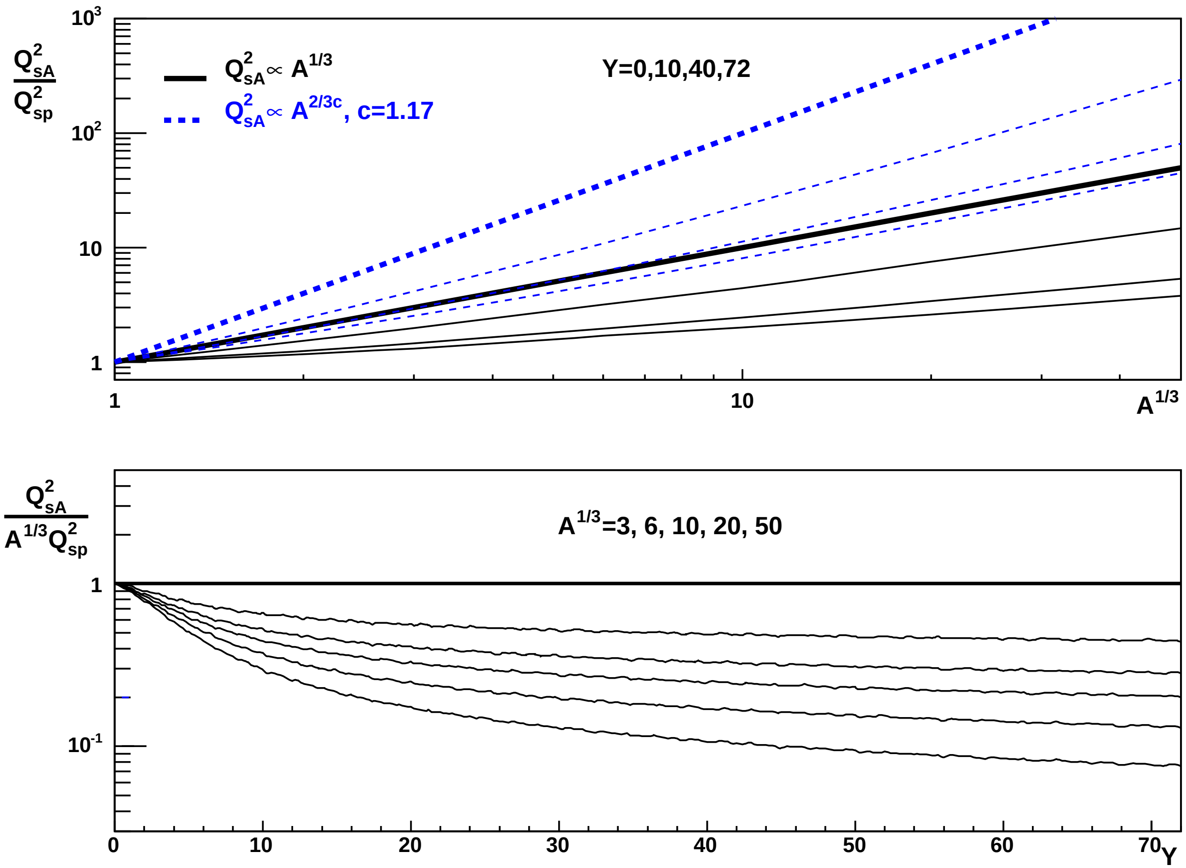

- For fixed α_s, the initial A-dependence is preserved.
- For running α_s, the A-dependence vanishes with increasing Y:

$$\ln \frac{Q^2_{sA}(Y)}{Q^2_{sp}(Y)} \simeq \frac{\ln^2 \left[Q^2_{sA}(Y=0)/\Lambda^2 \right]}{2\sqrt{(\Delta')^2 Y}}$$

(LR, SJNP45(87)150; M, NPA724(03)223; RW, NPA739(04)183); $1/\sqrt{Y}$ for $Y, A \gg 1$.

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- LHC: small x available to check saturation in pp and pA, fulfilling the requirements: small x (coherence, high density), $\langle Q_s^2 \rangle \gg \Lambda_{\rm QCD}^2$.

$p_t = 0.5$ GeV	$x_{1,2}^{y=0}$	$x_1^{y=3}$	$x_2^{y=3}$	$x_1^{y=5}$	$x_2^{y=5}$
200 GeV	$3 \cdot 10^{-3}$ (0.5)	$5 \cdot 10^{-2}$ (0.2)	10^{-4} (1.4)	0. 4 (0.1)	10^{-5} (2.7)
5.5 TeV	10^{-4} (1.4)	$2 \cdot 10^{-3}$ (0.6)	$5 \cdot 10^{-6}$ (3.26)	10^{-2} (0.4)	$6 \cdot 10^{-7}$ (6.0)

Multiplicities in AA

- Factorized form $\frac{dN_g^{AB}}{dY\, d\mathbf{p}_t d\mathbf{b}} \propto \frac{\alpha_S}{\mathbf{p}_t^2}$
$\times \int d\mathbf{k}\, \phi_A(Y, \mathbf{k}^2, \mathbf{b})\, \phi_B\left(Y, (\mathbf{k}-\mathbf{p}_t)^2, \mathbf{b}\right)$,
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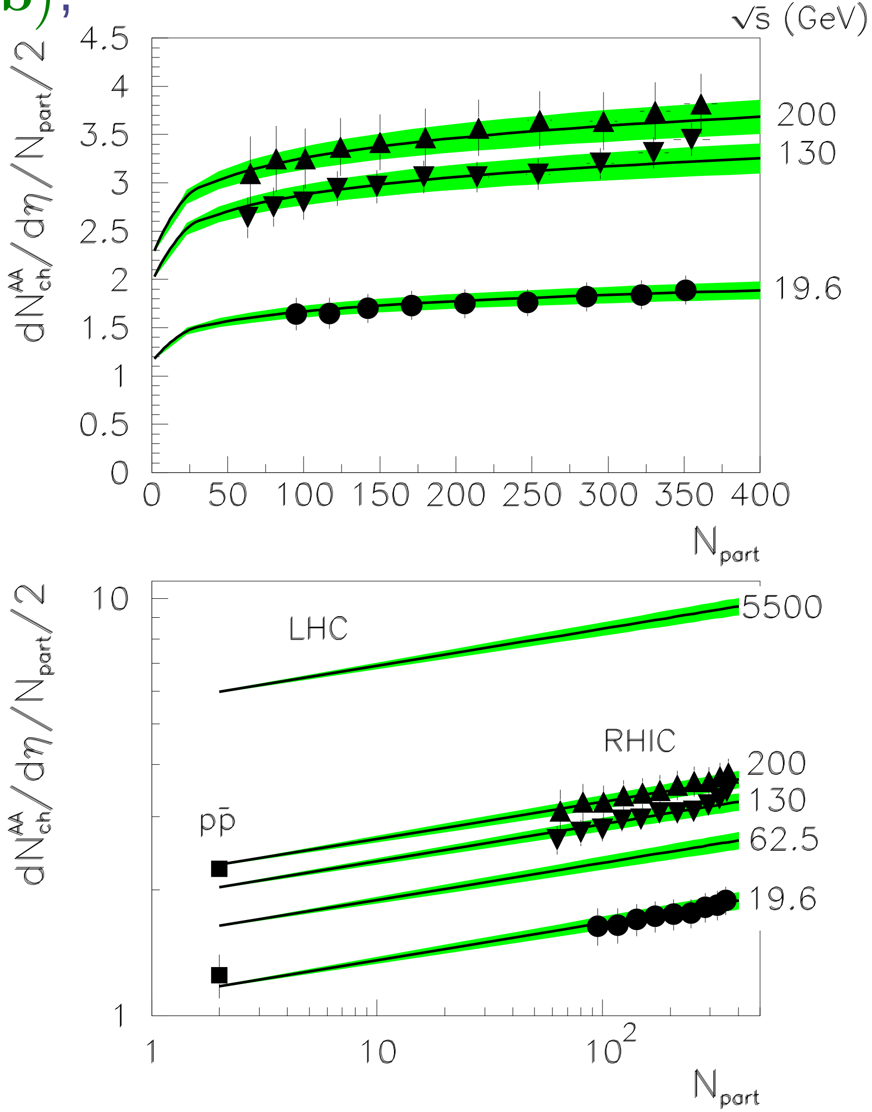
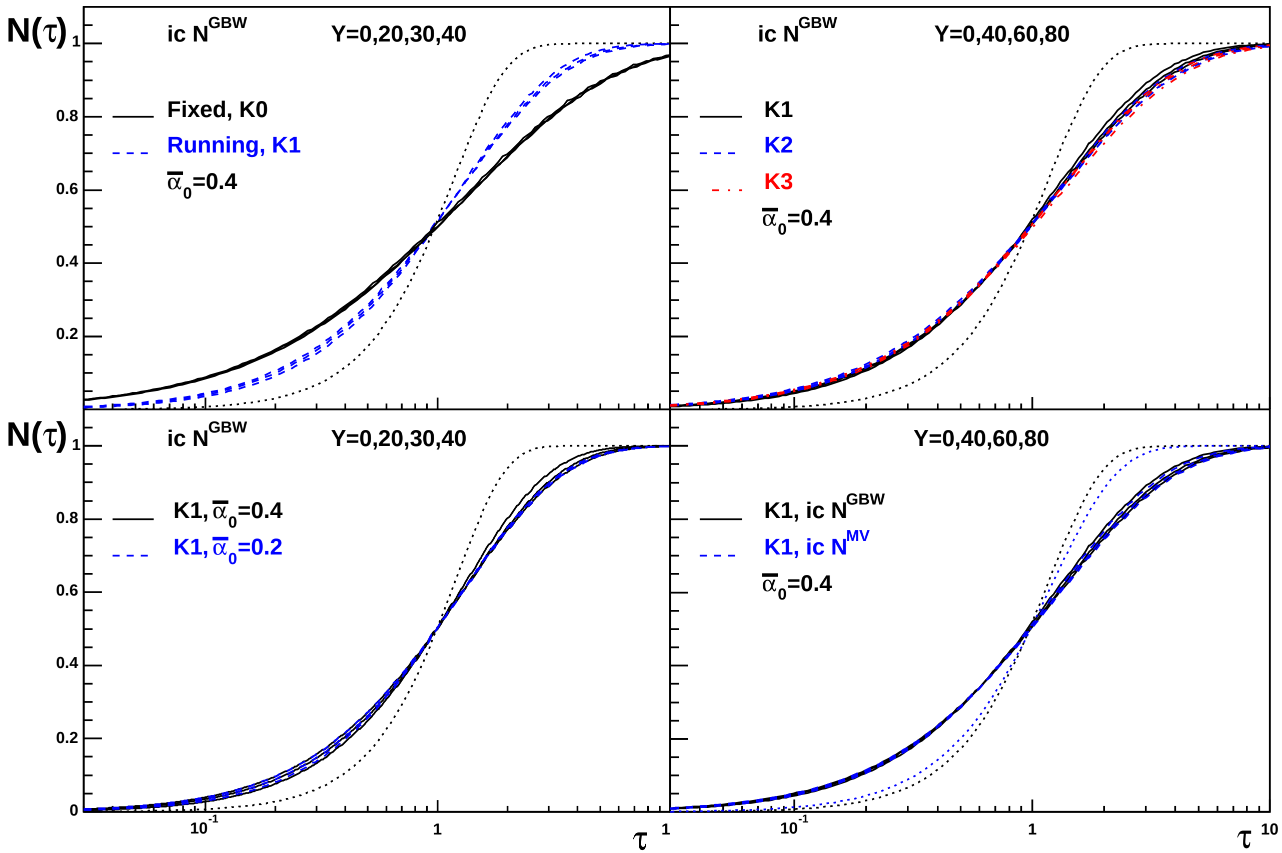


Scaling



N(τ)
ic N^GBW
Y=0,20,30,40
Fixed, K0
Running, K1
ᾱ0=0.4
ic N^GBW
Y=0,40,60,80
K1
K2
K3
ᾱ0=0.4
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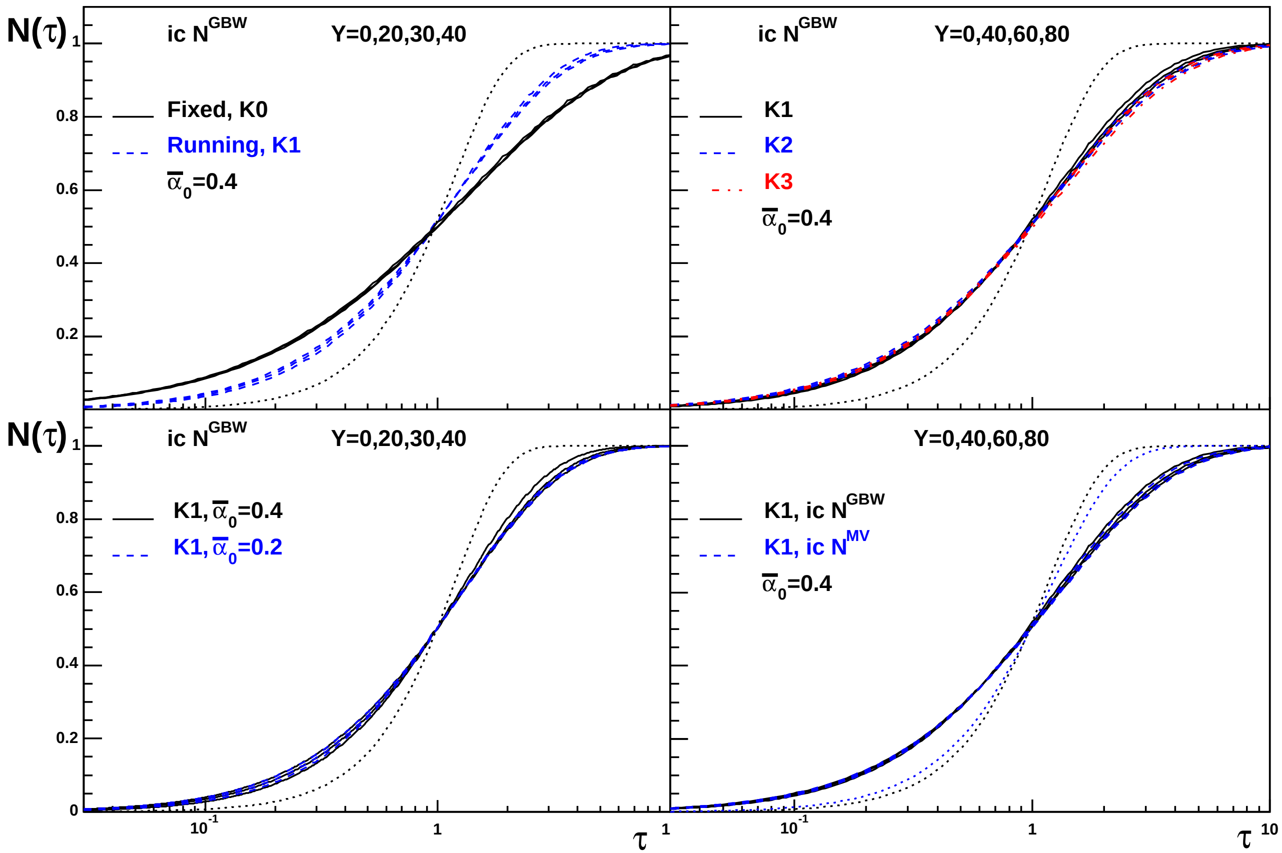
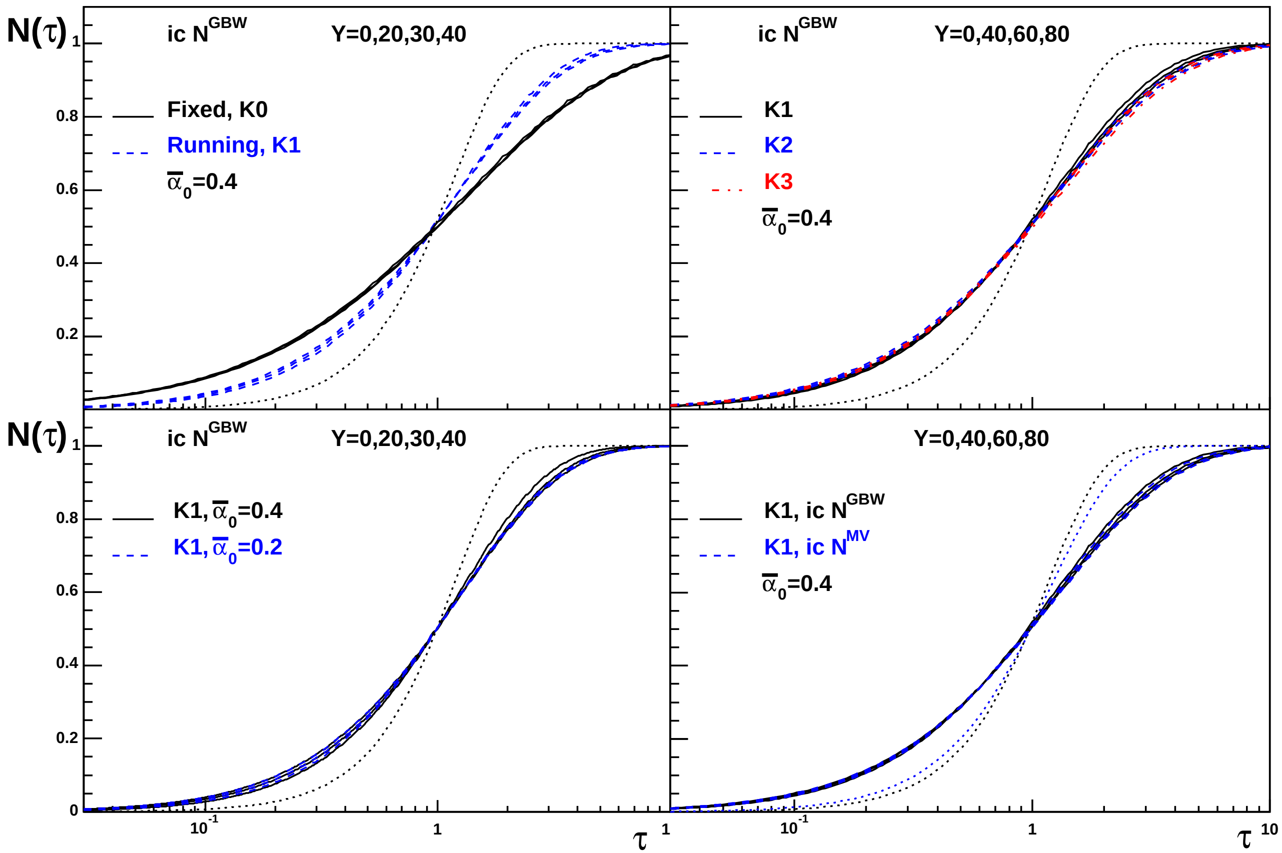
K1, ic N^GBW
K1, ic N^MV
ᾱ0=0.4
τ



- Scaling $N(Y,r) \equiv N(\tau = rQ_s(Y))$ for $Y \gg 1$ both for fixed (AB, EPJC20(01)517; L, EPJC21(01)513) and running (B, PLB576(03)115) α_s.

Scaling



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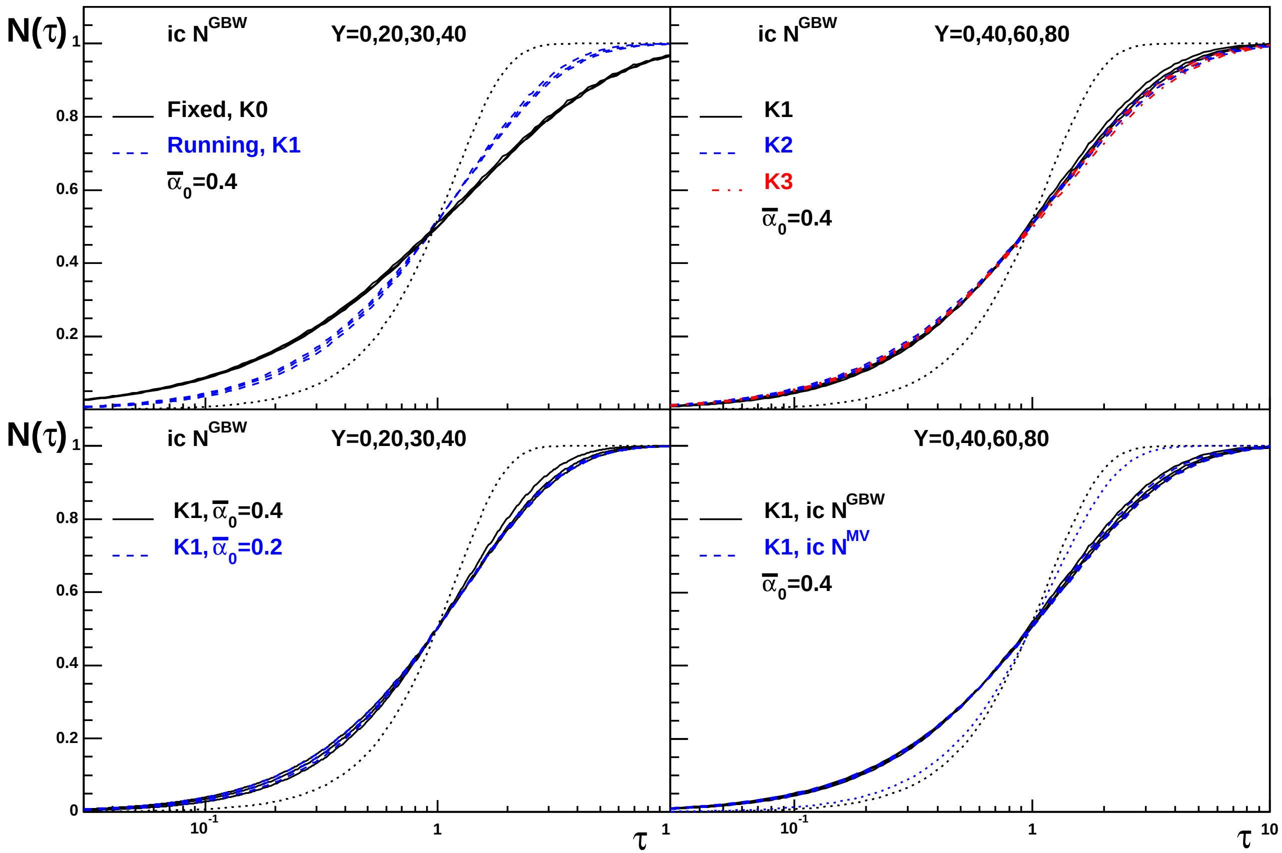

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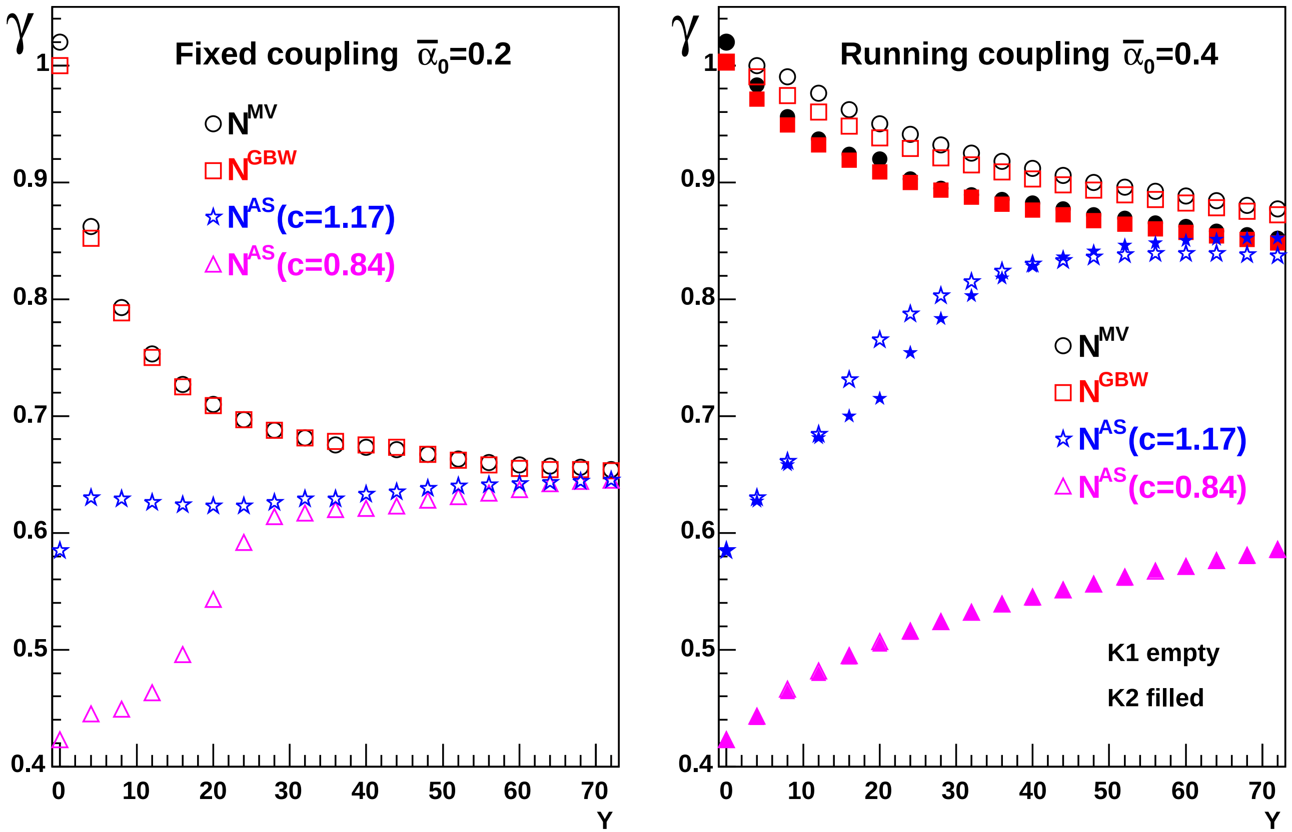
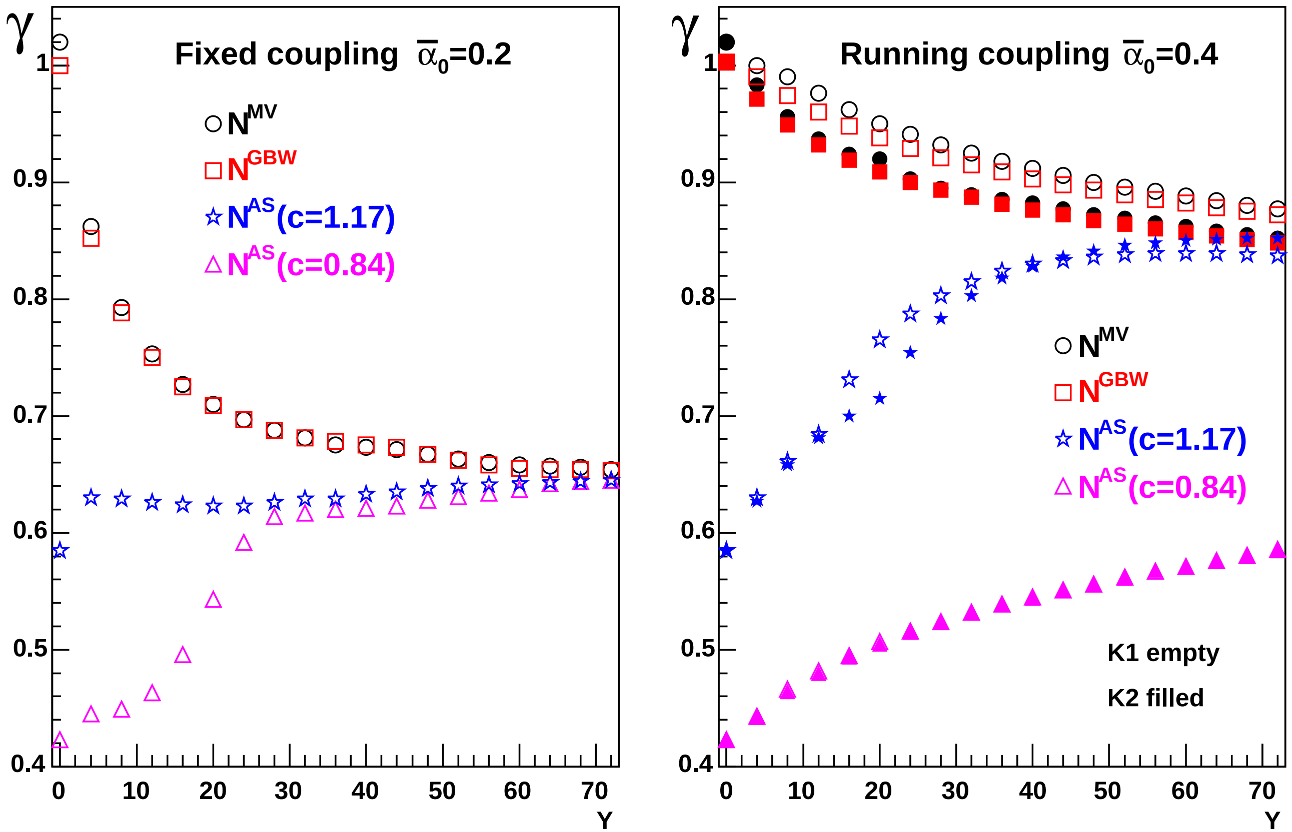
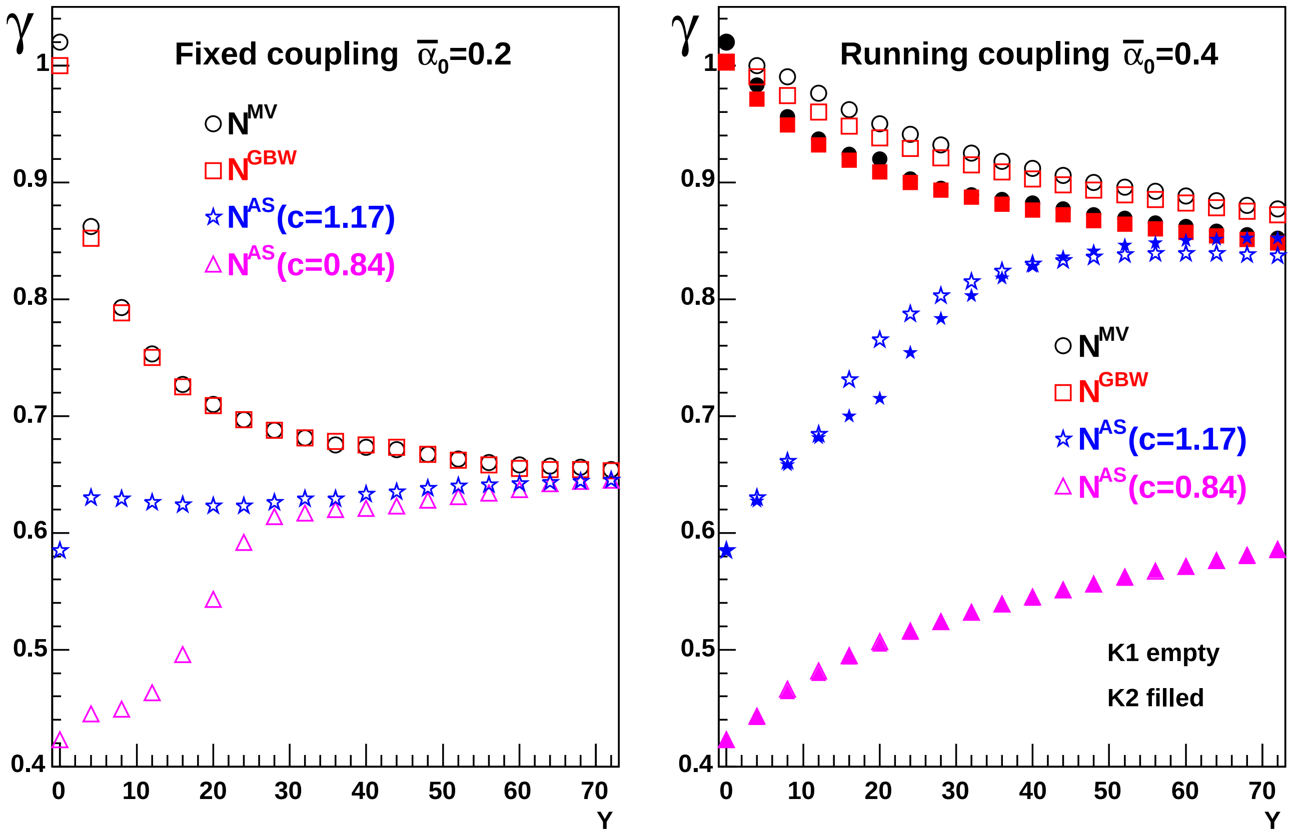
- Initial condition-independent: GBW, MV or AS $(1 - \exp\left[-(r\,Q_s)^c\right])$.
- Little dependence on details of the scale to run α_s (external, internal) or modifications of the kernel (exponential damping, kinematical cuts (CLSV, hep-ph/0408333)) or on the value $\alpha_s(Q=0)$ (fixed and running).



- Fits to $a\tau^{2\gamma}\left(\ln \tau^2 + d\right)$, $\tau = rQ_s$, (MT, NPB640(02)331), in $10^{-5} < \tau < 10^{-1}$; $1-\gamma \equiv$ 'anomalous dimension'.



- Fits to $a\tau^{2\gamma}\left(\ln \tau^2 + d\right)$, $\tau = rQ_s$, (MT, NPB640(02)331), in $10^{-5} < \tau < 10^{-1}$; $1-\gamma \equiv$ 'anomalous dimension'.
- $Y \to \infty$: $\gamma \simeq 0.65$ for fixed (IIM, NPA708(02)327; AAKSW, PRL92(04)082001) and $\simeq 0.85$ for running, unexpected (B, PLB576(03)115).



- Fits to $a\tau^{2\gamma}\left(\ln \tau^2 + d\right)$, $\tau = rQ_s$, (MT, NPB640(02)331), in $10^{-5} < \tau < 10^{-1}$; $1-\gamma \equiv$ 'anomalous dimension'.
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- Faster evolution for fixed than for running; for AS with $c = 0.84$, it takes a very long Y.