Some String Theory Technology

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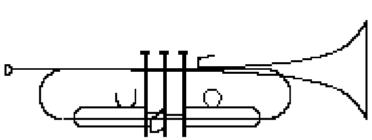


This Section's Goal

To introduce:

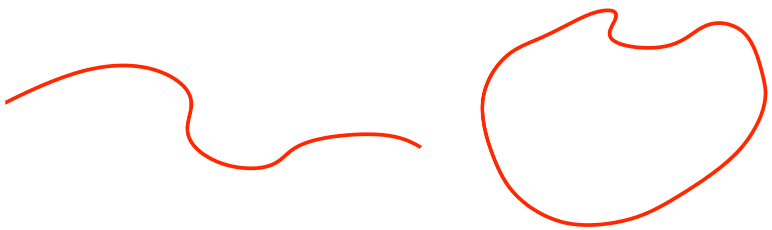
- the language
- the setting and tools
- the scope and limitations

To set the scene for:

- various breakthroughs
- various special topics
- various current events....

Describing Strings

Strings come in two broad varieties, open and closed:

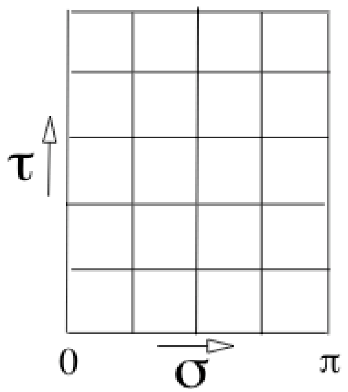


As they move in spacetime, they sweep out a two dimensional surface known as a “worldsheet”.

A starting point: tackle the description of the string’s allowed motion = allowed shapes of worldsheet.

Describing Strings

First parameterize the worldsheet with some coordinates: $\{\sigma, \tau\}$



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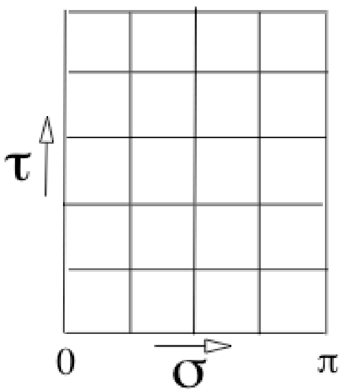
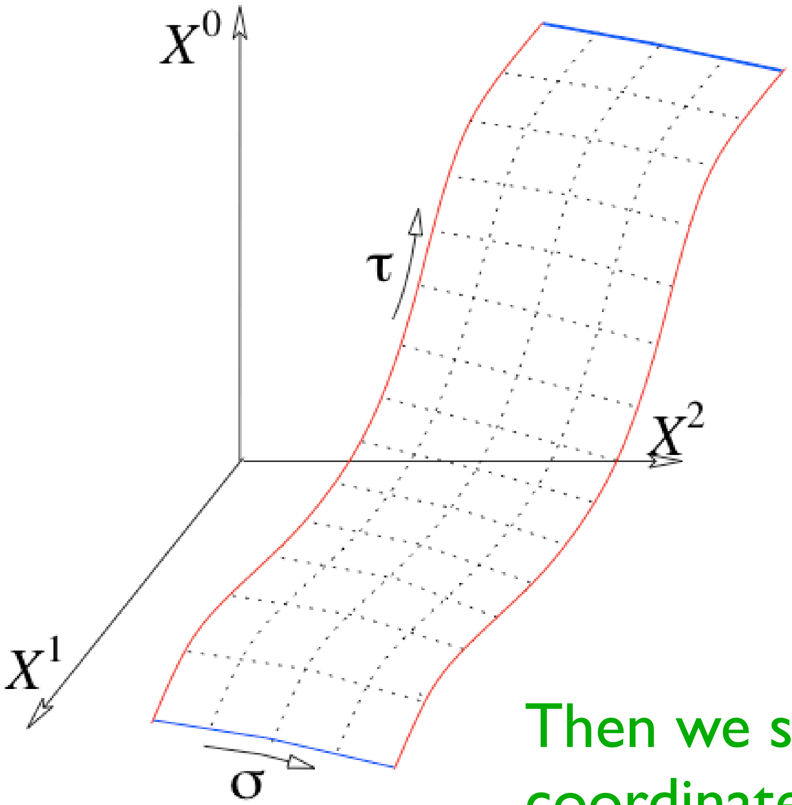


Then we say where the string is in spacetime, which has coordinates: $\{X^\mu\}, \mu = 0, 1, \ldots, D-1$

Describing Strings



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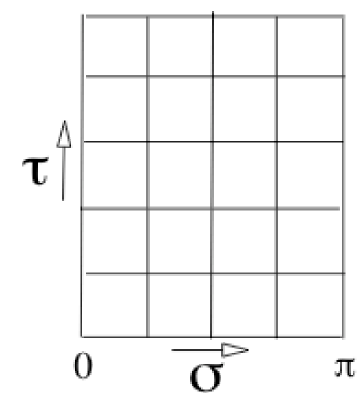


Then we say where the string is in spacetime, which has coordinates: $\{X^\mu\}, \mu = 0, 1, \ldots, D-1$

This implies some map(s): $X^\mu(\tau, \sigma)$

What are the allowed ones?

Describing Strings

Determine the allowed shapes by an action principle:

$$S = -T \int dA = -T \int d\tau d\sigma \left(-\det h_{ab}\right)^{1/2}$$

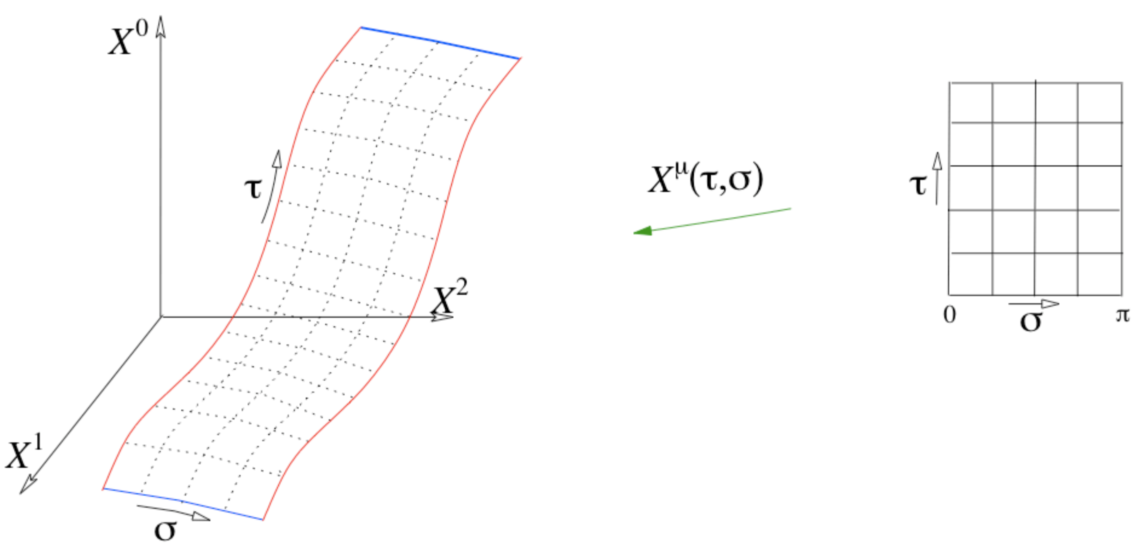
extremize the area swept out by worldsheet

$$h_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}$$

induced metric

$$(\sigma^1, \sigma^2) \equiv (\tau, \sigma)$$

$$(\partial_a \equiv \partial / \partial \sigma^a)$$



Tension: $T = \dfrac{1}{2\pi\alpha'}$

Sets a basic length scale: $\ell_s \sim \sqrt{\alpha'}$

Comparable to Planck length?

Aside: Planck Scale?

Where does the Planck scale come from?

Simple argument:

Quantum effects set a natural length scale: $$\ell_c = \frac{\hbar}{mc}$$

Gravity sets a natural length scale: $$\ell_s = \frac{Gm}{c^2}$$

At what mass do these length scales coincide?

$$m_p = \sqrt{\frac{\hbar c}{G}}$$

This length scale is the Planck length: $$\ell_p = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35}\mathrm{m}$$

Describing Strings

An equivalent action:

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma (-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

$\gamma_{ab}(\sigma, \tau)$ independent metric on worldsheet

This action is more pleasant to work with than the other one.

But they are equivalent!

Describing Strings

An equivalent action:

$$S = -\frac{1}{4\pi\alpha'}\int d\tau d\sigma (-\gamma)^{1/2}\gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$

Action has some interesting local (“gauge”) invariances:

World-sheet reparameterization invariance: $\sigma, \tau \longrightarrow \tilde{\sigma}(\sigma,\tau), \tilde{\tau}(\sigma,\tau)$

Weyl invariance: $\gamma_{ab} \to \gamma'_{ab} = e^{2\omega}\gamma_{ab}$

$\omega(\tau,\sigma)$ is function

So three functions altogether

Describing Strings

This allows us to choose a gauge:

"Conformal" gauge

$$\gamma_{ab} = \eta_{ab} e^{\phi} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} e^{\phi}$$

Theory is conformally invariant
(remnant of reparams and Weyl.)

ϕ drops out of the action, classically.

Equations of motion become:

$$\left(\frac{\partial^2}{\partial \sigma^2} - \frac{\partial^2}{\partial \tau^2} \right) X^{\mu}(\tau, \sigma) = 0$$

The two dimensional wave equation!

So the problem of the string's motion is just solutions to this, with boundary conditions.

Closed: Periodic
Open: Neumann or Dirichlet.

Describing Strings

Solutions:

$$X^\mu(\tau,\sigma) = x^\mu + 2\alpha' p^\mu \tau + i(2\alpha')^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma$$

(can also have DD and ND strings. See later.)

Open: Neumann - Neumann (NN)

standing waves

$$\begin{aligned} X^\mu(\tau,\sigma) &= X_R^\mu(\sigma^-) + X_L^\mu(\sigma^+) \\ X_R^\mu(\sigma^-) &= \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^- + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \alpha_n^\mu e^{-2in\sigma^-} \\ X_L^\mu(\sigma^+) &= \frac{1}{2}x^\mu + \alpha' p^\mu \sigma^+ + i\left(\frac{\alpha'}{2}\right)^{1/2} \sum_{n\neq 0} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2in\sigma^+} \end{aligned}$$

Closed: Periodic

Traveling waves, independent left- and right-moving

$\tilde{\alpha}_{-n}^\mu = (\tilde{\alpha}_n^\mu)^*$ etc...

Quantum Strings

Fourier modes become annihilation and creation operators.

There are D independent families of harmonic oscillators.
One for each boson in world-sheet theory.

Theory must be reparameterisation invariant: Virasoro constraints.
Succeeds in removing "ghosts" etc...

Spacetime Physics?

Example of closed string states:

$$\tilde{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1}|0>$$

oscillators add mass-energy +2

vacuum has mass-energy -2

massless states in spacetime

symmetric	$G_{\mu\nu}(x)$	"graviton"
antisymmetric	$B_{\mu\nu}(x)$	"Kalb-Ramond"
trace	$\Phi(x)$	"dilaton"

Note: vacuum gives a tachyon in spectrum. (More later)

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Example of open string states:

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oscillator adds mass-energy +1

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massless states in spacetime

$A_{\mu}(x)$ "photon"

Note: vacuum gives a tachyon in spectrum. (More later)

Spacetime Physics?

Consistency sometimes places a condition on D.

The condition comes about because theory has an anomaly in conformal invariance, with the following ingredients:

Counting D as number of bosons on world sheet

Each field X contributes +1: $$c = D - 1$$

There are Fadeev-Popov ghosts for fixing gauge, and they give: $$c = -26$$

The field ϕ does not in general decouple, and contributes: $$c = 1 + 3\left(\frac{26 - D}{3}\right)$$

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Comes about because ϕ has non-trivial world-sheet coupling

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Total vanishes, and so can have a string theory in any number of dimensions.

But one of them is different from others. Lorentz...?

Spacetime Physics?

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For Lorentz invariance, all dimensions must couple same way, resulting in:

"Critical Dimension" $D = 26$

Spacetime Physics?

What determines the dynamics of the spacetime fields?

$$G_{\mu\nu}(x)$$
$$B_{\mu\nu}(x)$$
$$\Phi(x)$$

Revisit worldsheet action (henceforth Euclidean):

$$S_\sigma = \frac{1}{4\pi\alpha'} \int d^2\sigma\, g^{1/2} \left\{ (g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X)) \partial_a X^\mu \partial_b X^\nu + \alpha' \Phi R \right\}$$

String now propagating in background of fields whose quanta it generates!

Model must still be conformally invariant...

String Perturbation Theory

Notice that the value of the dilaton couples to the Euler number of the worldsheet.

$$\chi = \frac{1}{4\pi}\int_{\mathcal{M}} d^2\sigma\, g^{1/2} R \;= 2-2h$$

# of handles

$$S_\sigma = \frac{1}{4\pi\alpha'}\int d^2\sigma\, g^{1/2}\,\left\{(g^{ab}G_{\mu\nu}(X) + i\epsilon^{ab}B_{\mu\nu}(X))\partial_a X^\mu \partial_b X^\nu + \alpha'\Phi R\right\}$$

So what? Well, in the path integral, amplitudes will be weighed by a topological factor: $e^{-\Phi\chi}$

$$\mathcal{Z} = \int \mathcal{D}X\mathcal{D}g\; e^{-S}$$

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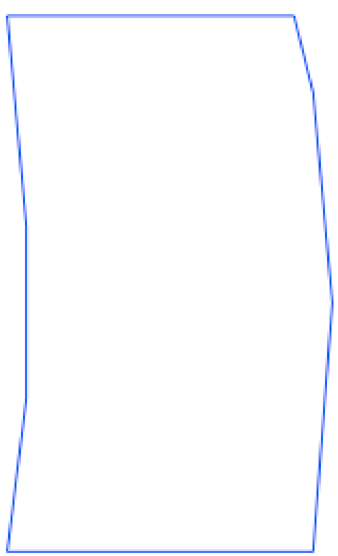
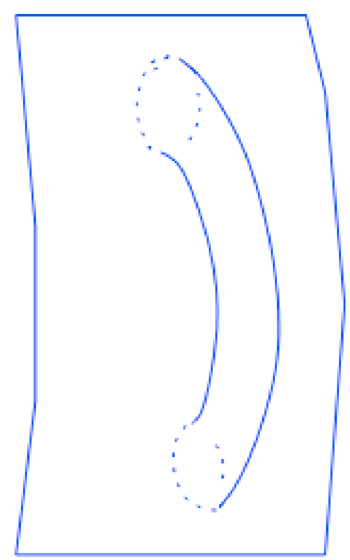
Look at different orders in perturbation theory:



$$\delta\chi = -2$$



Every vertex has additional factor of "closed string coupling":

$$g_s = e^{\Phi}$$

local value of coupling set by dynamical field...

Spacetime Physics

Trace of stress tensor for the model (complicated since background fields act as highly non-linear couplings):

$$G_{\mu\nu}(x)$$
$$B_{\mu\nu}(x)$$
$$\Phi(x)$$

$$T^a{}_a = -\frac{1}{2\alpha'}\beta^G_{\mu\nu} g^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{i}{2\alpha'}\beta^B_{\mu\nu}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}\beta^\Phi R$$

$$\begin{aligned}
\beta^G_{\mu\nu} &= \alpha'\left(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\Phi - \frac{1}{4}H_{\mu\kappa\sigma}H_\nu{}^{\kappa\sigma}\right) + O(\alpha'^2),\\
\beta^B_{\mu\nu} &= \alpha'\left(-\frac{1}{2}\nabla^\kappa H_{\kappa\mu\nu} + \nabla^\kappa\Phi H_{\kappa\mu\nu}\right) + O(\alpha'^2),\\
\beta^\Phi &= \alpha'\left(\frac{D-26}{6\alpha'} - \frac{1}{2}\nabla^2\Phi + \nabla_\kappa\Phi\nabla^\kappa\Phi - \frac{1}{24}H_{\kappa\mu\nu}H^{\kappa\mu\nu}\right) + O(\alpha'^2)
\end{aligned}$$

$$H_{\mu\nu\kappa} \equiv \partial_\mu B_{\nu\kappa} + \partial_\nu B_{\kappa\mu} + \partial_\kappa B_{\mu\nu}$$

Conformal invariance requires these to vanish...But this looks like a set of spacetime field equations!

Spacetime Physics

Those field equations can be derived from this spacetime action:

$G_{\mu\nu}(x)$
$B_{\mu\nu}(x)$
$\Phi(x)$

$$S = \frac{1}{2\kappa_0^2}\int d^D X (-G)^{1/2} e^{-2\Phi}\left[R + 4\nabla_\mu\Phi\nabla^\mu\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} + O(\alpha')\right]$$

some dimensionful numbers...

This is the low energy effective action for the string theory, at tree level.

Spacetime Physics

$$G_{\mu\nu}(x)$$
$$B_{\mu\nu}(x)$$
$$\Phi(x)$$

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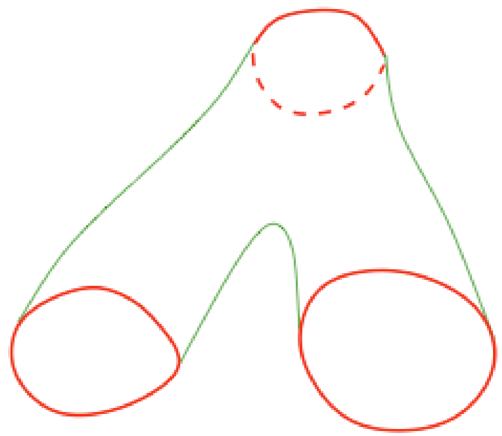
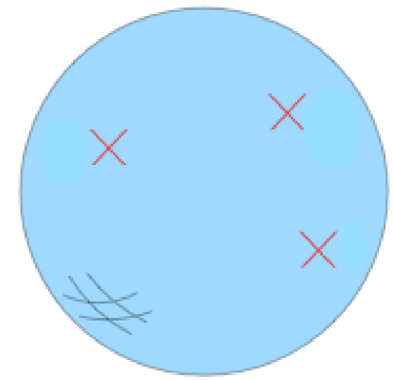
Notice the implicit power of the string coupling that appears. Everything was computed at the sphere level. Tree level in string pert. theory.

Language of Perturbation Theory

Sphere?

Can always choose conformal factor to map all external states to "vertex operators" at points....





g_s^{-2}

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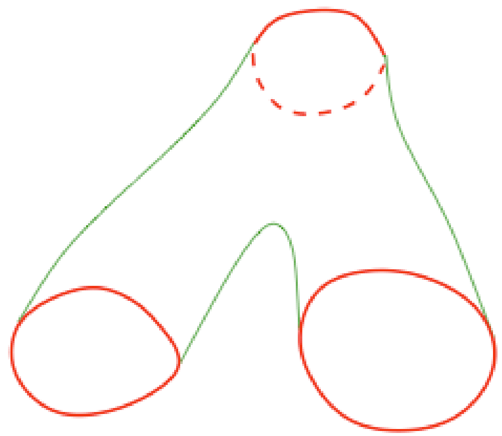
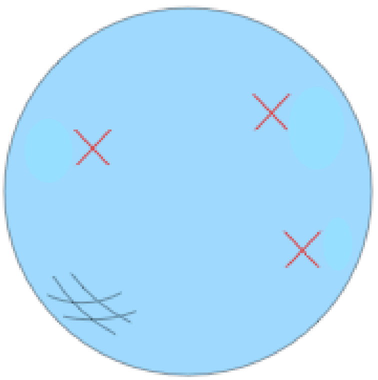


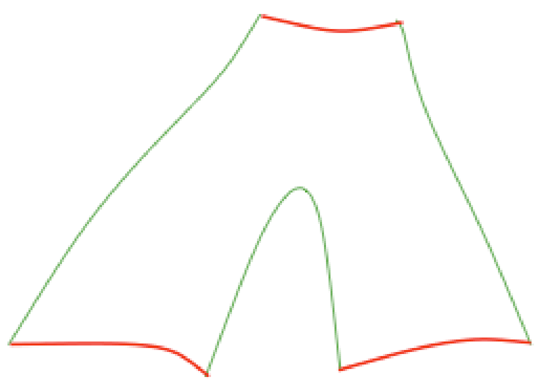


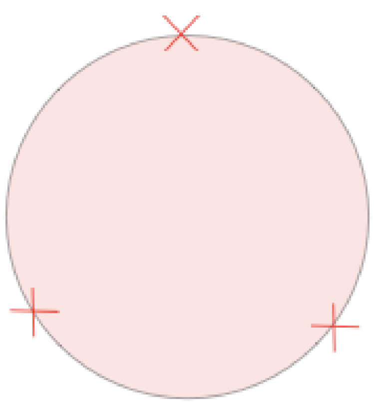
g_s^{-2}

Disc (see later)

Similarly for open strings....

...they become operators on boundary.

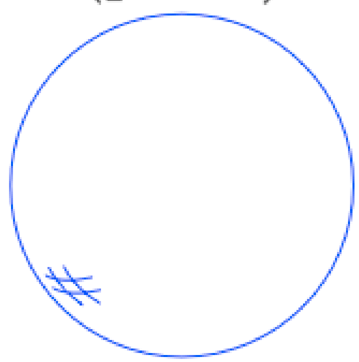
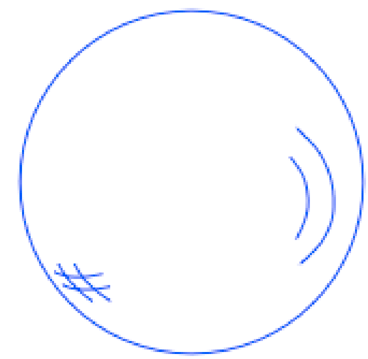
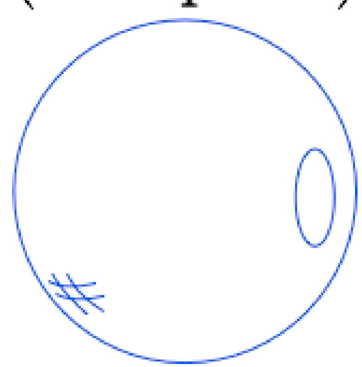
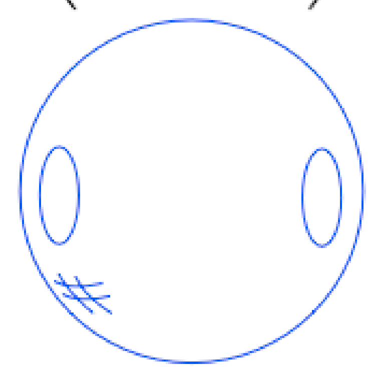




g_s^{-1}

Language of Perturbation Theory

Some more diagrams...

	g_s^{-2}	g_s^{-1}	g_s^{0}
closed oriented	sphere S^2 (plane) 	.	torus T^2 
open oriented	.	disc D_2 (half–plane) 	cylinder C_2 (annulus) 

Spacetime Physics

Can also rescale metric to present action differently:

set this to be asymptotic value of dilaton

$G_{\mu\nu}(x)$

$B_{\mu\nu}(x)$

$\Phi(x)$

$$\tilde{G}_{\mu\nu}(X) = e^{2\Omega(X)} G_{\mu\nu} = e^{4(\Phi_0 - \Phi)/(D-2)} G_{\mu\nu}$$

$$\begin{aligned} S \;=\; & \frac{1}{2\kappa^2} \int d^D X (-\tilde{G})^{1/2} \Big[R - \frac{4}{D-2} \nabla_\mu \tilde{\Phi} \nabla^\mu \tilde{\Phi} \\ & - \frac{1}{12} e^{-8\tilde{\Phi}/(D-2)} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} e^{4\tilde{\Phi}/(D-2)} + O(\alpha') \Big] \end{aligned}$$

Spacetime Physics

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$$S = \frac{1}{2\kappa^2} \int d^D X (-\tilde{G})^{1/2} \Big[R - \frac{4}{D-2} \nabla_\mu \tilde{\Phi} \nabla^\mu \tilde{\Phi} - \frac{1}{12} e^{-8\tilde{\Phi}/(D-2)} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} e^{4\tilde{\Phi}/(D-2)} + O(\alpha') \Big]$$

This is the low energy effective action in "Einstein frame". (Previous was "String frame").

Recall: $g_s = e^{\Phi}$

Closed string coupling sets Newton's constant $\kappa \equiv \kappa_0 e^{\Phi_0} = (8\pi G_N)^{1/2}$