Recent Advances in the CGC

(Helmut's suggestion !)

High–$k_\perp$ fluctuations & Pomeron loops in the approach towards saturation

Edmond Iancu

SPhT Saclay & CNRS

Based on: E.I., A. Mueller and S. Munier (hep–ph/041018)
E.I., D. Triantafyllopoulos, in preparation

... or About the surprisingly deep connection between

High–Energy QCD and Statistical Physics

... or How to go beyond BK–JIMWLK equations

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Outline (1)

- High–energy (“small–x") evolution in QCD is a classical stochastic process
  - Color Dipole Picture (Master equation)
  - CGC (Fokker–Planck equation: ‘JIMWLK’)

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  “Classical": Large separation in rapidity/time scales

  $\Longrightarrow$ Effective theory in three (or two) spatial dimensions

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- Not exactly equivalent ...
  - Color Dipole Picture : Unitarization without saturation
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  - Color Dipole Picture : Unitarization without saturation
  - CGC (JIMWLK) : Saturation (but no 'pomeron loops')
- ... but they both involve Mean Field aspects & Fluctuations
  - MFA should work better at/near saturation (unitarity): $k_\perp \lesssim Q_s$ (strong color fields, large occupation numbers)
  - Fluctuations are more important in the dilute regime at high momenta: $k_\perp \gg Q_s$

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- ... but they both involve Mean Field aspects & Fluctuations
- BK equation: the simplest MFA, common to both formalisms
  - Closed, non–linear equation. User friendly !
  - Solutions to BK: unitarity, geometric scaling

Outline (2)

- One could expect MFA (BK equation) to correctly describe the approach towards saturation ...

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Outline
Color Dipole Picture
Color Glass Condensate
Mean Field Approximation
Fluctuating pulled fronts
A new equation
Conclusions

- One could expect MFA (BK equation) to correctly describe the approach towards saturation ...
- ... but this is actually not true !
  - The growth of the saturation momentum is driven by high–$k_\perp$ fluctuations
  - BK evolution violates unitarity at intermediate steps (Mueller & Shoshi, 2004)

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- ... but this is actually not true !
- Deep analogy with problems in statistical physics
  - 'Fluctuating pulled fronts'
  - The growth of the saturation momentum is slowed down
  - Geometric scaling is violated

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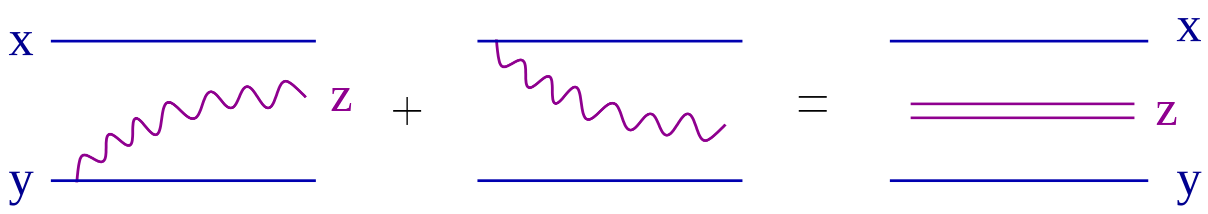
- The fluctuations are not correctly described by the JIMWLK, or Balitsky, equations !
- A Langevin equation for saturation with pomeron loops

Color Dipole Picture *(Mueller, 94)*

"Color Dipole" = A quark–antiquark pair in a color singlet state

- Dipole evolution $\Longleftrightarrow$ Dipole splitting
  - ▷ Leading–log $Y \equiv \ln 1/x$ (BFKL) + Large N_c



$$p(\boldsymbol{x},\boldsymbol{y}|\boldsymbol{z})\, d^2\boldsymbol{z}\, dY \;=\; \frac{\alpha_s N_c}{\pi}\, dY \;\times\; \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\,\frac{d^2\boldsymbol{z}}{2\pi}$$

- $P_N(Y) \equiv P_N(\boldsymbol{z}_1, \boldsymbol{z}_2, \dots, \boldsymbol{z}_{N-1}|\boldsymbol{x}_0, \boldsymbol{y}_0,\, Y)$

$$\frac{\partial P_N}{\partial Y} \;=\; -\left[\sum_{i=1}^{N}\int_{\boldsymbol{z}} p(\boldsymbol{z}_{i-1},\boldsymbol{z}_i|\boldsymbol{z})\right] P_N \;+\; \sum_{i=1}^{N-1} p(\boldsymbol{z}_{i-1},\boldsymbol{z}_{i+1}|\boldsymbol{z}_i)\, P_{N-1}$$

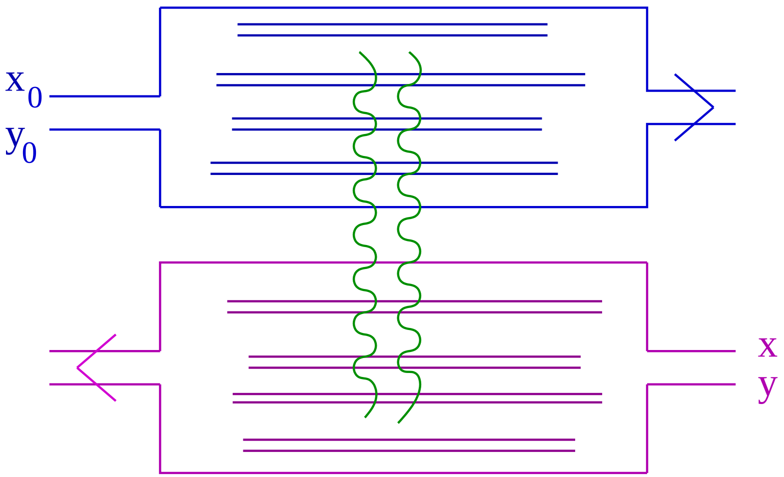
  - ▷ Master equation for a classical Markovian process

Dipole–Dipole Scattering

$$\text{COM frame}: \quad Y_1 = Y_2 = Y/2\,, \qquad Y = \ln s \quad \text{(rapidity)}$$

- Low energy: Single scattering ($T \ll 1$)

  Two gluon exchange between a pair of dipoles



$$T_{\text{one-scatt}}(r, r_0, Y) \approx \alpha_s^2\, n^2(r, r_0, Y/2) \sim \alpha_s^2\, \mathrm{e}^{\omega_{\mathbb{P}} Y}$$

$\omega_{\mathbb{P}} = (4 \ln 2)\alpha_s N_c/\pi$: BFKL intercept

- "One (BFKL) pomeron exchange"

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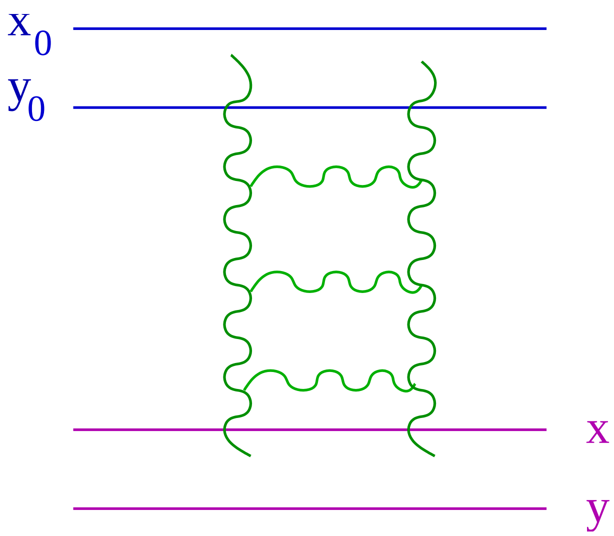

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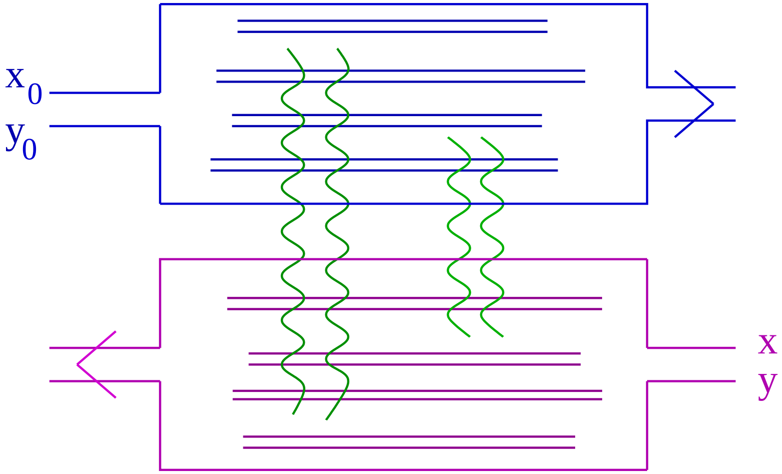
Multiple Scattering: Unitarization ...

- High energy: Multiple scattering ($T \equiv 1 - S \sim \mathcal{O}(1)$)

  Simultaneous scattering between several pairs of dipoles



$$S(Y) = \sum_{N,N'=1}^{\infty} \int d\Gamma_N P_N(Y/2) \int d\Gamma_{N'} P_{N'}(Y/2) \exp\left\{ - \sum_{i=1}^{N} \sum_{j=1}^{N'} T_0(i|j) \right\}$$

  Unitarization configuration by configuration : $S_{N\times N'} \leq 1$

- "Pomeron loops"

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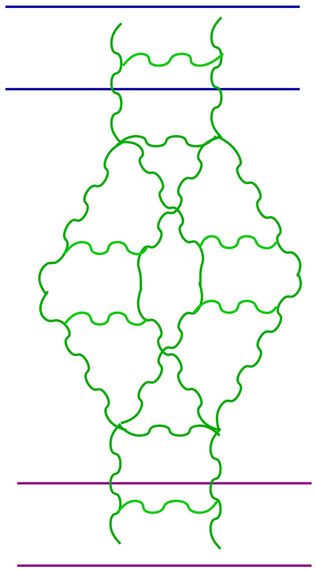

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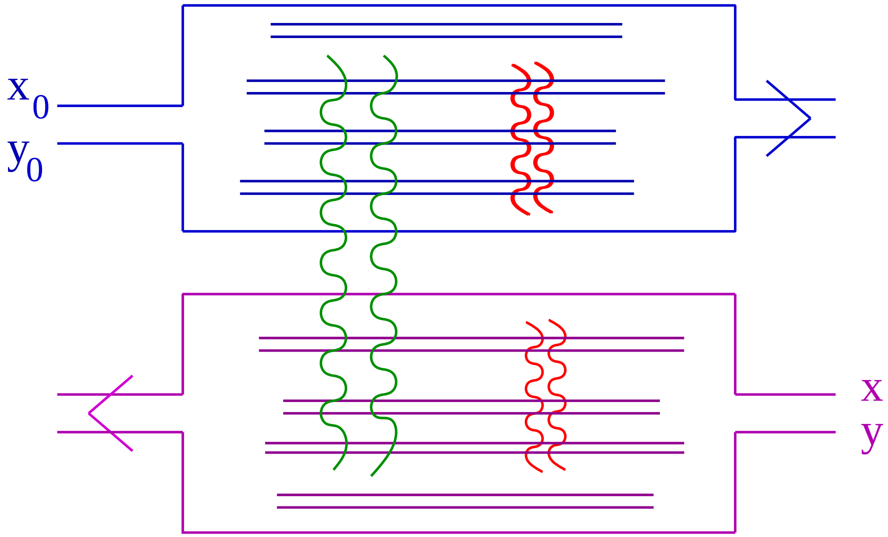
... without Saturation !

- Dipole picture neglects saturation effects

  (non–linear effects inside the wavefunction)



$$\alpha_s^2\, n^2(Y/2) \sim 1 \quad \text{but} \quad \alpha_s^2\, n(Y/2) \sim \alpha_s^2\, \mathrm{e}^{\omega_{\mathbb{P}} Y/2} \ll 1$$

▷ Restricted to the COM frame and to a finite energy range:

$$Y_c \lesssim Y \ll 2Y_c \quad \text{with} \quad Y_c \sim \frac{1}{\omega_{\mathbb{P}}} \ln \frac{1}{\alpha_s^2}$$

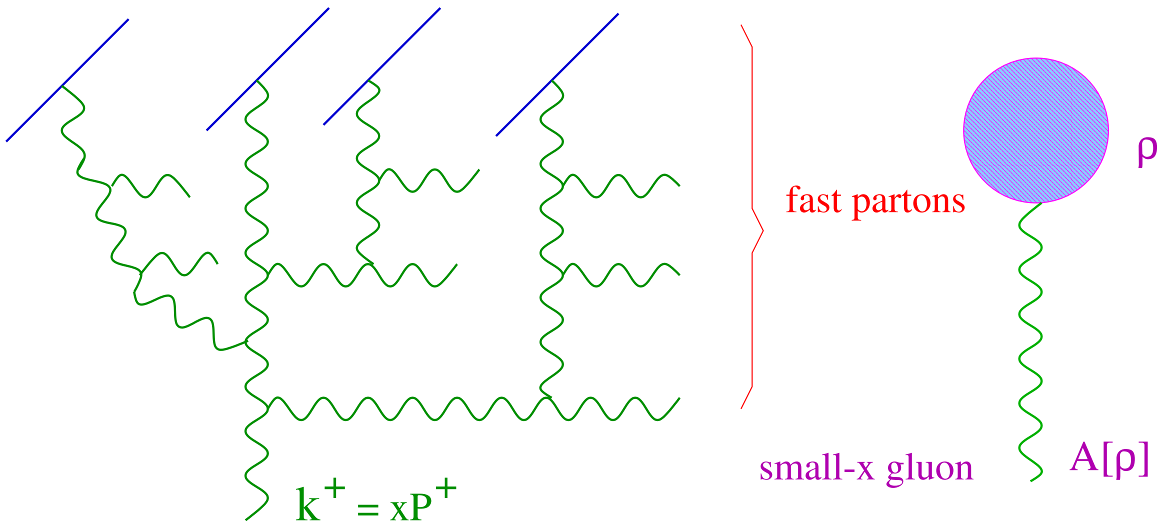
The **C**olor **G**lass **C**ondensate *(MV, BK, JIMWLK)*

- A classical, stochastic, effective theory for gluon saturation

  Small–x gluons $\longleftrightarrow$ Classical color fields radiated by 'sources' (partons) with larger values of x

- High gluon density $\longleftrightarrow$ Strong classical color fields

  $\Longrightarrow$ Non–linear effects leading to saturation ($A \sim 1/g$)



- Non–linear evolution : Quantum gluons rescatter off the classical background fields

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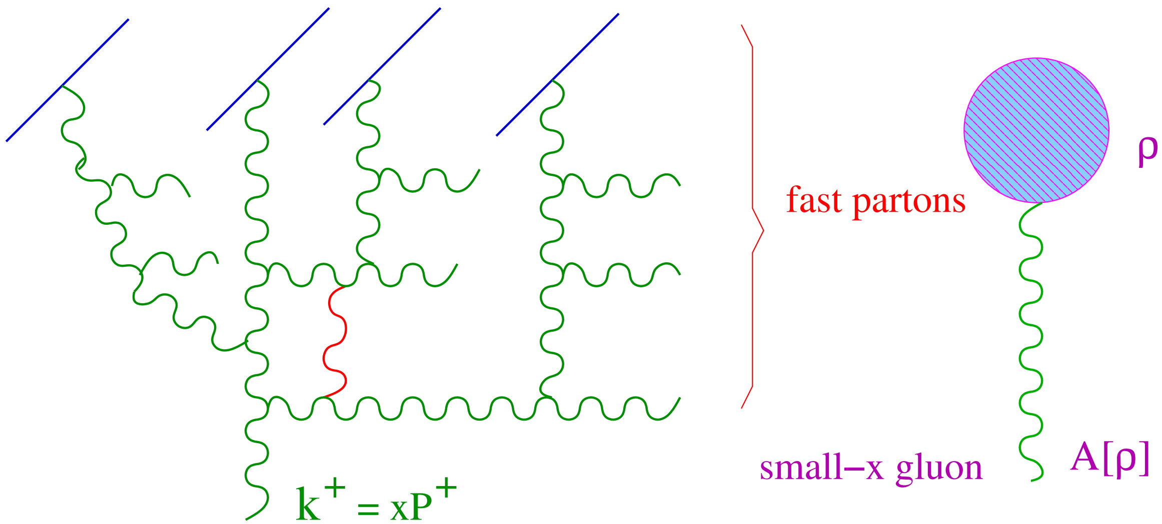
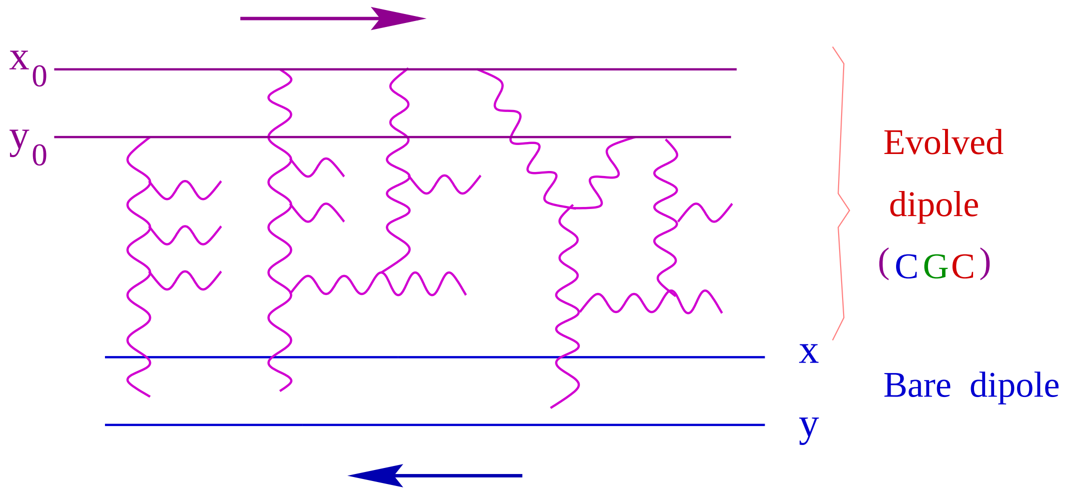


- No ('pomeron') loops : Sub-dominant so long as the classical fields are relatively strong ($A \gg 1$)

Dipole – CGC Scattering

- Dipole frame : the dipole is nearly at rest and unevolved



$$S_Y = \frac{1}{N_c}\Big\langle \mathrm{tr}\big(V^\dagger_{\boldsymbol{x}}\, V_{\boldsymbol{y}}\big)\Big\rangle_Y = \int \mathrm{D}[A^+]\, W_Y[A^+]\, \frac{1}{N_c}\,\mathrm{tr}\Big(V^\dagger_{\boldsymbol{x}}[A^+]\, V_{\boldsymbol{y}}[A^+]\Big)$$

$$V^\dagger_{\boldsymbol{x}}[A^+] \equiv \mathrm{P}\exp\Big(\mathrm{i}g\int dx^- A^+_a(x^-,\boldsymbol{x})t^a\Big) \qquad \text{(Wilson line)}$$

- $W_Y[A^+]$: probability distribution for the classical field A^+
- Unitarization via multiple scattering off the classical field

Non–linear evolution in CGC

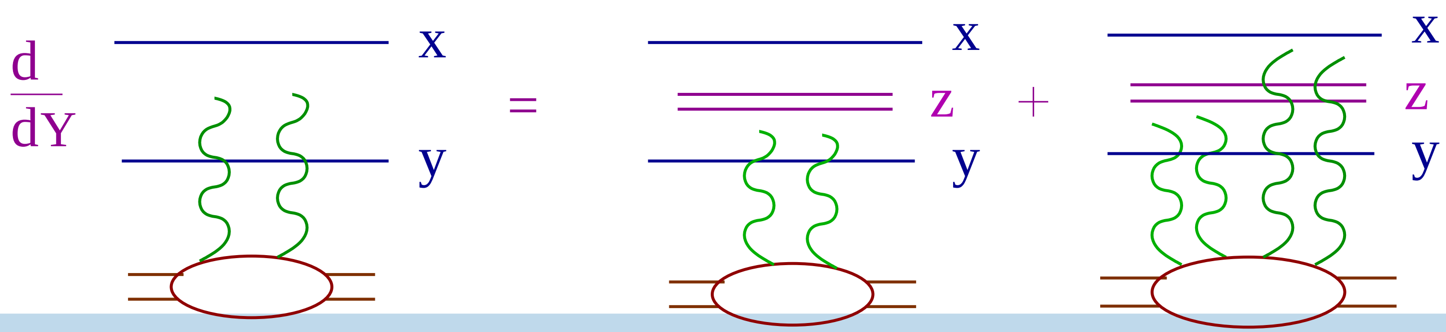
- JIMWLK equation (a functional Fokker–Planck eq.)

$$\frac{\partial W_Y[A]}{\partial Y} = \frac{1}{2}\int_{\boldsymbol{x},\boldsymbol{y}} \frac{\delta}{\delta A_Y^a(\boldsymbol{x})}\,\chi^{ab}(\boldsymbol{x},\boldsymbol{y})[A]\,\frac{\delta W_Y}{\delta A_Y^b(\boldsymbol{y})}$$

- Coupled equations for Wilson line correlators: Balitsky eqs.
- Dipole–CGC scattering amplitude: $T = 1 - S$, $S = \frac{1}{N_c}\mathrm{tr}(V_{\boldsymbol{x}}^\dagger V_{\boldsymbol{y}})$

$$\frac{\partial}{\partial Y}\langle T(\boldsymbol{x},\boldsymbol{y})\rangle_Y = \frac{\alpha_s N_c}{\pi}\int_{\boldsymbol{z}} \frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}$$

$$\Big\langle \underbrace{-T(\boldsymbol{x},\boldsymbol{y}) + T(\boldsymbol{x},\boldsymbol{z}) + T(\boldsymbol{z},\boldsymbol{y})}_{\text{2-point ftions}} + \underbrace{T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})}_{\text{3-point ftion}} \Big\rangle_Y$$



Balitsky–Kovchegov equation

- Mean field approximation $\Longrightarrow$ A closed equation !

$$\langle T(\boldsymbol{x},\boldsymbol{z})T(\boldsymbol{z},\boldsymbol{y})\rangle_Y \approx \langle T(\boldsymbol{x},\boldsymbol{z})\rangle_Y \, \langle T(\boldsymbol{z},\boldsymbol{y})\rangle_Y$$

  - Incoherent multiple scattering
  - Justified if CGC = Large nucleus ($A \gg 1$)
    & not too high energies *(Kovchegov, 99)*

- Numerous studies (analytic & numerical)
- The same universality class as the F–KPP equation
  (Munier, Peschanski, 03)

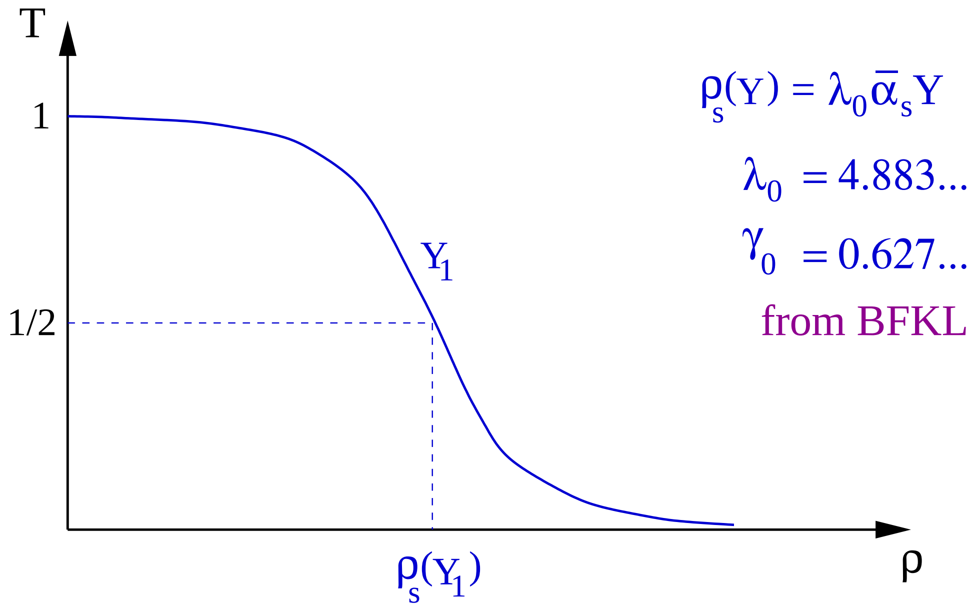
$$\partial_Y T(\rho,Y) = \underbrace{\partial_\rho^2 T(\rho,Y)}_{\text{diffusion}} + \underbrace{T(\rho,Y)}_{\text{growth}} \underbrace{- \; T^2(\rho,Y)}_{\text{recombination}}$$

  ▷ A large variety of situations in physics, chemistry, biology

- Two fixed points: $T=0$ (unstable) and $T=1$ (stable)
- “Traveling wave" : A front propagating into the unstable state

Traveling Wave

$T(r, Y) \equiv T(\rho, Y)$ with $\rho \equiv \ln \frac{1}{r^2 Q_0^2}$ (“small dipole" = “large ρ")



- $T \ll 1$: Linearized (BFKL) eq. : $T \sim r^{2\gamma}\, \mathrm{e}^{\omega Y} \sim \mathrm{e}^{-(\gamma\rho - \omega Y)}$
- $T \sim 1$: The non–linear term saturates the growth at $T = 1$
- $T = 1$ for $r = 1/Q_s(Y)$ or $\rho = \rho_s(Y)$ $(\equiv \ln Q_s^2(Y)/Q_0^2\ )$

$$Q_s^2(Y) \propto \mathrm{e}^{\lambda_0 \bar{\alpha}_s Y} \quad : \quad \text{Saturation momentum}$$

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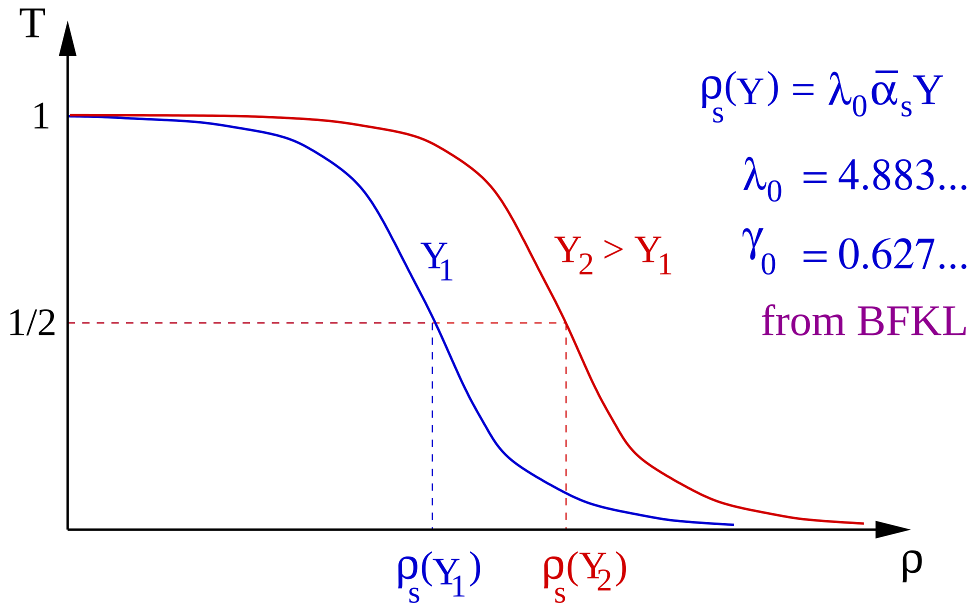

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$Q_s^2(Y) \propto \mathrm{e}^{\lambda_0 \bar{\alpha}_s Y}$: Saturation momentum

Geometric Scaling

- The shape of the front does not change in the course of the propagation $\Longrightarrow$ "Geometric scaling"

  $$T(\rho, Y) \simeq \mathrm{e}^{-\gamma_0(\rho-\rho_s(Y))} \equiv \left(r^2 Q_s^2(Y)\right)^{\gamma_0} \quad \text{for} \quad r \ll 1/Q_s(Y)$$

  (E.I., Itakura, McLerran, 02 ; Mueller, Triantafyllopoulos, 02)

- A natural explanation for a new scaling law identified in the HERA data for DIS at small–x

  (Staśto, Golec-Biernat, and Kwieciński, 2000)

- Relevant for the high–p_T suppression observed in d-Au collisions at RHIC

  (Kharzeev, Levin, McLerran, 02 ; E.I., Itakura, Triantafyllopoulos, 04)

- The saturation exponent $\lambda_0 = 4.88..$
  and the anomalous dimension $\gamma_0 = 0.63...$
  are correctly given by the linearized (BFKL) equation !

  WHY ?!

Pulled front & Fluctuations

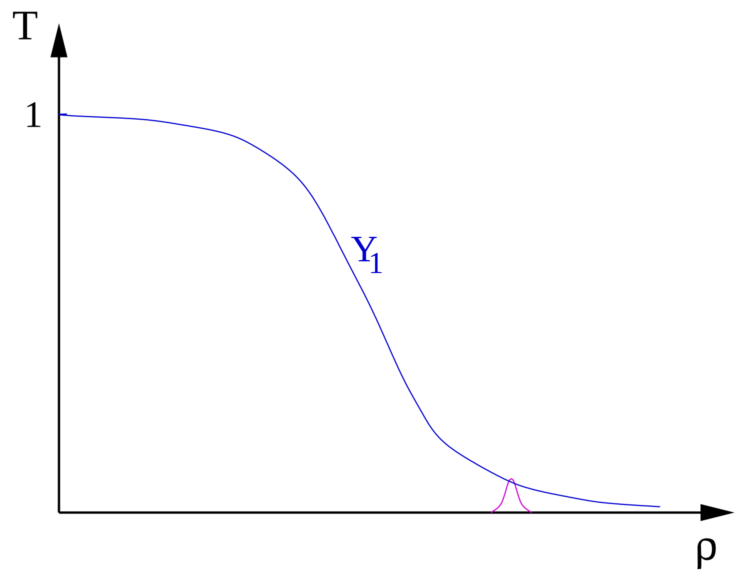
- The propagation of the front is driven by the growth and spreading of the small perturbations about the unstable state
  - The front is pulled along by its ‘leading edge’ ($T \ll 1$)
  - Specific to F–KPP equation !
- The propagation is governed by the linearized equation.
- The front properties (λ, γ) are strongly sensitive to small fluctuations !
  - Fluctuations $\left(\langle T^2 \rangle - \langle T \rangle^2\right)$ are important precisely in the leading edge, where $\langle T \rangle \ll 1$
- Mean field approximation is not reliable !
- Fluctuations due to the discreteness of the particle number

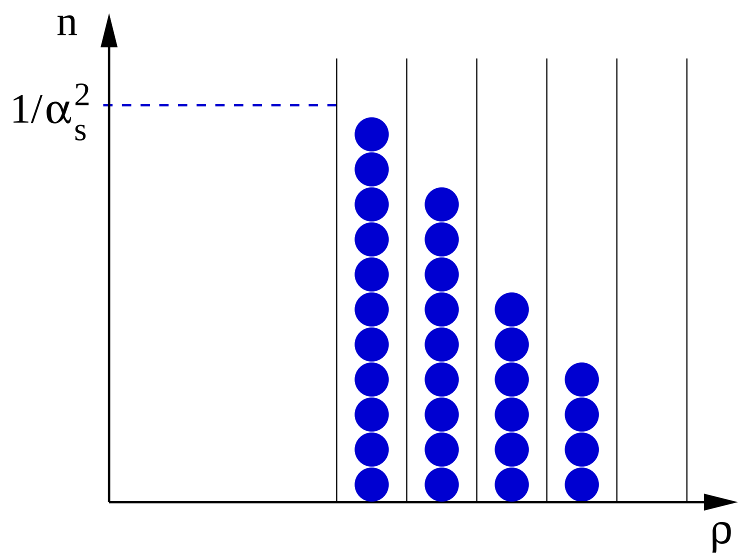
$$T(r, r_0, Y) \approx \alpha_s^2\, n(r, r_0, Y) \quad : \ \text{Discrete !}$$

$n(r, r_0, Y) =$ dipole occupation number = 0, 1, 2, ...

Saturation exponent with fluctuations

- Unitarity ($T \sim 1$) $\Longleftrightarrow$ Saturation ($n \sim 1/\alpha_s^2$)
- BK eq. : Front propagation is driven by growth in the tail.
- Discrete system : Diffusion of the dipoles in the foremost bin.





- There should be at least one dipole per bin for the growth to begin: $n \geq 1$, or $T \gtrsim \alpha_s^2$

$$\partial_Y T(\rho, Y) \; = \; D\partial_\rho^2 T(\rho, Y) \, + \, \Theta(T - \alpha_s^2)\big(T - T^2\big)$$

(Brunnet, Derrida, 97 – finite particle number version of F–KPP)

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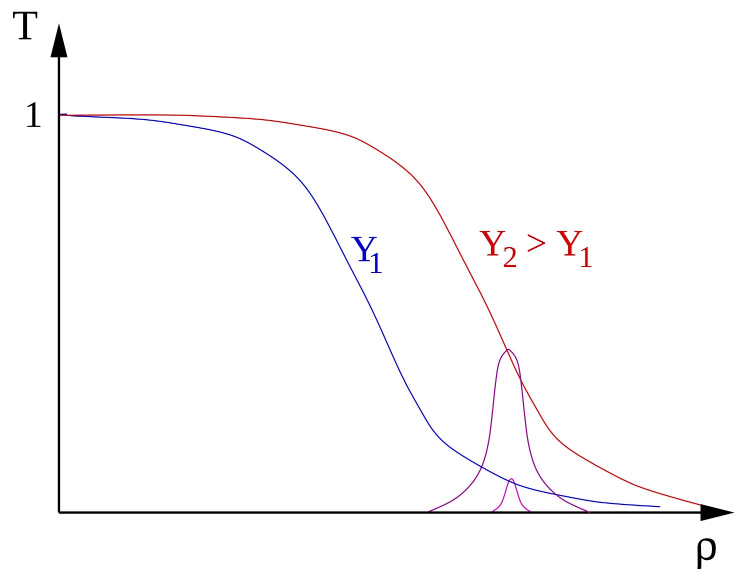

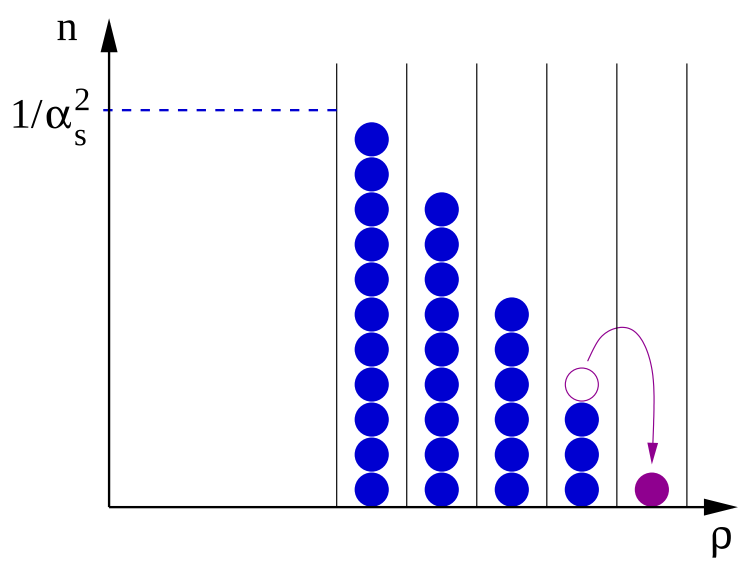

- The speed of the front (saturation exponent) for $\alpha_s \to 0$:

$$\lambda_s \equiv \frac{d\rho_s(Y)}{\bar{\alpha}_s\, dY} \approx \lambda_0 - \frac{D}{\ln^2(1/\alpha_s^2)}\,, \qquad \lambda_0 \approx 4.88, \quad D \approx 150\,(!)$$

(consistent with Mueller & Shoshi, 2004)

Front diffusion

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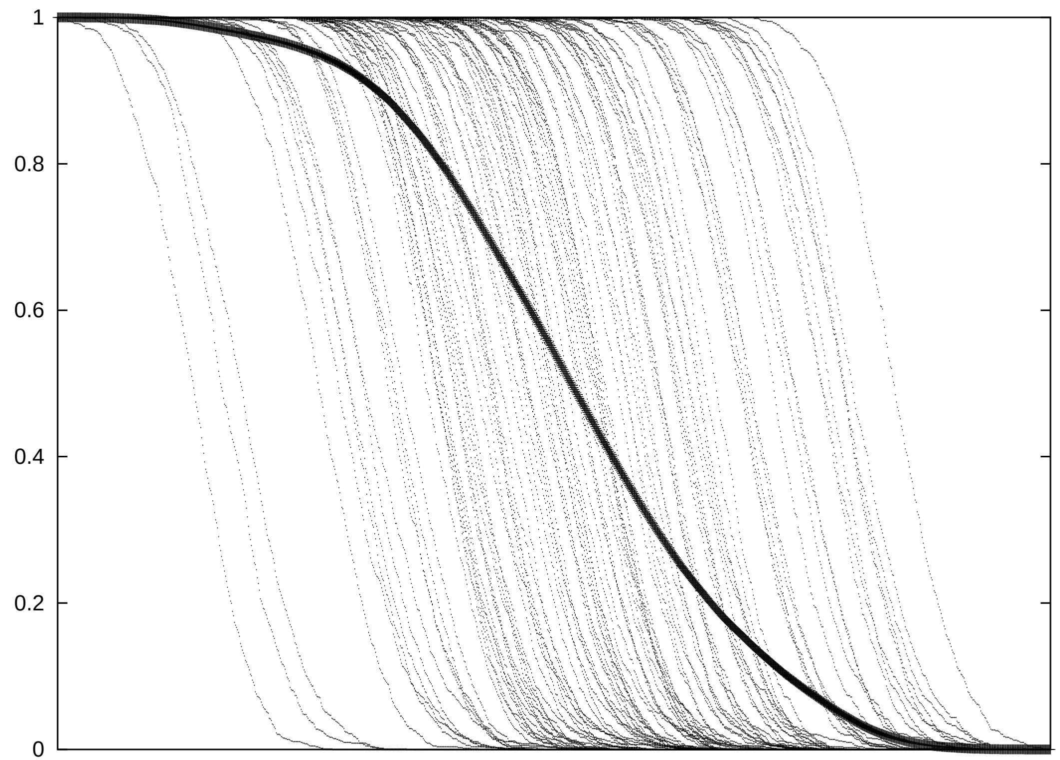
Color Dipole Picture
Color Glass Condensate
Mean Field Approximation
Fluctuating pulled fronts
- Pulled fronts
- Saturation exponent
- Front diffusion

A new equation
Conclusions

- The position $\rho_s(Y)$ of the front shows a diffusive wandering around its average value

$$\langle \rho_s(Y) \rangle = \lambda_s \bar{\alpha}_s Y, \quad \langle \rho_s^2 \rangle - \langle \rho_s \rangle^2 = D_{\rm front} \bar{\alpha}_s Y, \quad D_{\rm front} \sim \frac{1}{\ln^3(1/\alpha_s^2)}$$



- At large Y, geometric scaling is badly violated !

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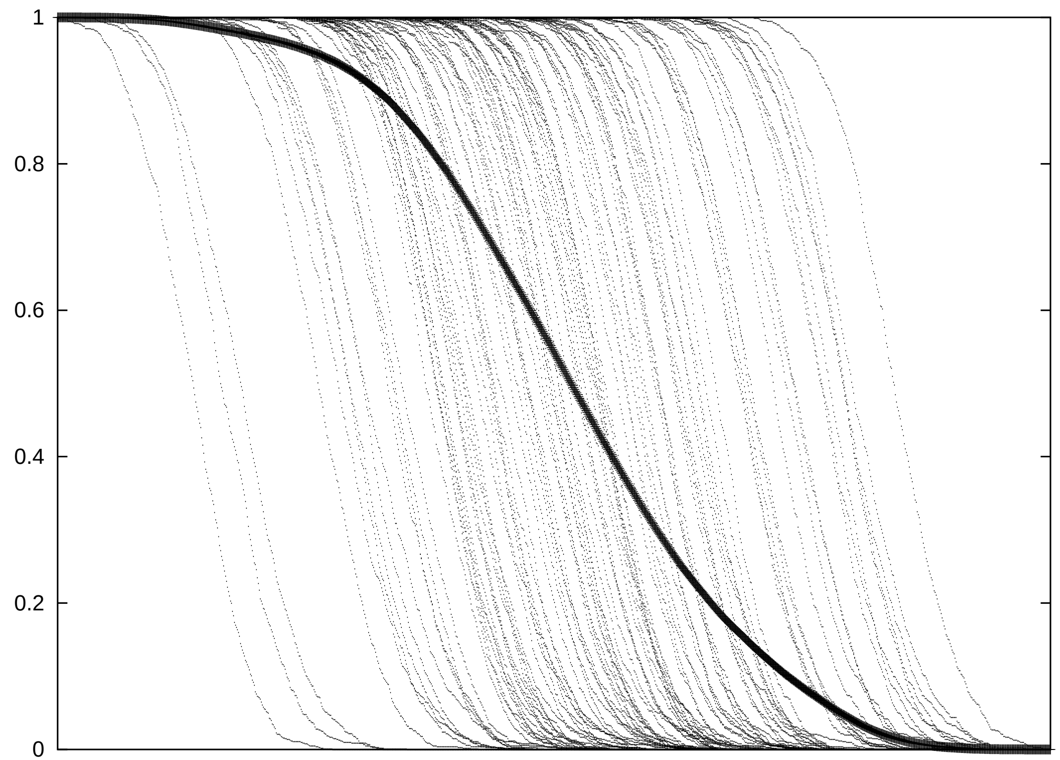


- Large Y ... but HOW large ??

Fluctuations + Saturation = Pomeron loops

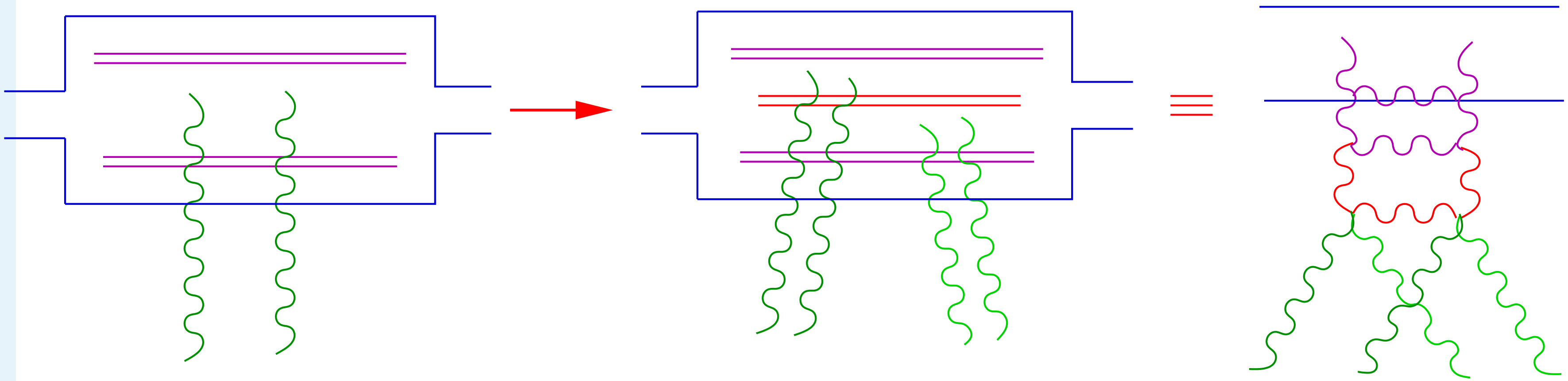
- A unified description of saturation with fluctuations:

  CGC for strong fields + Dipole picture in the dilute regime

$$\partial_Y T(\rho,Y) = \underbrace{\partial_\rho^2 T(\rho,Y)}_{\text{diffusion}} + \underbrace{T(\rho,Y)}_{\text{growth}} - \underbrace{T^2(\rho,Y)}_{\text{recomb.}} + \underbrace{\sqrt{\alpha_s^2 T}\,\eta(\rho,Y)}_{\text{noise}}$$

$$\langle \eta(\rho,Y)\rangle = 0, \qquad \langle \eta(\rho,Y)\eta(\rho',Y')\rangle = \delta(\rho-\rho')\,\delta(Y-Y')$$

- Noise term $\Longleftrightarrow$ Dipole multiplication in the dilute regime



$\partial_Y \langle T(r_1)T(r_2)\rangle \sim \alpha_s^2 \langle T(r_1+r_2)\rangle$: Dominant when $T < \alpha_s^2$

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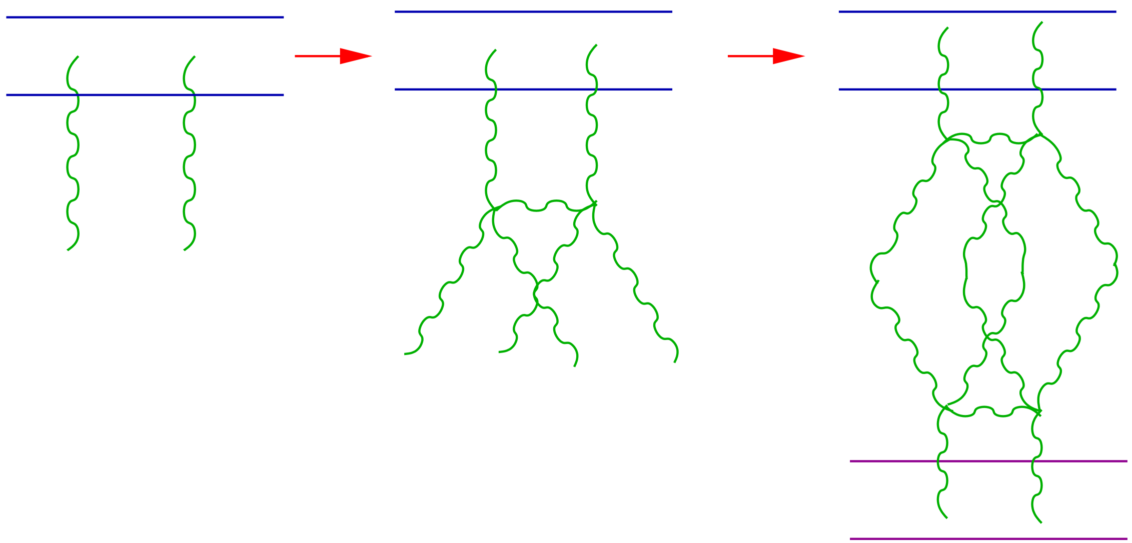
- Splitting + Recombination $\Longrightarrow$ Pomeron loops



Conclusions

- JIMWLK eq. itself is a kind of “mean field approximation"
- The effects of the fluctuations are huge !
  - very slow convergence of λ_s towards λ_0 when $\alpha_s \to 0$
  - lowest–order estimate: $\lambda_s < 0$ unless $\alpha_s < 0.05$!!
  - useless for practical applications
- Urgent need for better estimates & numerics
  - The Langevin equation is well suited for that !
- Exact solutions ??
  - conformal symmetry
- Enriching correspondence with numerous problems in statistical physics, chemistry, biology, ...
  - biological pattern formations, directed percolation, chemical reactions, spreading of epidemics, solar activity (dynamo waves in the sunspots), computer science (digital search trees and data compression) ...