The Color Glass Condensate:

A classical effective theory of high energy QCD

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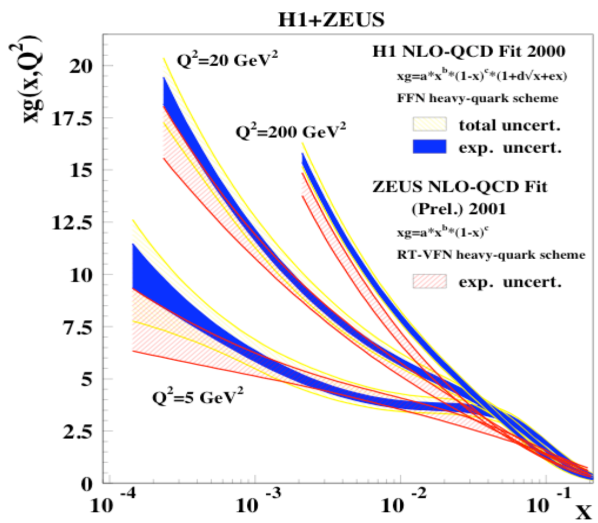
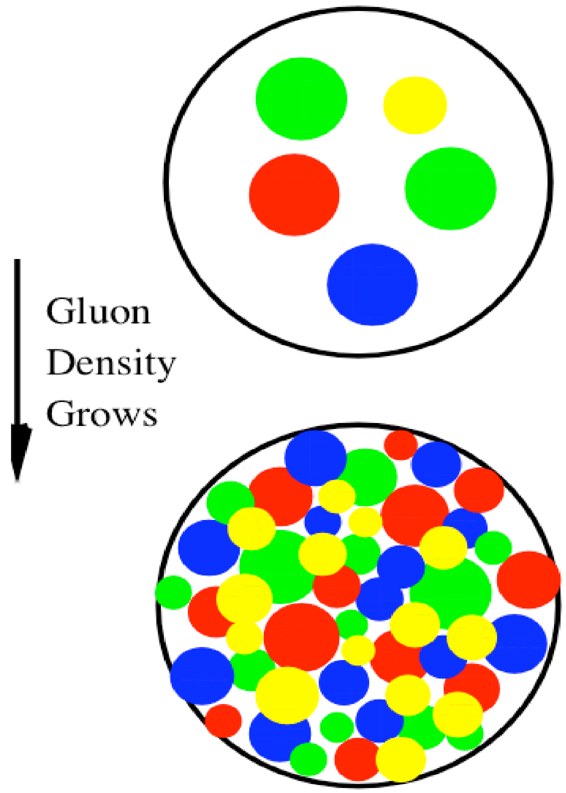
Outline of talk:

- A classical effective theory for high energy QCD
- Quantum evolution *a la* JIMWLK and BK
- Hadronic scattering and k_t factorization in the Color Glass Condensate
- What the CGC tells us about the matter produced in AA and dA collisions at RHIC.
- Thermalization and other open issues

Parton saturation at small x

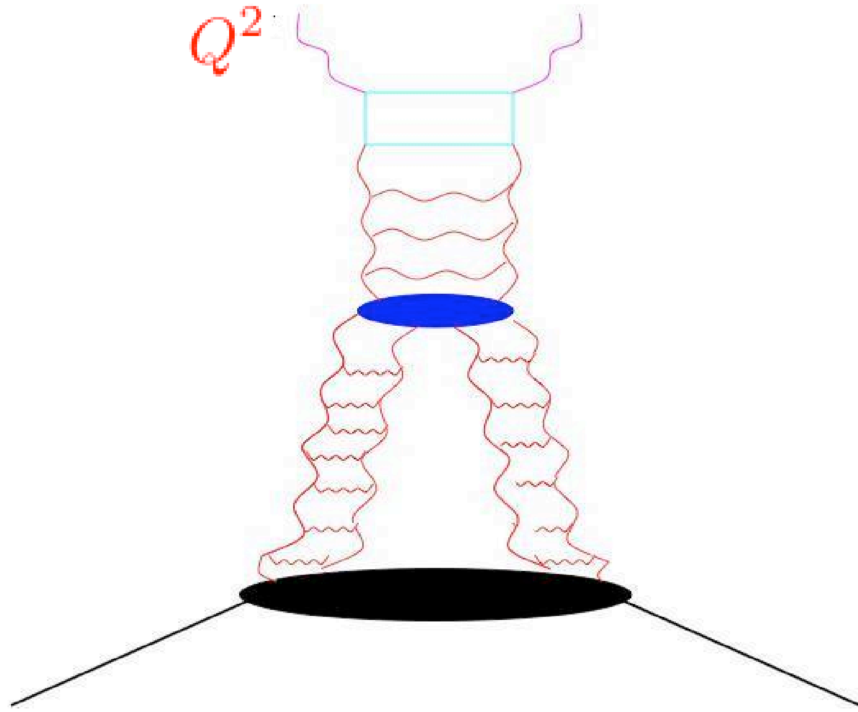




Phase space density grows rapidly-BFKL evolution breaks down when partons begin to overlap in transverse plane

Gluon density saturates at phase space density f= $1/\alpha_S$

Gluon recombination-higher twist effects



Gribov, Levin, Ryskin
Mueller, Qiu
Blaizot, Mueller

Recombination effects compete with DGLAP Bremsstrahlung effects when

$$\alpha_S x G(x, Q^2) \sim R^2 Q^2$$

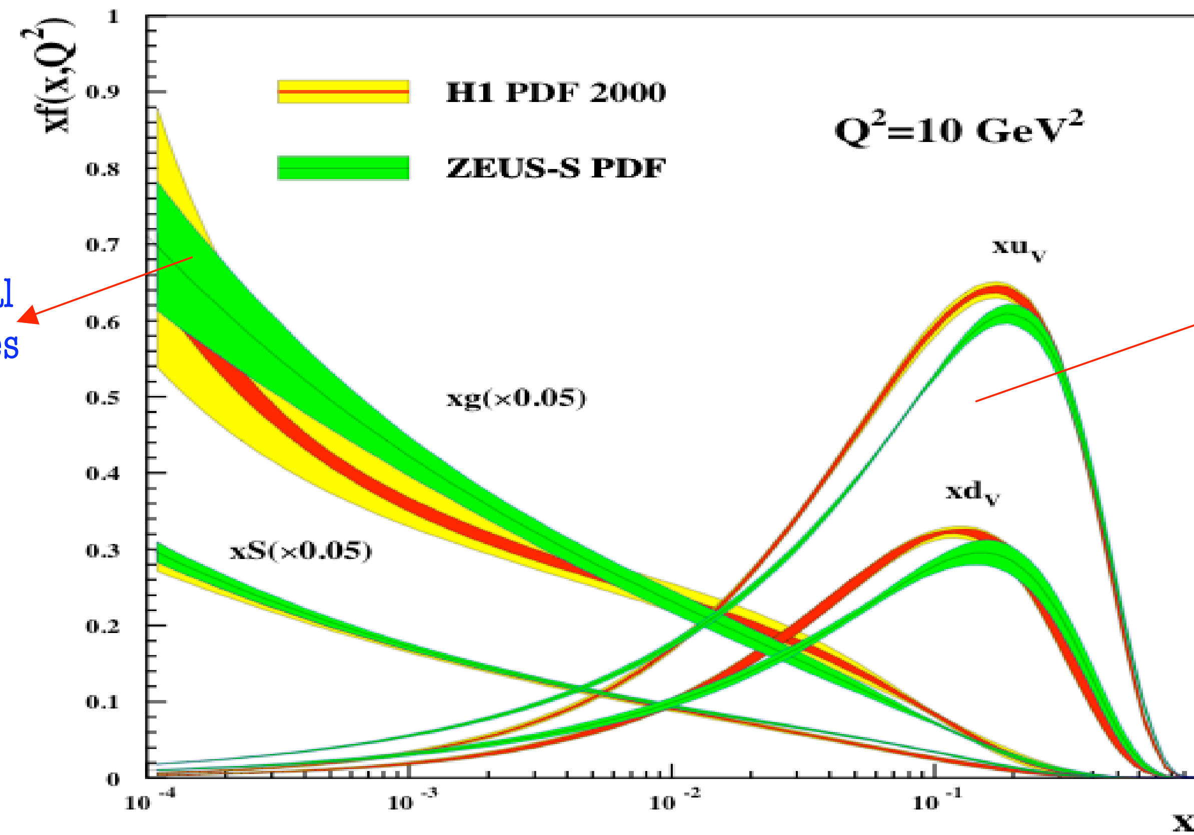
Saturation of the gluon density for $Q \equiv Q_s(x)$

Classical Effective Theory

McLerran, RV; Kovchegov;
Jalilian-Marian,Kovner,McLerran, Weigert

Born-Oppenheimer: separation of large x and small x modes

Dynamical
Wee modes



Valence modes-
are static
sources for wee
modes

In large nuclei, sources are Gaussian random sources MV, Kovchegov,
Jeon, RV

The effective action

Scale separating sources and fields

Generating functional:

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional describing distribution of the sources

$$S[A,\rho] = \frac{-1}{4}\int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c}\int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

Dynamical wee fields

Coupling of wee fields to classical sources

where $U_{-\infty,\infty}[A^-] = \mathcal{P}\exp\left(ig\int dx^+ A^{-,a}T^a\right)$

For large nuclei, $W_{\Lambda^+}[\rho] = \exp\left(-\int d^2x_\perp \frac{\rho^a\rho^a}{2\mu_A^2}\right)$ *with* $\mu_A^2 = \frac{g^2 A}{2\pi R_A^2} \propto A^{1/3}$

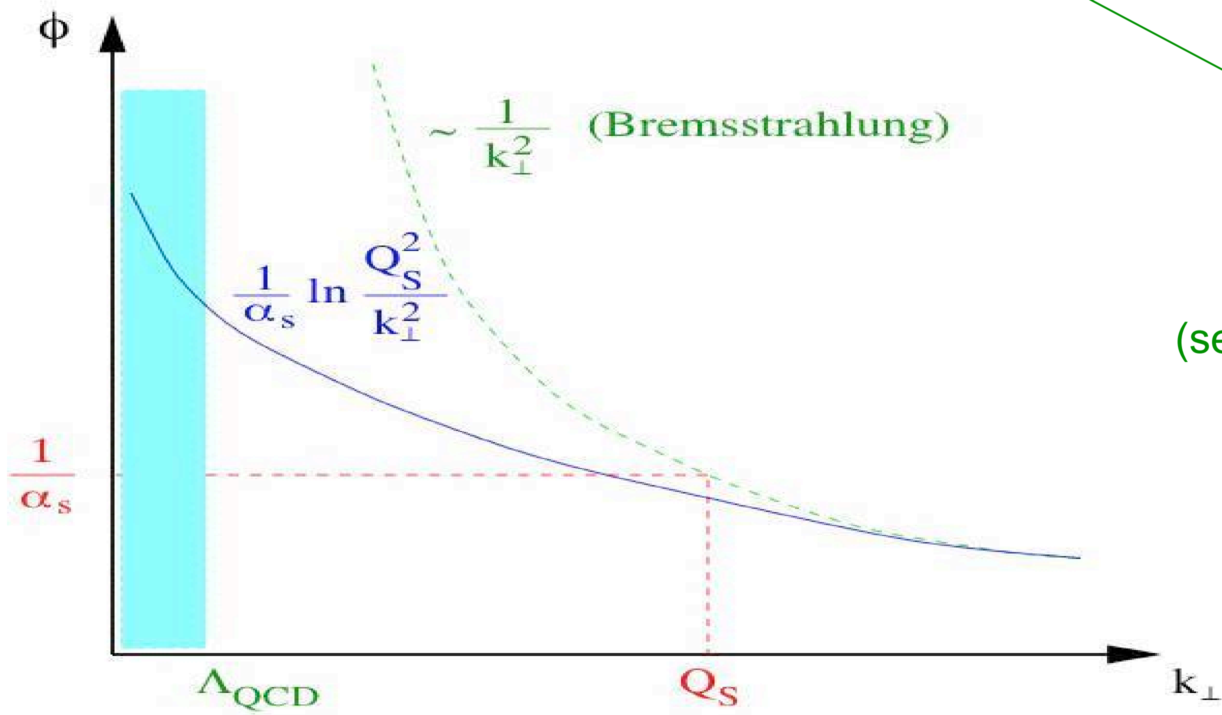
$\alpha_S(\mu_A^2) << 1$

Effective action describes a weakly coupled albeit non-perturbative system

Average over ρ^a to compute classical gluon distribution

$$< AA >_\rho = \int [d\rho] A_{\rm cl.}(\rho) A_{\rm cl.}(\rho) W_{\Lambda^+}[\rho]$$

Gaussian in MV



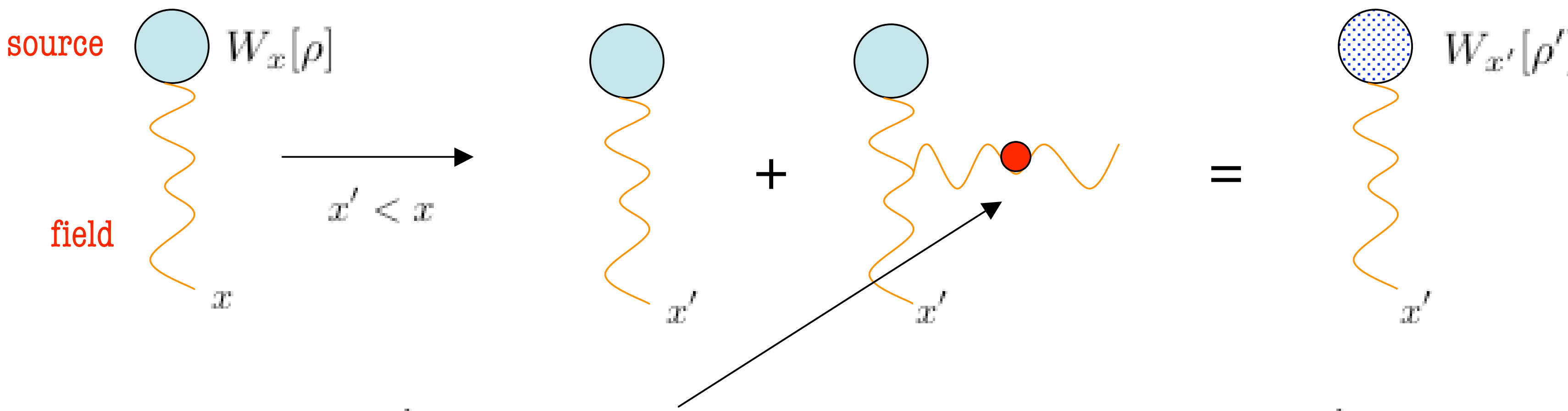
(see talk by Triantafyllopolous)

$$\phi = \text{gluon phase space density} = \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN}{\pi R^2 d^2k_\perp dy}$$

$$Q_s^2 \approx \alpha_S N_c \mu_A^2 \ln\left(\frac{Q_s^2}{\Lambda^2}\right) \sim A^{1/3} \ln A \approx A^{1/3} \text{ for A >> 1}$$

QUANTUM EVOLUTION VIA WILSONIAN RG

Evolution to small x => $W_{\Lambda^+}[\rho] \neq \exp\left(-\int d^2x_\perp \frac{\rho^a \rho^a}{2\mu^2}\right)$



$$\rho' = \rho + \delta\rho \quad => W_x[\rho] \rightarrow W_{x'}[\rho']$$

$W_x[\rho]$ *obeys a non-linear Wilson renormalization group equation in x-the JIMWLK equation*

(Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

Correlation Functions

JIMWLK => An infinite hierarchy of ordinary differential equations for gluon correlators $< A_1 A_2 \cdots A_n >_y$

Change of variables: $\rho^a \rightarrow \alpha^a \, ; \, \nabla^2 \alpha = \rho$

$$< O[\alpha] >_Y = \int [d\alpha] O[\alpha] W_Y[\alpha]$$

Brownian motion in functional space: Fokker-Planck equation!

Weigert; Iancu,Mclerran

$$=> \frac{\partial}{\partial Y} < O[\alpha] >_Y = < \frac{1}{2} \int_{x,y} \frac{\delta}{\delta \alpha_Y^a(x)} \chi_{x,y}^{ab} \frac{\delta}{\delta \alpha_Y^b(y)} O[\alpha] >_Y$$

"time"

"diffusion coefficient"

For the gluon density $< \alpha(x_\perp) \alpha(y_\perp) >_Y$ for $g\,\alpha << 1$

Recover the BFKL equation in low density limit

JIMWLK Eqns. are master equations-a la BBGKY hierarchy in Stat. Mech. -difficult to solve

- Numerical studies — Rummukainen, Weigert

- Mean field approximation of hierarchy-the BK equation closes the hierarchy . — Balitsky; Kovchegov

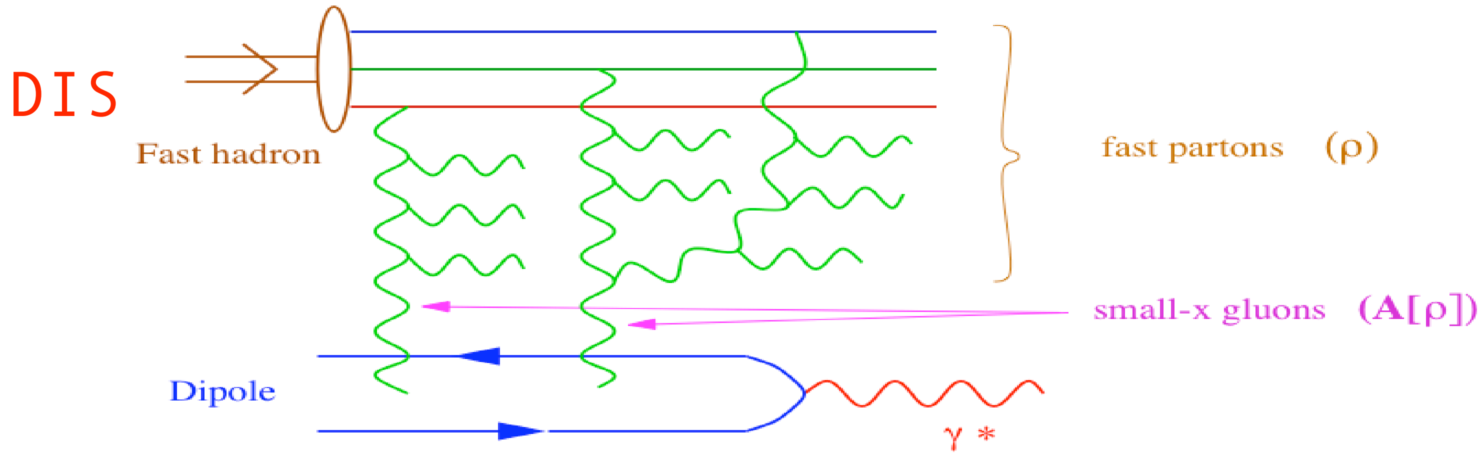
$$< \mathrm{Tr}(V_x V_z^\dagger)\mathrm{Tr}(V_z V_y^\dagger) > \longrightarrow < \mathrm{Tr}(V_x V_z^\dagger) > < \mathrm{Tr}(V_z V_y^\dagger) >$$

$$N_c \to \infty \; ; \; \alpha_S^2 A^{1/3} \to \infty$$

- Mean field solutions to JIMWLK deep in saturation regime. — Iancu, McLerran

The Balitsky-Kovchegov equation

I. Balitsky;
Y. Kovchegov



$$\sigma^{\gamma^* p}(x, Q^2) = \int_0^1 dz \int d^2 r |\psi(z, r; Q^2)|^2 \sigma_{\text{dipole}}(x, r)$$

where $\sigma_{\text{dipole}}(x, r) = 2 \int d^2 b \, (1 - S(x, r, b))$

with $S(x, r, b) = \frac{1}{N_c} < \mathrm{Tr} V^\dagger(x) V(y) >_Y \equiv 1 - \mathcal{N}_Y(r, b)$

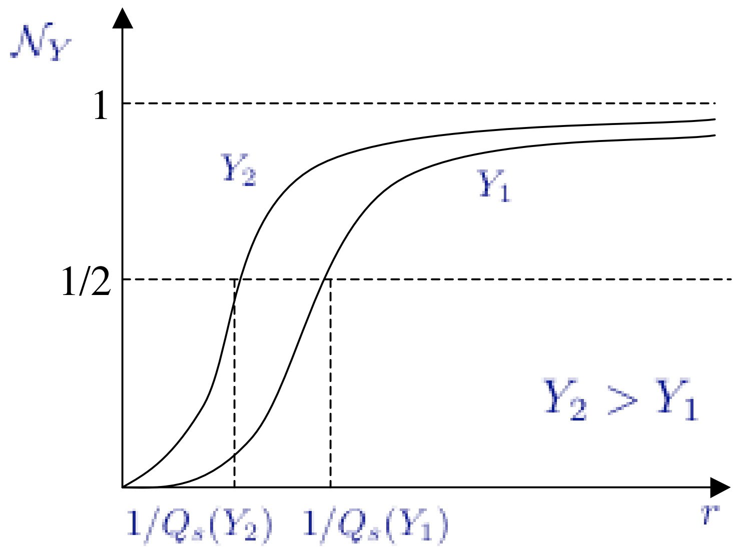
s-matrix — amplitude

$$V^\dagger(x) = \mathcal{P} \exp\left(ig \int dx^- \alpha_a(x^-, x) T^a \right)$$

In hadron rest frame-Mueller dipole picture

BK: Evolution eqn. for the dipole cross-section

$$\frac{\partial \mathcal{N}_Y(x,y)}{\partial Y} = \bar{\alpha}_s \int_z \frac{(x-y)^2}{(x-z)^2(y-z)^2} \{\underbrace{\mathcal{N}_Y(x,z) + \mathcal{N}_Y(z,y) - \mathcal{N}_Y(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{N}_Y(x,z)\mathcal{N}_Y(z,y)}_{\text{Non-linear}}\}$$



- From saturation condition,

  $\mathcal{N} = 1/2$ when $r \sim 1/Q_s(Y)$ =>

  $$\boxed{Q_s^2(Y) \approx Q_0^2\, e^{\lambda Y} \ \text{with}\ \lambda \sim 4.8\,\alpha_s}$$

- For large dipole, $(r >> 1/Q_s(Y))$

  $$\mathcal{N}_Y(r) \approx 1 - \kappa \exp\left(-\frac{1}{4c}\ln^2(r^2 Q_s^2(Y))\right)$$ Levin,Tuchin

  $c \approx 4.8$

Close analogy to theory of travelling waves -> approx. asymptotic solution Munier-Peschanski

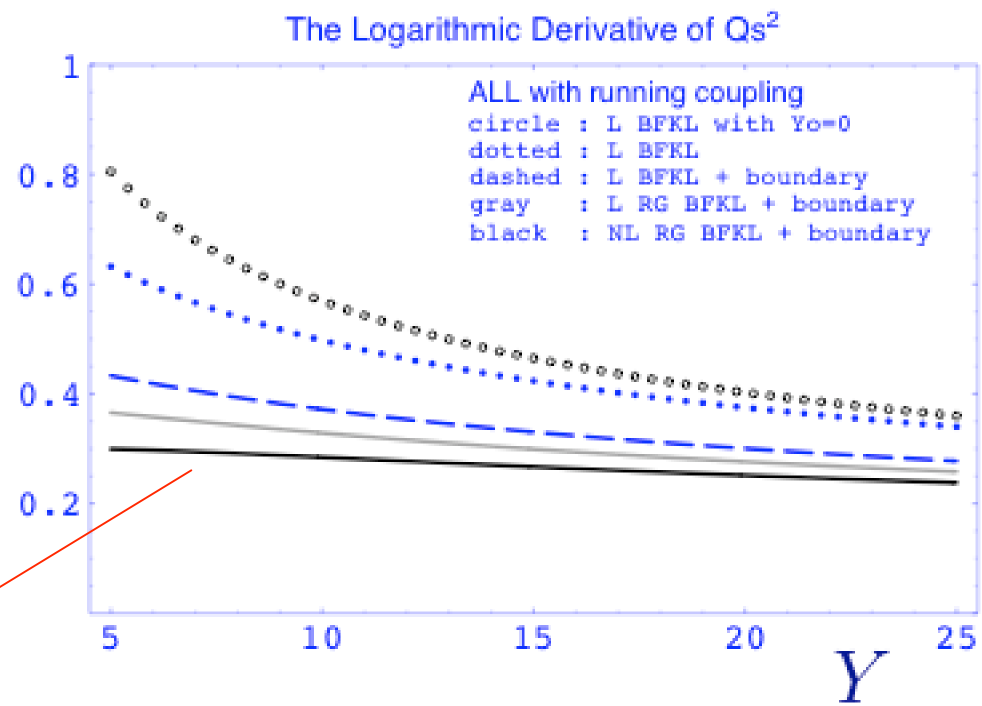
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$

LO BFKL+ running coupling: $Q_s^2 = \Lambda_{\mathrm{QCD}}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$

Re-summed NLO BFKL + CGC:

$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$



Triantafyllopolous

Very close to HERA result!

Synopsis of CGC numerics

Numerical simulations of BK-eqn display Geometrical Scaling
(Armesto,Braun; Golec-Biernat,Stasto,Motyka;
Albacete,Armesto,Salgado,Kovner,Wiedemann)

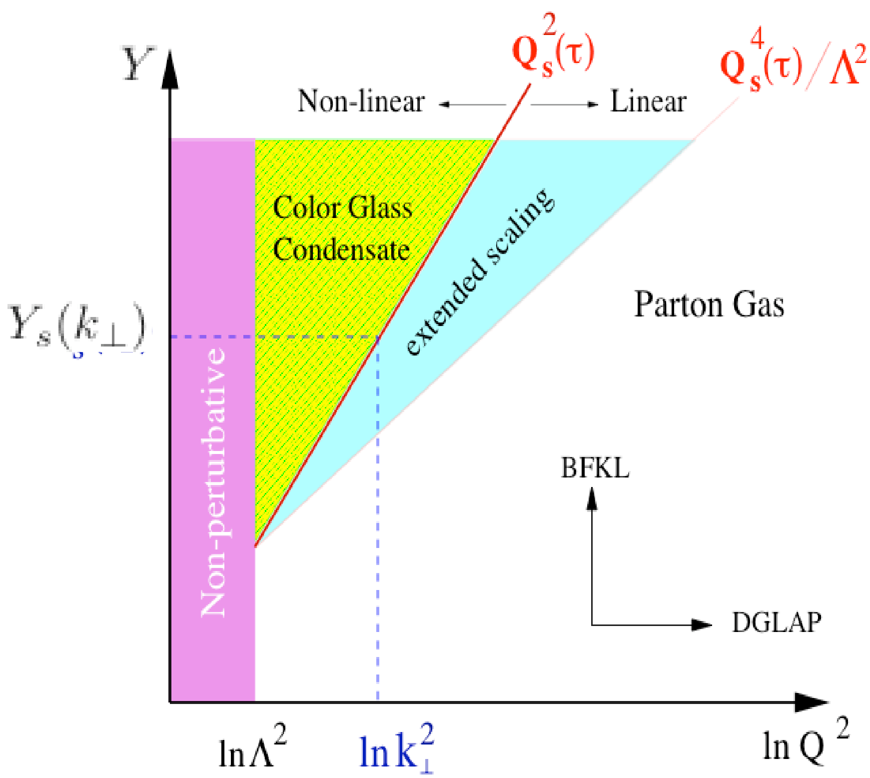
Infrared diffusion pathology of BFKL is cured.

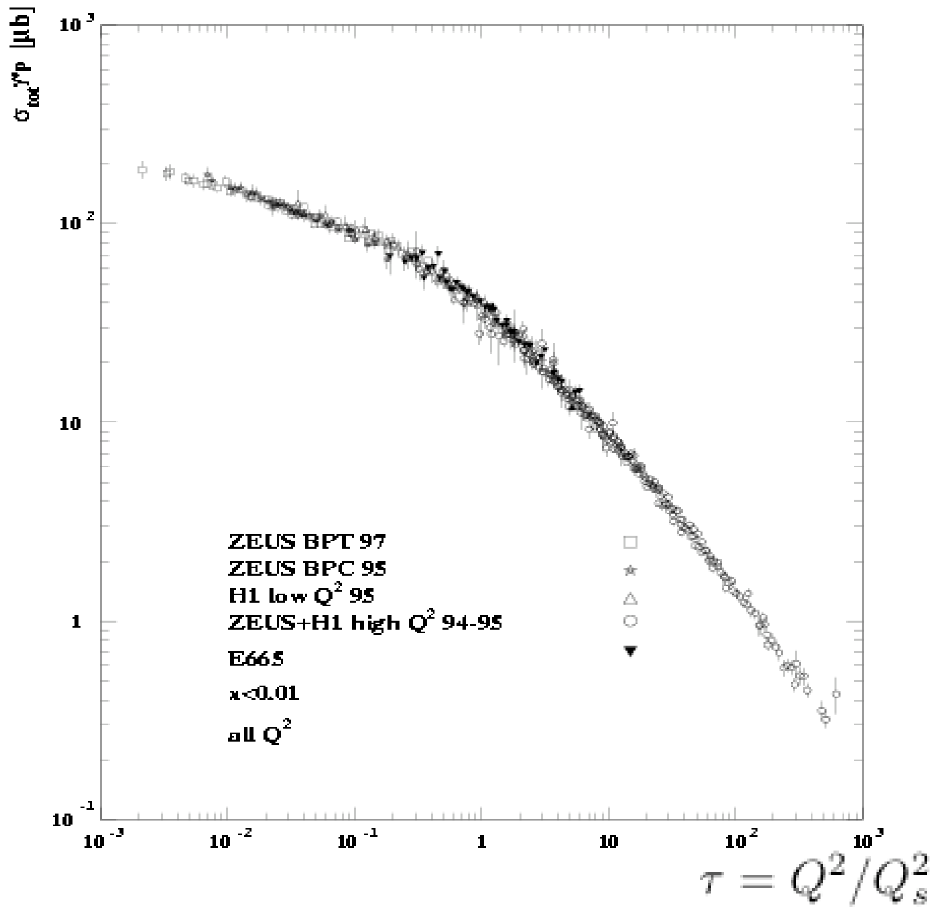
State of the art: numerical simulations of JIMWLK n-point correlators
(Rummukainen & Weigert)

Running coupling effects important

Geometrical scaling at HERA

(Golec-Biernat,Kwiecinski,Stasto)





NOVEL REGIME OF QCD EVOLUTION AT HIGH ENERGIES

- Impact parameter and fluctuations-the "Derrida deconstruction"

  Mueller, Shoshi
  Iancu, Mueller, Munier

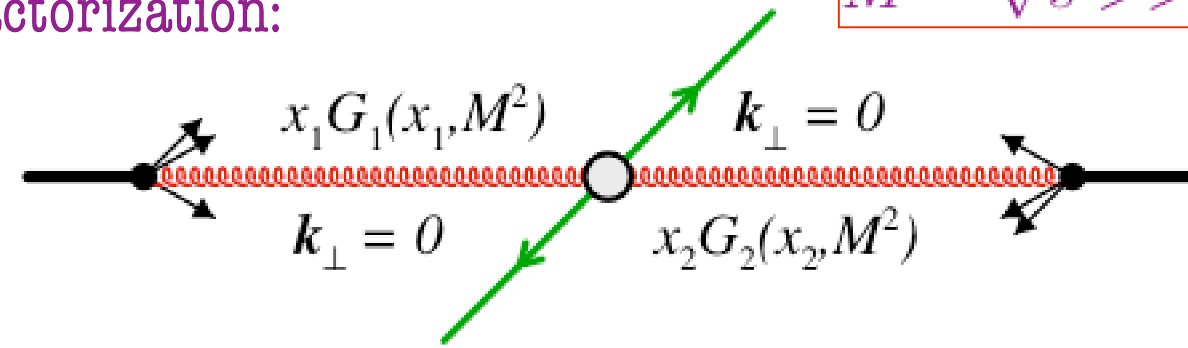
  (see Iancu talk)

Hadron & Nuclear Scattering at high energies

I: Universality: collinear versus k_t factorization

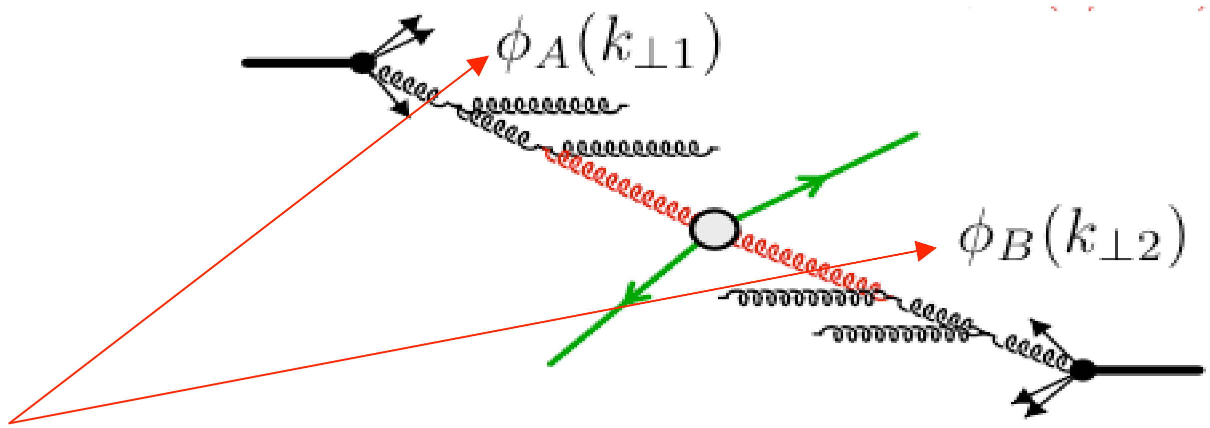
Collinear factorization:

$M \sim \sqrt{s} >> \Lambda_{\rm QCD}$



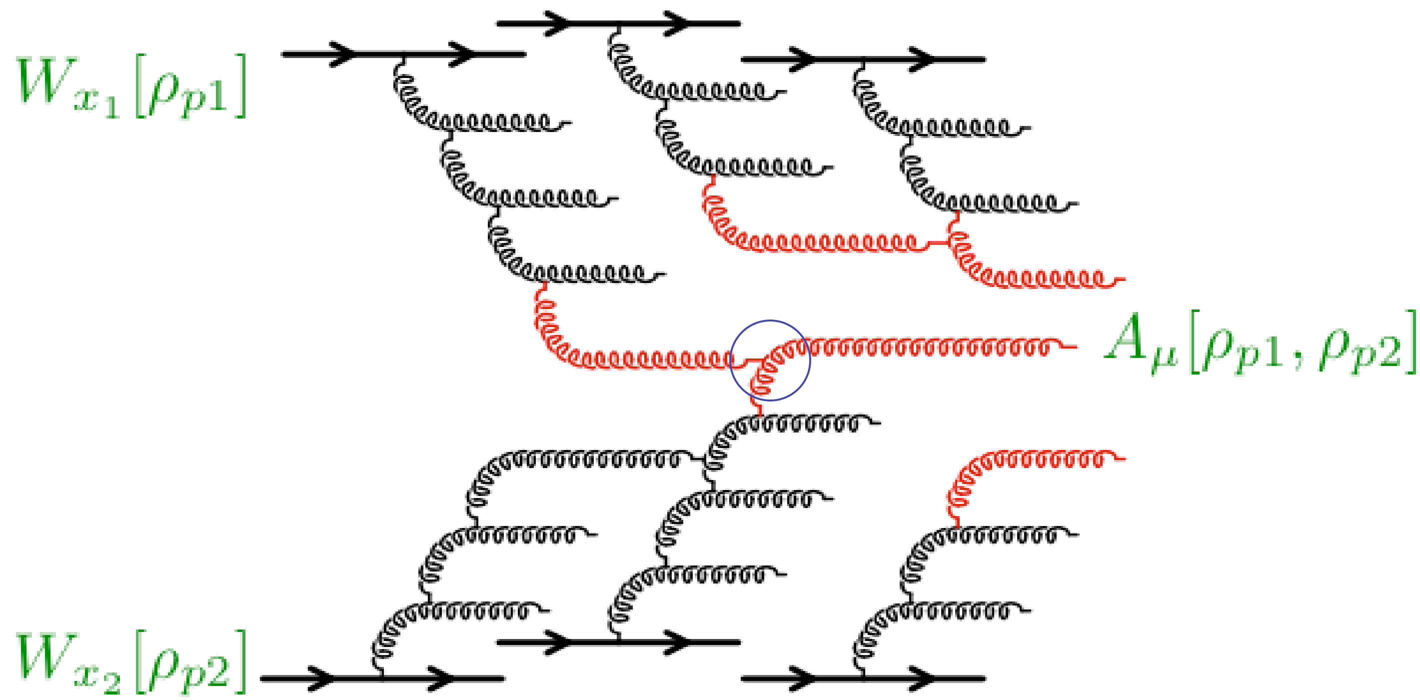
k_t factorization:

$\Lambda_{\rm QCD} << M << \sqrt{s}$



Are these objects universal? Very important for extraction of "gluon" distributions.

Hadronic collisions in the CGC framework

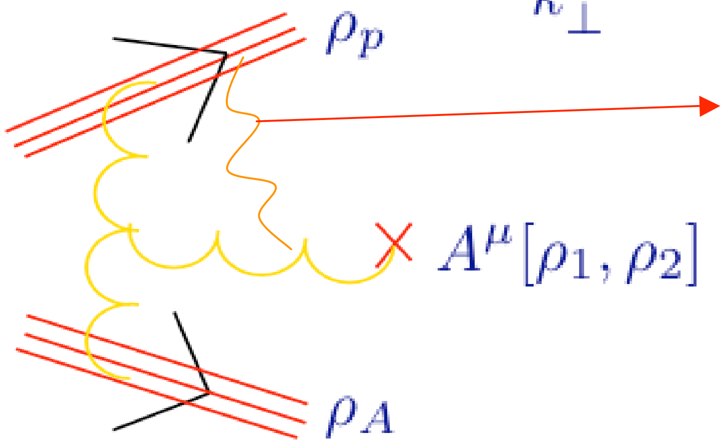


Solve Yang-Mills equations for two light cone sources: ρ_{p1} & ρ_{p2}

For observables $O\big(A_\mu(\rho_{p1}, \rho_{p2})\big)$ average over $W_{x1}[\rho_{p1}]$ & $W[\rho_{p2}]$

Systematic power counting for scattering in the CGC

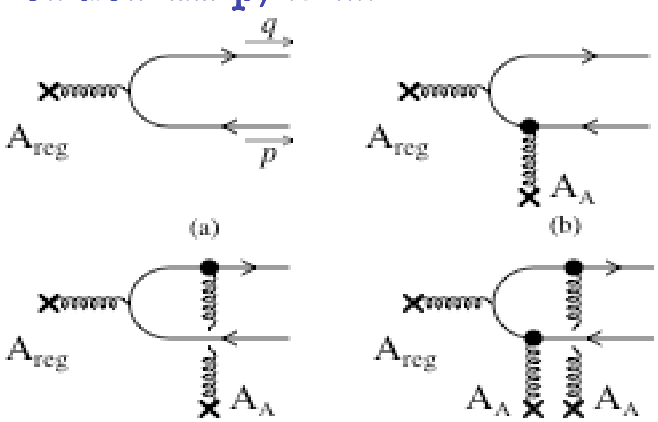
- Inclusive gluon production k_t factorization in p/D-A collisions to lowest order in $\frac{\rho_p}{k_\perp^2}$ but all orders in $\frac{\rho_A}{k_\perp^2}$ Kovchegov-Mueller



Breaks down at next order

Krasnitz,RV; Balitsky

- Inclusive pair production NO k_t factorization even at lowest order in p/D-A Blaizot, Gelis, RV



$A_{reg} = O(\rho_p\, \rho_A^n)\ ;\ n \to \infty$

Results depend on 2-point (dipole) 3-point & 4-point Wilson line correlators

(Talks by Gelis, Tuchin, Fujii)

THE DEMISE OF THE PARTON DISTRIBUTION ?

- Dipoles (and multipole) operators may be more relevant observables at high energies

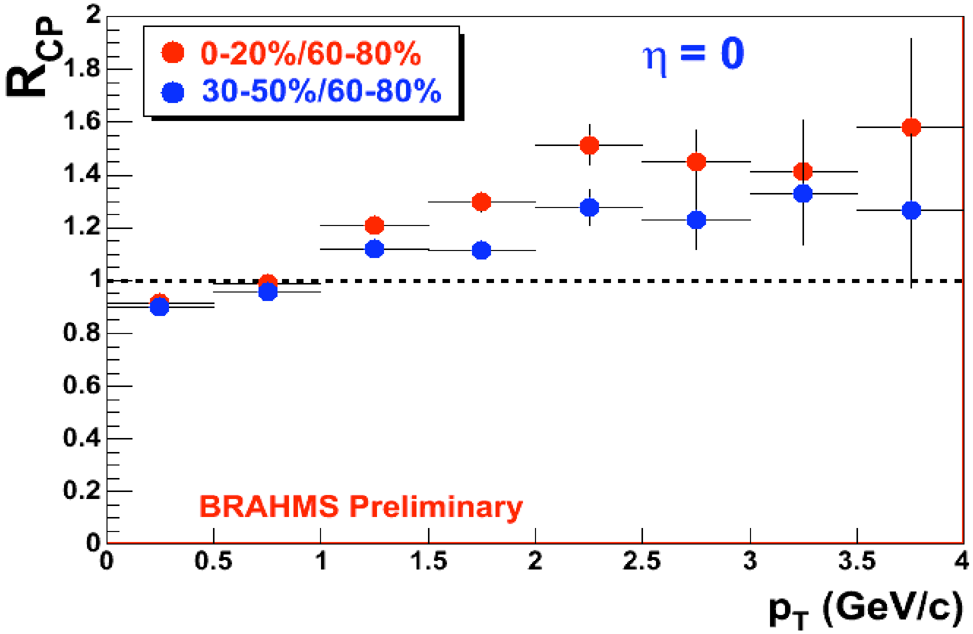
Jalilian-Marian, Gelis;
Kovner, Wiedemann
Blaizot, Gelis, RV

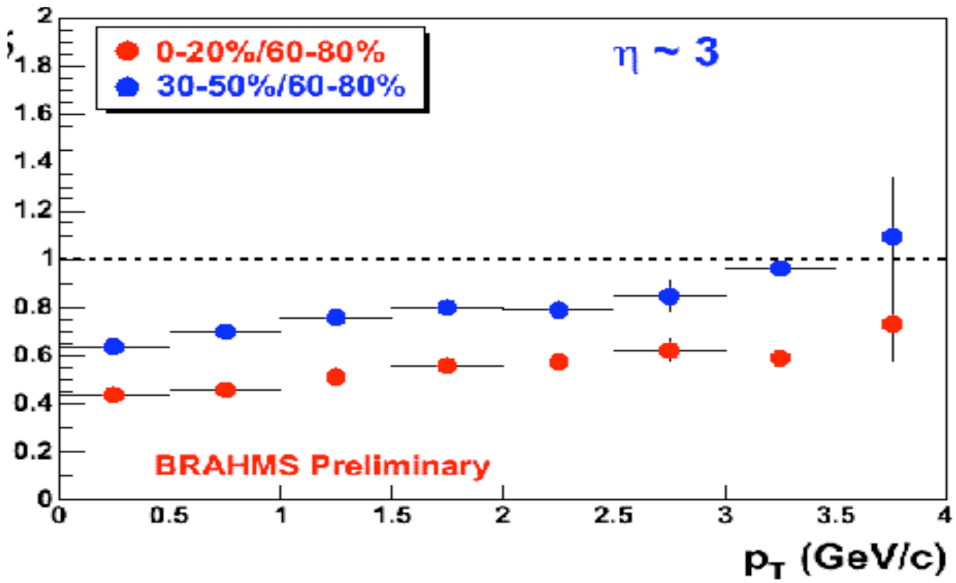
- Are universal-process independent.

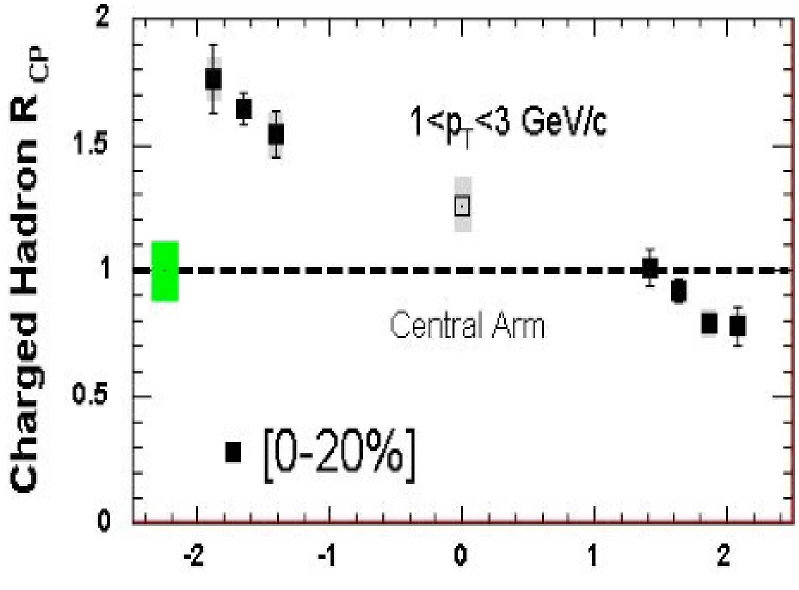
- RG running of these operators - detailed tests of high energy QCD.

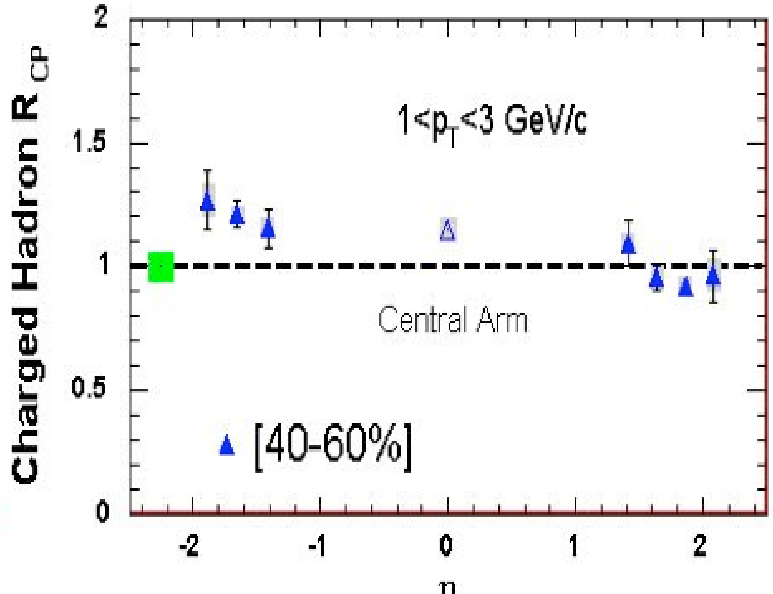
(See talks by Salgado & Jalilian-Marian)

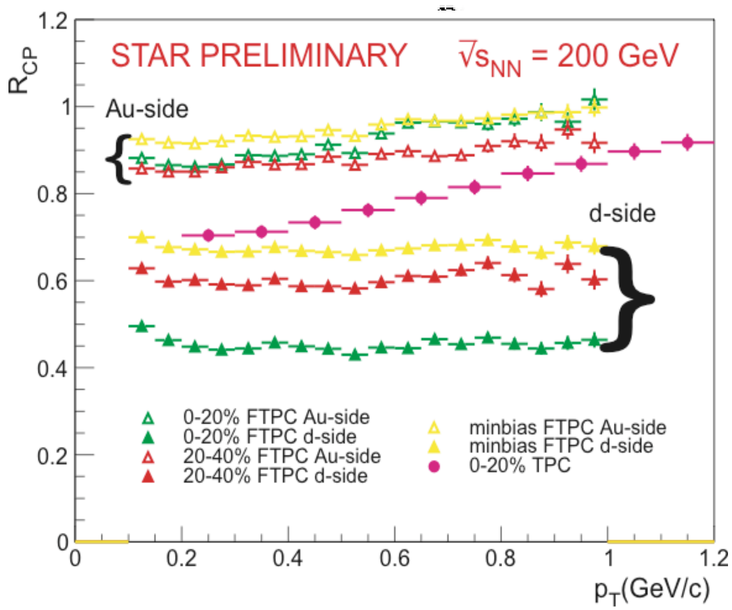
RHIC DATA ON THE CRONIN EFFECT

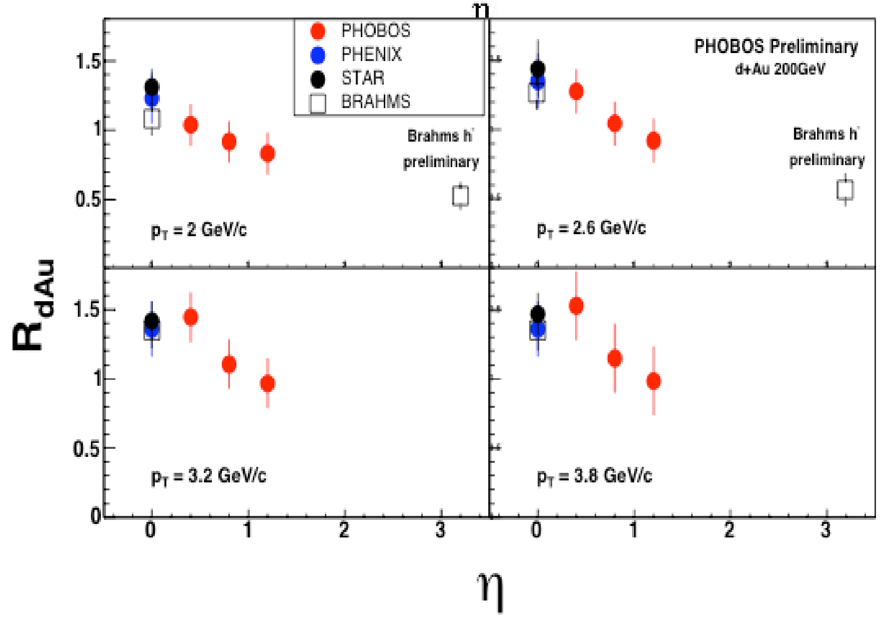




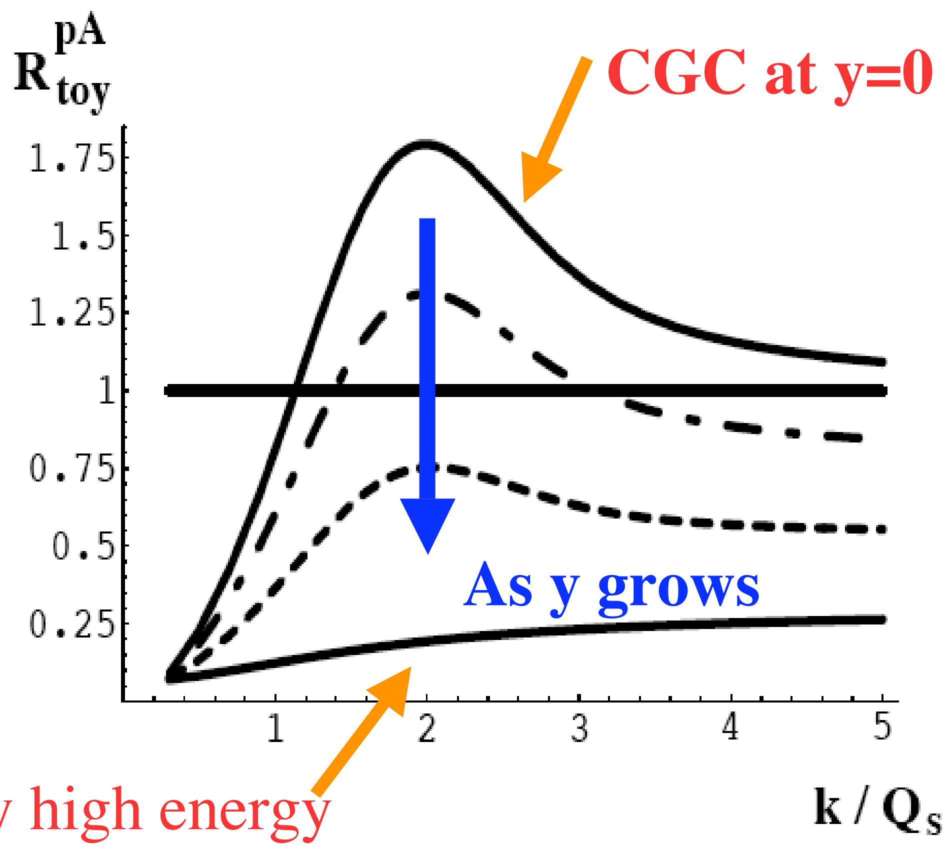

Charged Hadron R CP
1<pT<3 GeV/c
Central Arm
[0-20%]


Charged Hadron R CP
1<pT<3 GeV/c
Central Arm
[40-60%]
η


RCP
STAR PRELIMINARY
√sNN = 200 GeV
Au-side
d-side
0-20% FTPC Au-side
0-20% FTPC d-side
20-40% FTPC Au-side
20-40% FTPC d-side
minbias FTPC Au-side
minbias FTPC d-side
0-20% TPC
pT(GeV/c)


PHOBOS
PHENIX
STAR
BRAHMS
PHOBOS Preliminary
d+Au 200GeV
Brahms h-
preliminary
pT = 2 GeV/c
pT = 2.6 GeV/c
pT = 3.2 GeV/c
pT = 3.8 GeV/c
RdAu
η

Compute R_pA



Dumitru,Jalilian-Marian
Jalilian-Marian, Gelis
Accardi

Inversion of centrality
Dependence due to softening
Of the spectrum at small x

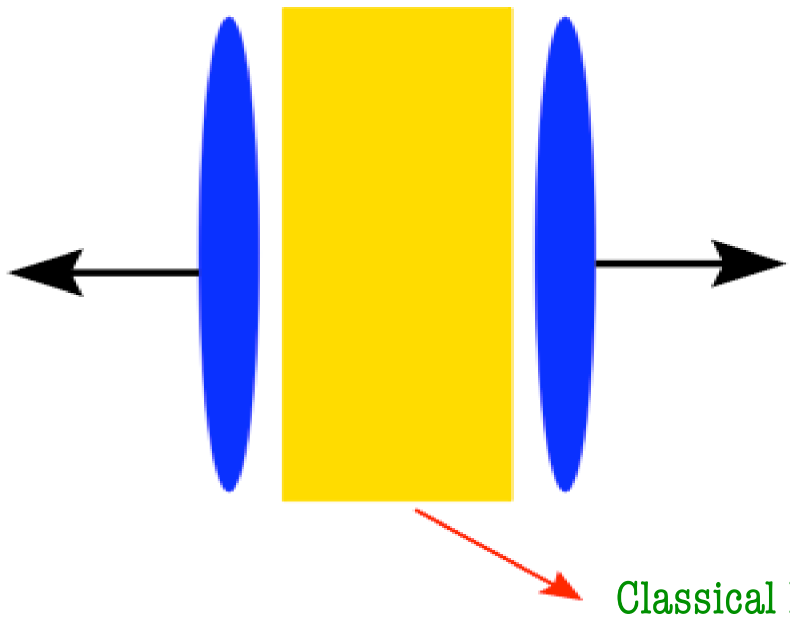
Kharzeev,Kovchegov,Tuchin
Baier,Kovner,Wiedemann;
Albacete,Armesto,Salgado,Kovner,Wiedemann
Blaizot,Gelis,RV
Iancu,Itakura,Triantafyllopolous

Broadening of azimuthal correlations due to CGC

Kharzeev,Levin,McLerran

- Other tests-photons and di-leptons in forward region (Talks by Baier & Gay-Ducati)

Colliding Sheets of Colored Glass at High Energies



Krasnitz,Nara,RV;

Lappi

Classical Fields with occupation # f= $\frac{1}{\alpha_S}$

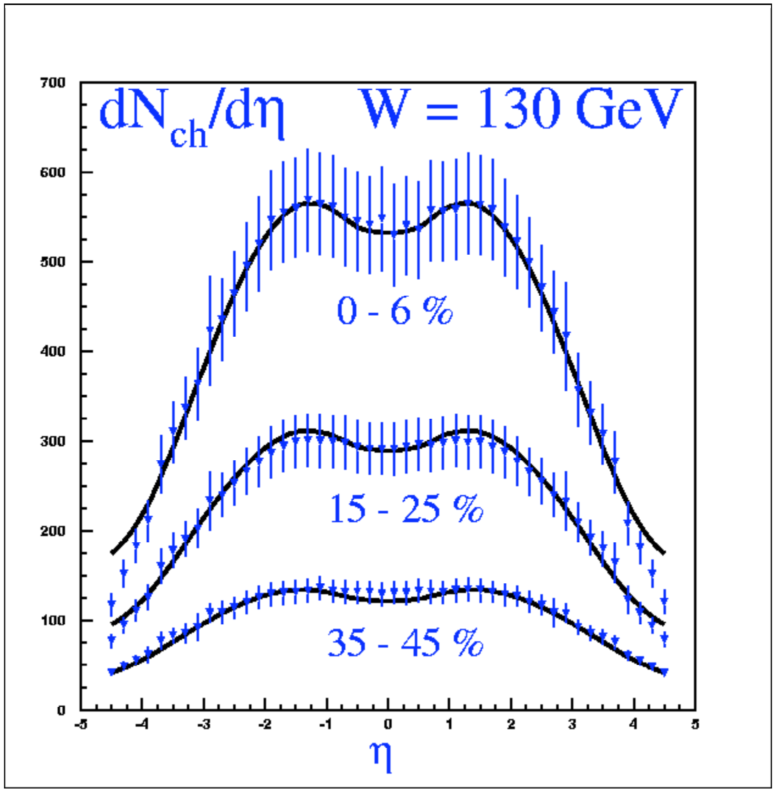
Initial energy and multiplicity of <u>produced</u> gluons depends on Q_s

$$\frac{1}{\pi R^2}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2} Q_s^3 \qquad \frac{1}{\pi R^2}\frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

Classical approach breaks down at late times when f <<1

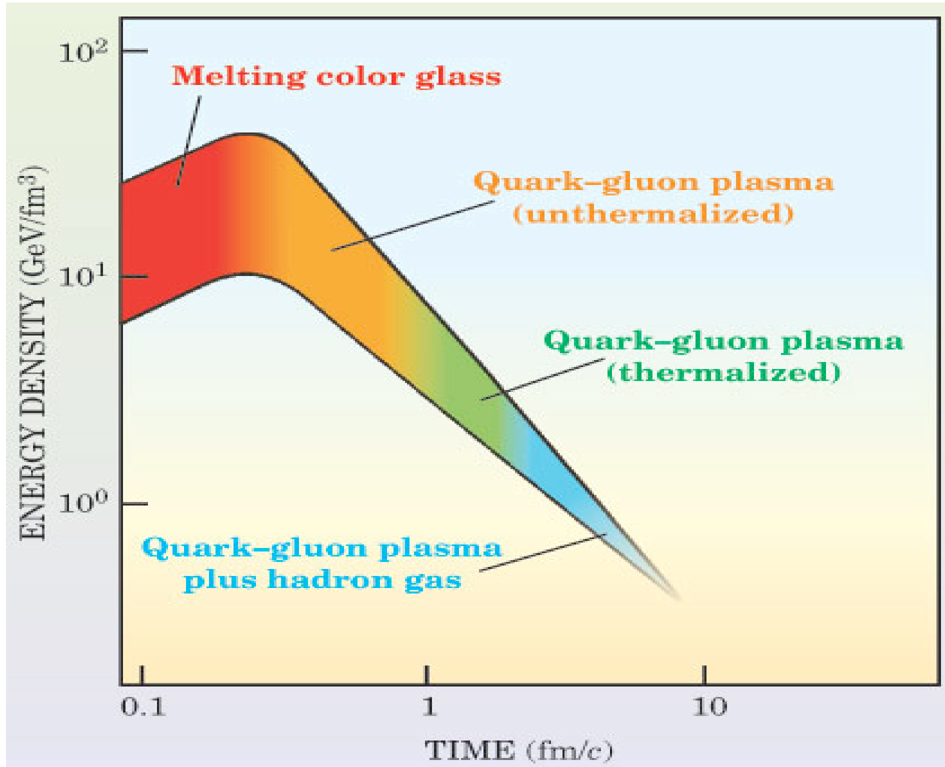
$$\tau >> \frac{1}{Q_s} \text{ but } \tau << R$$

- Successful KLN phenomenology Kharzeev,Levin,Nardi



- CGC+Hydro ? Hirano, Nara

- Requires rapid thermalization-is it possible in weak coupling approaches?



McLerran, Ludlam

In bottom up scenario, $\tau \approx \frac{1}{\alpha_S^{13/5}} \frac{1}{Q_s}$ ~ 2-3 fm at RHIC

Baier,Mueller,Schiff,Son

Exciting possibility - non-Abelian "Weibel" instabilities may speed up thermalization - simple estimates: Isotropization time ~ $\frac{1}{Q_s}$ ~ 0.3 fm

Mrowczynski
Arnold,Lenaghan,Moore,Yaffe
Romatschke, Strikland

Open questions

- Are there contributions in high energy QCD beyond JIMWLK?
- What is the domain of validity of BK?
- Are "dipoles" the correct degrees of freedom at high energies?
- Do we have a consistent phenomenological picture?
- Can we understand thermalization from first principles?