From Classical to Quantum Saturation in the Nuclear Wavefunction

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Based on : E.Iancu, K.Itakura, D.N.T., Nucl. Phys. **A 742** (2004) 182

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- Classical Saturation
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Saturation-Unitarity (1/3)

- The wavefunction of a generic fast moving hadron described by the “Gluon occupation factor” $\equiv$ $\#$ of gluons/transverse phase space/rapidity/color/spin

$$\varphi_H = \frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN}{dY\, d^2b_\perp\, d^2k_\perp}$$

- $\varphi_H \sim 1/\alpha_s$: maximal density allowed by mutual interactions $\rightsquigarrow$ Gluons (effectively) overlap in phase space $\rightsquigarrow$ saturate

Saturation-Unitarity (2/3)

- A strong field $\mathcal{A}$ associated with the hadronic wavefunction
  Assumes maximal value $1/g$ at saturation

$$\mathcal{A} \sim \sqrt{a^\dagger a} \sim \sqrt{\varphi_H} \sim 1/g$$

- Scattering amplitudes are of order $\mathcal{O}(1)$
  For example in dipole-hadron scattering

$$\mathcal{N}_{xy} = 1 - S_{xy} = 1 - \frac{1}{N_c} \left\langle \mathrm{tr}\, \left(V_x^\dagger V_y \right) \right\rangle$$

$$V_x^\dagger = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^- \mathcal{A}^+(x^-, x_\perp) \right]$$

Note : Average $\langle \cdots \rangle$ over possible configurations of color sources

Saturation-Unitarity (3/3)

Under what circumstances saturation and unitarity limits are reached, and strong color fields are created?

- In a large nucleus with $A \gg 1$:
  There are $A \times N_c$ valence quarks
  $\rightsquigarrow$ large number of radiated gluons
- At small Bjorken-$x \leftrightarrow$ very high energy:
  Successive gluon emissions - Resum ladder diagrams
  $\rightsquigarrow$ high density gluonic system

- Fast moving partons with momentum p^+ $\longrightarrow$ Large lifetime

$$\Delta x^+ \sim 1/p^- = 2p^+/p_\perp^2$$

- "Frozen" sources for slow moving gluons with momenta $k^+ \ll p^+$
- Solve Classical Yang-Mills equation $\rightsquigarrow \mathcal{A}(\rho)$ for given source ρ

$$\left(D_\nu F^{\nu\mu}\right)_a(x) = \delta^{\mu+}\rho_a(x) \qquad \underline{\text{Non}-\text{linear}}$$

- Calculate observable $\mathcal{O}(\mathcal{A}) = \mathcal{O}(\rho)$

$$\langle O[\rho]\rangle_Y = \int \mathcal{D}\rho\, W_Y[\rho] O[\rho]$$

$W_Y[\rho]$ = probability distribution of color sources at rapidity Y

- Color sources : The $A \times N_c$ valence quarks in a nucleus
- Uncorrelated for transverse separations $\Delta x_\perp \lesssim 1/\Lambda_{QCD}$
  $\rightsquigarrow$ Gaussian weight-function

$$W_{\rm MV} \propto \exp\left[-\frac{1}{2}\int^{1/\Lambda} d^2x_\perp \, \frac{\rho_a(x_\perp)\rho_a(x_\perp)}{\mu_A^2}\right]$$

$\mu_A^2 = 2\alpha_s A/R_A^2 \sim A^{1/3}\Lambda^2$ = color charge squared/transverse area

- Classical model : Weight function does not depend on rapidity
- Could be realized in providing initial conditions

The JIMWLK Equation (RGE)

- Increase rapidity $\rightsquigarrow$ More gluons included in the source
- Resum $\bar{\alpha}_s Y$ terms in presence of a strong color field
- Weight function depends on Y, satisfies the RGE

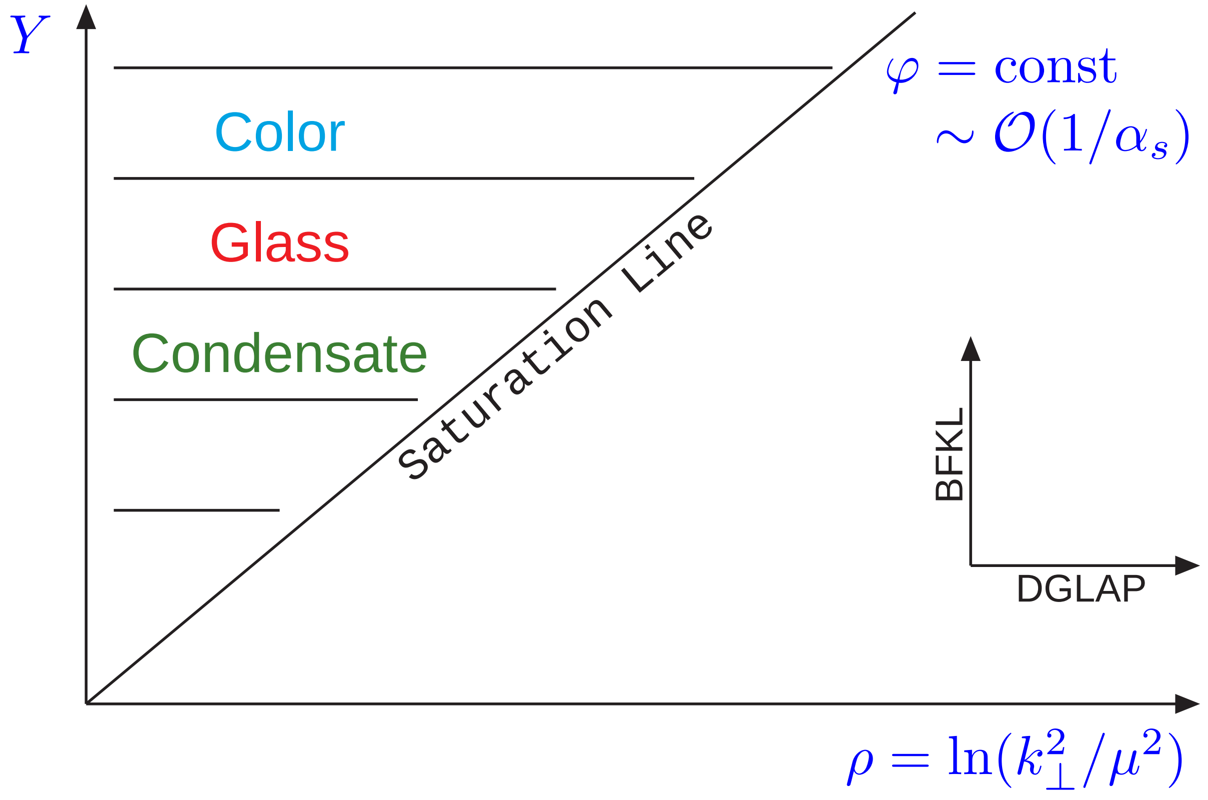
$$\frac{\partial}{\partial Y} W_Y[\rho] = \frac{1}{2} \frac{\delta}{\delta \rho_Y^a(x_\perp)} \chi^{ab}(x_\perp, y_\perp)[\rho] \frac{\delta}{\delta \rho_Y^b(y_\perp)} W_Y[\rho]$$

- “Observables” satisfy non-linear evolution equations, e.g.

$$\frac{\partial}{\partial Y} \left\langle \mathrm{tr}\left(V_x^\dagger V_y\right)\right\rangle_Y = \bar{\alpha}_s \int \frac{d^2 z_\perp}{2\pi} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2} \times \left\langle \frac{1}{N_c} \mathrm{tr}\left(V_x^\dagger V_z\right) \mathrm{tr}\left(V_z^\dagger V_y\right) - \mathrm{tr}\left(V_x^\dagger V_y\right) \right\rangle_Y$$

- Reduces to BFKL Equation in weak field, low density, limit

For given Y, modes with $k_\perp \lesssim Q_s(Y)$ will be saturated



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Energy dependence of saturation scale

Determined by linear dynamics with appropriate boundary conditions

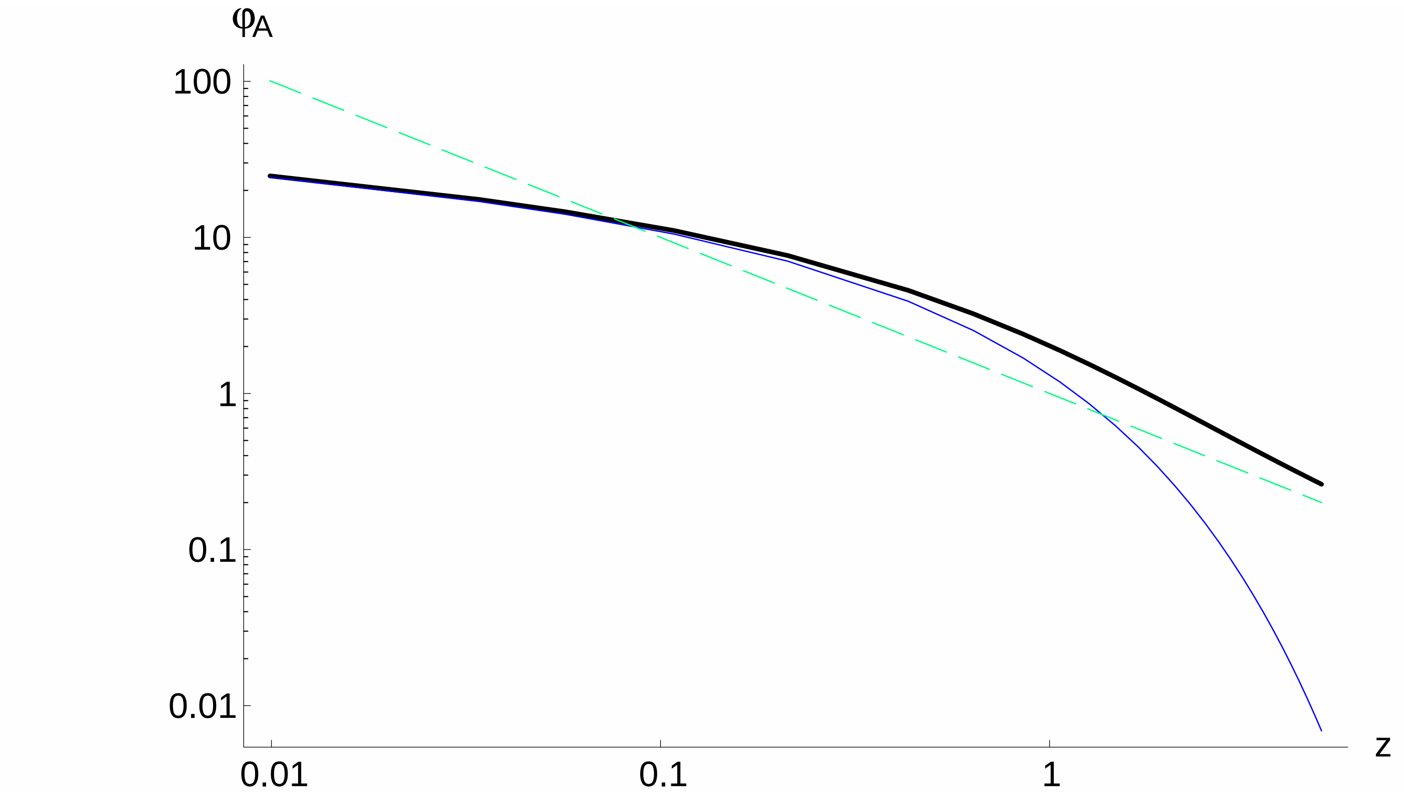
Leading asymptotic behavior

- Fixed Coupling : $Q_s^2(Y) = \# Q_s^2(0) \exp\left[\dfrac{\chi(\gamma_s)}{\gamma_s}\bar{\alpha}_s Y\right]$
- Running Coupling : $Q_s^2(Y) = \# \Lambda^2 \exp\left[\sqrt{\dfrac{2\chi(\gamma_s)}{b\gamma_s}Y + \ln^2 \dfrac{Q_s^2(0)}{\Lambda^2}}\right]$
  - $\chi(\gamma)$ = Eigenvalue of BFKL equation
  - $1/2 < \gamma_s \simeq 0.628 < 1$ associated anomalous dimension

Large $Y \Rightarrow Q_s^2(Y) \gg \Lambda^2 \to$ weak coupling techniques justified

- MV model: High color charge density $\rightsquigarrow$ Non-linear effects
  The saturation scale is set by the density
  $$Q_s^2(A) \approx \Lambda^2 A^{1/3} (\times \ln A^{1/3}) \gg \Lambda^2$$
- The gluon occupation factor reads
  $$\varphi_A(k_\perp) = \frac{1}{\alpha_s}\Gamma(0,z) + \varphi_A^{\rm twist}(z), \qquad z \equiv k_\perp^2/Q_s^2(A)$$
  - Parametrically enhanced term $\sim 1/\alpha_s$ dominates for all $z \lesssim 1$
    Gluon occupation factor in the CGC phase : $\varphi_A^{\rm sat}(z)$
    Compact distribution : Falls exponentially at large z
    Diverges only logarithmically at $z \ll 1$
  - Twist term is relevant only for high momentum tail
    Contains bremsstrahlung spectrum $\varphi_{BS} \propto 1/z$

Classical Saturation (2/3)



- *Black (thick) line : Gluon occupation factor* $\varphi_A(z)$
- *Blue (solid) line : Saturation contribution* $\varphi_A^{\rm sat}(z)$
- *Green (dashed) line: Bremsstrahlung spectrum* $\varphi_{BS}(z)$

Classical Saturation (3/3)

- In MV model color sources are uncorrelated $\rightsquigarrow$ Sum-rule

$$\int^{Z} dz \, [\varphi_A(z) - \varphi_{BS}(z)] \xrightarrow[Z\to\infty]{} 0$$

"Summing" nucleons $\rightsquigarrow$ Integrated gluon distribution for $Q^2 \gg Q_s^2(A)$

- Repulsive interactions redistribute gluons in momenta:
Two spectra are equal at a scale $Q_c^2(A)$

$$\Lambda^2 \ll Q_c^2(A) \approx \alpha_s Q_s^2(A) \ll Q_s^2(A)$$

Gluons in excess in bremsstrahlung spectrum at $k_\perp \lesssim Q_c(A)$
$\rightsquigarrow$ Gluons located at $k_\perp \sim Q_s(A)$ in MV spectrum

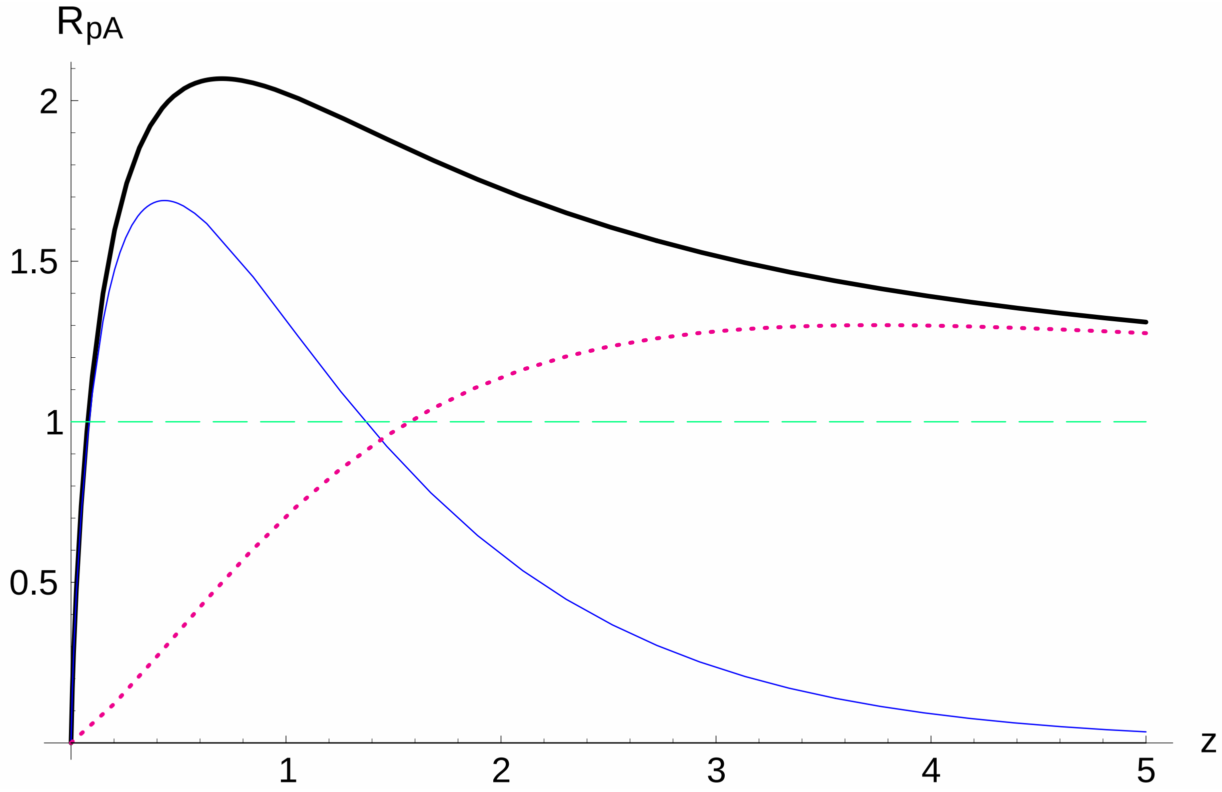
- <u>The MV spectrum is enhanced around the saturation scale</u>

The Cronin Effect (1/2)

- As an immediate consequence, consider the ratio

$$\mathcal{R}_{\rm pA} \equiv \frac{\varphi_A}{A^{1/3}\varphi_p} = \frac{\varphi_A}{\varphi_{BS}} = z\,\varphi_A$$

- Behaves as
  - $\mathcal{R}_{\rm pA} \ll 1$ if $z \ll 1$
  - $\mathcal{R}_{\rm pA} \sim \mathcal{O}(1/\alpha_s) \gg 1$ if $z \sim 1$
  - $\mathcal{R}_{\rm pA} \to 1^+$ if $z \gg 1$
- Maximum: $z_0 = 0.435 + \mathcal{O}(\alpha_s), \quad \mathcal{R}_{\max} = 0.281/\alpha_s + \mathcal{O}(\text{const})$
- Compact CGC distribution $\rightsquigarrow$ "Pronounced" peak
- Maximal value increases with A (since $1/\alpha_s = \ln Q_s^2(A)/\Lambda^2$)



$$\mathcal{R}_{\rm pA}(z) = \mathcal{R}_{\rm pA}^{\rm sat}(\alpha_s, z) + \mathcal{R}_{\rm pA}^{\rm twist}(z)$$

Quantum Saturation (1/2)

- Start to add small-x gluons $\rightsquigarrow$ Evolve in rapidity Y
  Correlations among color sources are induced
- General solution is not known; only in certain regions

$$\varphi(k_\perp, Y) = \begin{cases} \dfrac{1}{\alpha_s} \ln \dfrac{Q_s^2(Y)}{k_\perp^2} & \text{if } k_\perp^2 \ll Q_s^2(Y) \\ \dfrac{1}{\alpha_s} \left(\dfrac{Q_s^2(Y)}{k_\perp^2}\right)^{\gamma_s} \left(\ln \dfrac{k_\perp^2}{Q_s^2(Y)} + \Delta\right) & \text{if } k_\perp^2 \gtrsim Q_s^2(Y) \\ \dfrac{Q_0^2}{k_\perp^2} \, \mathrm{I}_0 \left(\sqrt{4\bar{\alpha}_s Y \ln \dfrac{k_\perp^2}{Q_0^2}}\right) & \text{if } k_\perp^2 \gg Q_s^2(Y) \end{cases}$$

- Scaling around saturation momentum

Quantum Saturation (2/2)

- Can do first (nonlinear) step in evolution, valid for $Y \ll 1/\alpha_s$

$$\varphi_A(k_\perp, Y) = \frac{1}{\alpha_s}\Gamma(0,z) + \varphi_A^{\rm twist}(z) + Y\Delta[\Gamma(0,z)]$$

- Evolution of compact piece contains power-law tails
  Generated from evolution kernel
- Extrapolate: When $Y \gtrsim 1/\alpha_s$ all components are “mixed” and unlike classical model
  - $\rightsquigarrow$ NO compact distribution for $k_\perp^2 \lesssim Q_s^2(Y)$
  - $\rightsquigarrow$ NO parametric separation between solutions below and above saturation line

The Cronin Ratio (1/4)

Various kinematical regimes $\rightsquigarrow$ not trivial study. Basic features are

- Proton is "less saturated" than nucleus $\rightsquigarrow$
  More transverse space for proton $\rightsquigarrow$ evolves faster
  e.g. evolving along $k_\perp^2 = Q_s^2(A,Y)$
  $$d\mathcal{R}_{\rm pA}/dY < 0 \qquad \& \qquad \mathcal{R}_{\rm pA} \xrightarrow[Y\to\infty]{} (\alpha_s A^{-1/3})^{1-\gamma_s}$$
  High p_T suppression
- Fixed Y, and extremely high momenta
  $$\mathrm{d}\mathcal{R}_{\rm pA}/\mathrm{d}k^2 > 0 \quad \text{for} \quad k^2 \gg Q_s^2(A,Y) \quad \& \quad \mathcal{R}_{\rm pA} \xrightarrow[k^2\to\infty]{} 1^-$$

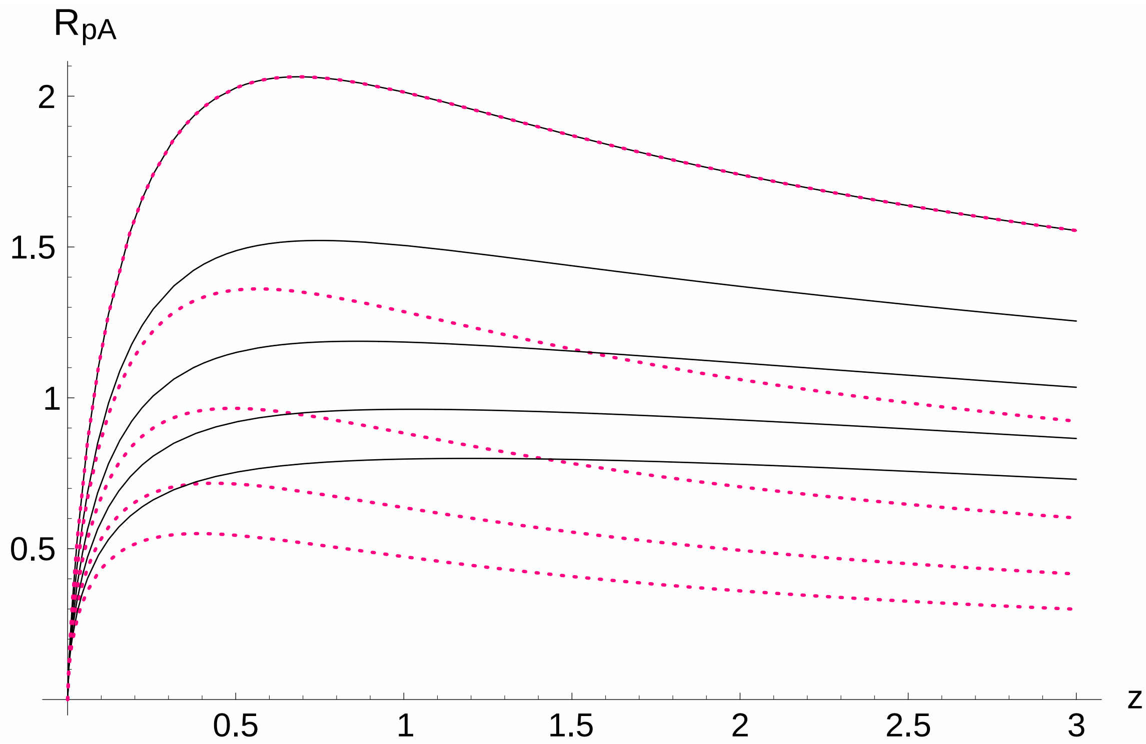
The Cronin-Ratio (2/4)

- Sum-rule breaks down due to correlations. Peak still exists
- Maximum is 1, when $\varphi_p = 1/(\alpha_s A^{1/3}) \ll 1$ is still a "dilute system"
  Evolution is DGLAP-like. Indeed

$$\mathcal{R}_{\rm pA}^{\rm max} = \mathcal{O}(1) \quad \text{when} \quad Y \simeq \frac{1}{4}\ln^2(1/\alpha_s) \ll 1/\alpha_s$$

  suppression is very fast
- For large Y, due to nuclear evolution, the peak flattens out

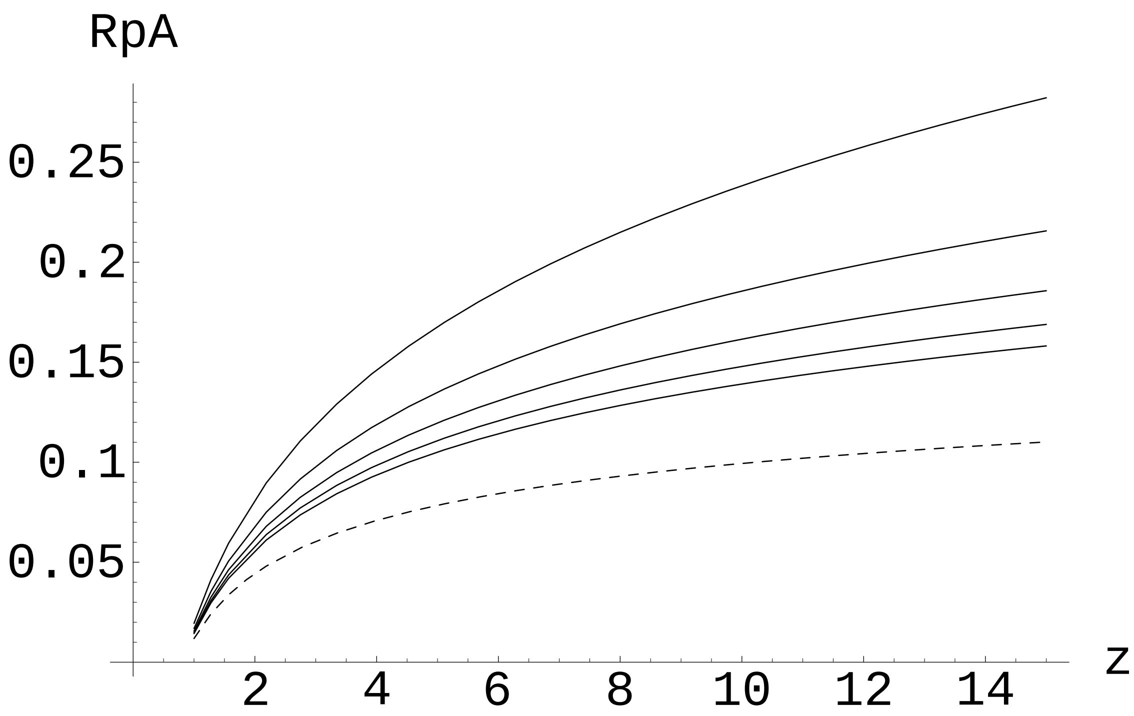
$$d\mathcal{R}_{\rm pA}/dk_\perp^2 > 0 \quad \text{when} \quad Y \gtrsim 1/\alpha_s$$



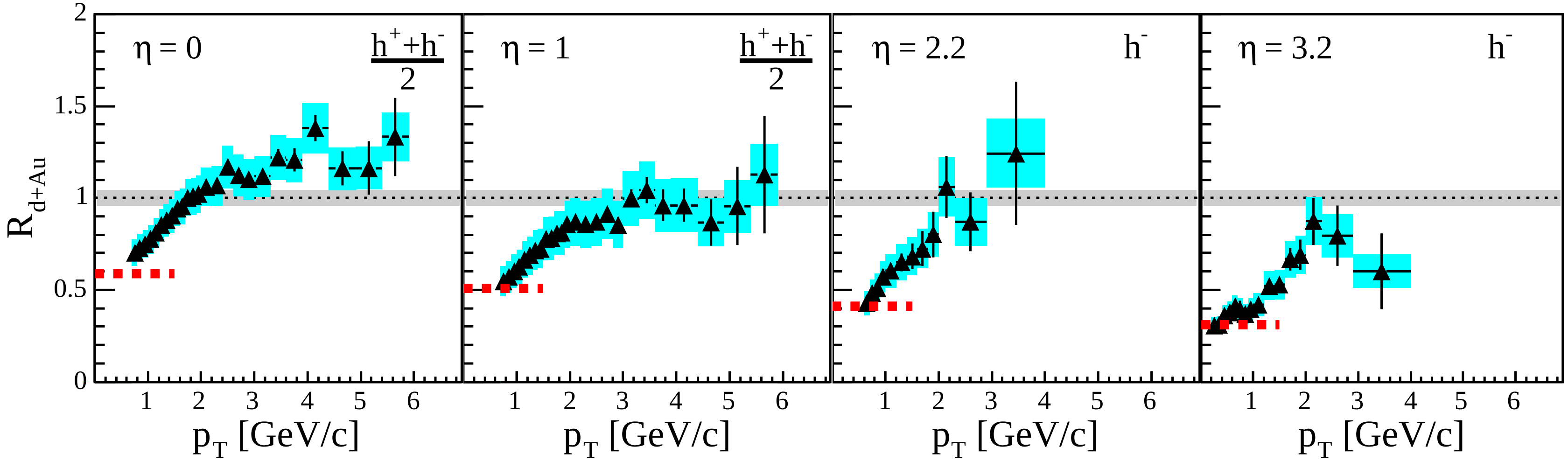
Cronin ratio for $z \lesssim 1$*. Top to bottom :* $\Delta Y = 0, 1/2, 1, 3/2, 2$

- *Black (solid) lines : Evolved nuclear wavefunction*
- *Red (dotted) lines : Unevolved one (MV)*
- *Proton wavefunction : Full DLA solution*

The Cronin Ratio (4/4)



- *Cronin ratio for* $z \gtrsim 1$. *Top to bottom :* $\alpha_s \Delta Y = 0.75 + 0.3\, n, \;\; n = 0,1,2,3,4$



- Gold Nucleus is probed at the x-value

$$x_{Au} \sim \frac{2p_T}{\sqrt{s_{NN}}} \exp[-\eta]$$

Conclusion

- Differences between classical (MV) and quantum saturation
- Different picture of the Cronin ratio at $Y = 0$ and $Y \neq 0$
- Extended previous discussions
- Explained results obtained from numerical solutions
  (+ Running coupling analysis : Not much different)
- “Rough” qualitative agreement with RHIC Data