Relating lA, pA and AA data through geometric scaling

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Dipole model for DIS

- The total $\gamma^* h$ cross section is in the dipole model

$$\sigma_{T,L}^{\gamma^* h}(x, Q^2) = \int d\mathbf{r} \int_0^1 dz |\Psi_{T,L}^{\gamma^*}(Q^2, \mathbf{r}, z)|^2 \, \sigma_{\rm dip}^h(\mathbf{r}, x) \,,$$

  - $|\Psi_{T,L}^{\gamma^*}(Q^2, \mathbf{r}, z)|^2$ is the $\gamma^* \to q\bar{q}$ wave function
  - $\sigma_{\rm dip}^h(\mathbf{r}, x)$ is the $q\bar{q} - h$ total cross section

$$\sigma_{\rm dip}^h(\mathbf{r}, x) = 2 \int d\mathbf{b} \, N_h(\mathbf{r}, x; \mathbf{b})$$

- The evolution in $y \equiv \log x_0/x$ of $N_h(\mathbf{r}, x; \mathbf{b})$ can be computed from QCD. In saturation models

  [e.g. the Balitsky–Kovchegov equation see talk by G. Milhano]

  - A saturation scale $Q_{\rm sat}^2$ appears
  - $N_h(\mathbf{r}, x; \mathbf{b}) = N_h(\mathbf{r} Q_{\rm sat}(x, \mathbf{b})) \longrightarrow$ **Geometric scaling**

Is this geometric scaling present in data??

Scaling of the DIS cross section

⇛ For massless quarks, (no impact parameter dependence)

$$|\Psi^{\gamma^*}_{T,L}(Q^2, \mathbf{r}, z)|^2 = Q^2 f(r^2 Q^2) \quad \Longrightarrow \quad \sigma^{\gamma^* h}_{T,L}(x, Q^2) = \sigma^{\gamma^* h}_{T,L}(Q^2/Q^2_{\rm sat}(x))$$

$$\boxed{N_h(\mathbf{r} Q_{\rm sat})}$$

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- With impact parameter: suppose all the b dependence can be scaled–out by the radius R_h: $\bar{b} = b/\sqrt{\pi R_h^2}$

$$\int d^2 b \, N_h \, (r \, Q_{\rm sat,h}(x, b)) \quad \longrightarrow \quad \pi R_h^2 \int d^2 \bar{b}, N_h(r \, Q_{\rm sat,h} \, (x, \bar{b}))$$

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⇛ If these two rescalings are exact

$$\frac{\sigma^{\gamma^* h}_{T,L}(Q^2/Q^2_{\rm sat,h}(x))}{\pi R_h^2}$$

is a universal function for any h = proton or nuclei

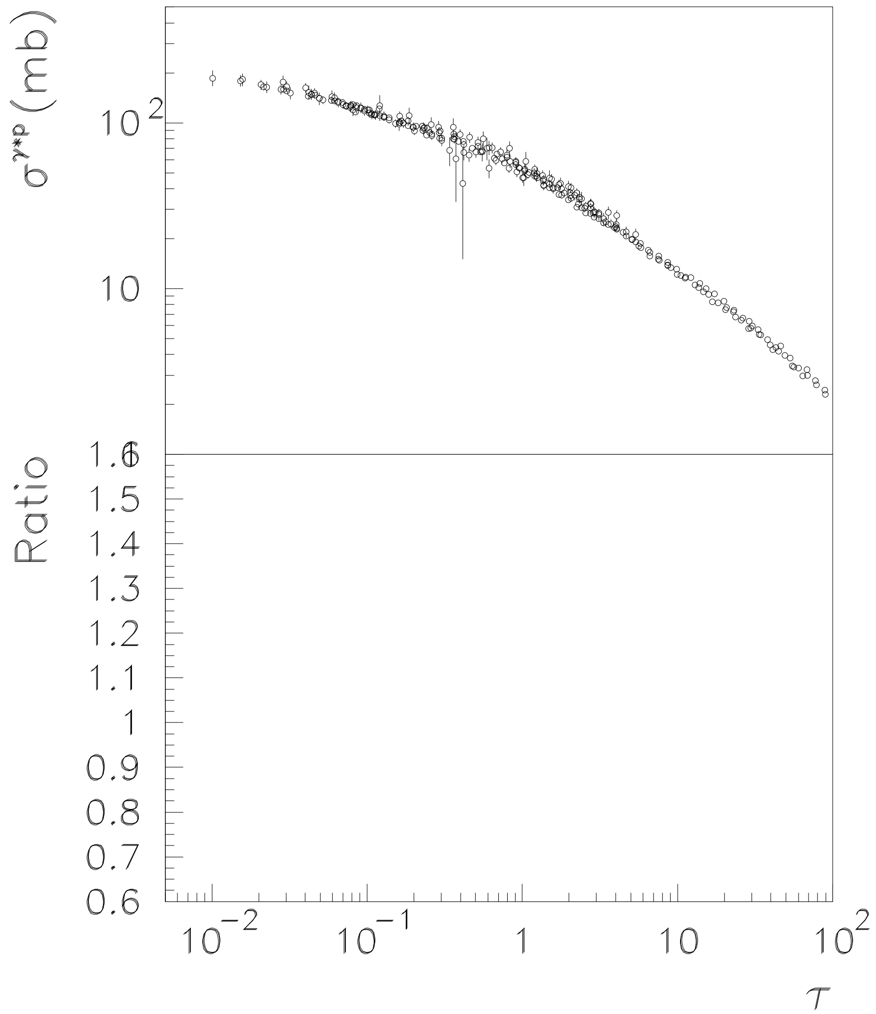
Geometric scaling in lepton-proton data

⇛ All lepton-proton data with $x \leq$0.01 only function of

$$\tau_p = \frac{Q^2}{Q^2_{\rm sat}}$$

$$Q^2_{\rm sat} = \left(\frac{x_0}{x}\right)^\lambda \;\; ; \;\; \lambda = 0.288$$

Stasto, Golec-Biernat, Kwiecinski PRL86, 596 (2001); Golec-Biernat, Wusthoff PRD59, 014017 (1999)



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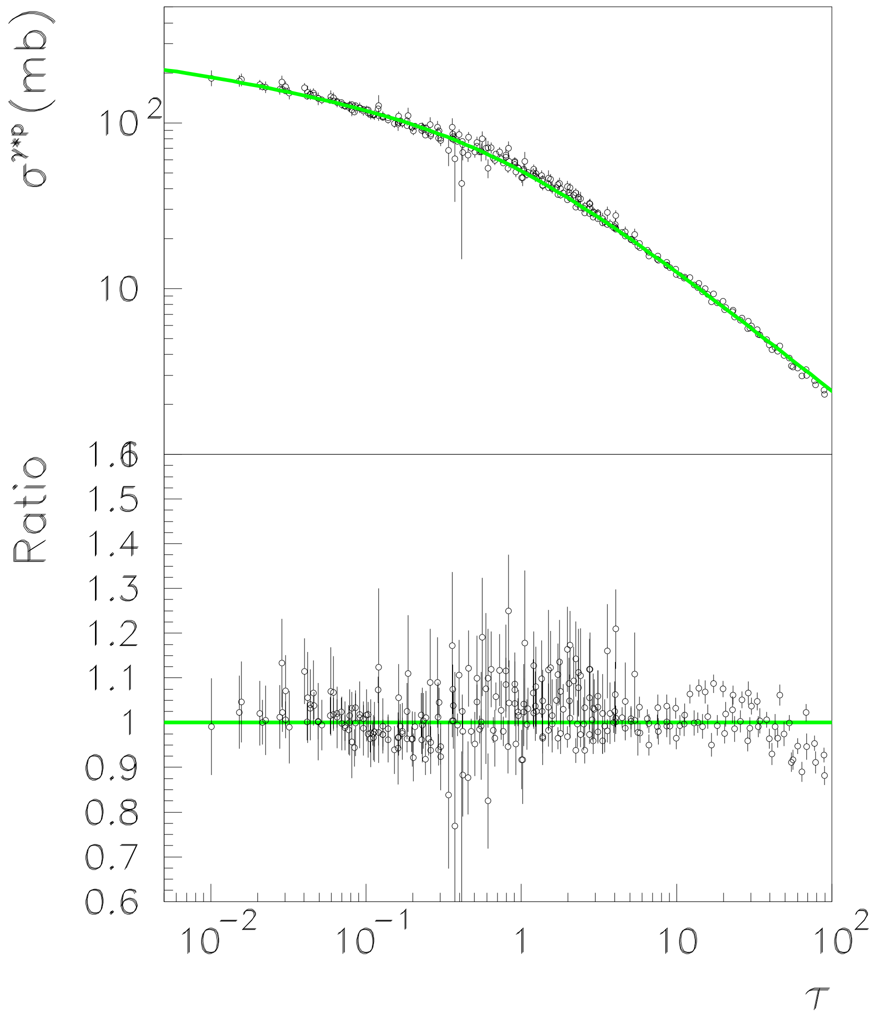
- We fit this scaling function to

$$\Phi(\tau) = \bar{\sigma}_0 \left[\gamma_E + \Gamma\left(0,\xi\right) + \ln \xi\right] \; ,$$

$$\xi = \frac{a}{\tau^b} \; ; \quad a = 1.868 \; , \quad b = 0.746$$

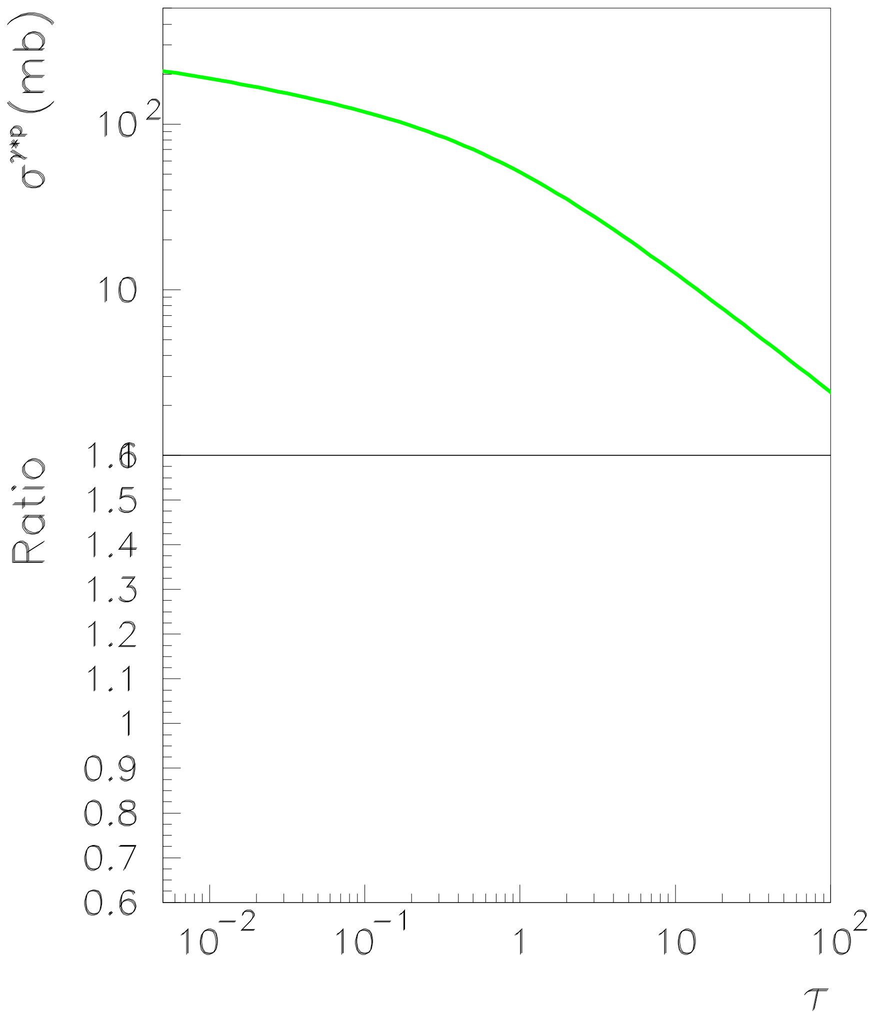


lepton–nucleus data

Geometric scaling in lepton-nucleus data

⇛ Scaling when

$$\frac{\sigma^{\gamma^* A}(\tau)}{\pi R_A^2} = \frac{\sigma^{\gamma^* p}(\tau)}{\pi R_p^2} .$$



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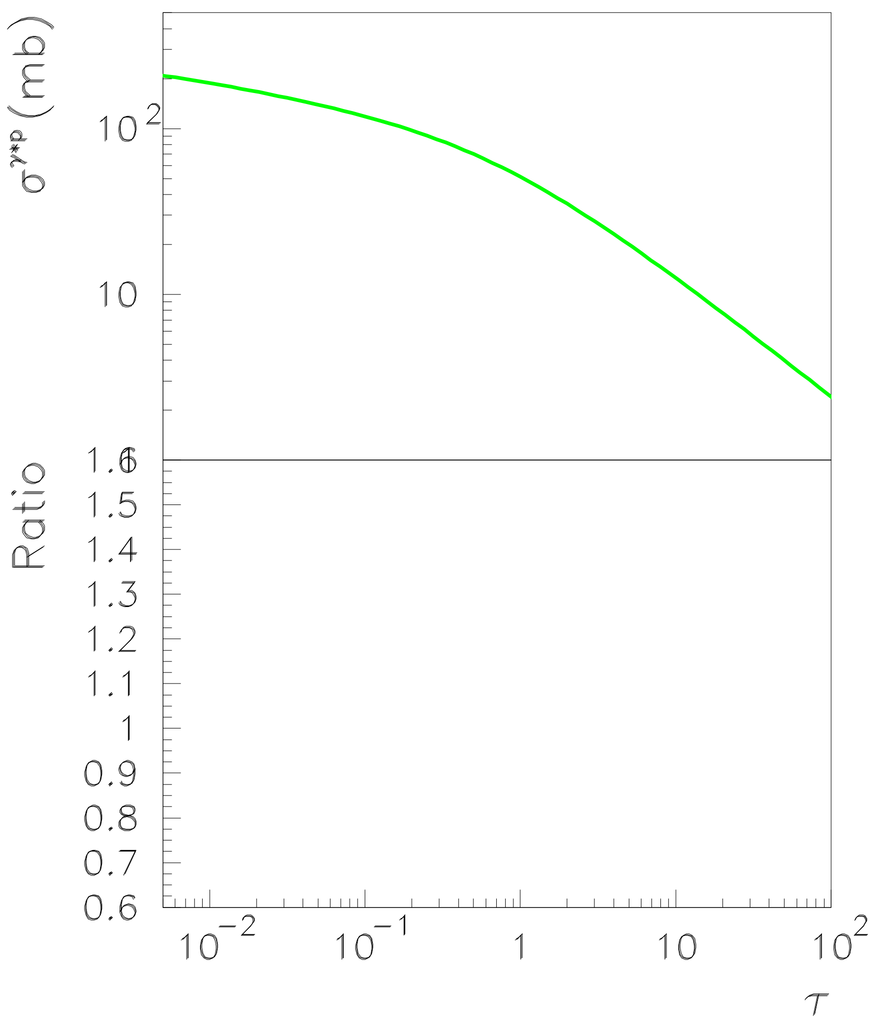
- Scaling when

$$\frac{\sigma^{\gamma^* A}(\tau)}{\pi R_A^2} = \frac{\sigma^{\gamma^* p}(\tau)}{\pi R_p^2}.$$

- We define

$$Q^2_{\rm sat,A} = Q^2_{\rm sat,p}\left(\frac{A R_p^2}{R_A^2}\right)^{1/\delta}$$

$$R_A = 1.12\,A^{1/3} - 0.86\,A^{-1/3}$$



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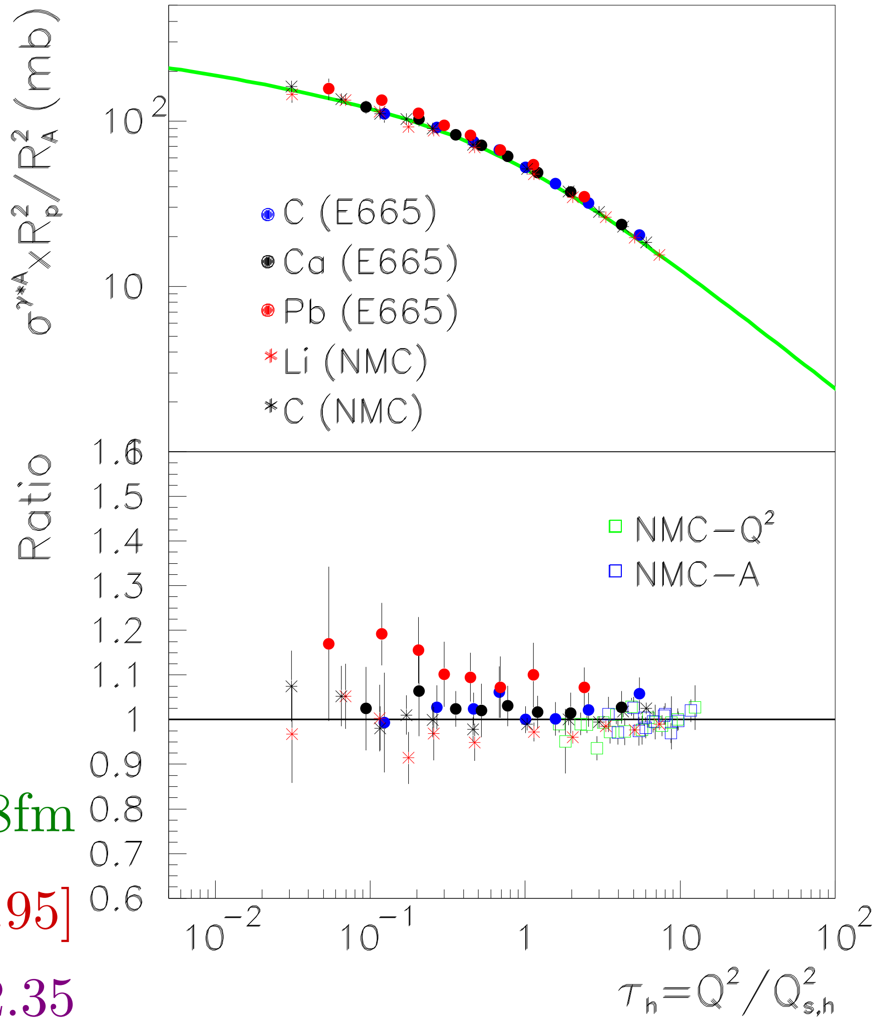
⇛ R_p, δ free parameters

$\boxed{\delta = 0.79 \pm 0.02}$ $R_p = 0.70 \pm 0.08\mathrm{fm}$

$\Rightarrow Q^2_{\rm sat} \sim A^{4/9}$ $[\chi^2/\mathrm{dof} = 0.95]$

$\delta = 1$ $[Q^2_{\rm sat} \sim A^{1/3}] \Rightarrow \chi^2/\mathrm{dof} = 2.35$



Some consequences in AA and dAu

Multiplicities in AA collisions

⇛ Factorization formula Gribov, Levin, Ryskin Phys Rep 100, 1 (1983) ...

$$\frac{dN_g^{AB}}{dyd^2p_tdb} \propto \frac{\alpha_S}{p_t^2} \int d^2k \, \phi_A(y,k^2,b) \, \phi_B\left(y,(\mathbf{k}-\mathbf{p}_t)^2,b\right) \ ,$$

where

$$\phi_h(y,k^2,b) = \int \frac{d^2r}{2\pi r^2} \, \exp\{i\mathbf{r}\cdot\mathbf{k}\} \, N_h(r^2,x;b)$$

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⇛ Geometric scaling: $\phi_h(y,k^2,b) = \phi(k^2/Q^2_{\rm sat,h})$

$$\left.\frac{dN_g^{AA}}{dy}\right|_{y\sim 0} \propto Q^2_{\rm sat,A}\pi R_A^2\int \frac{d\mathbf{s}}{\mathbf{s}^2}d\tau d\bar{\mathbf{b}}\;\phi(\tau^2)\,\phi\left((\tau-\mathbf{s})^2\right)$$

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⇛ But factorization not really needed, only geometric scaling.

Multiplicities and geometric scaling

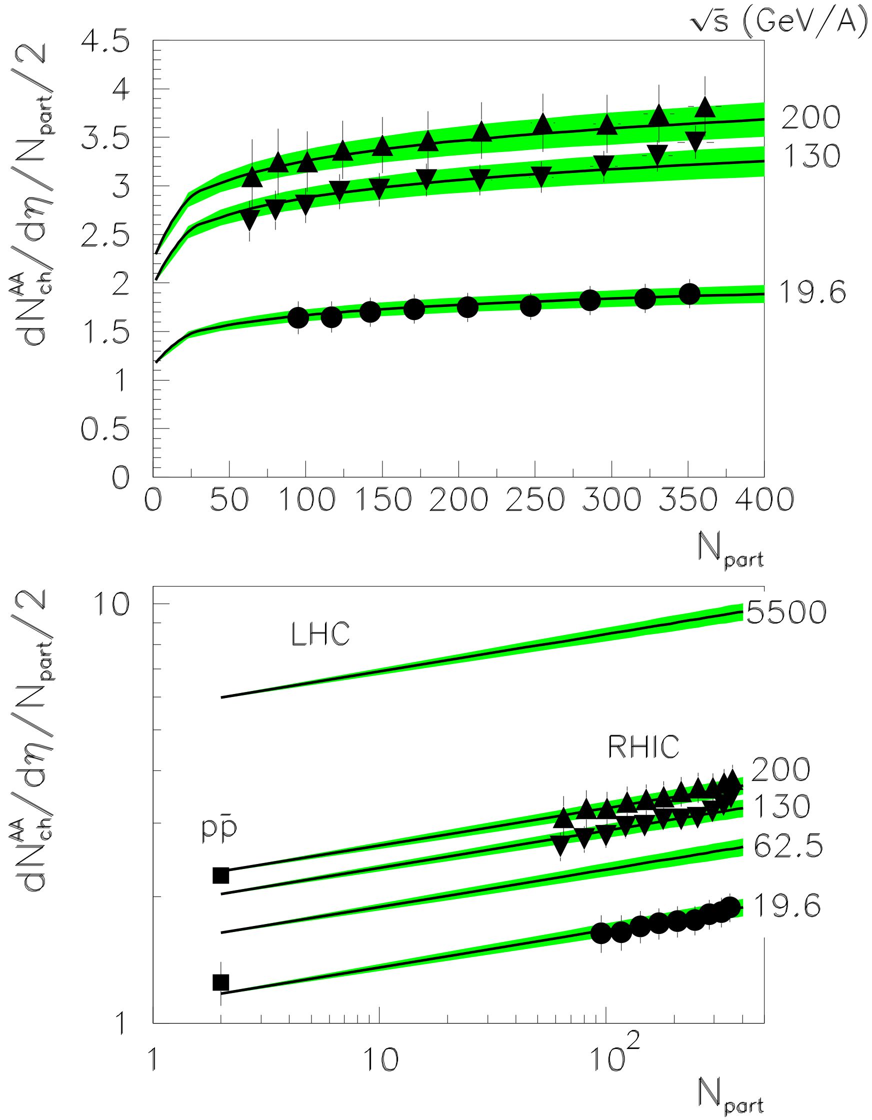
⇛ The multiplicities

$$\frac{1}{N_{\rm part}}\frac{dN^{AA}}{d\eta}\bigg|_{\eta\sim 0} = N_0\sqrt{s}^{\lambda} N_{\rm part}^{\frac{1-\delta}{3\delta}}\ .$$

⇛ $\lambda \;\rightarrow$ Energy dependence (lepton-proton data)

⇛ $\delta \;\rightarrow N_{\rm part}$ dependence (lepton-nucleus data)

⇛ Notice that centrality and energy dependence factorize.



PHOBOS: PRC65, 061901 (2002); nucl-ex/0405027

High-p_t at forward rapidities

- Forward rapidities $\longrightarrow$ testing ground for saturation.
- To check geometric scaling
  - Use the factorization formula

$$\frac{dN_g^{AB}}{dyd^2p_t db} \propto \frac{\alpha_S}{p_t^2} \int d^2k\, \phi_A(y,k^2,b)\, \phi_B\left(y,(\mathbf{k}-\mathbf{p}_t)^2,b\right)\ ,$$

  - Suppose
    $\phi_A(k=Q/2) \simeq \Phi(\tau_A)$; $\tau_A = k^2/4\bar{Q}^2_{\mathrm{sat,A}}$; $\bar{Q}^2_{\mathrm{sat,A}} = \frac{N_c}{C_F}\bar{Q}^2_{\mathrm{sat,A}}$
  - Taking $\phi_d \sim 1/k_t^n$, $n \gg 1$

$$\frac{\frac{dN^{\mathrm{dAu}}_{c_1}}{N_{\mathrm{coll}_1} d\eta d^2p_t}}{\frac{dN^{\mathrm{dAu}}_{c_2}}{N_{\mathrm{coll}_2} d\eta d^2p_t}} \approx \frac{N_{\mathrm{coll}_2}\phi_A(p_t/Q_{\mathrm{s,c_1}})}{N_{\mathrm{coll}_1}\phi_A(p_t/Q_{\mathrm{s,c_2}})} \approx \frac{N_{\mathrm{coll}_2}\Phi(\tau_{c_1})}{N_{\mathrm{coll}_1}\Phi(\tau_{c_2})}$$

    where $c1$, c_2 are two centrality classes.
- I.e. we make ratios of the DIS scaling function at the appropriate τ.

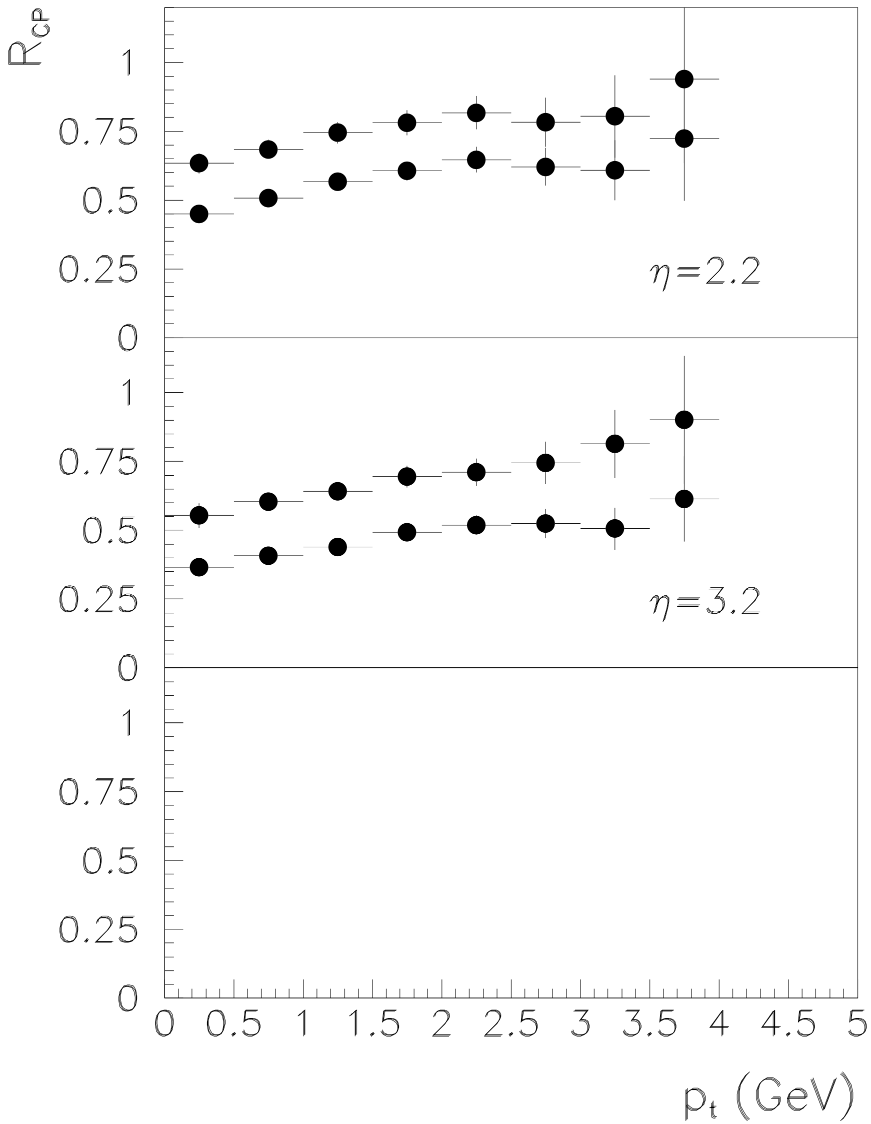
Geometric scaling and dAu data

- RHIC has measured p_t spectra of particles produced at forward rapidities (very small-x)

$$x \sim \frac{\exp\{-y\}}{\sqrt{s}}$$

- Strong suppression found (predicted from small-x evolution)

data: BRAHMS, nucl-ex/0403005



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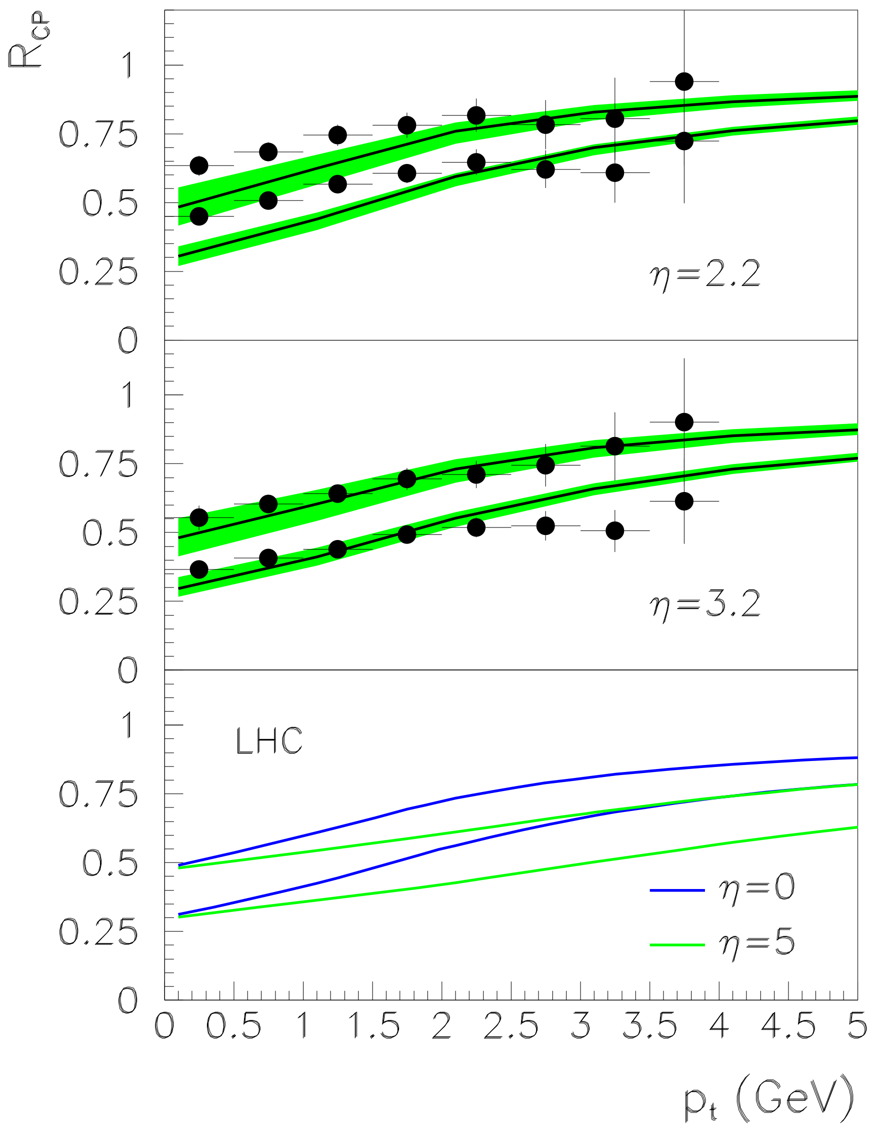
⇛ Comparison with the scaling curve found from lepton-hadron data gives reasonable agreement.

- $N_{\rm coll} = 13.6 \pm 0.3,\ 7.9 \pm 0.4,\ 3.3 \pm 0.4$
- $Q^2_{\rm sat,c} \propto N_{\rm coll}^{1/\delta}$



Comments on related results

Freund, Rummukainen, Weigert and Schafer PRL 90, 222002 (2003) studied lA data in a similar analysis.

- ⇒ Different conclusion: $Q^2_{\mathrm{sat,A}}$ grows slower than $A^{1/3}$
- ⇒ Differences (main)
  - Kinematics: $x \leq 0.1$
  - Geometry $R_A \sim A^{1/3}$
  - Normalization $F_2^A/A\, A^{\epsilon}$, $\epsilon \simeq 0.1$

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$Q^2_{\rm sat,A} \sim A^{1/3} \log A$ is sometimes taken Kharzeev, Levin, Nardi, ...

⇒ We have been unable to fit lepton-A data with the log term.

Relation to solutions of BK

Behavior of the saturation scale

See talk by G. Milhano

- Energy dependence:
  - fixed $\alpha_S \longrightarrow Q^2_{\rm sat} \sim x^{-d\,\alpha_s}$
  - running $\alpha_S \longrightarrow Q^2_{\rm sat} \sim \exp\left[\Delta'\sqrt{X - \log x}\right]$
- A–dependence
  - fixed $\alpha_S \longrightarrow$ same as initial conditions.
  - running $\alpha_S \longrightarrow$ A–dependence decreases with x.

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  - running $\alpha_S \longrightarrow$ A–dependence decreases with x.
- b-dependence of the saturation scale using BK??

Conclusions

- Two scales: dipole and hadron size. Once they are scaled by $Q_{\rm sat}$ and R_h one obtains a universal curve. $\longrightarrow$ Geometric scaling.
- The geometric scaling found in lp data has been extended to the nuclear case
  - $Q^2_{\rm sat,A}$ grows faster than $A^{1/3}$.
- This growth can explain the increase of multiplicities with $N_{\rm part}$ in AA at central rapidities.
- Suppression of particle production at forward rapidities also agree with the scaling law.
- These facts are in qualitative agreement with saturation approaches $\longrightarrow$ numerical coincidence??
  - Motivation for a more quantitative study (BK evolution...)