# Asymptotic behaviour of the saturation scale in non-linear QCD

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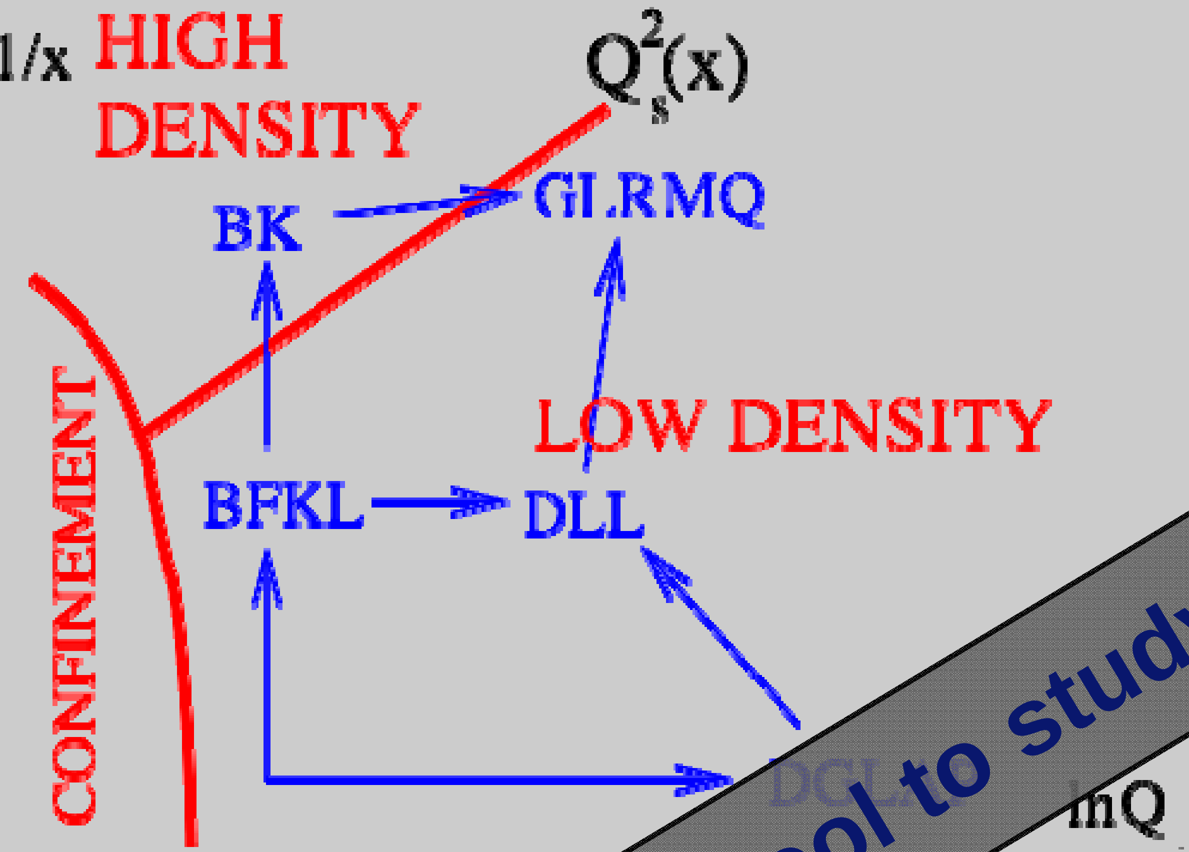
(with J. Albacete, N. Armesto, C. Salgado, and U. Wiedemann)

- introduction (BK equation with running coupling)
- evolution and saturation
- rapidity dependence of saturation scale
- nuclear size dependence of saturation scale
- prospects




## BK as main tool to study non-linear evolution

- evolution with rapidity of structure of dilute hadronic objects is perturbatively described by BFKL equation
- as object becomes dense, non-linear effects (e.g. recombination) ought to be accounted for
- need non-linear generalization of BFKL

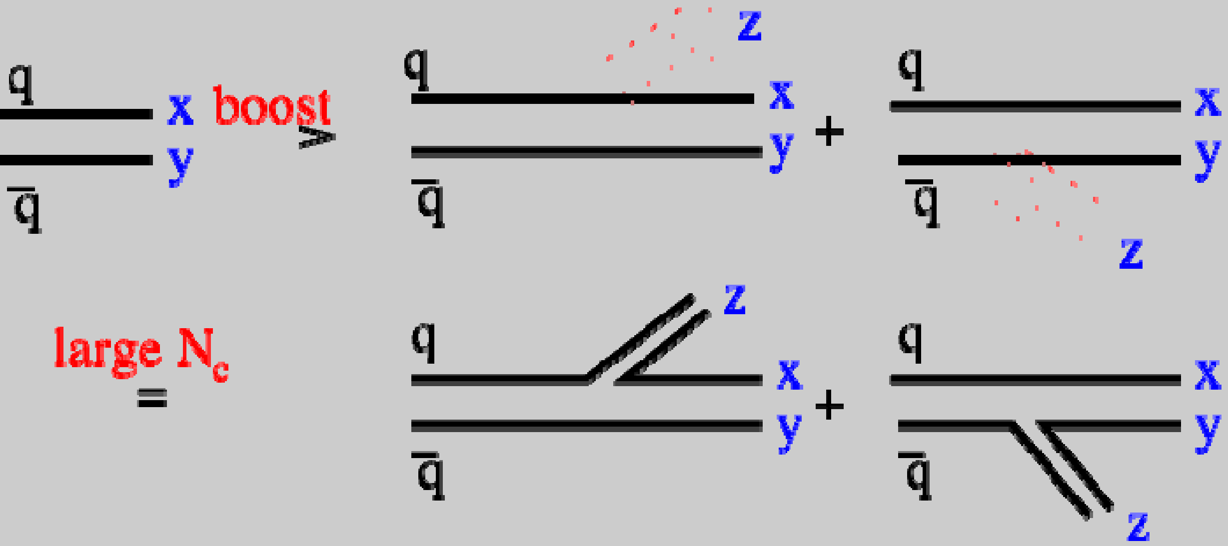
**Balitsky-Kovchegov (BK) equation**

Balitsky (1996)

Kovchegov (1999)

Kovner, Wiedemann (2001) [S-matrix app]

[rapidity evolution of scattering probability $N(x, y; Y)$ of $q\bar{q}$ dipole with hadronic target]




$$\vec{r} = \vec{x} - \vec{y}$$
$$\vec{r}_1 = \vec{x} - \vec{z}$$
$$\vec{r}_2 = \vec{y} - \vec{z}$$

homogeneous target with radius much larger than any dipole size

↪ neglect impact parameter dependence

$$\frac{\partial N(r,Y)}{\partial Y} = \int \frac{d^2 z}{2\pi} K(\vec{r},\vec{r}_1,\vec{r}_2)\Big[N(r_1,Y)+N(r_2,Y)-N(r,Y)-N(r_1,Y)N(r_2,Y)\Big]$$

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \bar{\alpha}_s \frac{r^2}{r_1^2 r_2^2}, \qquad \bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}$$

BFKL kernel:

probability of glu

- full analytic solution not known
  - analytical approximations (saddle point, 2nd order kernel expansion)
  - numerical studies
- BK derived at LO in $\alpha_s \ln(s/s_0)$ for fixed coupling
- NLL contributions known to play important role in BFKL
  - NLL programme just started
  - pragmatic approach
    - *first step*: introduce running coupling
    - *second step*: estimate further corrections

use different implementation prescriptions to check for sensitivity

## ee distance scales in kernel

$$K(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s N_c}{\pi}\,\frac{r^2}{r_1^2 r_2^2}$$

'external' scale: size of parent dipole

two 'internal' scales: size of offspring dipoles

- external size as running scale: $\alpha_s \rightarrow \alpha_s(r)$

$$\hookrightarrow \quad K_1(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{\alpha_s(r) N_c}{\pi}\,\frac{r^2}{r_1^2 r_2^2}$$

- internal sizes as running scale

*:: recast kernel in dipolar form as WW probability for gluon emission ::*

$$K(\vec{r},\vec{r}_1,\vec{r}_2) \equiv \frac{N_c}{4\pi^2}\left|\frac{g_s\vec{r}_1}{r_1^2} - \frac{g_s\vec{r}}{r_2^2}\right.$$

$$\hookrightarrow \quad K_2(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{N_c}{4\pi^2}\left|\frac{g_s(r_1)\vec{r}_1}{r_1^2} - \frac{g_s(r_2)\vec{r}_2}{r_2^2}\right|^2$$

- check on sensitivity to Coulomb tails

impose short range interactions :: *suppress large size dipole emission*

:: *weight gluon emission by Yukawa-like terms*

$$\hookrightarrow \quad K_3(\vec{r},\vec{r}_1,\vec{r}_2) = \frac{N_c}{4\pi^2}\left|\frac{e^{-\mu r_1/2} g_s(r_1)\vec{r}_1}{r_1^2} - \frac{e^{-\mu r_2/2} g_s(r_2)\vec{r}_2}{r_2^2}\right|^2$$

$$\mu = \Lambda_{QCD}$$

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$$\alpha_s(r) = \alpha_s(k=2/r) = \frac{12\pi}{\beta_0 \ln\left(\frac{4}{r^2\Lambda_{QCD}^2}+\lambda\right)}, \quad \beta_0 = 11N_c - 2N_f\,, \quad N_f = 3$$

$$\alpha_s(r=\infty) = \alpha_0$$

- functional form in configuration space given by Golec-Biernat–Wüsthoff model fixed x

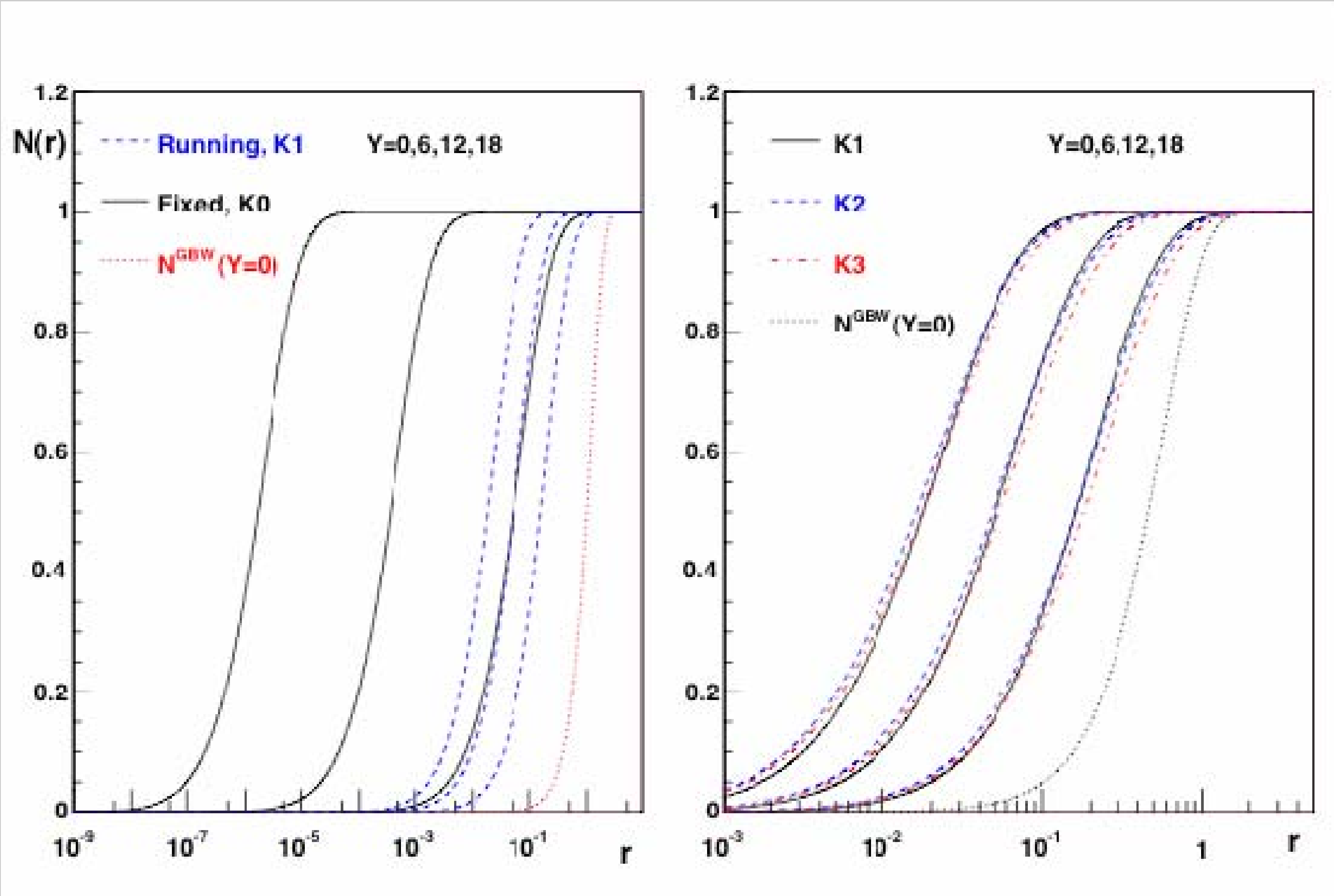
$$\hookrightarrow \quad N^{GBW}(r) = 1 - \exp\left[-\frac{r^2 Q_s'^2}{4}\right]$$

- form given by McLerran-Venugopalan model

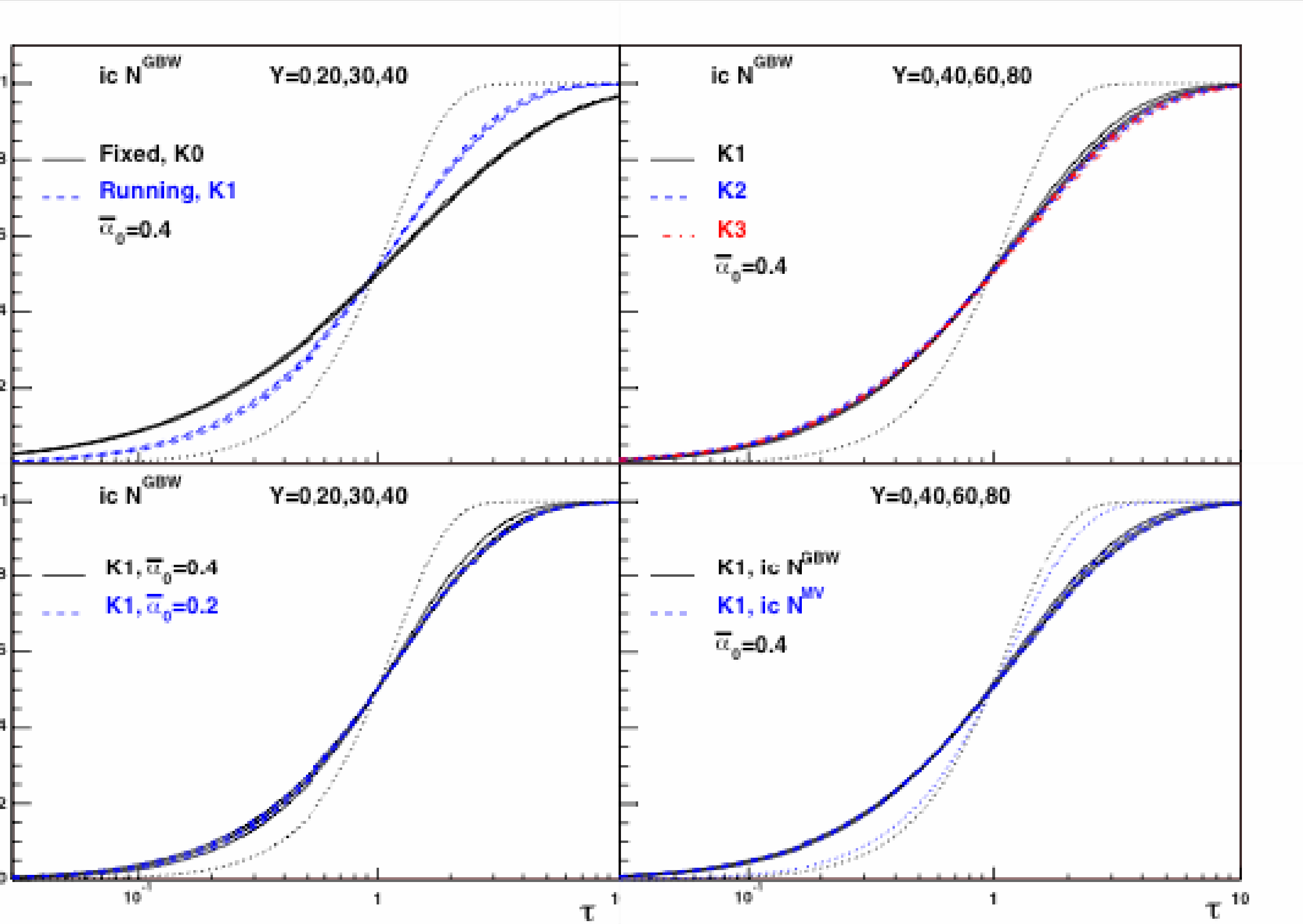
$$\hookrightarrow \quad N^{MV}(r) = 1 - \exp\left[-\frac{r^2 Q_s'^2}{4} \ln\left(\frac{1}{r^2 \Lambda_{QCD}^2} + e\right)\right]$$

- a third i.c. [its usefulness will become apparent]

$$\hookrightarrow \quad N^{AS}(r) = 1 - \exp\left[-(r\, Q_s')^c\right]$$




- f.c. evolution much faster than r.c.
- essentially insensitive to specific implementation of r.c. (small differences understood)
- further NLL ‘corrections’ (kinematical constraints)




$Y \to \infty$ limit

$N(r;Y) \longrightarrow N(\tau \equiv r\,Q_s(Y$

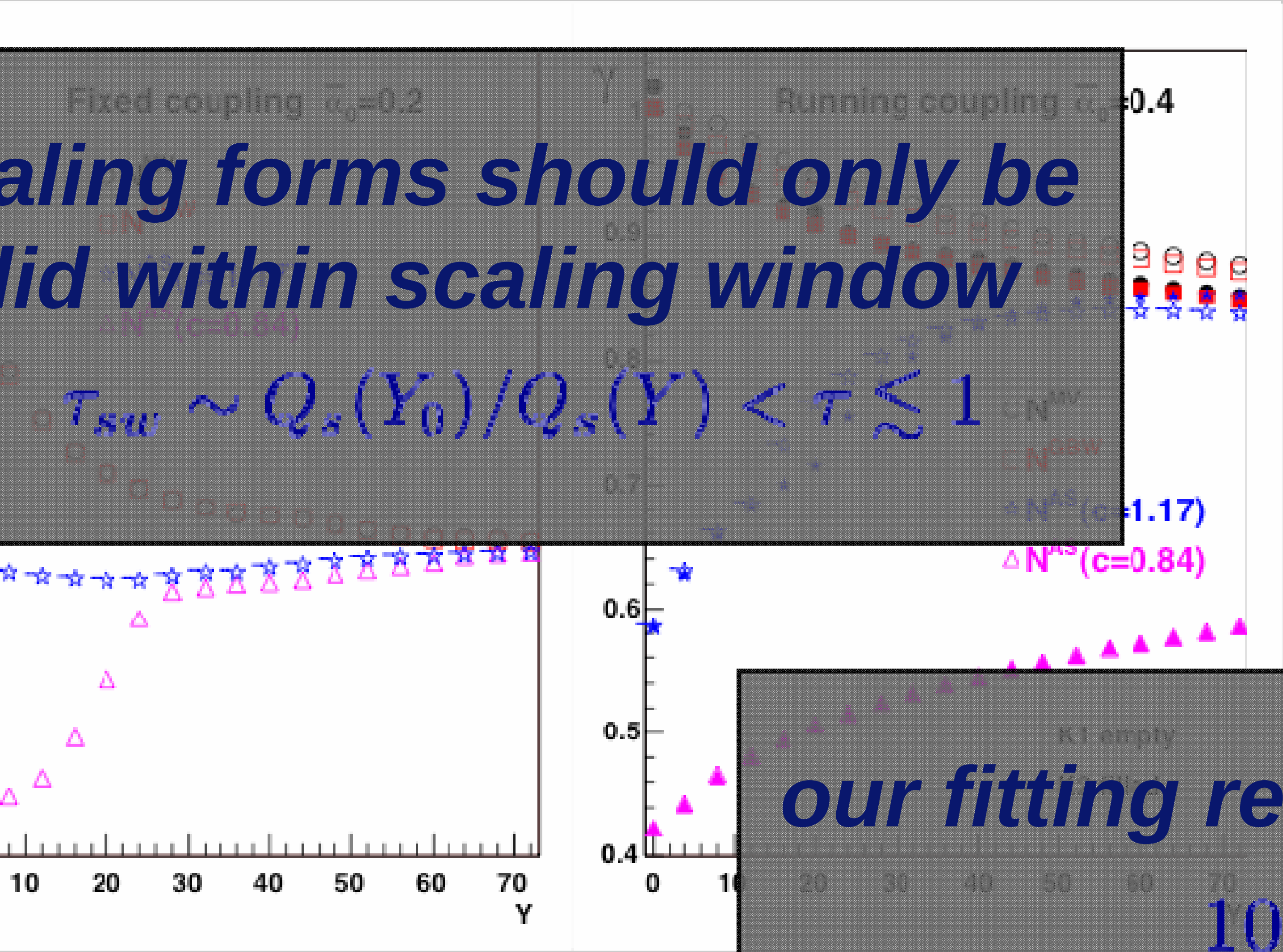
:: scaling ::

$Q_s(Y)$: saturation moment

*[transverse momentum below which unintegrated gluon distribution is saturated]*

$N(r = 1/Q_s(Y); Y) = \kappa\,,$

- at f.c. scaling quantified in several numerical works and confirmed analytical
- in r.c. case, broken scale invariance of BFKL kernel
  - unclear if scaling persists
- r.c. solutions tend to universal scaling forms with increasing rapidity




***aling forms should only be lid within scaling window***

$$\tau_{sw} \sim Q_s(Y_0)/Q_s(Y) < \tau \lesssim 1$$

$(1-\gamma) \longrightarrow$ anomalous dim

*leading large momentum behaviour of unintegrated gluon distribution*

$$N_{AS} \propto r^{2\gamma}, \; r \to 0$$

*to allow for 'from belo*

*initial anomalous dimensions*

***our fitting region***

$$10^{-5} < \tau < 10^{-1}$$

***and insensitive to lower end (accurate within 10% up to highest available transverse***

$$\ldots) = a\tau^{2\gamma}\left(\ln \tau^2 + \delta\right) \qquad f^{2)}(\ldots$$

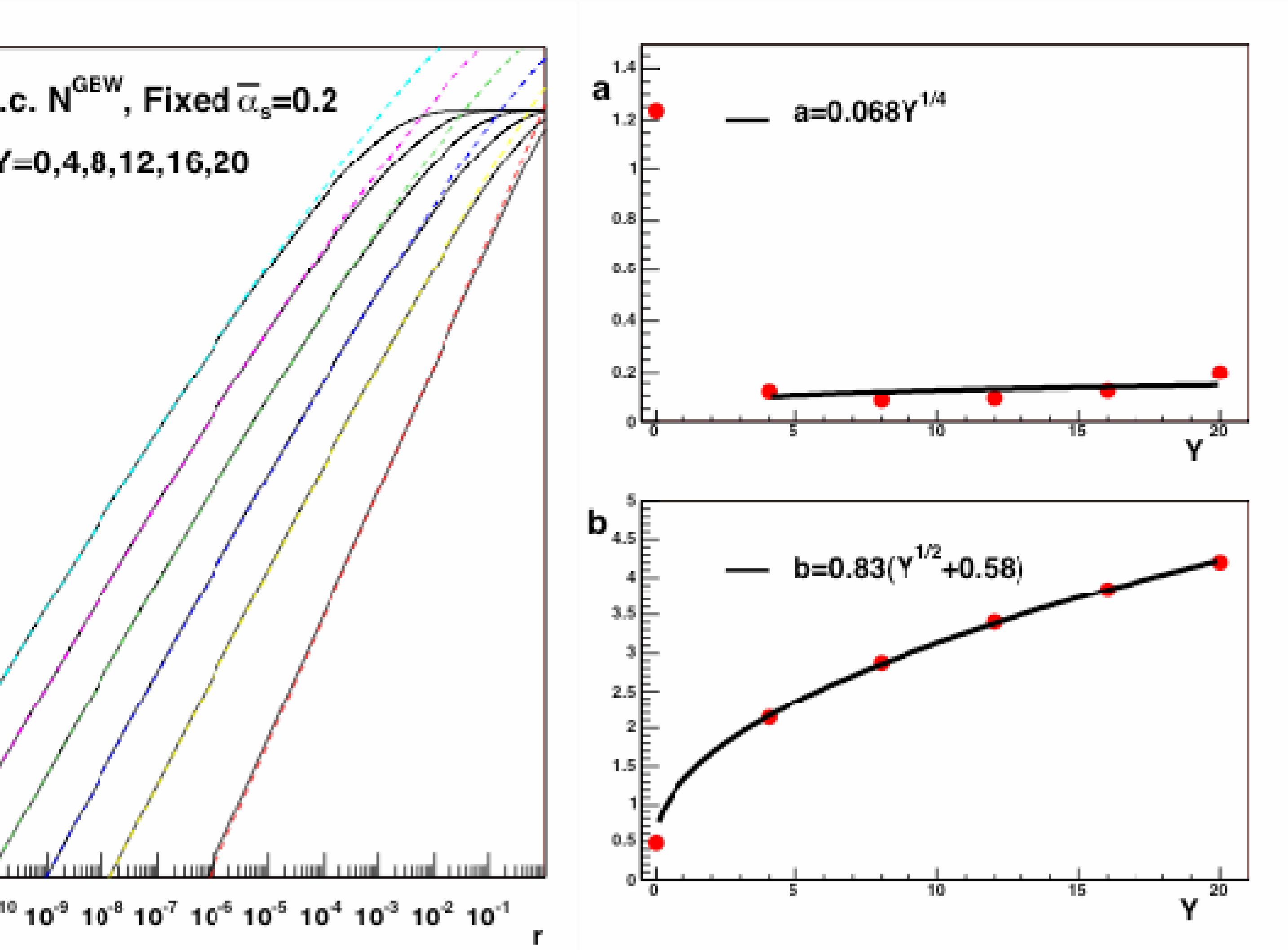
$\simeq 0.65$

[$\gamma = 0.628$ theo]

$\gamma \sim 0.85$

[Mueller, Triantafyllopoulos

[Ba...]




- coefficients follow expected Y-behaviour
- accurate within 10%
- fit of scaling form within scaling window only yields same limiting anomalous dimension

for $\tau < \tau_{sat}$ dipole scattering probability should return to DLL form

$$\hookrightarrow \quad N^{DLL} = a(Y)\, r^2 \left[-\ln(r^2\Lambda^2)\right]^{-3/4} \exp\left[b(Y)\sqrt{-\ln(r^2\Lambda^2)}\right]$$

$$a(Y) \propto Y^{1/4}$$

- in the scaling region and for large $Y$ (where $Q_s(Y) \gg \Lambda_{QCD}$ ) the rapidity dependence of the saturation scale is determined by

[Iancu, Itakura and McLerran (2

$$\frac{\partial \ln\left[Q_s^2(Y)/\Lambda^2\right]}{\partial Y} = d\,\bar{\alpha}_s$$

the numerical value of

$$d = \int \frac{d^2\tau d^2\tau_1}{2\pi^2} \frac{1}{\tau_1^2 \tau_2^2} \left[N(\tau_1) + N(\tau_2) - N(\tau) - N(\tau_1)N(\tau_2)\right]$$

can only be found once the scaling solution $N(\tau)$ is known.

several analytical approaches will be compared with our numerical results

[Iancu, Itakura, McLerran (2002)]

- for fixed coupling ( $\bar{\alpha}_s \to \bar{\alpha}_0 = \text{const.}$ ), $Q_s^2$ grows exponentially with rapidity

$$Q_s^2(Y) = Q_0^2 \exp\left[\Delta Y\right]$$

$$\Delta = d\alpha_0$$

$$Q_0^2 = Q_s^2(Y = 0)$$

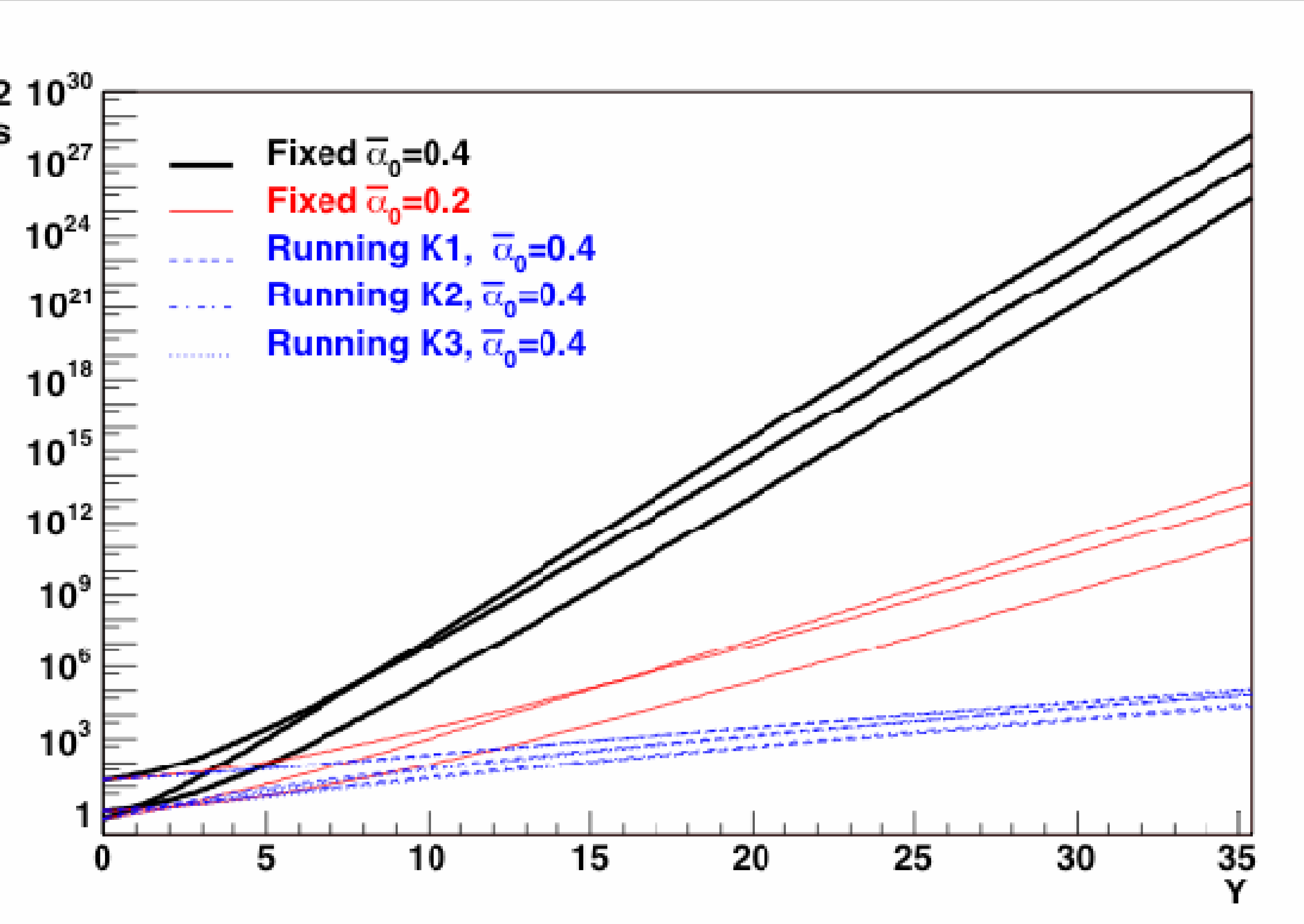
- for running coupling the momentum scale is expected to be $\sim Q_s$
  - d is dominated by the $r \sim 1$ region
  - numerical results also show that typical gluon transverse momentum

thus $\bar{\alpha}_s \longrightarrow \bar{\alpha}_s(Q_s(Y))$

$$Q_s^2(Y) = \Lambda^2 \exp\left[\Delta' \sqrt{Y + X}\right]$$

$$(\Delta')^2 = 24 N_c d/\beta_0$$

$$X = (\Delta')^{-2} \ln\left(Q_0^2/\Lambda^2\right)$$




- faster rise for f.c.
- f.c. : excellent agreement with exponential for high enough Y

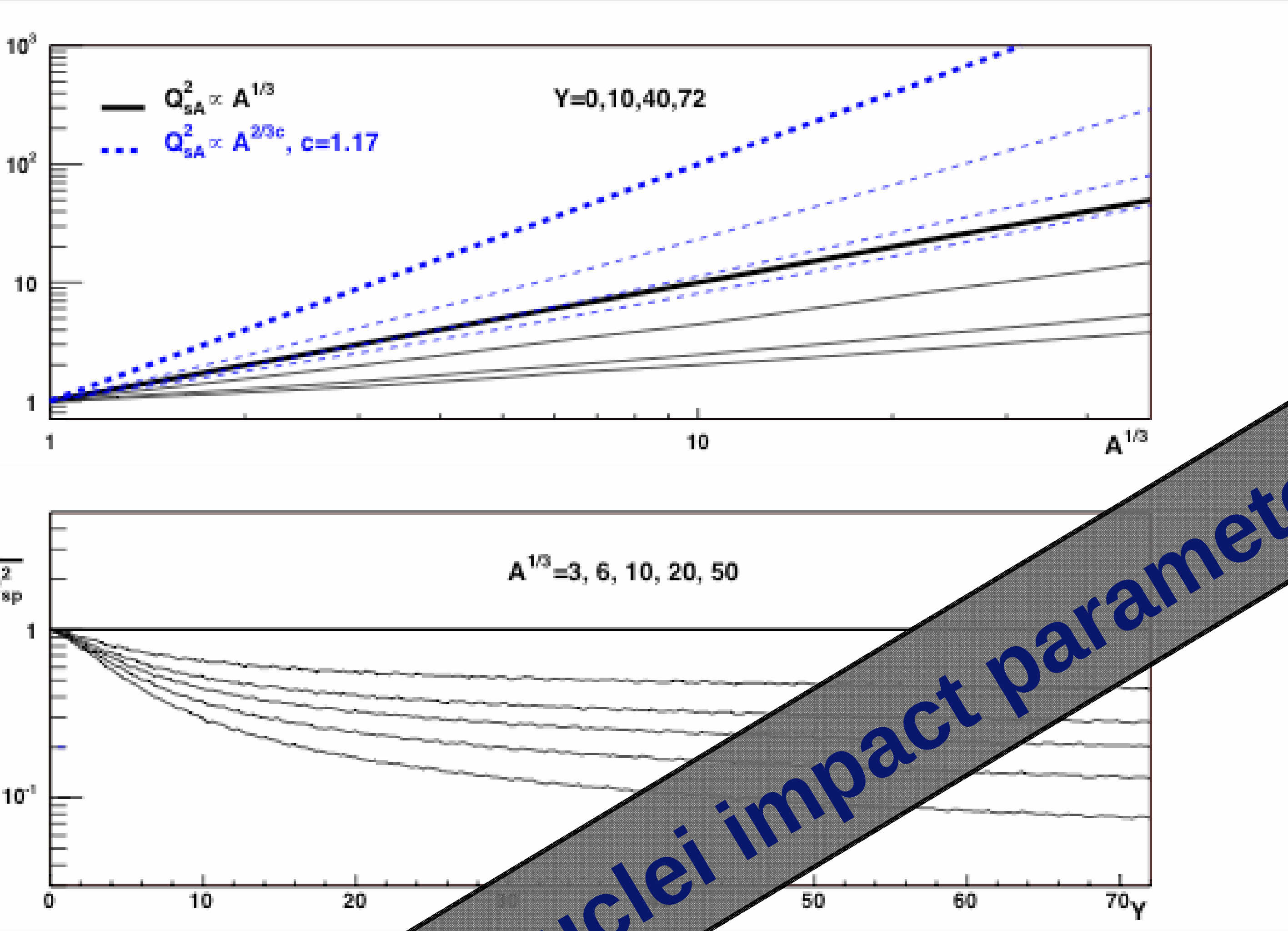
t $Q_s^2(Y) = Q_0^2 \exp[d\bar{\alpha}_0 Y]$

f.c. $d \simeq 4.57$ *[agreement with earlier numerical results]*
*[ $d = 4.88$ analytical]*

fit $Q_s^2(Y) = \Lambda^2 \exp\left[\Delta' \sqrt{Y + X}\right]$

r.c. $\Delta' \simeq 3.2$ *[$\Delta' = 3.6$ analyti*

*good fit over entire Y-region*




r.c. over wide Y-range

$$\ln \frac{Q^2_{sA}(Y)}{Q^2_{sp}(Y)} \sim Y^{-0.4}$$

… and Y

$$\ln \frac{Q^2_{sA}(Y)}{Q^2_{sp}(Y)} \sim \frac{1}{\sqrt{Y}}$$

[Mueller (2003)]

*at extremely large Y all hadronic targets look the same*

**for realistic nuclei impact parameter likely to play role**

nuclear size enters initial condition

- does BK evolution modify/preserve initial dependence?

…tation invariance of f.c. BK allows for scaling-out of any nuclear dependence

- running coupling plays important role
- dominant NLL effect (?)
- good agreement between numerics and analytical studies

- impact parameter dependence will be important (running)
- need NLL-BK