Proton-nucleus collisions in the color glass condensate framework

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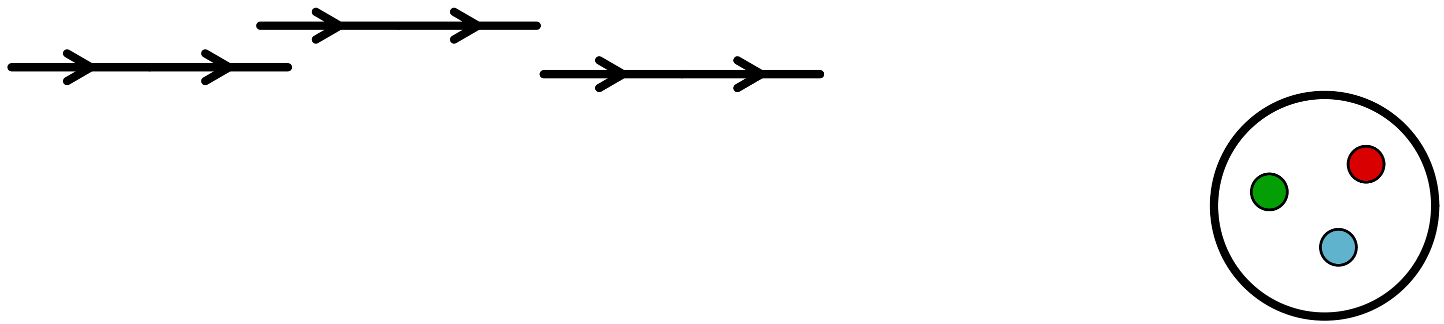
CEA / DSM / SPhT

Outline

- Evolution and saturation
- Color Glass Condensate
- $\boldsymbol{k}_\perp$-factorization
- Proton-nucleus collisions
  - Classical color field
  - Quark production

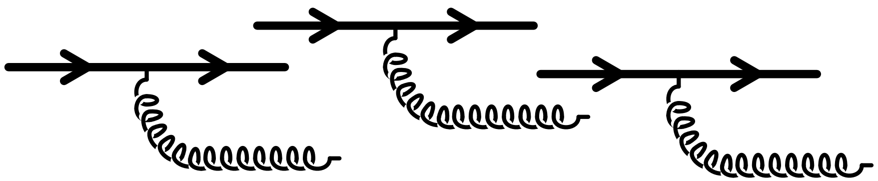
- FG, Venugopalan, hep-ph/0310090
- Blaizot, FG, Venugopalan, hep-ph/0402256, 0402257
- Fujii, FG, Venugopalan, work in progress

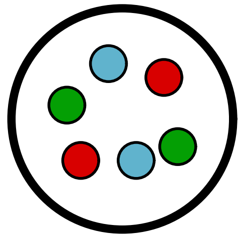
Evolution and saturation (1/5)



▷ at low energy, only valence quarks are present in the hadron wave function

Evolution and saturation (2/5)



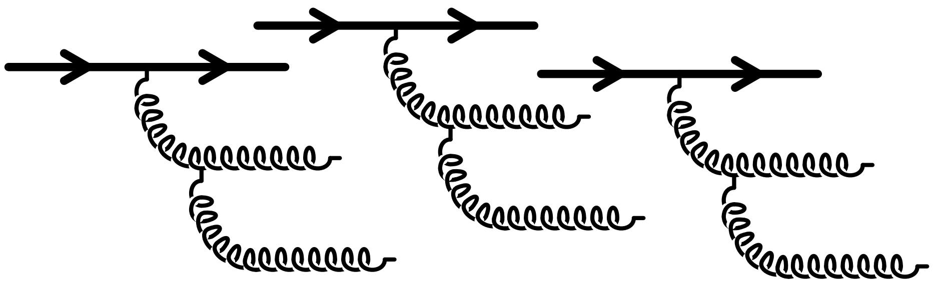


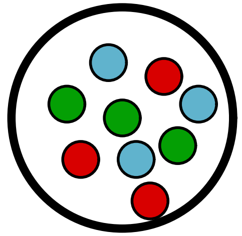
▷ when energy increases, new partons are emitted

▷ the emission probability is $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon

▷ at small-x (i.e. high energy), these logs need to be resummed

Evolution and saturation (3/5)

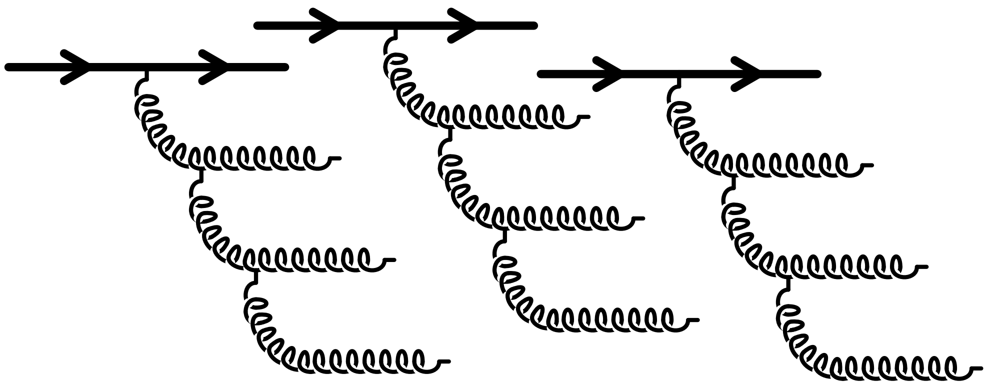


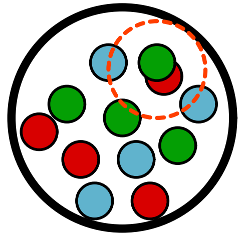


▷ as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)
Kuraev, Lipatov, Fadin (1977), Balitsky, Lipatov (1978)

SPhT

Evolution and saturation (4/5)

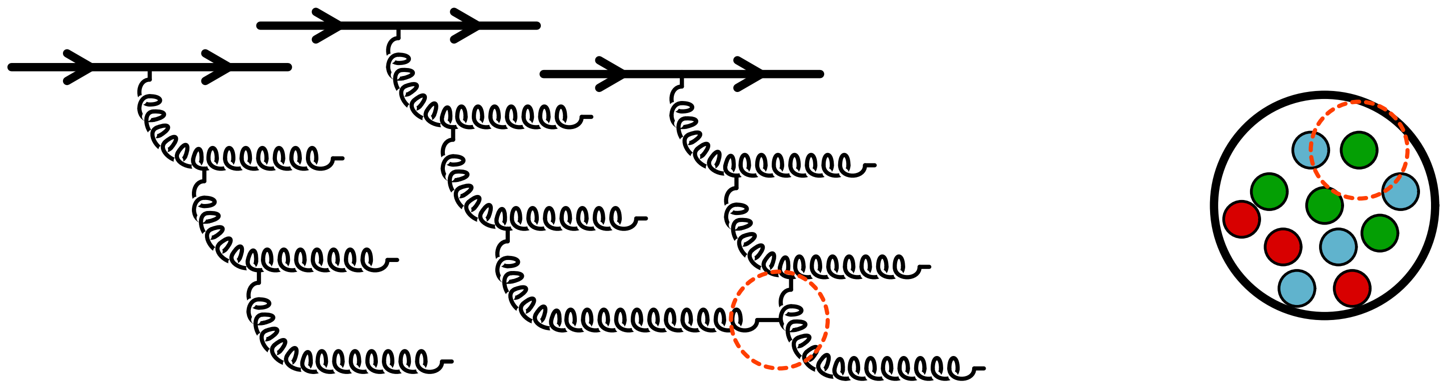




▷ eventually, the partons start overlapping in phase-space

SPhT

Evolution and saturation (5/5)



▷ parton recombination becomes favorable

▷ after this point, the evolution is non-linear:

the number of partons created at a given step depends non-linearly on the number of partons present previously

Balitsky (1996), Kovchegov (1996,2000)

Jalilian-Marian, Kovner, Leonidov, Weigert (1997,1999)

Iancu, Leonidov, McLerran (2001)

Color glass condensate (1/2)

McLerran, Venugopalan (1994)
Iancu, Leonidov, McLerran (2001)

- Small x modes have a large occupation number
  ▷ they are described by a classical color field
- Large x modes are described by "frozen" color sources ρ_a
- The classical field obeys Yang-Mills equations:

$$[D_\nu, F^{\nu\mu}]_a = \delta^{\mu +}\delta(x^-)\rho_a(\vec{\boldsymbol{x}}_\perp)$$

- The color sources ρ_a are random, and their distribution is described by a functional $W_{x_0}[\rho]$, where x_0 is the separation between "small x" et "large x".

Color glass condensate (2/2)

- The distribution $W_{x_0}[\rho_a]$ evolves with x_0 (new modes are included into $W_{x_0}[\rho_a]$ when x_0 decreases):

$$\frac{\partial W_{x_0}[\rho]}{\partial \ln(1/x_0)} = \frac{1}{2}\int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\rho_a(\vec{\boldsymbol{x}}_\perp)}\chi_{ab}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)\frac{\delta}{\delta\rho_b(\vec{\boldsymbol{y}}_\perp)}W_{x_0}[\rho]$$

  ▷ χ_{ab} depends on ρ to all orders $\Rightarrow$ non-linear evolution in ρ
  ▷ reduces to BFKL in the low density regime

- Observables are calculated in the presence of the classical field, and then averaged over the configurations of the sources ρ_a:

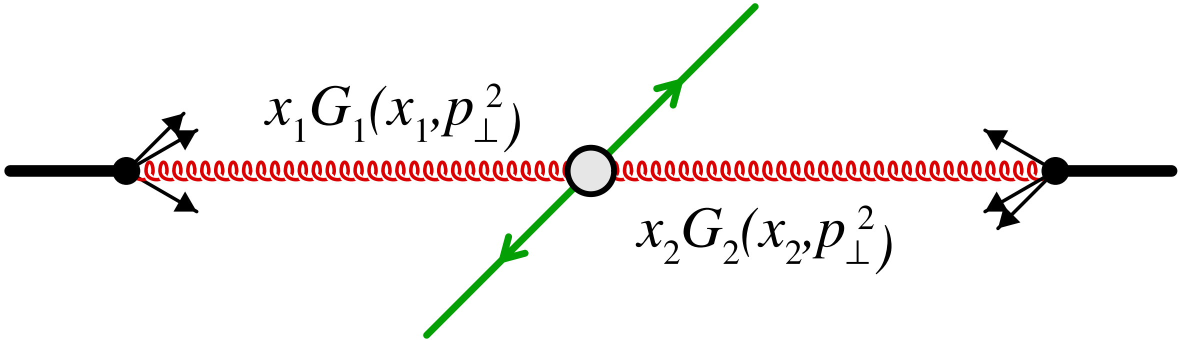
$$\langle\mathcal{O}\rangle = \int [D\rho_a]\, W_{x_0}[\rho_a]\, \mathcal{O}[\rho_a]$$

Note: this average restores gauge invariance

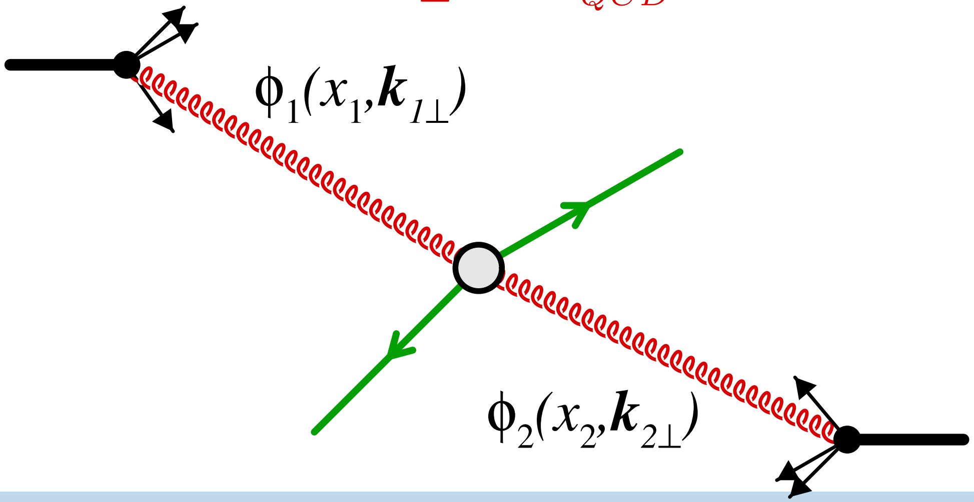
Collinear vs. Kt-factorization

Collins, Ellis (1991), Catani, Ciafaloni, Hautmann (1991)

- Collinear factorization : $s \sim p_\perp^2 \gg \Lambda_{QCD}^2$



- $\boldsymbol{k}_\perp$-factorization : $s \gg p_\perp^2 \gg \Lambda_{QCD}^2$



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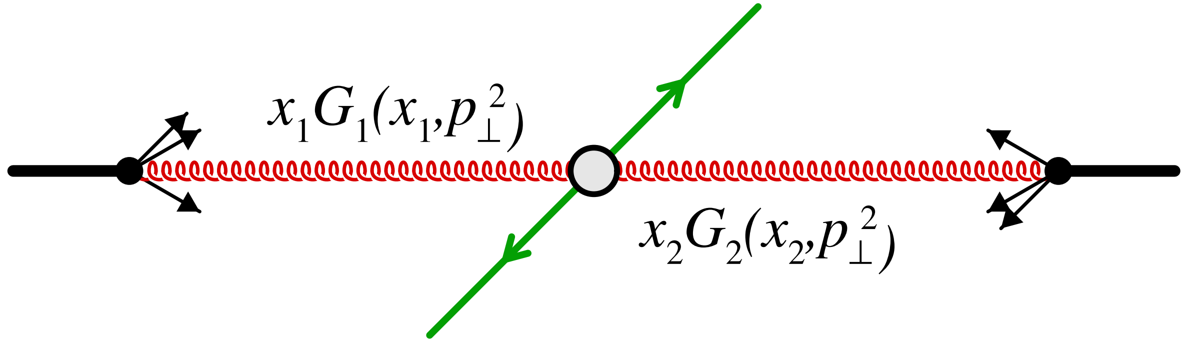
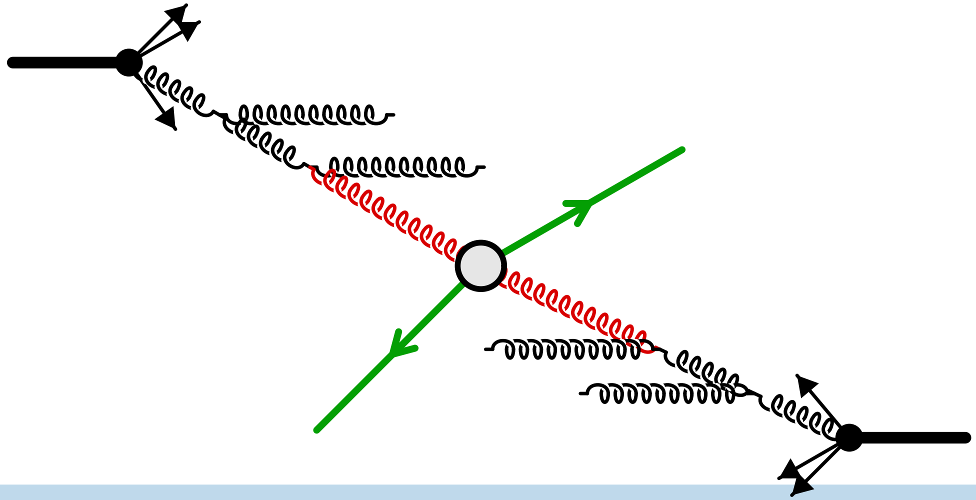
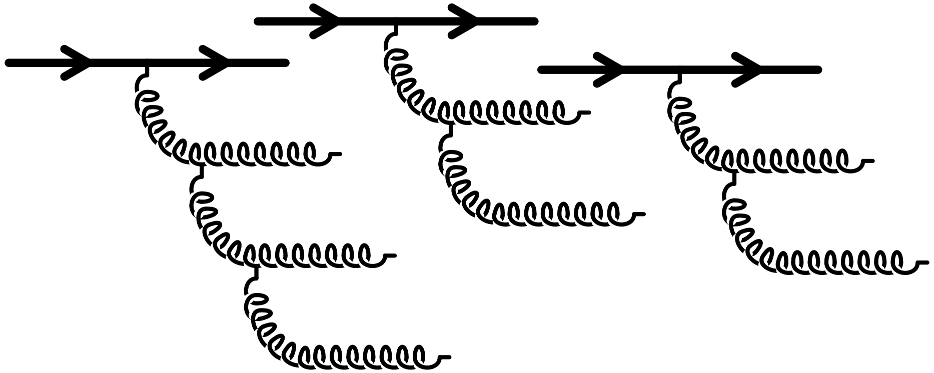


- $\boldsymbol{k}_\perp$-factorization + BFKL: resum $\left[\alpha_s \ln(s/p_\perp^2)\right]^n$

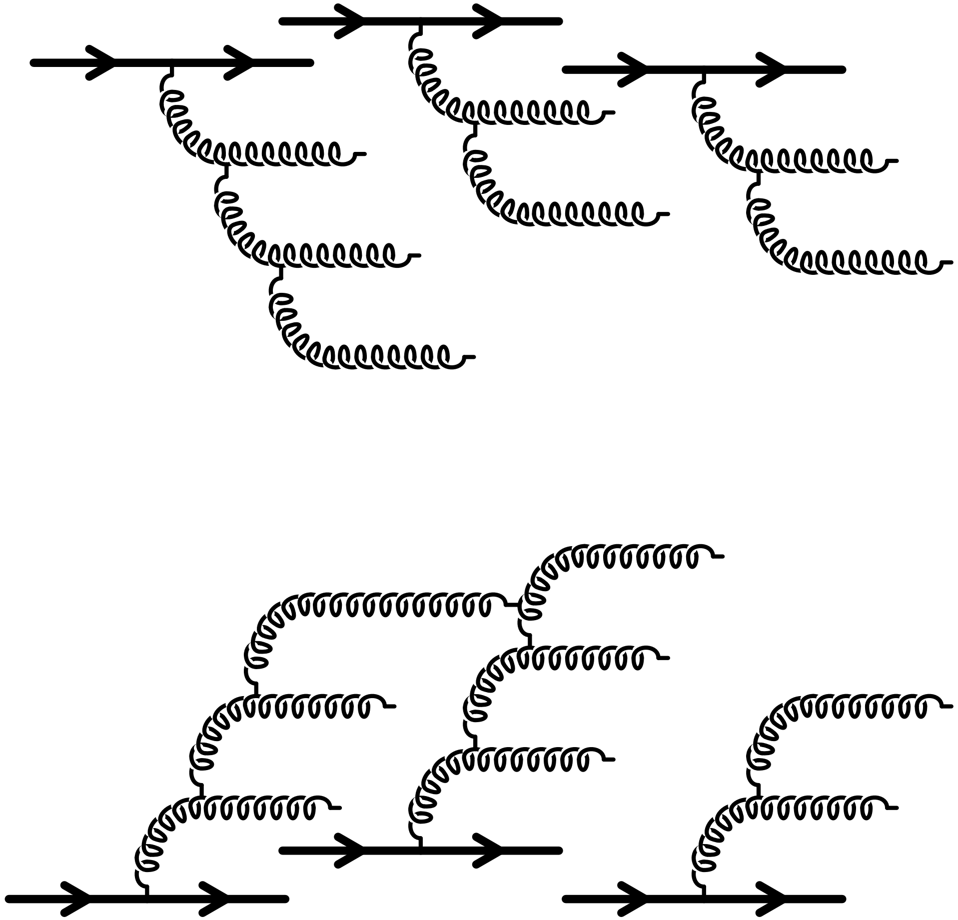


Quark production: strategy (1/4)



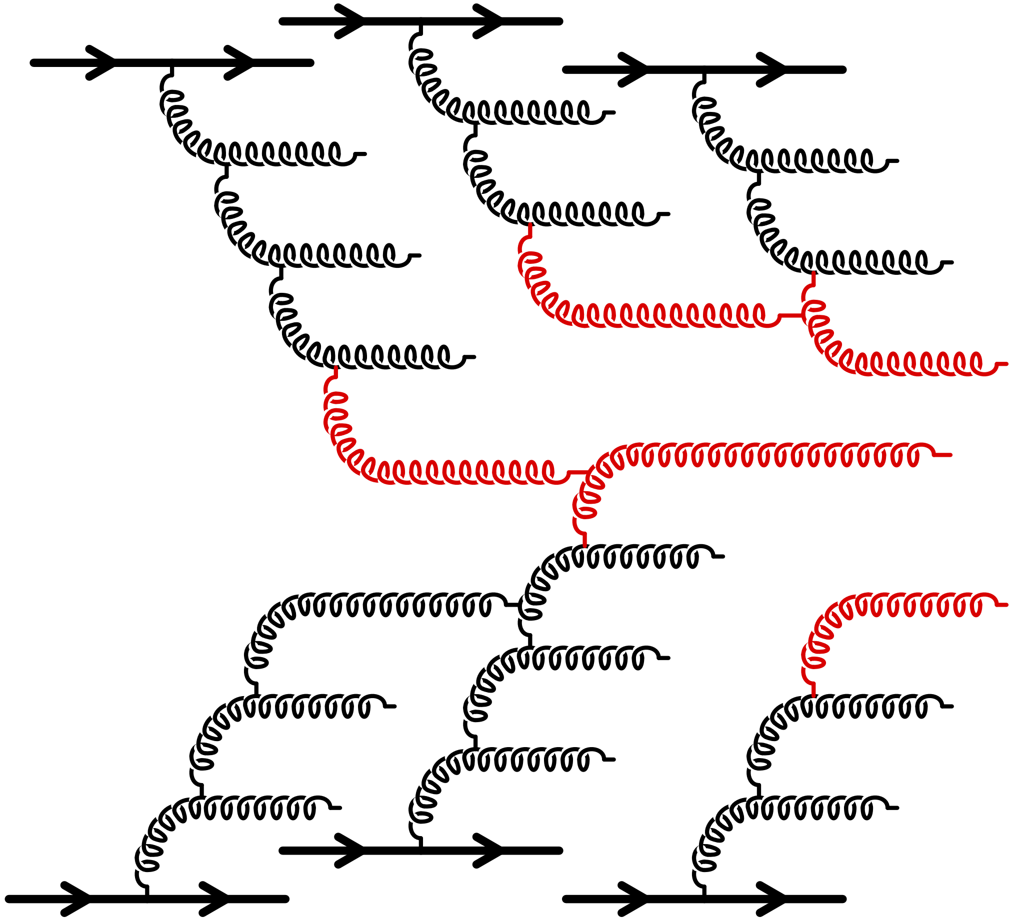
▷ get $W_{x_1}[\rho_1]$ for the first projectile

Quark production: strategy (2/4)



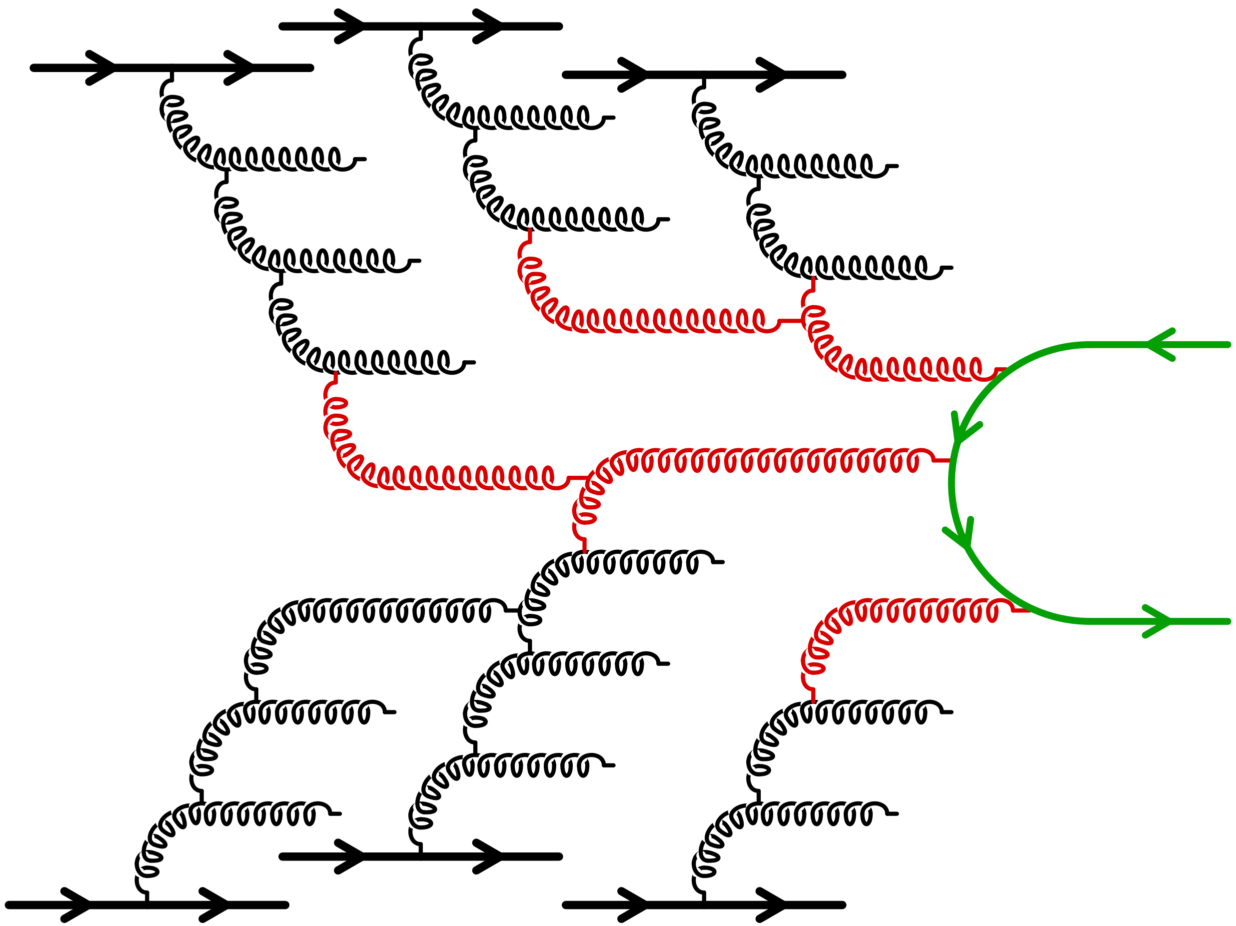
▷ get $W_{x_2}[\rho_2]$ for the second projectile

Quark production: strategy (3/4)



▷ solve the Yang-Mills equations for the sources ρ_1, ρ_2

Quark production: strategy (4/4)



▷ compute the quark propagator in the classical field

Proton-nucleus: Gauge field (1/2)

Blaizot, FG, Venugopalan (2004)

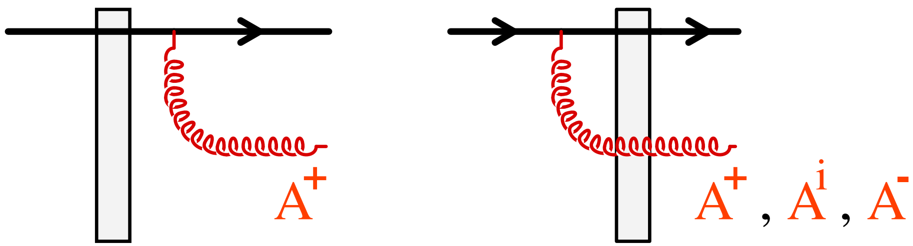
- ρ_p is a weak source, ρ_A is a strong source
  $\Rightarrow$ we want the pair production amplitude to first order in ρ_p and to all orders in ρ_A
- Yang-Mills equations:

$$[D_\mu, F^{\mu\nu}] = J^\nu \quad , \qquad [D_\nu, J^\nu] = 0$$

$$J^\nu|_{\text{lowest order}} = \delta^{\nu+}\delta(x^-)\rho_p(\boldsymbol{x}_\perp) + \delta^{\nu-}\delta(x^+)\rho_A(\boldsymbol{x}_\perp)$$

$$\partial_\mu A^\mu = 0$$

- (Very sketchy) diagrammatic interpretation:



Proton-nucleus: gauge field (2/2)

- Solution in the Lorenz gauge $(\partial_\mu A^\mu = 0)$:

$$A^\mu(k) = \frac{ig}{k^2} \int_{\vec{\boldsymbol{k}}_{1\perp}} \Big\{ C_U^\mu \Big[U(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2 \delta(\vec{\boldsymbol{k}}_{2\perp}) \Big] + C_V^\mu \Big[V(\vec{\boldsymbol{k}}_{2\perp}) - (2\pi)^2 \delta(\vec{\boldsymbol{k}}_{2\perp}) \Big] \Big\} \frac{\rho_p(\vec{\boldsymbol{k}}_{1\perp})}{\boldsymbol{k}_{1\perp}^2}$$

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$$C_U^+ \equiv -\boldsymbol{k}_{1\perp}^2/k^- \;,\; C_U^- \equiv (\boldsymbol{k}_{2\perp}^2 - \boldsymbol{k}_\perp^2)/k^+ \;,\; C_U^i \equiv -2\boldsymbol{k}_1^i$$

$$C_V^+ \equiv 2k^+ \;,\; C_V^- \equiv -2k^- + 2\boldsymbol{k}_\perp^2/k^+ \;,\; C_V^i \equiv 2\boldsymbol{k}^i$$

$$\text{Note :}\quad C_U^\mu + C_V^\mu/2 = L^\mu \quad (\text{Lipatov's effective vertex})$$

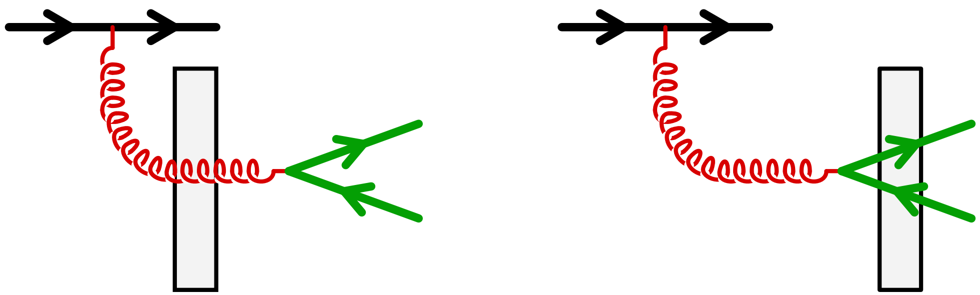
$$U(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\; \mathcal{P}\, e^{ig\int_{z^+} A_A^-(z^+,\vec{\boldsymbol{x}}_\perp)\cdot T}$$

$$V(\vec{\boldsymbol{k}}_{2\perp}) \equiv \int_{\vec{\boldsymbol{x}}_\perp} e^{-i\vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{x}}_\perp}\; \mathcal{P}\, e^{i\frac{g}{2}\int_{z^+} A_A^-(z^+,\vec{\boldsymbol{x}}_\perp)\cdot T}$$

Proton-nucleus: quark production (1/8)

- Popular wisdom says:

  "The pair is produced either before or after the collision with the nucleus. The production of the pair inside the nucleus is suppressed by $s^{-1/2}$*"*



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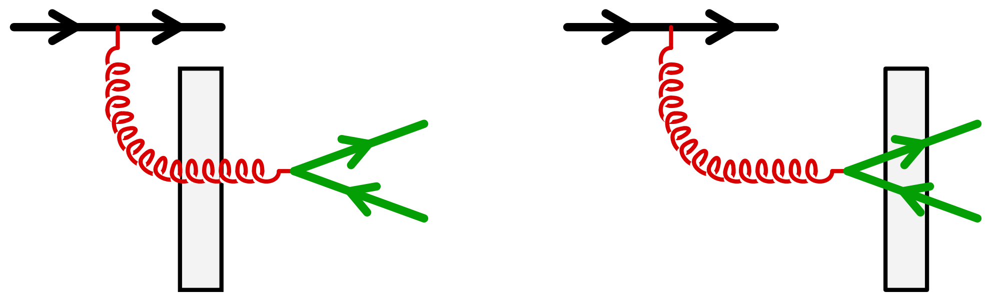


- ...but does not apply to gauge dependent quantities
  - True for the amplitude if the classical field A^μ inside the nucleus remains bounded when $s \to \infty$
  - This is not the case in covariant gauge...
  - One must split the field into a singular part (proportional to $\delta(x^+)$) and a regular part (no $\delta(x^+)$)

Proton-nucleus: quark production (2/8)

- Regular contributions to the amplitude:

$$\mathcal{M}_F^{\rm reg} = g^2 \int\limits_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{p,a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int\limits_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

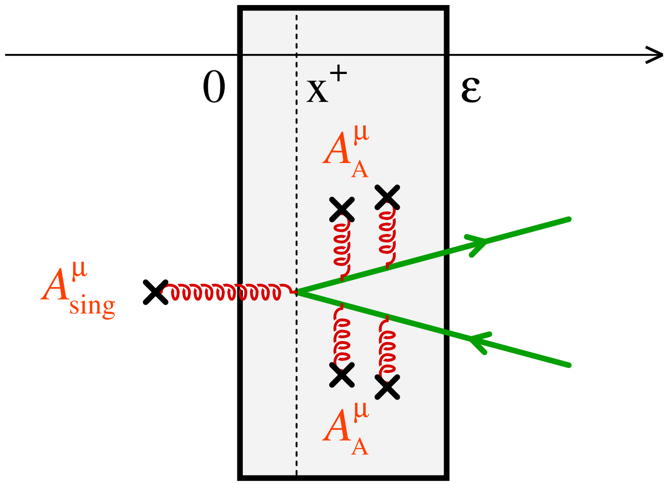
$$\times\overline{u}(\vec{\boldsymbol{q}})\Big\{\frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+[\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]}{2p^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2]+2q^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

$$+t^b\Big[\frac{\not{C}_U(p+q,\vec{\boldsymbol{k}}_{1\perp})}{(p+q)^2}U_{ba}(\vec{\boldsymbol{x}}_\perp)-\frac{\gamma^+}{p^++q^+}V_{ba}(\vec{\boldsymbol{x}}_\perp)\Big]\Big\}v(\vec{\boldsymbol{p}})$$

- **Notes:**
  - $\widetilde{U}$ is a Wilson line in the fundamental representation
  - the Wilson line V is still there !

Proton-nucleus: quark production (3/8)

- Singular diagram:



- Corresponding term in the amplitude:

$$\mathcal{M}_F^{\rm sing} = g^2 \int_{\vec{\boldsymbol{k}}_{1\perp}} \frac{\rho_{p,a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int_{\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{x}}_\perp}$$
$$\times \frac{\overline{u}(\vec{\boldsymbol{q}})\gamma^+ t^b v(\vec{\boldsymbol{p}})}{p^+ + q^+} \left[V_{ba}(\vec{\boldsymbol{x}}_\perp) - U_{ba}(\vec{\boldsymbol{x}}_\perp)\right]$$

Proton-nucleus: quark production (4/8)

- Total amplitude:

$$\mathcal{M}_F = g^2 \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_\perp} \frac{\rho_{p,a}(\vec{\boldsymbol{k}}_{1\perp})}{k_{1\perp}^2} \int_{\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp} e^{i(\vec{\boldsymbol{p}}_\perp+\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})\cdot\vec{\boldsymbol{y}}_\perp}$$

$$\times\overline{u}(\vec{\boldsymbol{q}})\Big\{[\widetilde{U}(\vec{\boldsymbol{x}}_\perp)t^a\widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp)]T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) + [t^b U_{ba}(\vec{\boldsymbol{x}}_\perp)]\not{L}\Big\}v(\vec{\boldsymbol{p}})$$

with

$$T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp) \equiv \frac{\gamma^+(\not{q}-\not{k}+m)\gamma^-(\not{q}-\not{k}-\not{k}_1+m)\gamma^+}{2p^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp)^2+m^2] + 2q^+[(\vec{\boldsymbol{q}}_\perp-\vec{\boldsymbol{k}}_\perp-\vec{\boldsymbol{k}}_{1\perp})^2+m^2]}$$

- **Notes:**
  - the V's cancel between regular and singular contributions
  - $\overline{u}(\vec{\boldsymbol{q}})\left[\not{C}_U(p+q,\vec{\boldsymbol{k}}_{1\perp}) - \gamma^+\frac{(p+q)^2}{p^++q^+}\right]v(\vec{\boldsymbol{p}}) = \overline{u}(\vec{\boldsymbol{q}})\not{L}v(\vec{\boldsymbol{p}})$

Proton-nucleus: quark production (5/8)

- Pair production cross-section:

$$\frac{d\sigma_{q\bar{q}}}{d^2\vec{\boldsymbol{p}}_\perp d^2\vec{\boldsymbol{q}}_\perp dy_p dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A} \int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}} \frac{\delta(\vec{\boldsymbol{p}}_\perp + \vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{k}}_{1\perp} - \vec{\boldsymbol{k}}_{2\perp})}{\boldsymbol{k}_{1\perp}^2 \boldsymbol{k}_{2\perp}^2}$$

$$\times\Big\{ \int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}'_\perp)\Big]\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp)$$

$$+ \int_{\vec{\boldsymbol{k}}_\perp} \mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$

$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_p(\vec{\boldsymbol{k}}_{1\perp})$$

▷ $k_\perp$-factorization valid on the proton side, but not for the nucleus: one needs three different “distributions” in order to describe the nucleus

Proton-nucleus: quark production (6/8)

- Nuclear "gluon distributions":

$$\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp}) = \frac{k_{2\perp}^2}{4\alpha_s N} \int\limits_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} e^{i\vec{\boldsymbol{k}}_{2\perp}\cdot(\vec{\boldsymbol{x}}_\perp - \vec{\boldsymbol{y}}_\perp)} \, \mathrm{tr}\left\langle U(\vec{\boldsymbol{x}}_\perp) U^\dagger(\vec{\boldsymbol{y}}_\perp)\right\rangle$$

$$\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = \frac{k_{2\perp}^2}{2\alpha_s N} \int\limits_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp, \vec{\boldsymbol{z}}_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp + (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp - \vec{\boldsymbol{k}}_{2\perp}\cdot\vec{\boldsymbol{z}}_\perp\right]} \times \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp) t^a \widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp) t^b U_{ab}^\dagger(\vec{\boldsymbol{z}}_\perp)\right\rangle$$

$$\phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp, \vec{\boldsymbol{k}}'_\perp) = \frac{k_{2\perp}^2}{2\alpha_s N} \int\limits_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp, \vec{\boldsymbol{x}}'_\perp, \vec{\boldsymbol{y}}'_\perp} e^{i\left[\vec{\boldsymbol{k}}_\perp\cdot\vec{\boldsymbol{x}}_\perp - \vec{\boldsymbol{k}}'_\perp\cdot\vec{\boldsymbol{x}}'_\perp + (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}_\perp)\cdot\vec{\boldsymbol{y}}_\perp - (\vec{\boldsymbol{k}}_{2\perp} - \vec{\boldsymbol{k}}'_\perp)\cdot\vec{\boldsymbol{y}}'_\perp\right]} \times \mathrm{tr}\left\langle \widetilde{U}(\vec{\boldsymbol{x}}_\perp) t^a \widetilde{U}^\dagger(\vec{\boldsymbol{y}}_\perp) \widetilde{U}(\vec{\boldsymbol{y}}'_\perp) t^a \widetilde{U}(\vec{\boldsymbol{x}}'_\perp)\right\rangle$$

Proton-nucleus: quark production (7/8)

- Sum rules and $k_\perp$-factorization:
  - Sum rule:

$$\int_{\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp} \phi_A^{q\bar{q},q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp,\vec{\boldsymbol{k}}'_\perp) = \int_{\vec{\boldsymbol{k}}_\perp} \phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = \phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$$

  - $k_\perp$-factorization is possible if one can neglect the $\vec{\boldsymbol{k}}_\perp$ dependence in $T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)$
  - this happens if the $Q\bar{Q}$ pair has a small transverse size (compared to the typical inverse $k_\perp$ in the nucleus, i.e. Q_s^{-1})
  - **examples:** heavy quarks, large invariant mass

Proton-nucleus: quark production (8/8)

- Single quark production cross-section:

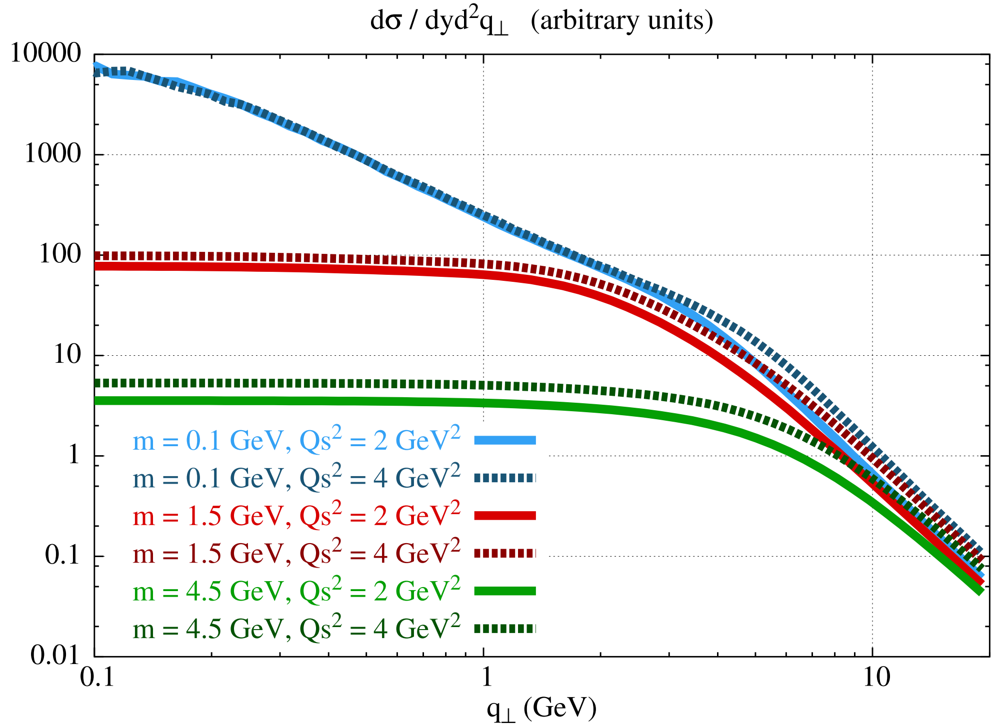
$$\frac{d\sigma_q}{d^2\vec{\boldsymbol{q}}_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^4 d_A}\int\frac{dp^+}{p^+}\int_{\vec{\boldsymbol{k}}_{1\perp},\vec{\boldsymbol{k}}_{2\perp}}\frac{1}{\boldsymbol{k}_{1\perp}^2\boldsymbol{k}_{2\perp}^2}$$
$$\times\Big\{\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp})(\not{p}-m)T^*_{q\bar{q}}(\vec{\boldsymbol{k}}_{2\perp})\Big]\frac{C_F}{N}\phi_A^{q,q}(\vec{\boldsymbol{k}}_{2\perp})$$
$$+\int_{\vec{\boldsymbol{k}}_\perp}\mathrm{tr}\Big[(\not{q}+m)T_{q\bar{q}}(\vec{\boldsymbol{k}}_\perp)(\not{p}-m)\not{L}^* + \mathrm{h.c.}\Big]\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp)$$
$$+\mathrm{tr}\Big[(\not{q}+m)\not{L}(\not{p}-m)\not{L}^*\Big]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})\Big\}\varphi_p(\vec{\boldsymbol{k}}_{1\perp})$$

  - $\phi_A^{q,q}$ is the analogue of $\phi_A^{g,g}$ for the fundamental representation
  - still no $k_\perp$-factorization on the nucleus side
  - contains only 2-point and 3-point correlators

Proton-nucleus: numerics (1/3)

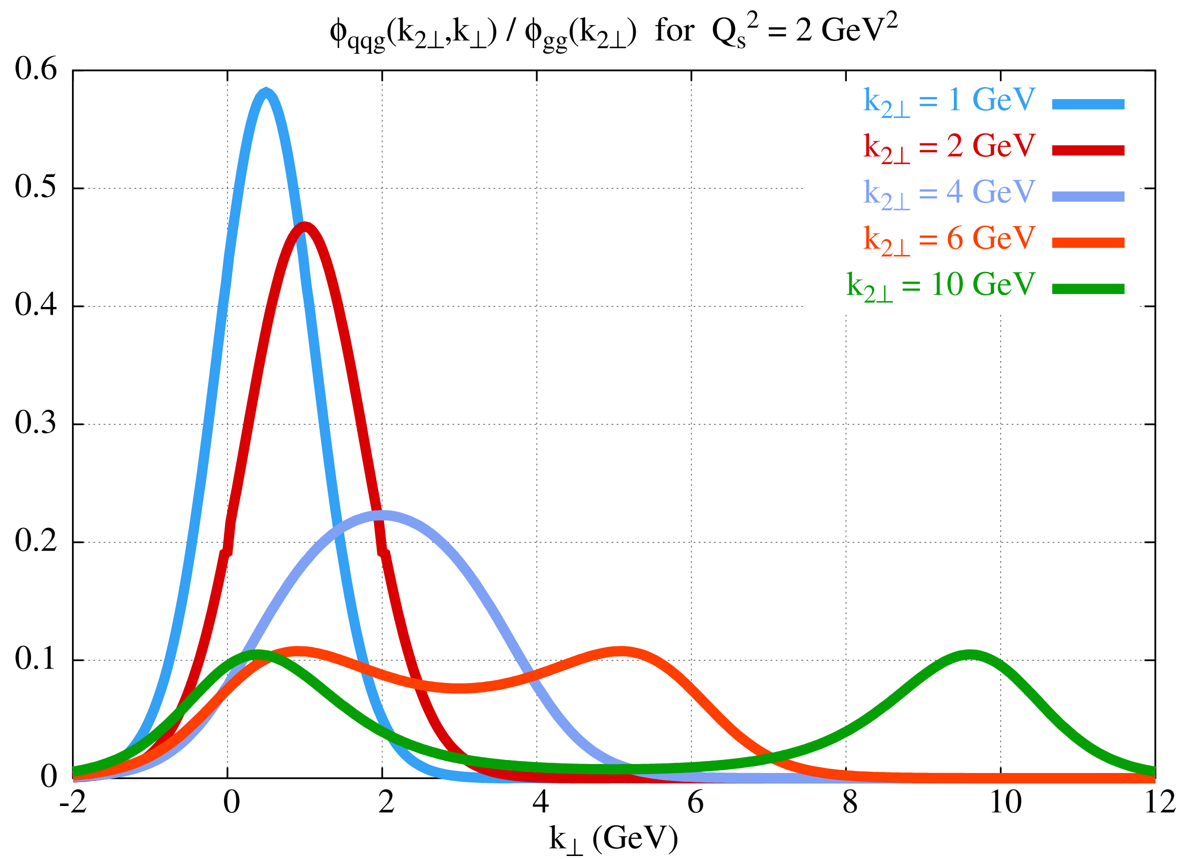
- Single quark cross-section in the approximation of $\boldsymbol{k}_\perp$-factorization: (Fujii, FG, Venugopalan)

$$\phi_A^{q\bar{q},g}(\vec{\boldsymbol{k}}_{2\perp}|\vec{\boldsymbol{k}}_\perp) = (2\pi)^2 \frac{1}{2}\left[\delta(\vec{\boldsymbol{k}}_\perp) + \delta(\vec{\boldsymbol{k}}_\perp - \vec{\boldsymbol{k}}_{2\perp})\right]\phi_A^{g,g}(\vec{\boldsymbol{k}}_{2\perp})$$



Proton-nucleus: numerics (2/3)

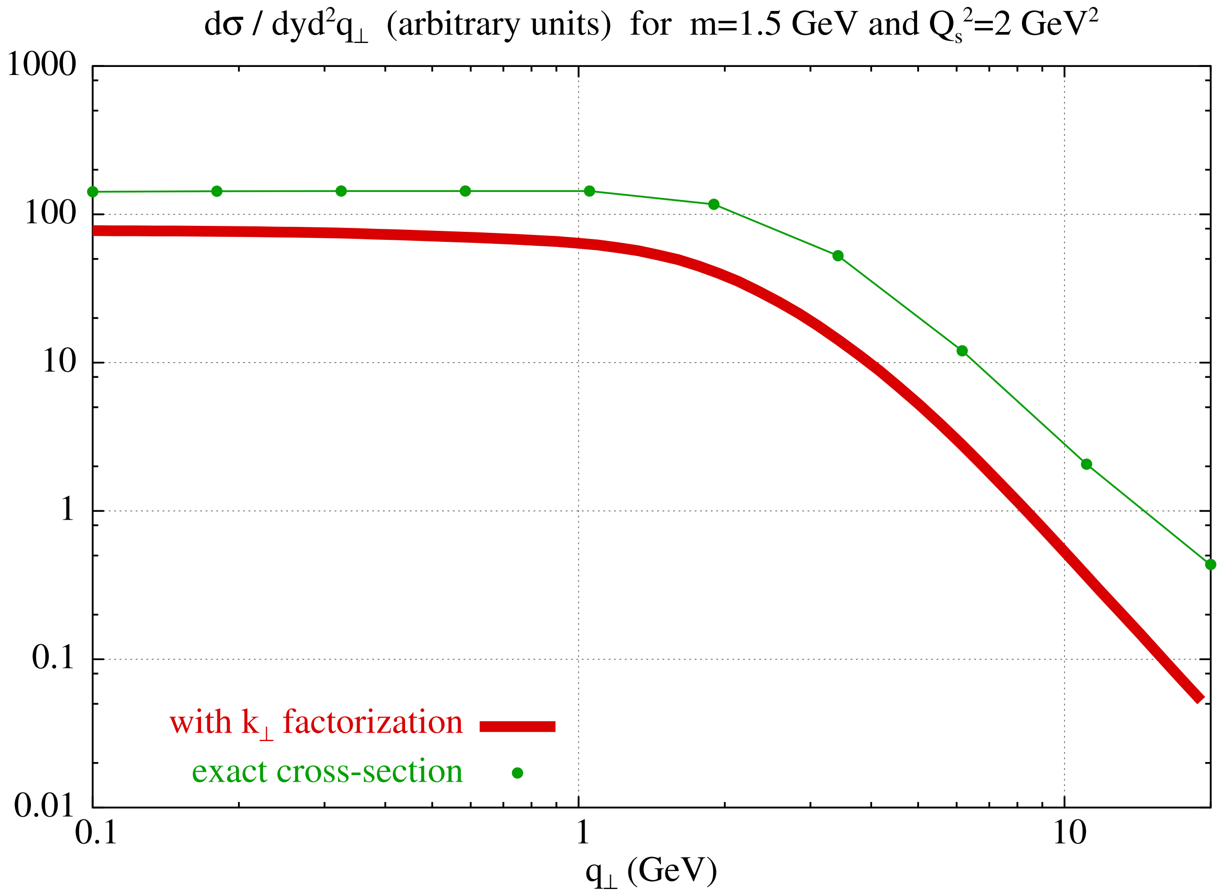
- Exact 3-point function in the MV model:



▷ far from a sum of two δ functions, strong broadening

Proton-nucleus: numerics (3/3)

- Exact single quark cross-section: (preliminary)



▷ the breaking of $\boldsymbol{k}_\perp$-factorization is an important effect

Conclusions

- At leading order in the classical sources:
  - The semi-classical approach is equivalent to: pQCD + $k_\perp$-factorization + BFKL
- Higher orders in the classical sources
  - proton-nucleus collisions:
    - can be solved analytically
    - $k_\perp$-factorization is broken for the nucleus - important correction at moderate momentum or quark mass
    - one needs 2, 3 and 4 point functions
  - quark production in nucleus-nucleus collisions (FG, Kajantie, Lappi: hep-ph/0409058 + work in progress):
    - must be treated numerically
    - can be reduced to solving Dirac's equation in a classical background field