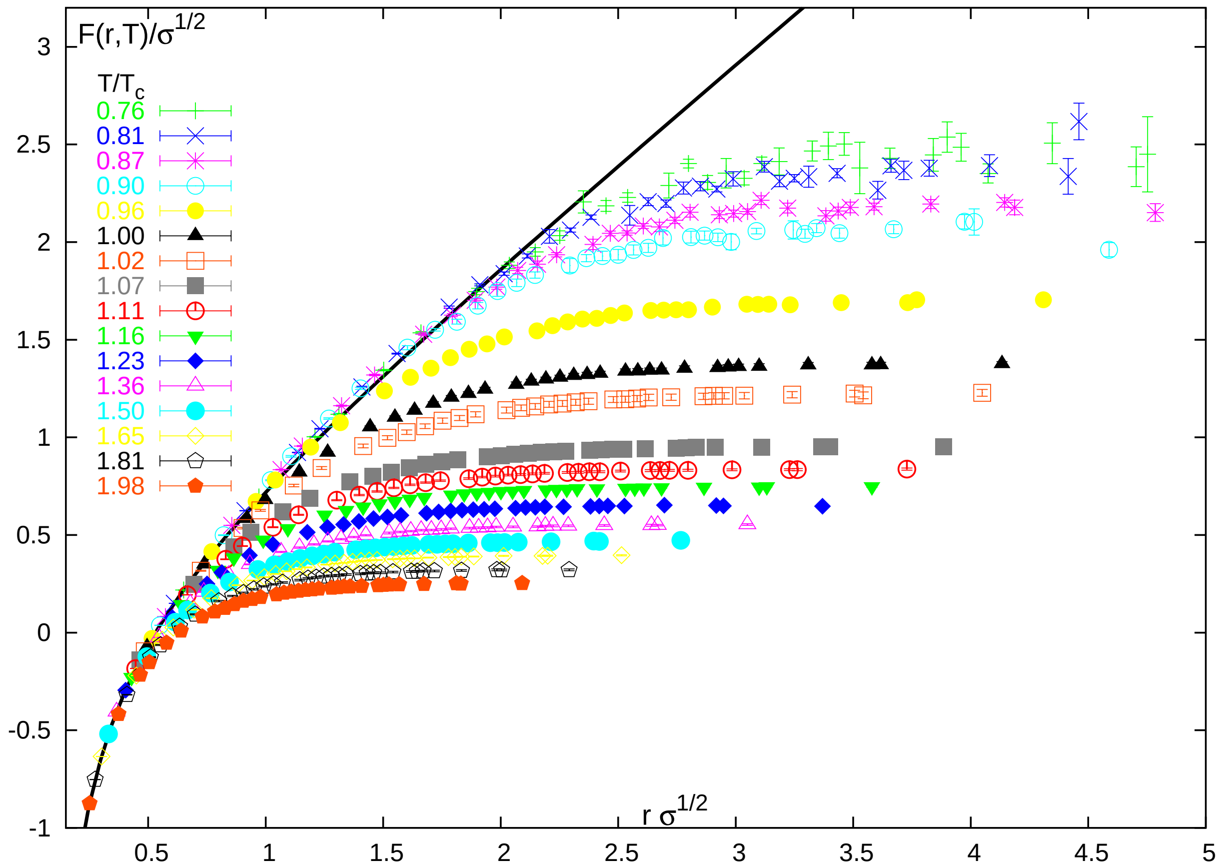


How does $F(r,T)$ depend on r and T?

Heavy Quark Interaction in Finite Temperature QCD

SD, O. Kaczmarek, F. Karsch, & H. Satz

Outline:

(i) String breaking and medium induced screening in the Debye-Hückel Theory

(ii) Understanding the lattice data in terms of screening

(iii) Conclusions

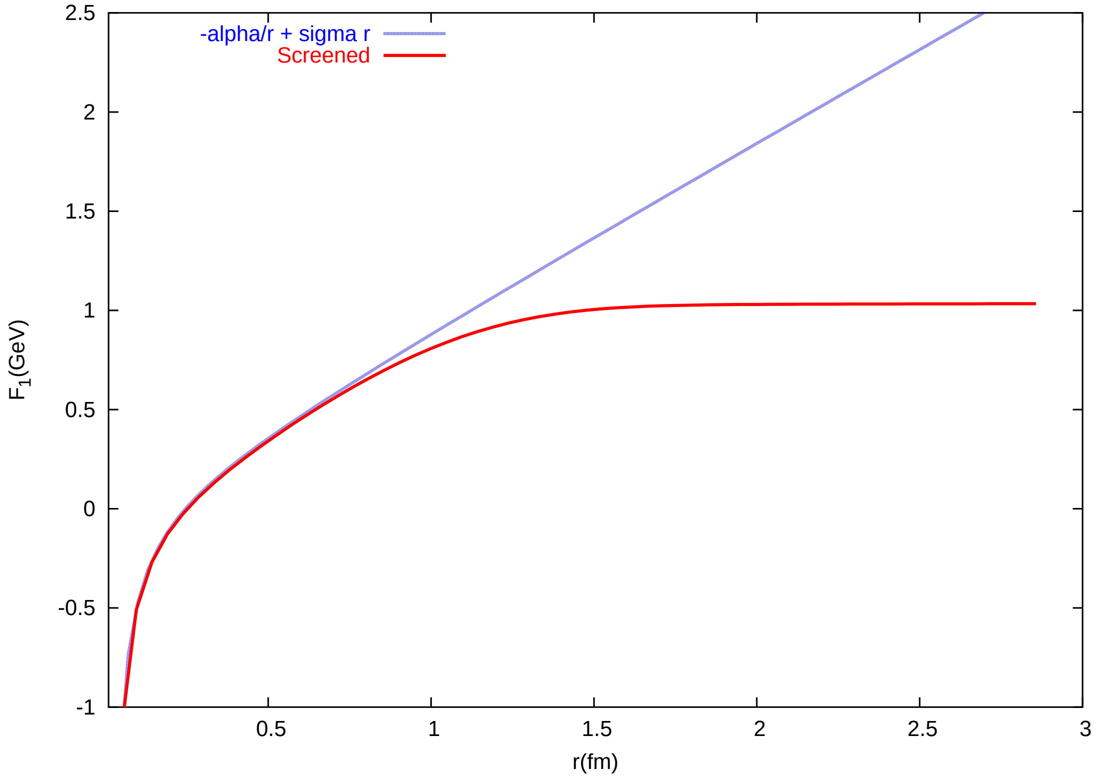
In the string model, the free energy of a heavy quark-antiquark at $T = 0$ is given by

$$F(r, T = 0) = \sigma r - \frac{\alpha}{r}.$$

It describes the charmonium and bottomonium spectrum quite well in terms of α and σ.

The string breaks at $r = r_0$ and results in saturation or flattening of the free energy for $r > r_0$

$$F(r > r_0, T = 0) = \sigma r_0 - \frac{\alpha}{r_0}$$



Estimating r_0 from

$$\begin{aligned} 2M_D &= M_{J/\Psi} + \mathrm{BindingEnergy} \\ &= 2m_c + F(r_0, T=0) \\ F(r_0, T=0) &= 2(M_D - m_c) \end{aligned} \tag{1}$$

we get

$$r_0 \simeq \frac{2(M_D - m_c)}{\sigma} \simeq \frac{1.2\ GeV}{\sigma} \simeq 1.5\ fm$$

Similarly, one can look at bottomonium, with M_B and m_b, and get same value of r_0, so that r_0 is universal.

Screening:

At finite T medium effects screen the heavy quark-antiquark interaction. This leads to the modification of the free energy $F(r,T)$,

$$F(r,T) = \sigma r f_s(r,T) - \frac{\alpha}{r} f_c(r,T)$$

Since for $r \ll 1/T$, the $\bar{Q}Q$ does not see the medium

$$f_s(r,T) = f_c(r,T) = 1, \qquad r \to 0$$

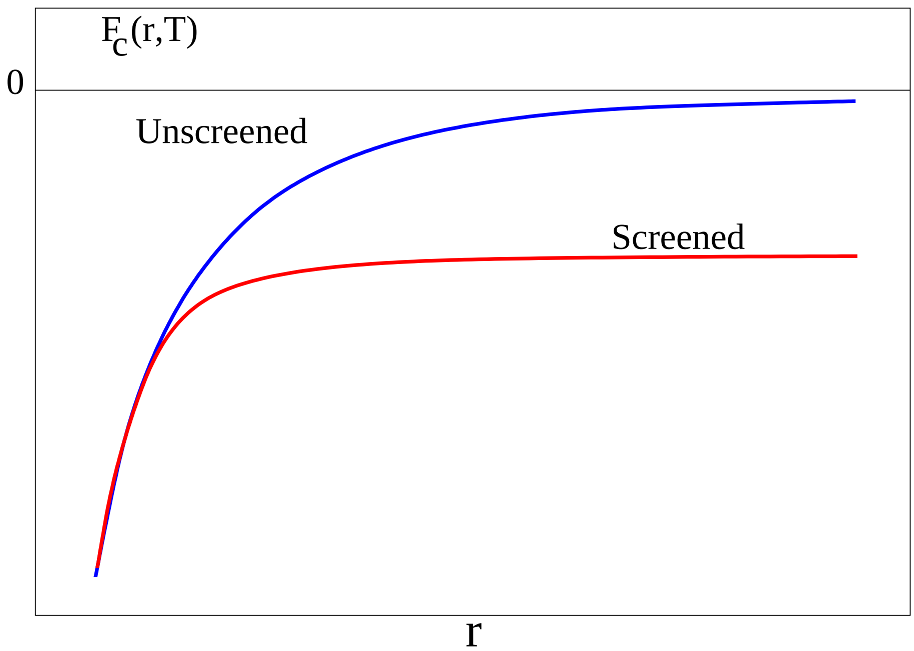
Coulomb screening: $f_c(r,T)$

Apart from the boundary condition $f_c(r,T) = 1$ for small r, the Coulomb part of the free energy must approach

$$F_c(r,T) = -\alpha\mu, \qquad r \to \infty$$

where $\mu(1/\mu)$ is the screening mass(radius). This corresponds to the change in free energy of the system due to the presence of two static charges. Both boundary conditions at $r = 0$ and $r = \infty$ are satisfied by

$$f_c(r,\mu) = [exp(-\mu r) + \mu r]$$



String Screening: $f_s(r,T)$

In Debye-Hückel theory, a free energy

$$F_s = \sigma r^{\nu}$$

in d-dimensions satisfies the Poisson equation

$$-\frac{\nabla^2 F_s}{r^{\nu+d-2}} + \frac{(\nu+d-2)}{r^{\nu+d-1}}\vec{\nabla} F_s \cdot \hat{r} = 4\pi\sigma\delta(r)$$

However, the heavy quark polarizes the medium, leading to the following substitution

$$\delta(r) \longrightarrow \delta(r) + AF_s$$

Here A, with dimension $d-1$, contains the effect of the medium. Defining the screening mass $\mu = (4\pi\sigma A)^{1/(\nu+d)}$ one obtains

$$\frac{1}{r^{\nu+d-2}}\frac{d^2 F_s}{dr^2} + \frac{1-\alpha}{r^{\nu+d-1}}\frac{dF_s}{dr} - \mu^{\nu+d} F_s = -4\pi\sigma\delta(r)$$

For our case $\nu = 1$ and $d = 3$. The solution to this equation is [V. V. Dixit, Mod. Phys. Lett. A5 (1990) 227.]

$$F_s(r,T) = \sigma r f_s(\mu r)$$

where the screening function $f_s(\mu r)$ is given by

$$f_s(x = \mu r) = \frac{1}{x}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{x}}{2^{3/4}\Gamma(3/4)} K_{1/4}\left(x^2\right)\right]$$

The screening function has the required behavior

$$\begin{aligned} f_s(x) &= 1, & x \to 0 \\ f_s(x) &= \frac{1}{x}\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} \simeq \frac{1.046}{x} & x \to \infty \end{aligned}$$

So instead of rising linearly, the free energy flattens off to

$$F_s(r,\mu) = \frac{\sigma}{\mu}\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)}, \qquad r \to \infty$$

Putting together the Coulomb and String screened free energies together we get the following free energy for a quark-antiquark in the medium

$$F(r,T) = \frac{\sigma}{\mu}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{\mu r}}{2^{3/2}\Gamma(3/4)} K_{1/4}\left((\mu r)^2\right)\right] - \frac{\alpha}{r}\left[e^{-\mu r} + \mu r\right]$$

The temperature dependence enters through the screening mass $\mu(T)$. For very small r or very large T, α will depend on r or T, respectively. For our calculations we take it as a constant

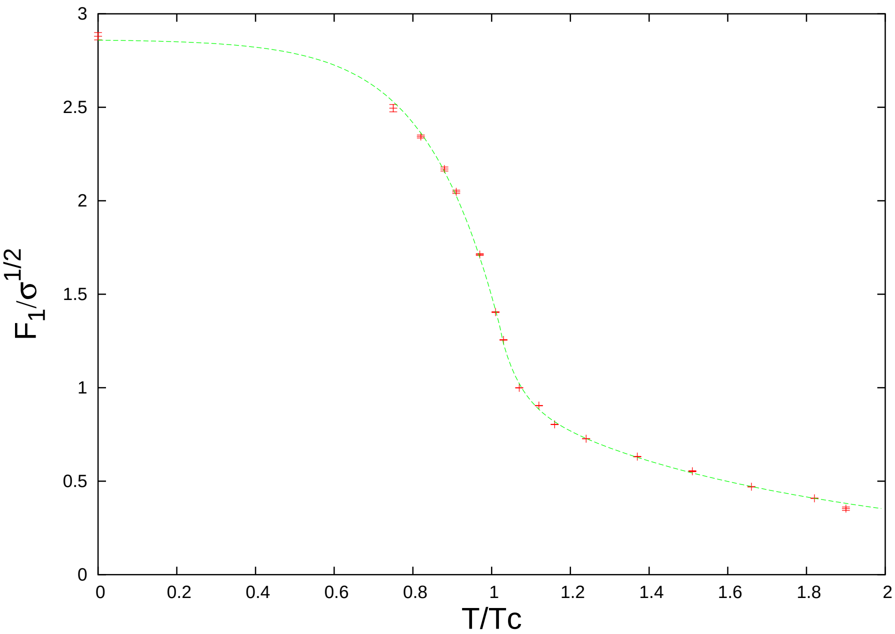
Fitting F(r,T)

For small r the free $F(r,T)$ is independent of $\mu(T)$. On the other hand $F(r=\infty,T)$ depends only on $\mu(T)$ as a variable. So we obtain the screening mass $\mu(T)$ from the large distance behavior of $F(r,T)$, by solving

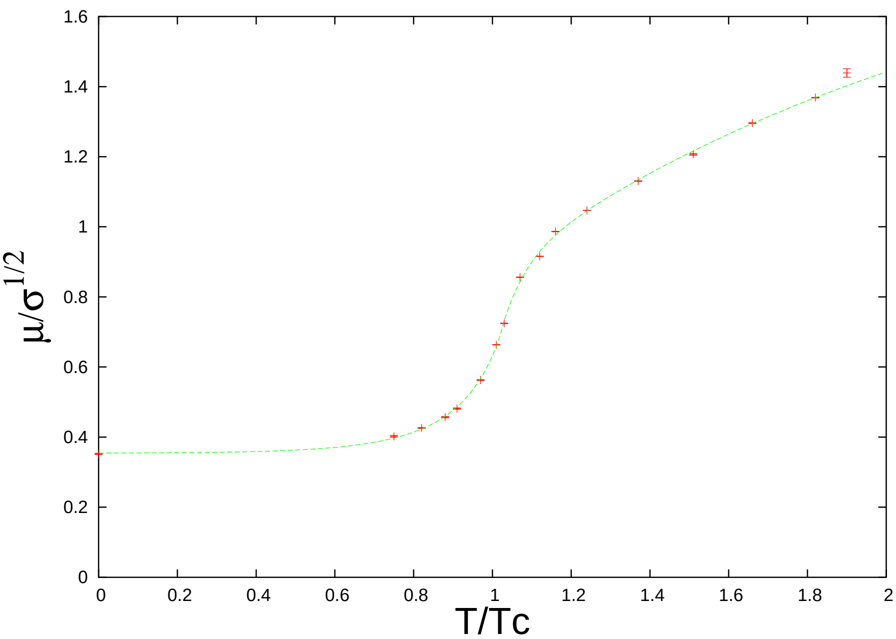
$$F(T) = \frac{\sigma}{\mu}\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \alpha(T)\mu(T)$$

Which gives,

$$\mu(T) = -\frac{F(T)}{2\alpha} + \left[\left(F(T)^2 + 4\sigma\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)}\right)\Big/ 4\alpha^2\right]^{1/2}$$



$F_1/\sigma^{1/2}$
T/Tc
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.5
1
1.5
2
2.5
3



$\mu/\sigma^{1/2}$
T/Tc
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6

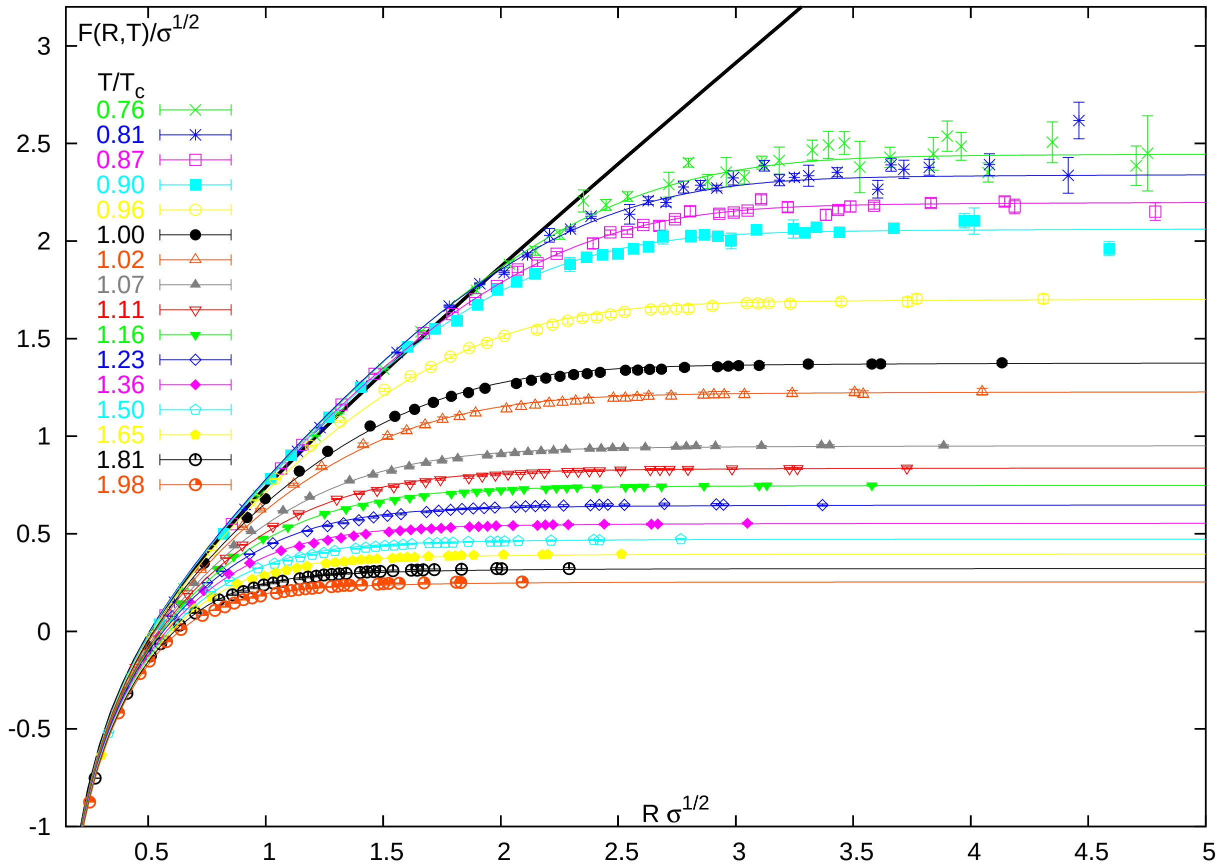
However the fit curves with $\mu(T)$ obtained from large r behavior of $F(r,T)$ are good only at small r and large r. To improve the fit curves we introduced another temperature dependent parameter κ as in the following

$$K_{1/4}\left(x^2\right) \longrightarrow K_{1/4}\left(x^2 + \kappa x^4\right)$$

With this,

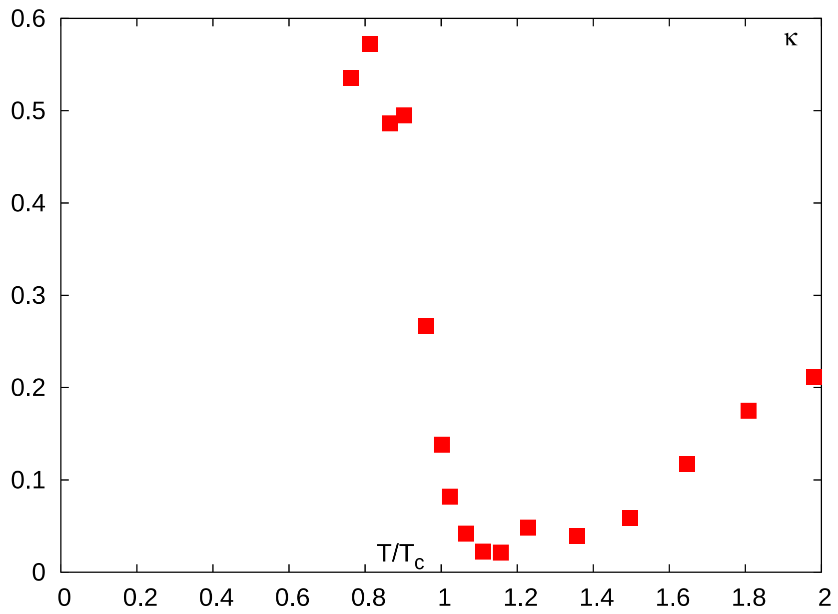
$$F(r,T) = \frac{\sigma}{\mu}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{\mu r}}{2^{3/2}\Gamma(3/4)} K_{1/4}\left((\mu r)^2 + \kappa(\mu r)^4\right)\right] - \frac{\alpha}{r}\left[e^{-\mu r} + \mu r\right]$$

With $\mu(T)$ obtained from large r, we fit the free energies in terms of the parameter κ.



The solid curves show the resulting fits.

It is seen that the screened Cornell potential provides a good description of the temperature dependence of the free energy from lattice QCD. The screened free energy has only terms of two temperature dependent parameters $\mu(T)$ and $\kappa(T)$.



$\kappa(T)$ is significant only below T_c. It sharply drops above T_c.

The non-zero values above T_c may be a result of normalization difficulties at small distance, as well as a reflection of the r, T dependence of the coupling constant $\alpha(r, T)$.

Conclusions

Screened Cornell potential describes the lattice heavy quark-antiquark free energy quite well.

The $F(r,T)$ has only two temperature dependent parameters $\mu(T)$ and $\kappa(T)$.