# Static quark anti-quark free energy in full QCD

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# Static quark anti-quark free energy in full QCD

1) Introduction

   Details of the calculations

2) Renormalized free energies

   Short vs. large distance behaviour

   Running coupling and screening

4) Connection to entropy and energy

   $F(r,T) = E(r,T) - TS(r,T)$

5) The renormalized Polyakov loop

6) Conclusions & outlook

   N-point correlation functions ($qqq$)

   Heavy quark free energies at finite density

# Details of the calculation

$N_f = 2$ with $m/T = 0.4$ ($m_\pi/m_\rho \approx 0.7$ at $T_c$)

Lattice size: $16^3 \times 4$ , $T_c \approx 170$MeV

(generated by the Bielefeld-Swansea collaboration)

Symanzik improved gauge action and
p4-improved staggered fermion action

Physical scale is determined by string tension

$T$=0 potential obtained from Wilson loops

[F. Karsch, E. Laermann and A. Peikert, Nucl. Phys . B 605 (2001) 579]

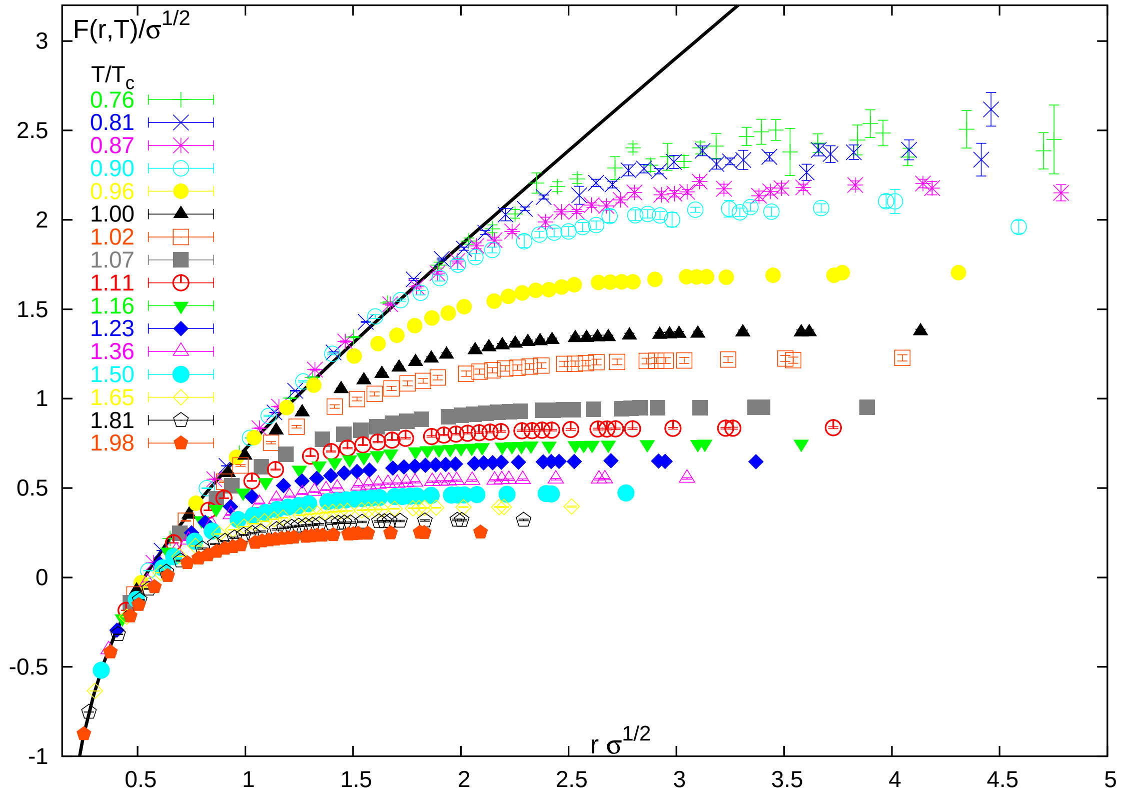
Coulomb gauge fixing [O. Philipsen, Phys. Lett. B535 (2002) 138]

$$-\ln\left(\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) \tilde{\mathrm{Tr}} L^\dagger(\mathbf{y})\rangle\right) = \frac{F_{\bar{q}q}(r,T)}{T}$$

$$-\ln\left(\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) L^\dagger(\mathbf{y})\rangle\right)\Big|_{GF} = \frac{F_1(r,T)}{T}$$

$$-\ln\left(\frac{9}{8}\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) \tilde{\mathrm{Tr}} L^\dagger(\mathbf{y})\rangle - \frac{1}{8}\langle \tilde{\mathrm{Tr}} L(\mathbf{x}) L^\dagger(\mathbf{y})\rangle\Big|_{GF}\right) = \frac{F_8(r,T)}{T}$$

# Heavy quark free energy




Renormalization of $F(r,T)$

by

matching with $T$=0 potential

$$e^{-F_1(r,T)/T} = \left(Z_r(g^2)\right)^{2N_\tau} \langle \mathrm{Tr}\, (L_x L_y^\dagger) \rangle$$

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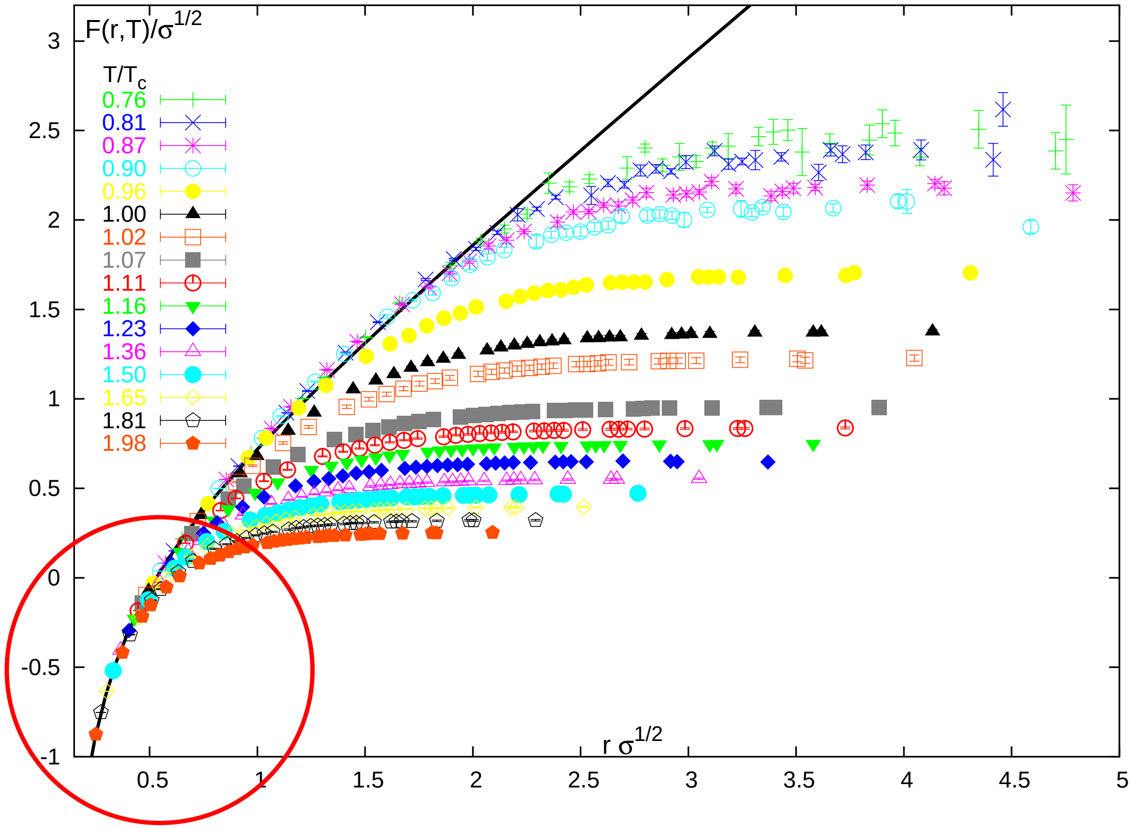


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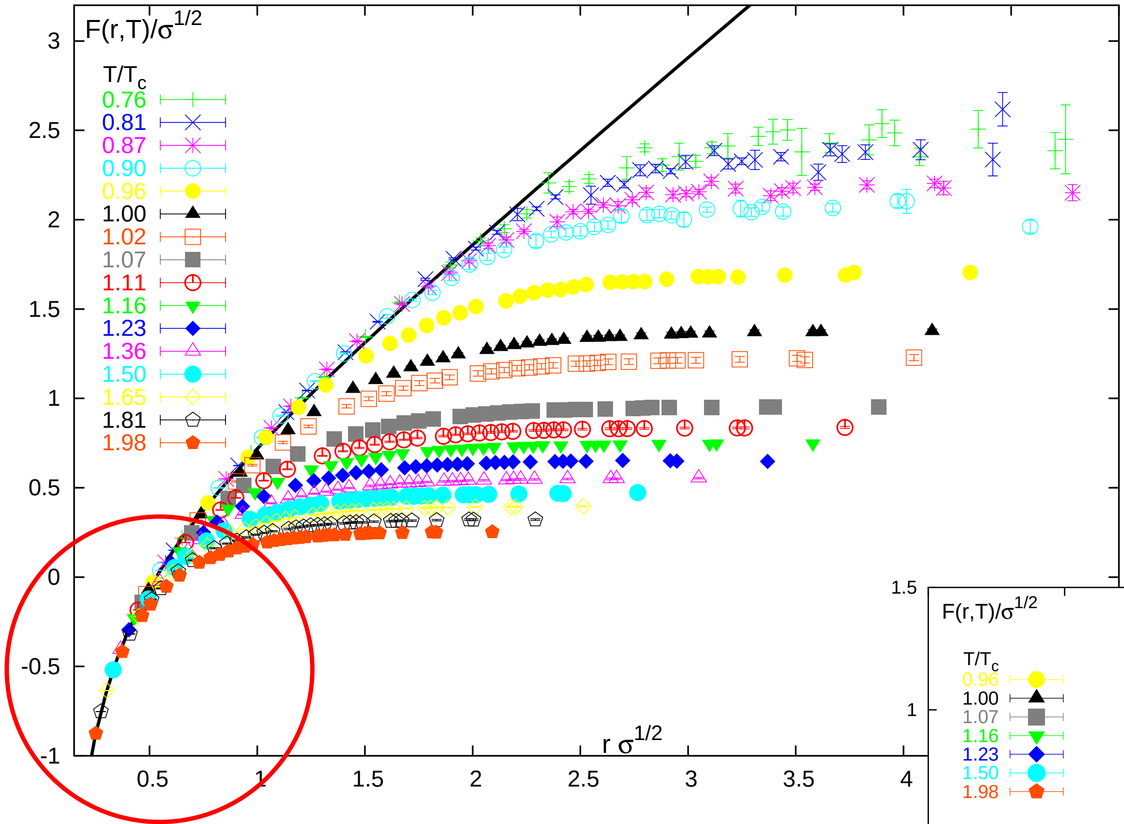
$T$-independent

$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$

# Heavy quark free energy

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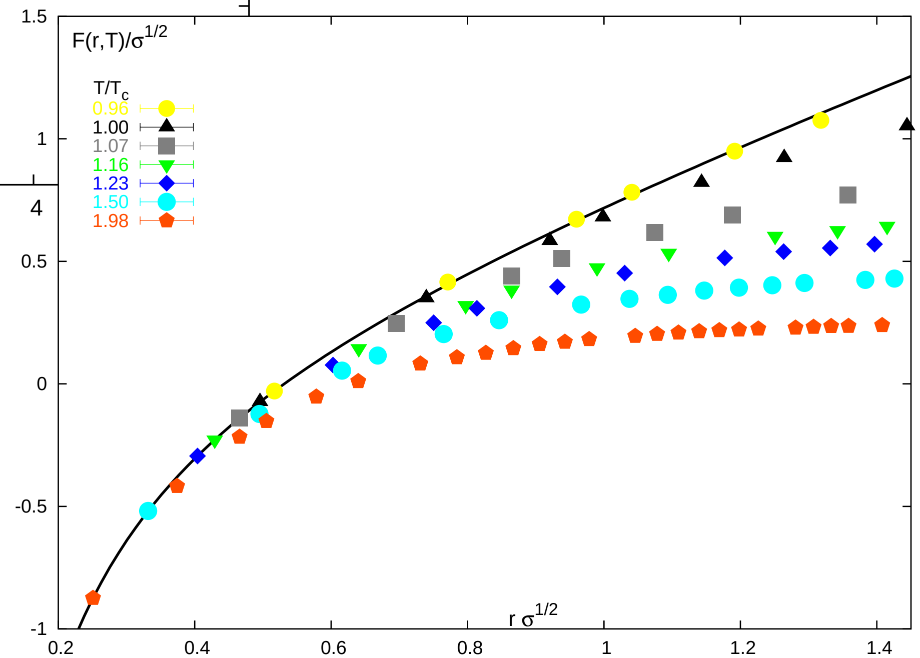


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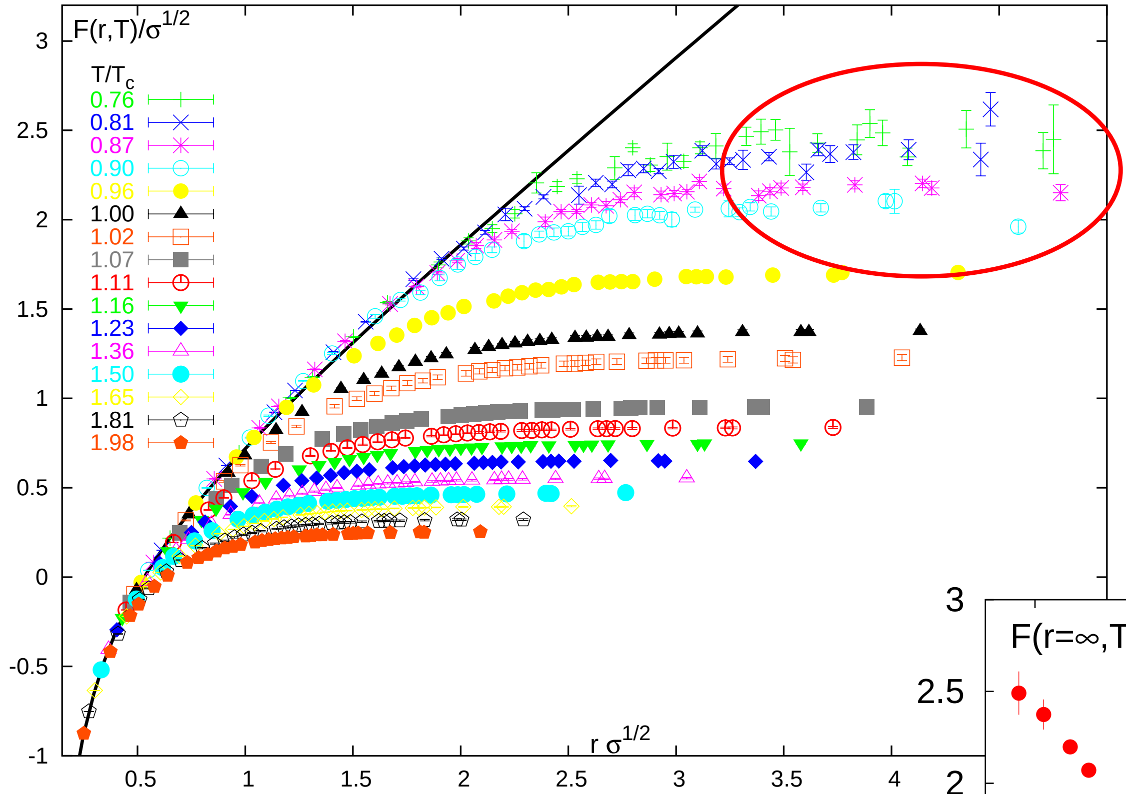
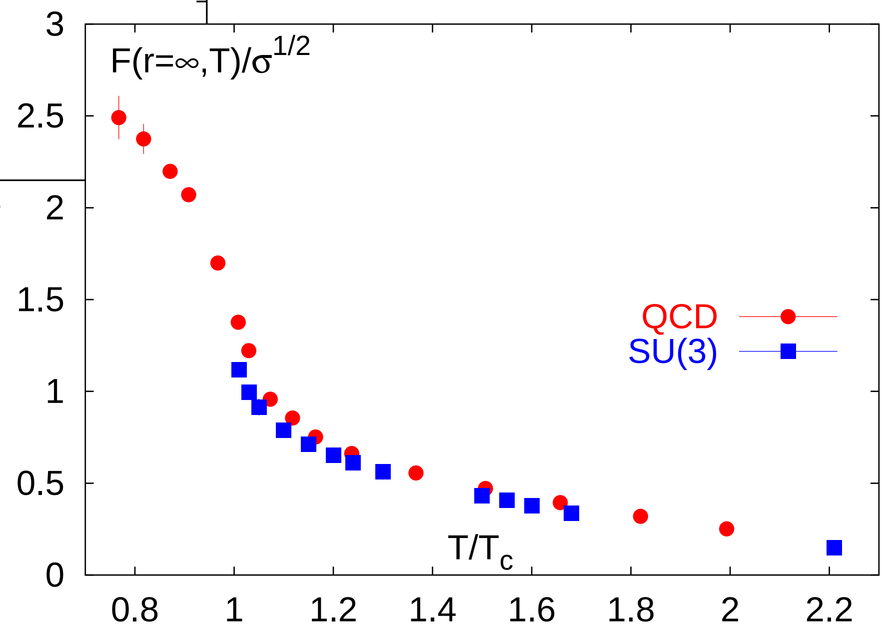



String breaking

$T < T_c$

$F(r\sqrt{\sigma} \gg 1, T) < \infty$

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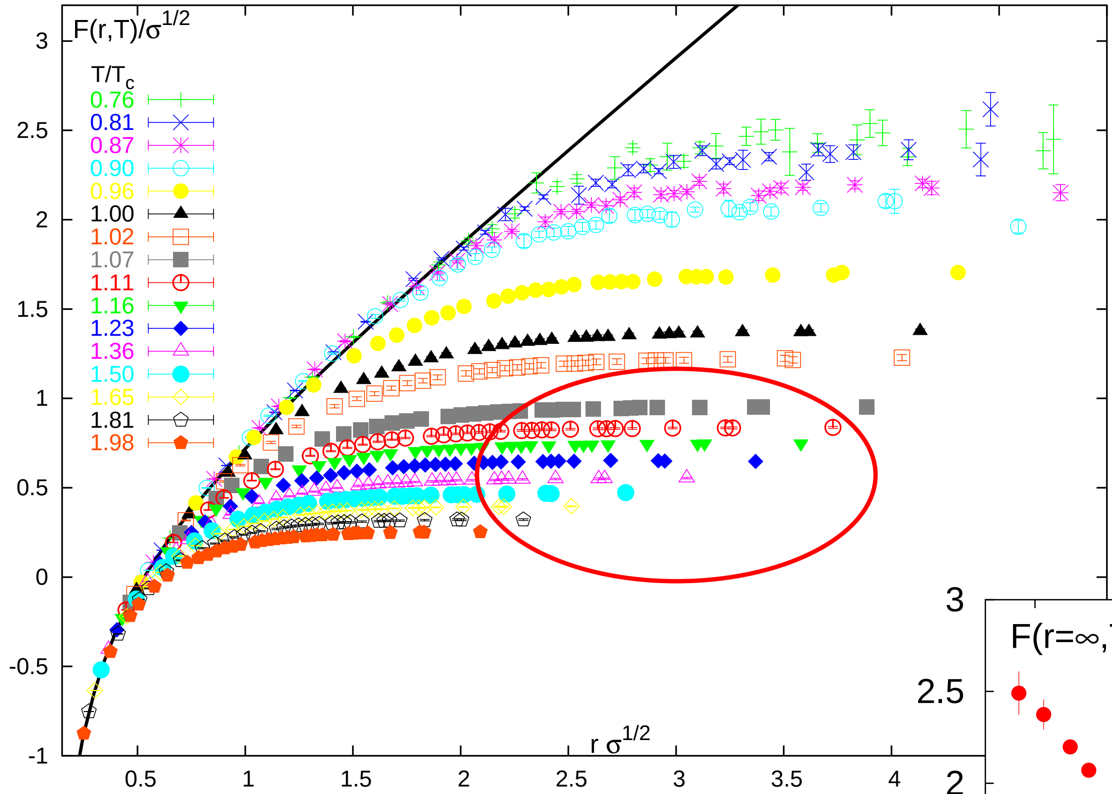


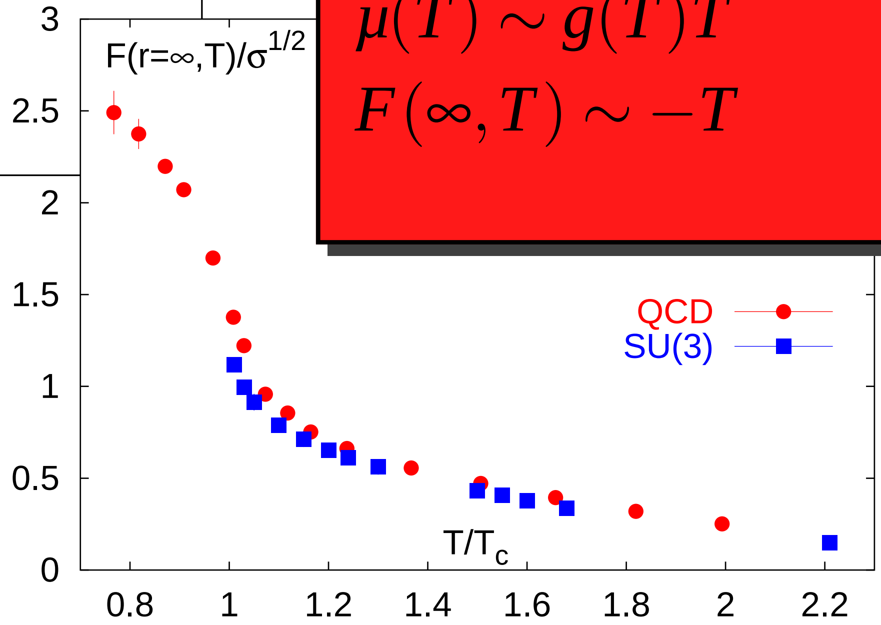


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**high-T physics**

$rT \gg 1$ ; screening

$\mu(T) \sim g(T)T$

$F(\infty, T) \sim -T$

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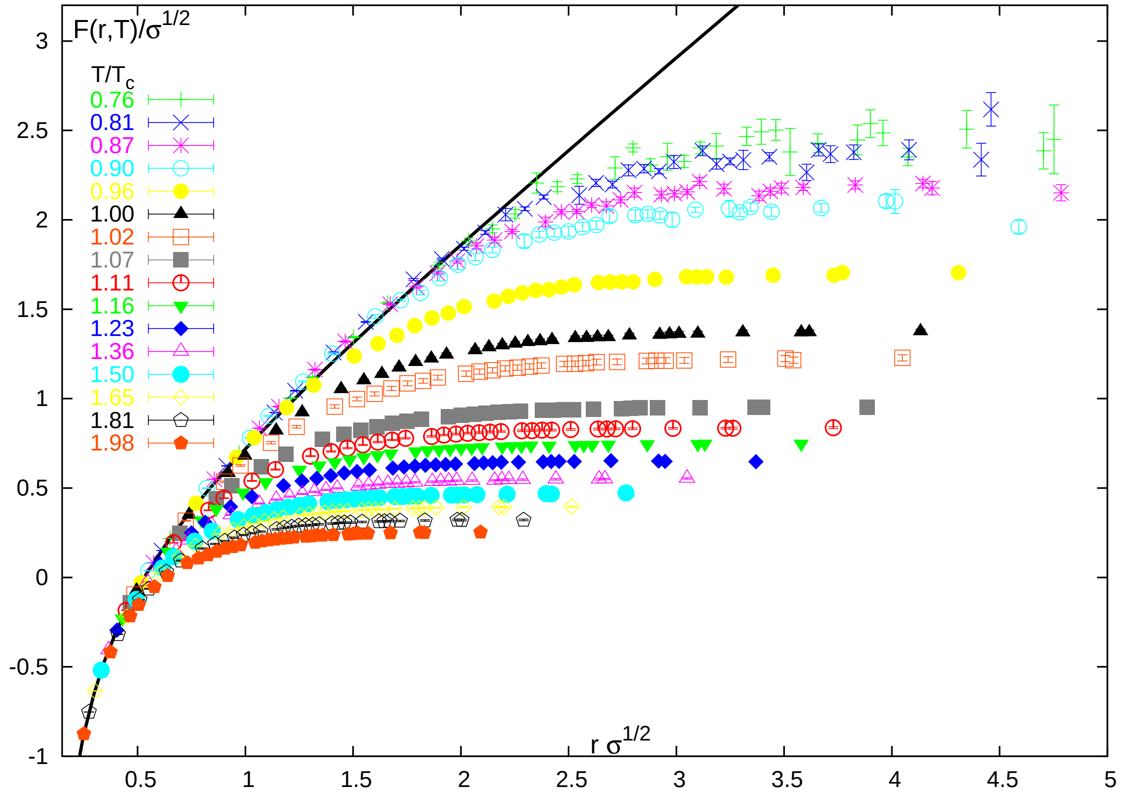
# Heavy quark free energy

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**Complex $r$ and $T$ dependence**

Small vs. large distance behavior

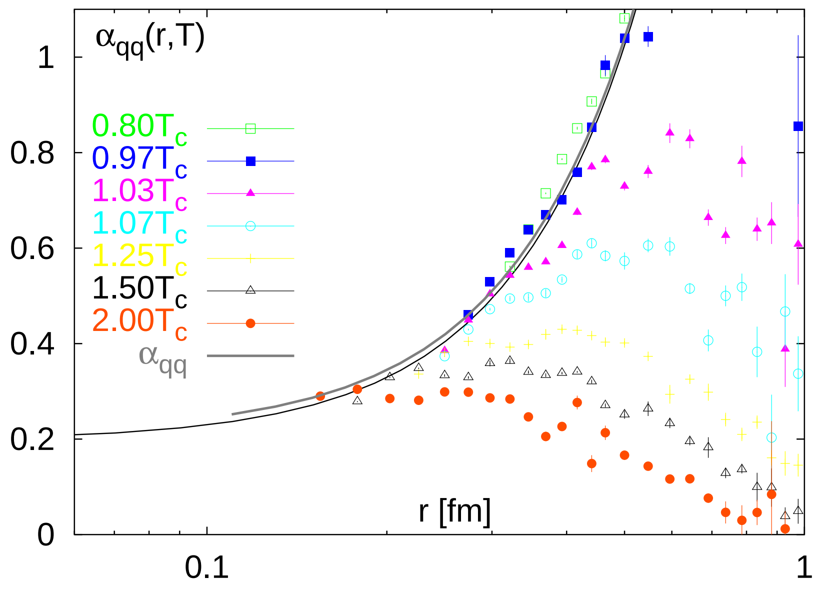
Running coupling vs. screening

# Running coupling vs. Screening

Effective running coupling:

$$\alpha_{qq}(r,T) = \frac{3r^2}{4} \frac{\mathrm{d}F_1(r,T)}{\mathrm{d}r}$$

$\longrightarrow$ Talk by Felix Zantow

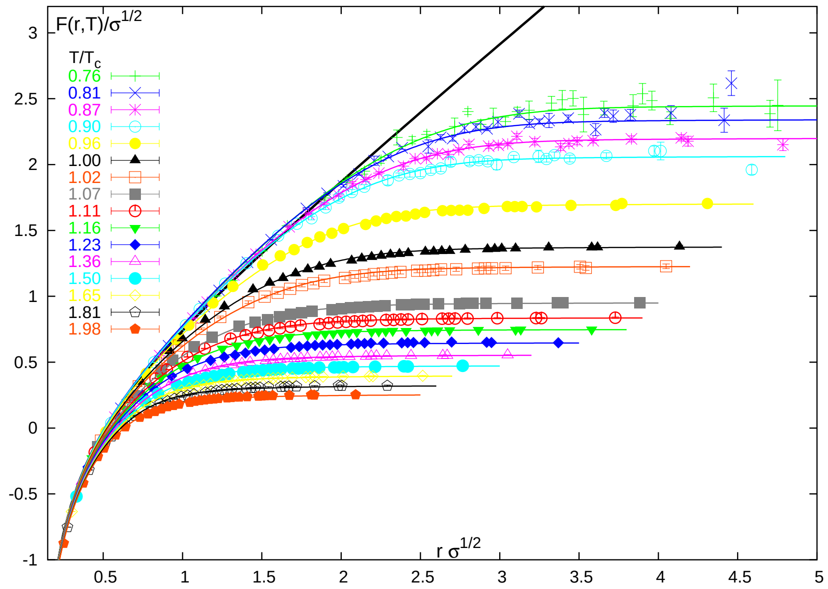



Screening dominates at large $r$

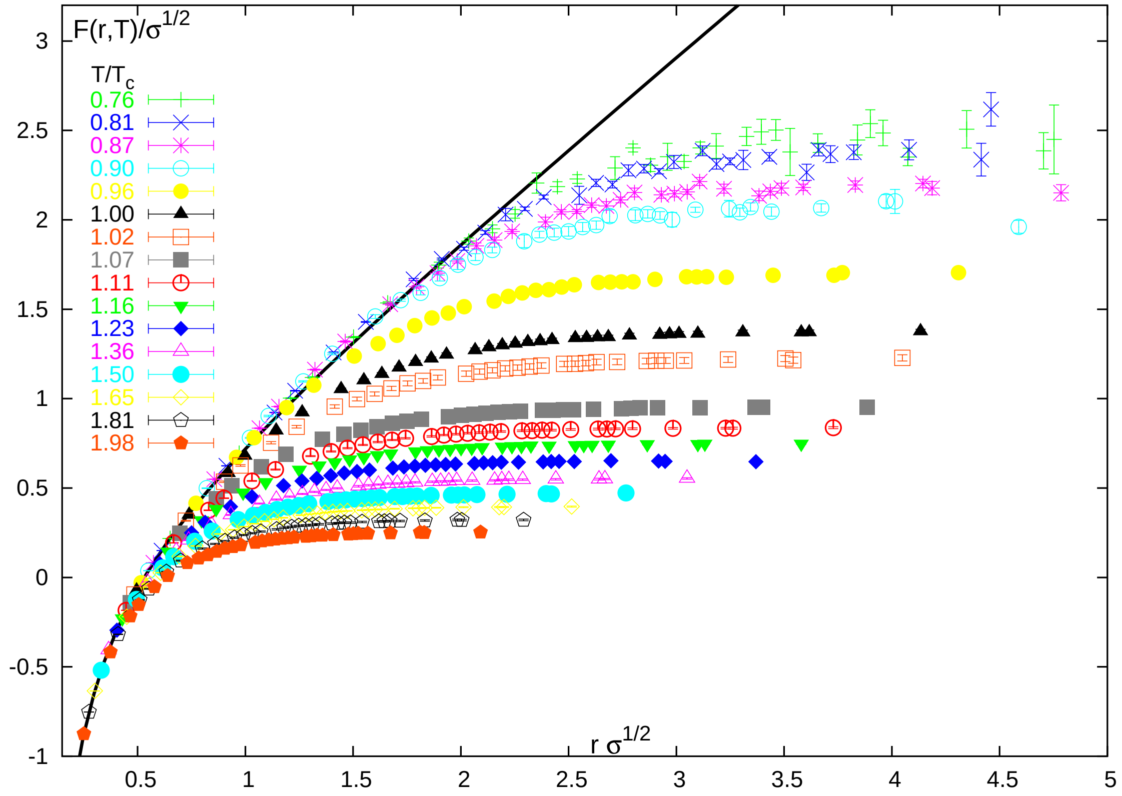
Large lattices needed
to extract screening properties

Suitable Ansatz to describe the data

$\longrightarrow$ Talk by Sanatan Digal



# Heavy quark free energy $F = E - TS$




Expectation values ↔ Free energies

$$\langle \mathrm{Tr}\, L_{\vec{x}} L_{\vec{y}}^{\dagger} \rangle = \frac{\int \mathcal{D}U \ \mathrm{Tr}\, L_{\vec{x}} L_{\vec{y}} e^{-S}}{\int \mathcal{D}U e^{-S}} = Z_{q\bar{q}}/Z_0 = e^{-(F_{q\bar{q}}(r,T) - F_0(T))}$$

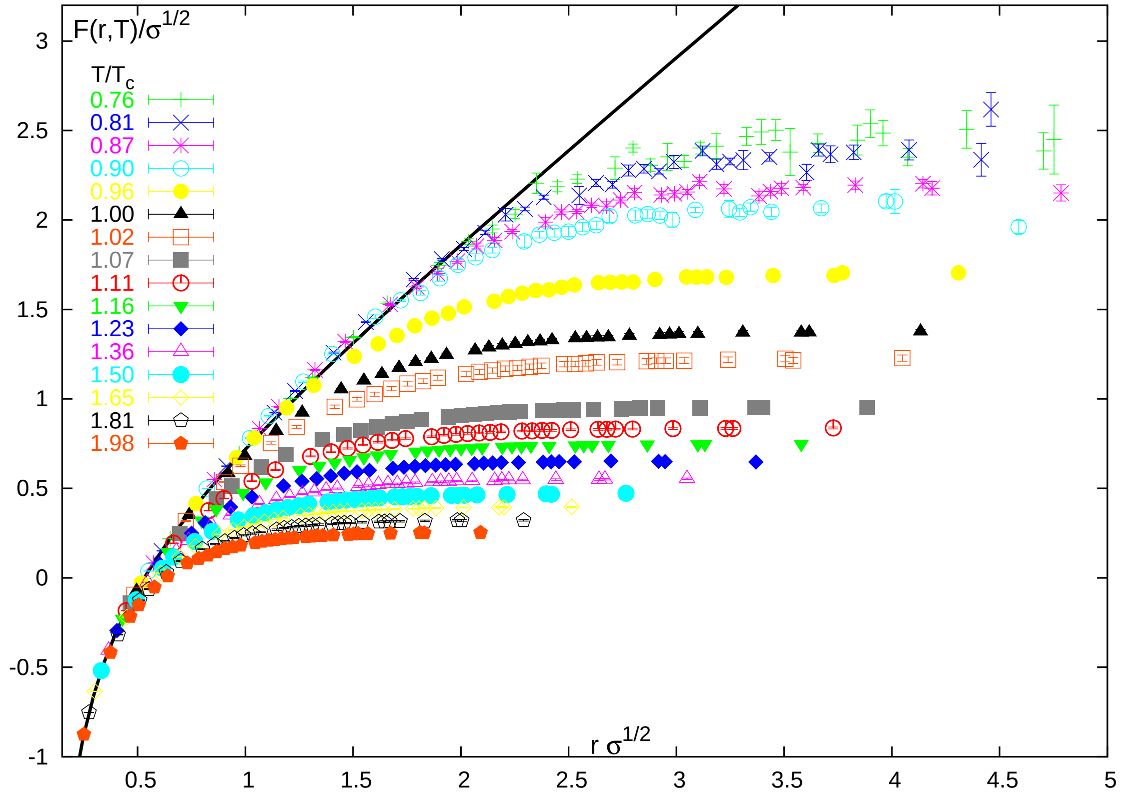
$$F_{q\bar{q}}(r,T) = -\log \langle \mathrm{Tr}\, L_{\vec{x}} L_{\vec{y}}^{\dagger} \rangle + F_0(T)$$

Free energies not only determined by internal energy

## Energy and entropy contributions

$$F(r,T) = E(r,T) - TS(r,T)$$

# Heavy quark free energy $F = E - TS$

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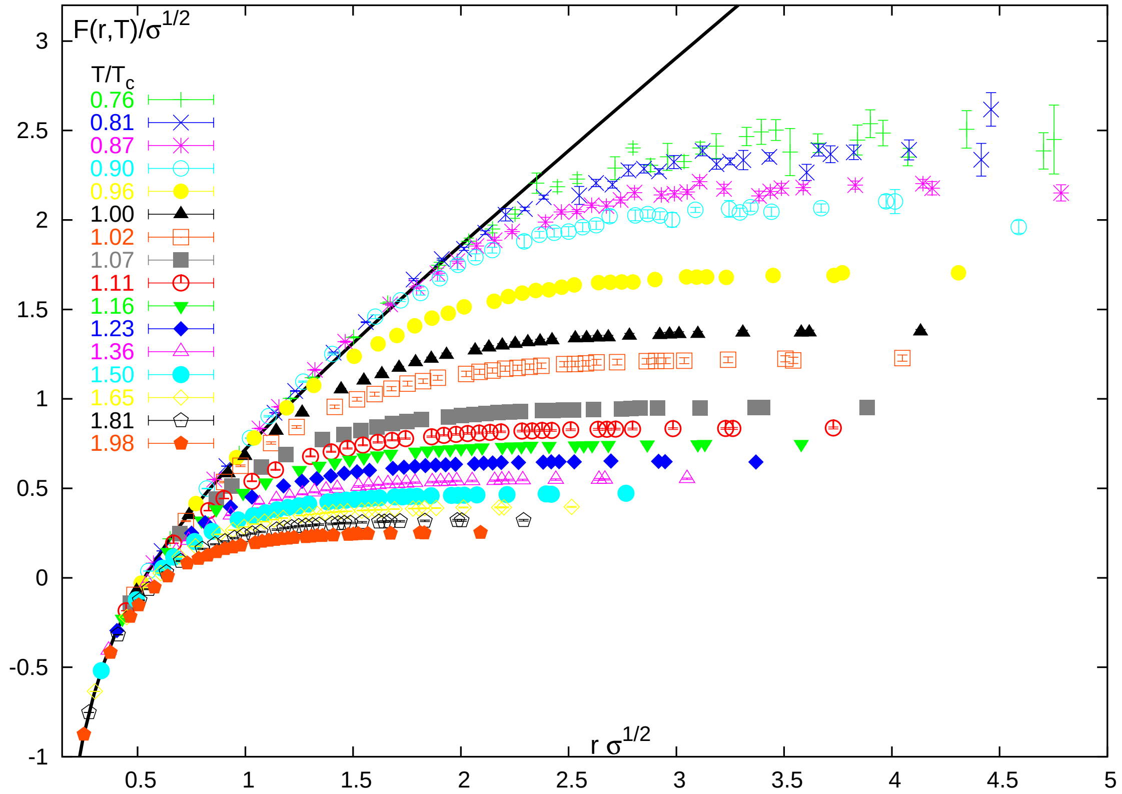


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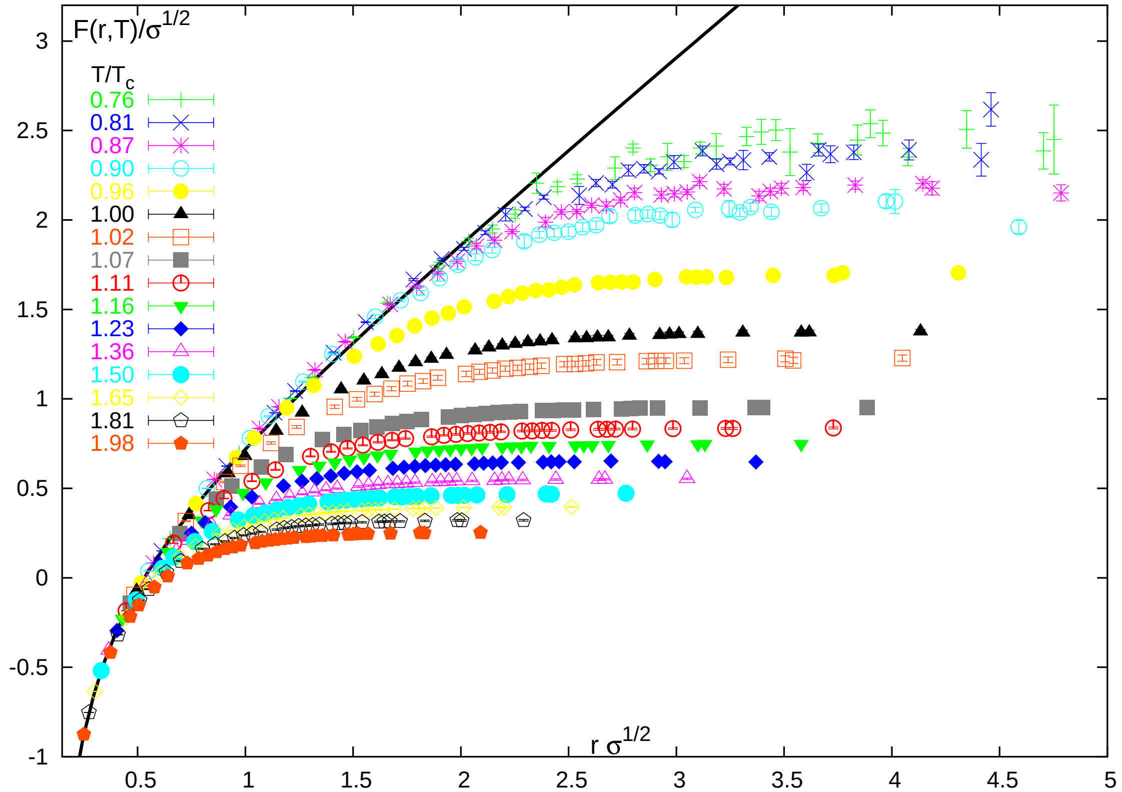


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$r \ll 1/\sqrt{\sigma}$

$F(r,T) \sim g^2(r)/r$

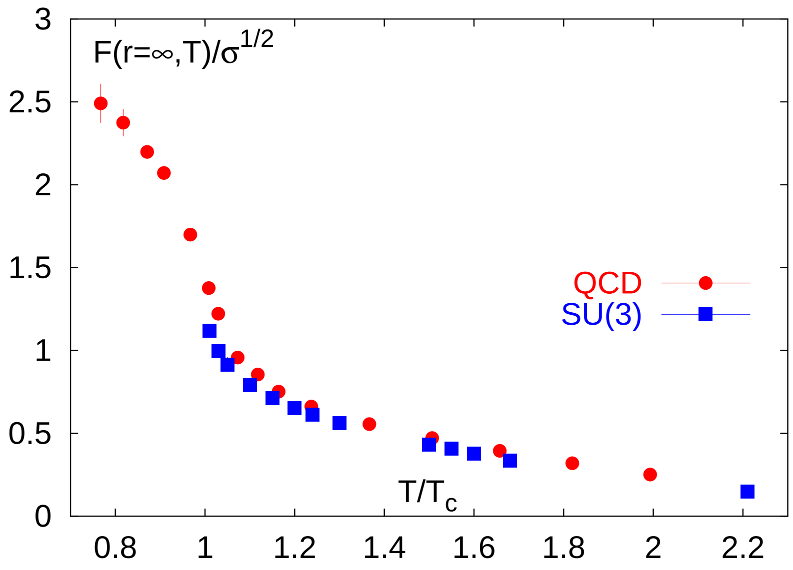
(energy dominated)

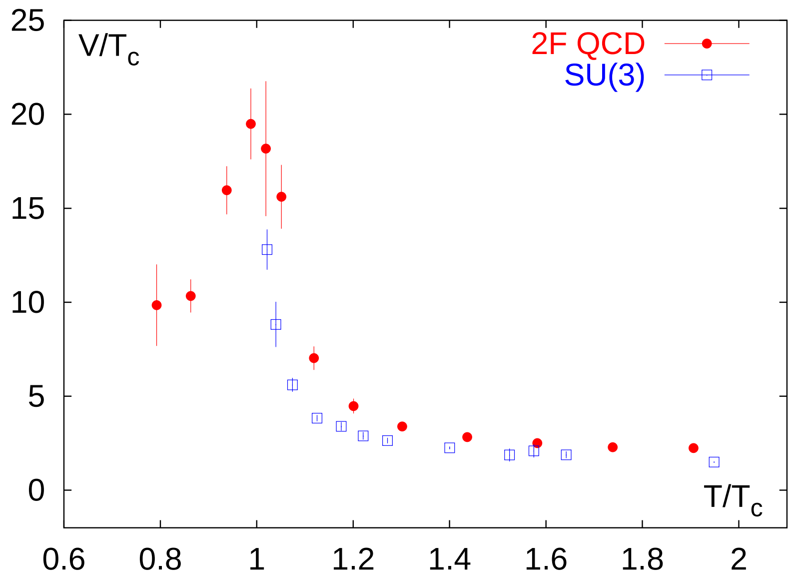
Entropy contributions play a role at fi nite T

Separation of energy/entropy contributions

$$F(r,T) = E(r,T) - TS(r,T)$$

# Energy contributions for $r \to \infty$



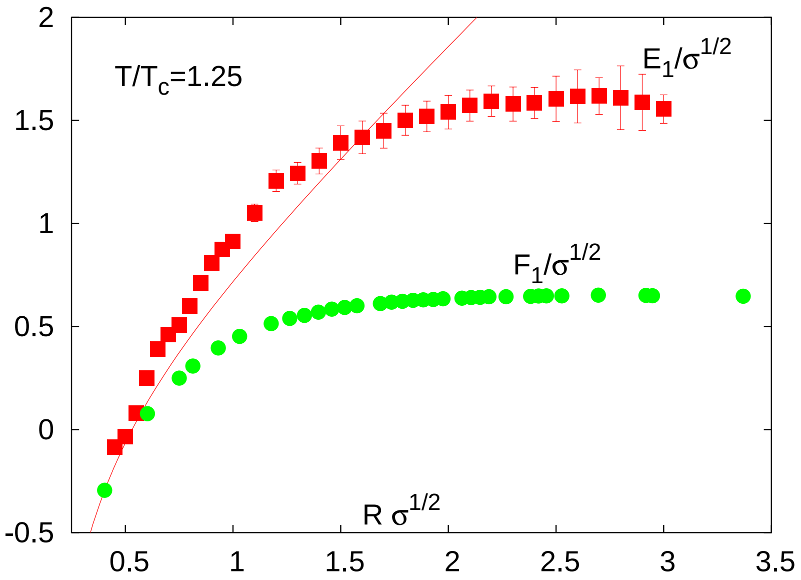

Energy contributions at infinite quark separation

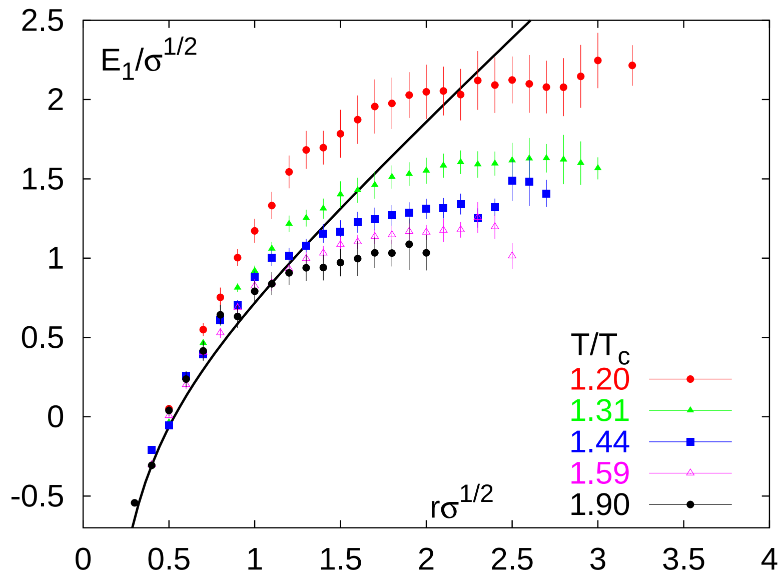
$$E(r = \infty, T) = -T^2 \frac{\partial F(r = \infty, T)/T}{\partial T}$$

Finite *string breaking energy* below $T_c$

Peak in the energy near $T_c$

# Seperating free and internal energy




Seperation of free energy and internal energy

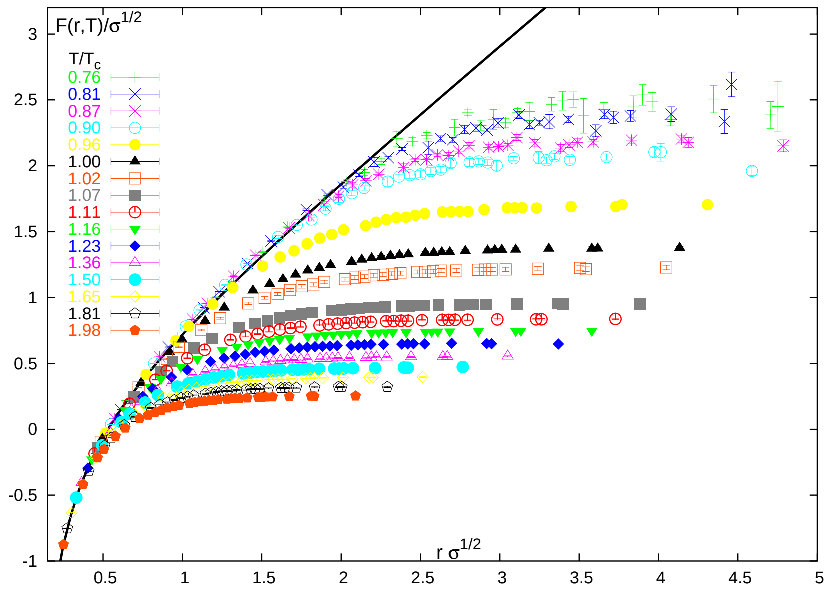
$$E_1(r,T) = -T^2 \frac{\partial F_1(r,T)/T}{\partial T}$$

Screening of $E_1(r,T)$

Enhancement of internal energy compared to free energy

# Renormalized Polyakov loop

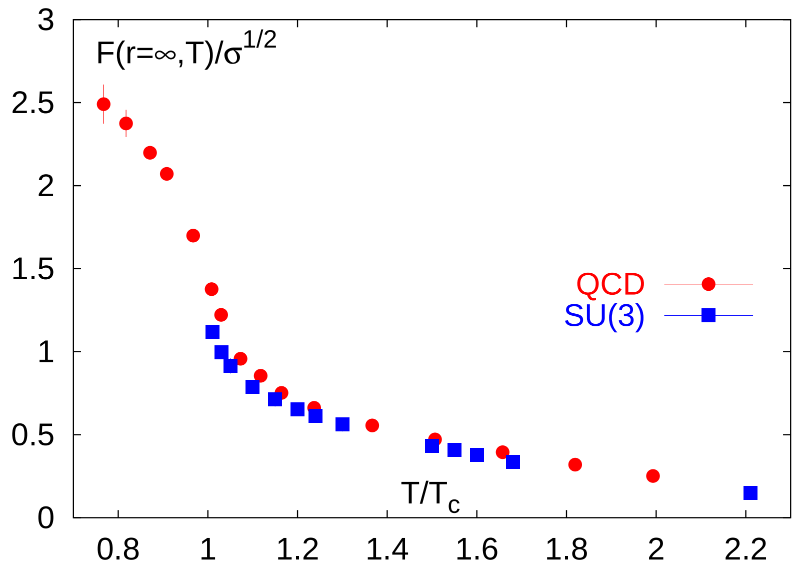
$$L_{\text{ren}} = \exp\left(-\frac{F(r=\infty,T)}{2T}\right)$$




Defined by long distance behaviour of $F(r,T)$.

Renormalized by renormalization of $F$ at small distances.

$$L_{\text{ren}} = \left(Z_R(g^2)\right)^{N_t} L_{\text{lattice}}$$



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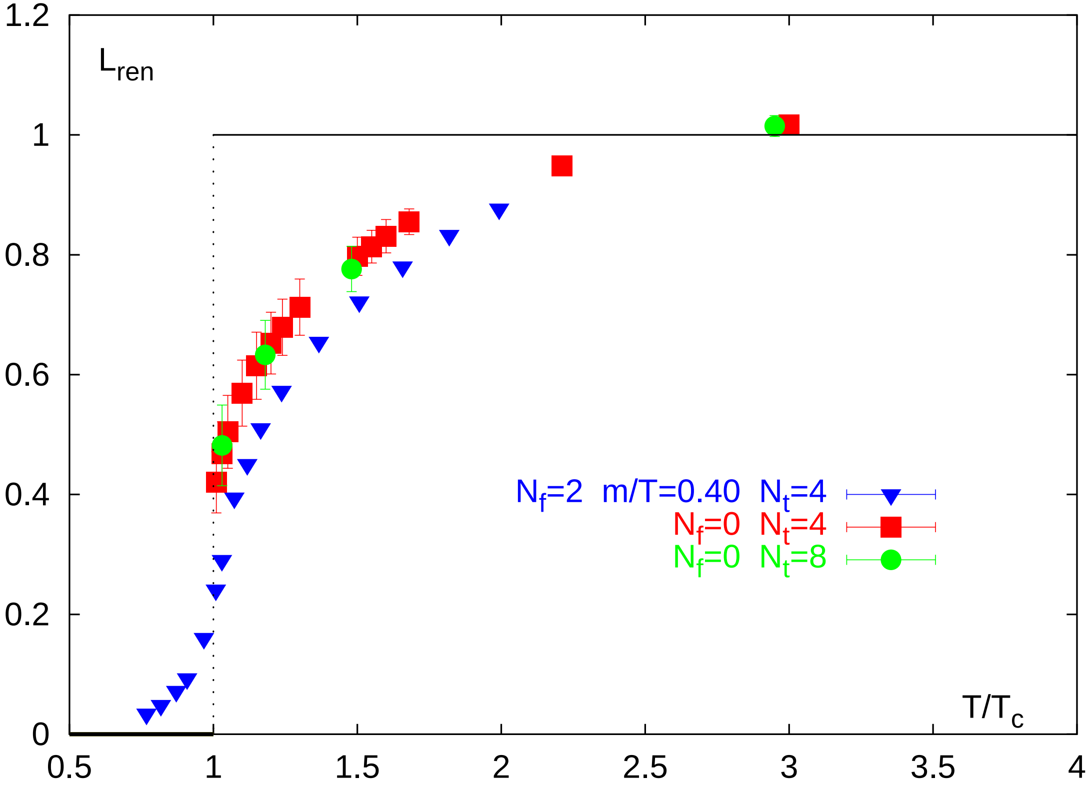


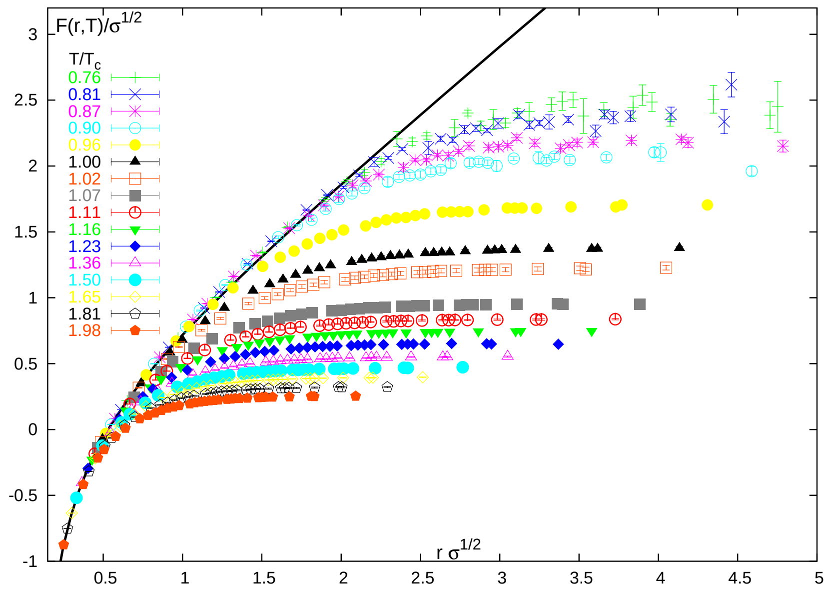


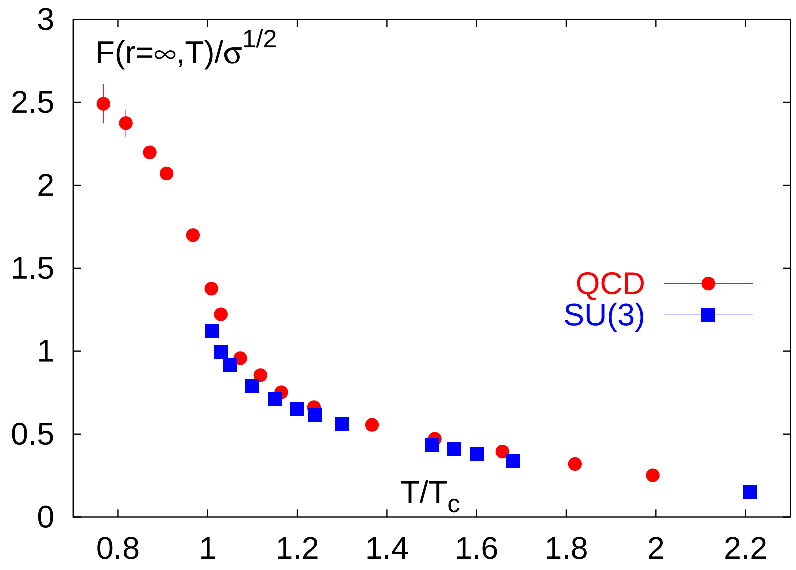

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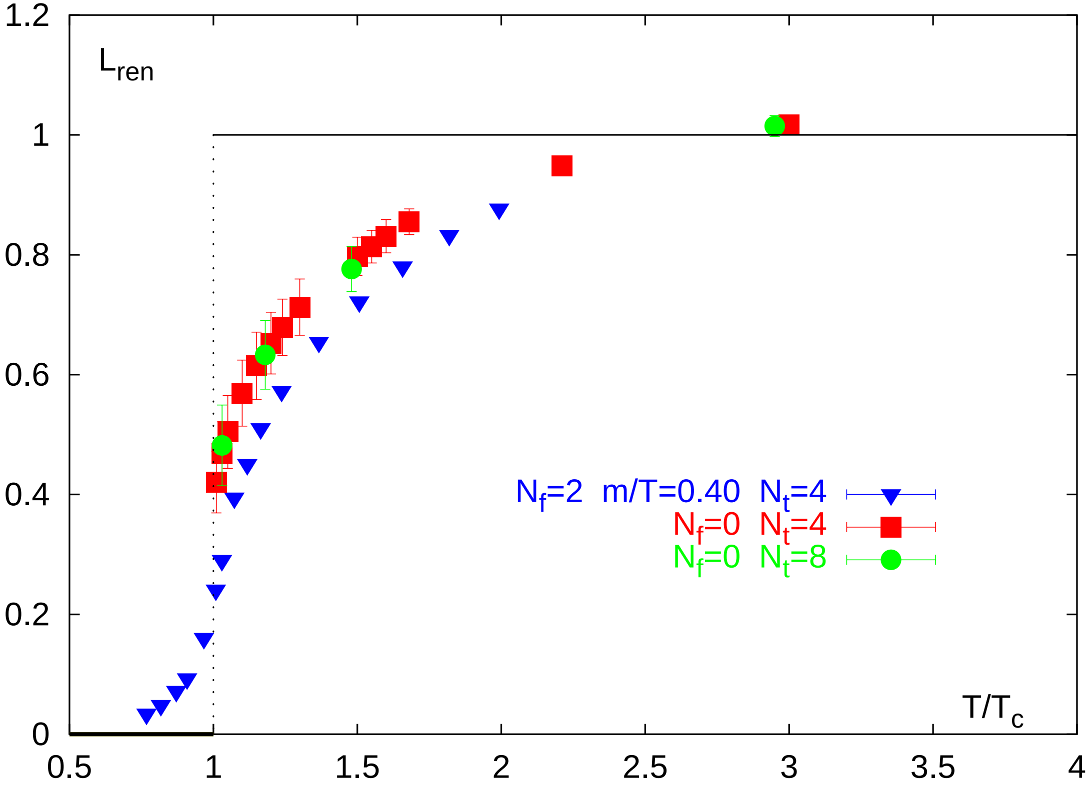


Quenched QCD:

$L_{\mathrm{ren}} = 0$ for $T < T_c$.

Finite gap at $T_c$

Full QCD:

$L_{\mathrm{ren}}$ finite for all $T$.

Strong increase near $T_c$.

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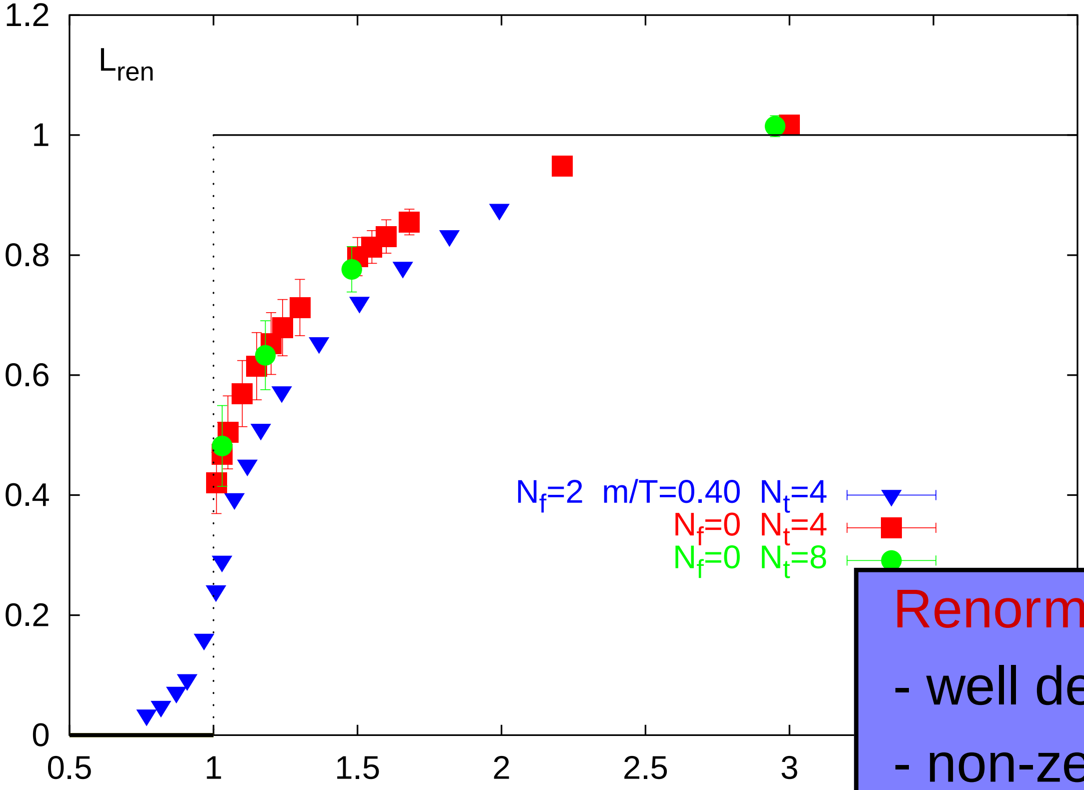


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Full QCD:

$L_{\text{ren}}$ finite for all $T$.

Strong increase near $T_c$.

Renormalized Polyakov loop

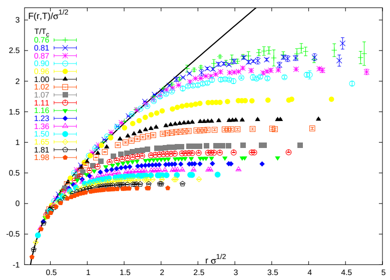
- well defined in quenched and full QCD
- non-zero for finite quark mass
- strong increase near $T_c$

# Conclusions

Renormalized free energies

Zero-$T$ behaviour at small $r$

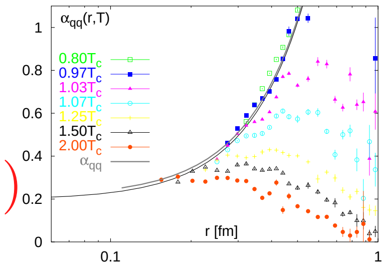
Complex $r$ and $T$ dependence




Short vs. long distance behaviour

$T$-independent at small $r$, running coupling $g(r)$

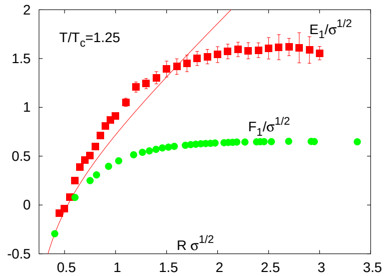
Screening properties at large $r$, running coupling $g(T)$




Free energies, internal energy and entropy

Entropy contributions play a role at finite $T$

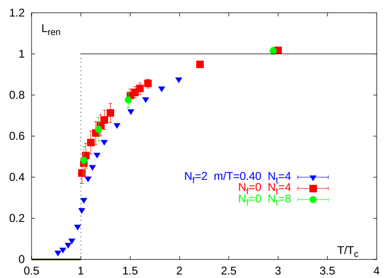
Potential energy larger than free energy




Renormalized Polyakov loop

Defined by long distance properties of $F$

Well behaved in the continuum limit



# Outlook

Short distance behaviour

- More detailed analysis of $g(r)$ and zero-$T$ behaviour
- Smaller lattice spacings needed

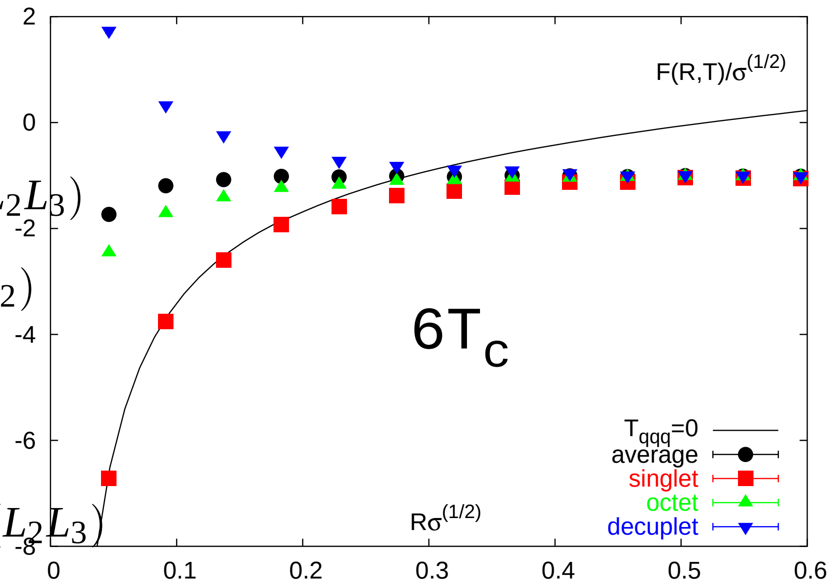
Long distance behaviour

- Extraction of screening properties and masses
- Larger lattices needed, $rT \gg 1$

N-point correlation functions

- Check of renormalization procedure
- 3-quark free energies

# 3-quark free energies [K. Hübner, O. Vogt, O.K.]




Different color channels: $3 \otimes 3 \otimes 3 = 10 \oplus 8' \oplus 8 \oplus 1$

$$\exp\left(-F^{1}_{qqq}/T\right) = \frac{1}{6}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 - 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3) - 9\,\mathrm{Tr}\,L_2\mathrm{Tr}\,(L_1L_3) - 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2) + 3\,\mathrm{Tr}\,(L_1L_2L_3) + 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle$$

$$\exp\left(-F^{8}_{qqq}/T\right) = \frac{1}{24}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3) - 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2) - 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle$$

$$\exp\left(-F^{8'}_{qqq}/T\right) = \frac{1}{24}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2) - 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3) - 3\,\mathrm{Tr}\,(L_1L_2L_3)\Big\rangle$$

$$\exp\left(-F^{10}_{qqq}/T\right) = \frac{1}{60}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3) + 9\,\mathrm{Tr}\,L_2\mathrm{Tr}\,(L_1L_3) + 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2) + 3\,\mathrm{Tr}\,(L_1L_2L_3) + 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle$$

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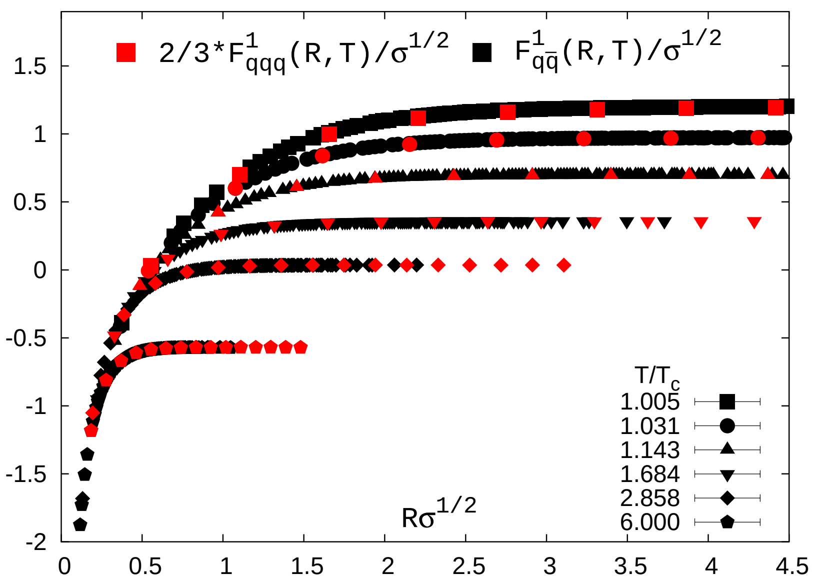


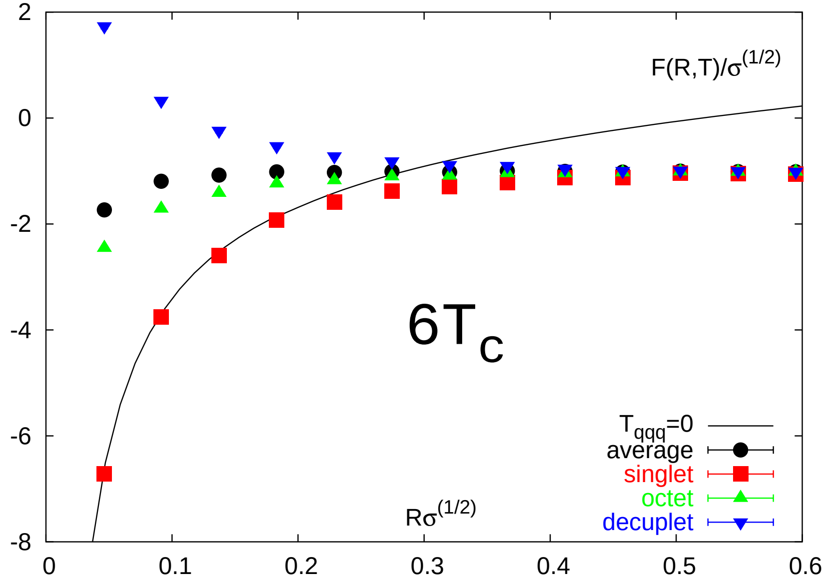


Renormalization of 3-quark Polyakov loop correlation functions

$$F_{qqq}(r,T) = \left(Z_R(g^2)\right)^{3N_\tau} \langle f\left(L_x, L_y, L_z\right)\rangle$$

Comparison with $q\bar{q}$ free energies

$$F_{qqq}(r) = \sum_{\langle qq \rangle} F_{qq}(r) = \frac{3}{2} F_{q\bar{q}}(r)$$

Renormalization of n-point functions with the same $Z_R(g^2)$

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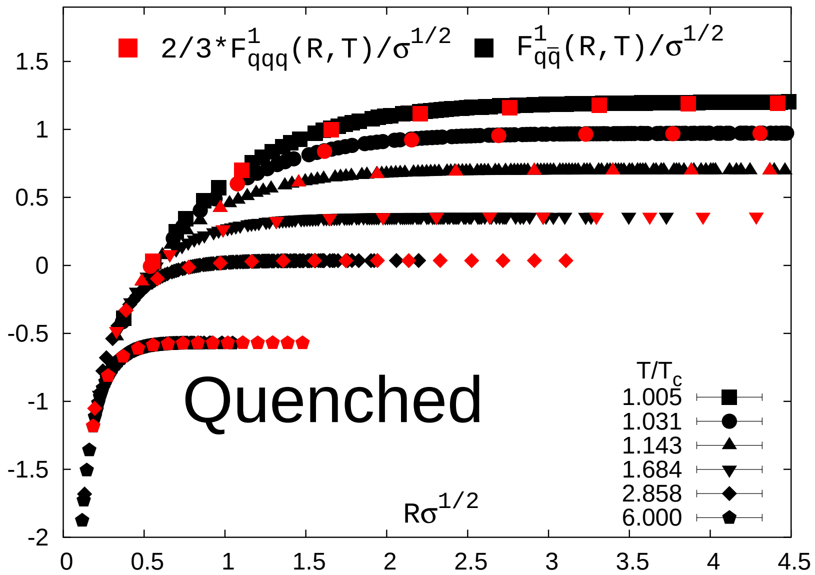


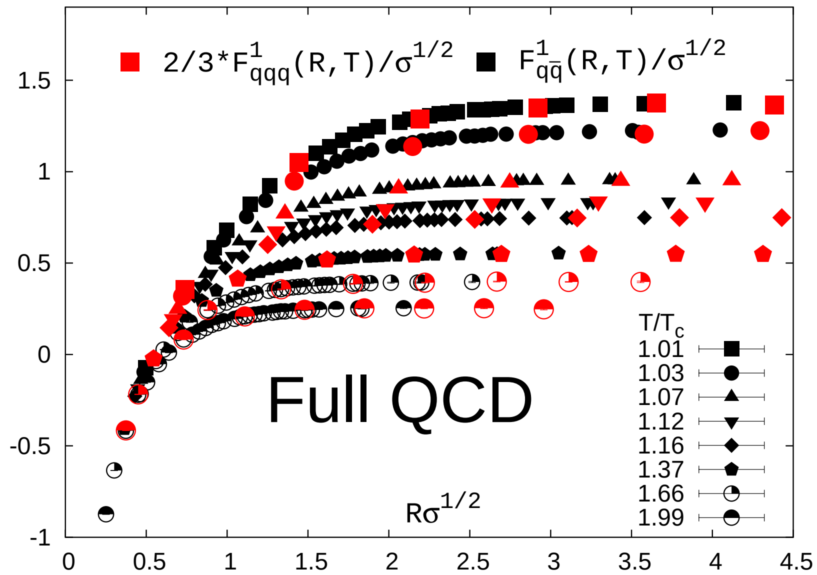


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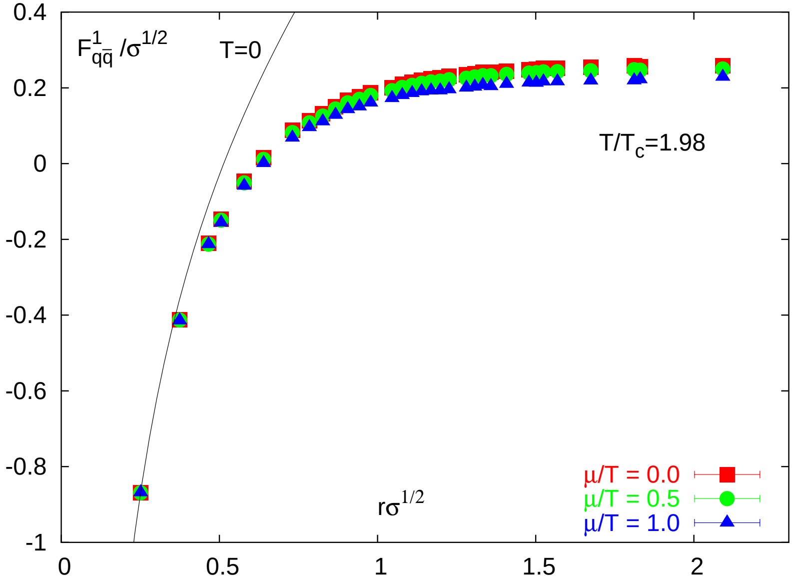
N-point correlation functions

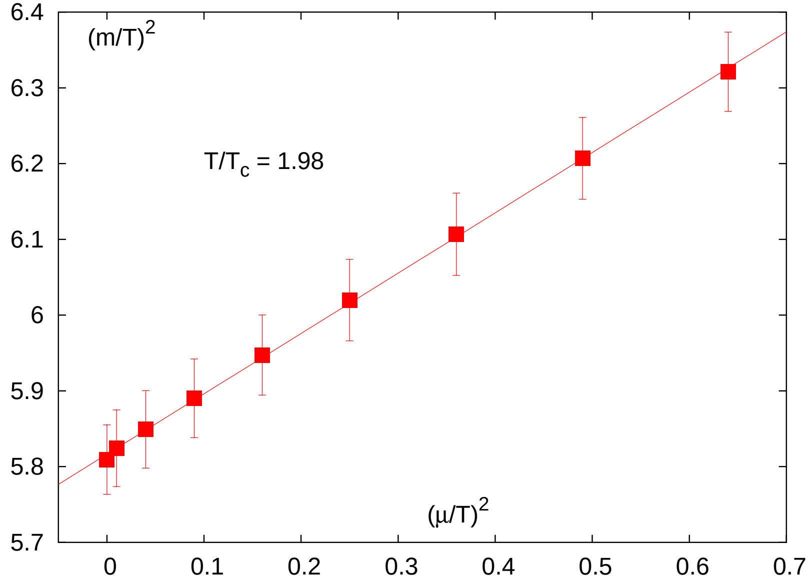
- Check of renormalization procedure
- 3-quark free energies

Extension to finite density

- Taylor expansion of correlation functions/free energies

# Heavy quark free energies at finite density [M. Döring, S. Ejiri, O.K.]




Taylor expansion of the correlation functions in $\mu$

$$
\begin{aligned}
F_1(r,T,\mu)/T &= C_0(r) + C_2(r)\mu^2 + C_4(r)\mu^4 \\
C_0(r) &= -\log\langle \mathrm{Tr}\,(L_x L_y^\dagger)\rangle_0 = F_1(r,T)/T
\end{aligned}
$$

Enhancement of $m/T$ with increasing $\mu$

$$
\text{perturbation theory :}\quad \left(\frac{m}{T}\right)^2 = \left(\left(\frac{N_c}{3}+\frac{N_f}{6}\right) + \frac{N_f}{2\pi^2}\left(\frac{\mu}{T}\right)^2\right) g^2
$$

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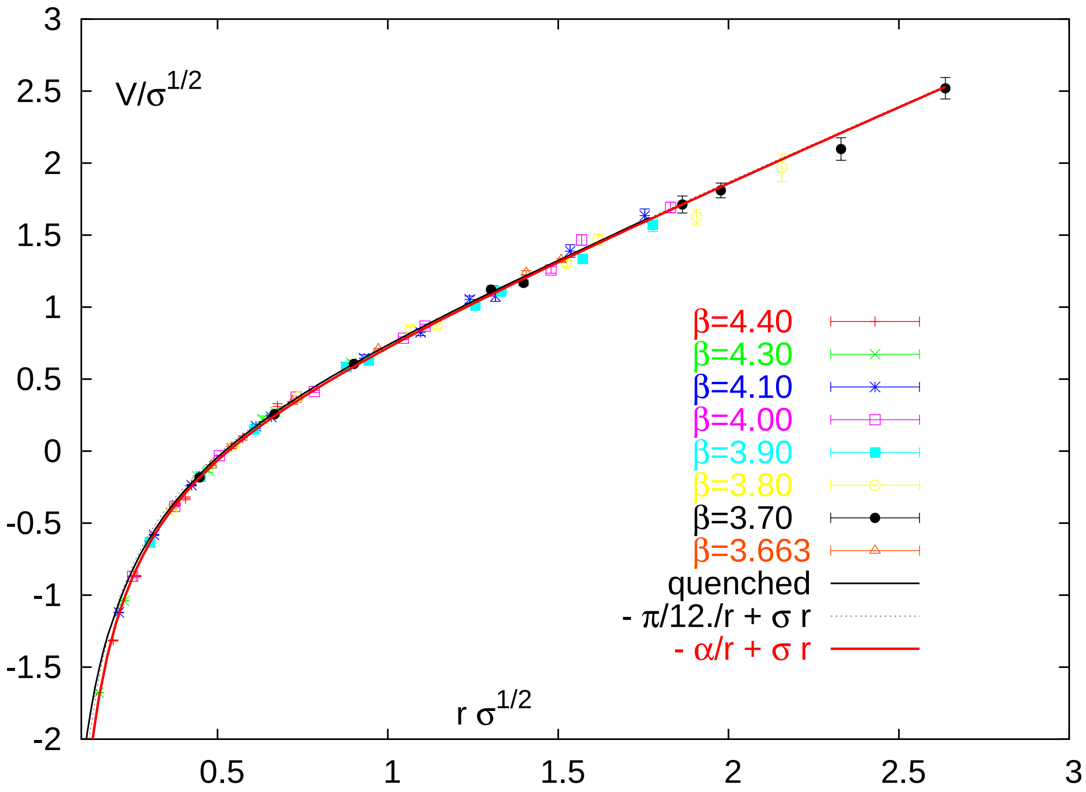
- Taylor expansion of correlation functions/free energies

Application in potential models

- Charmonium wave functions and suppression pattern

# Details of the calculation and $T$=0 Potential




Matching of $V(r)$ at large $r$

String model: $V(r) = -\frac{\pi}{12r} + \sigma\, r$ , for large $r$

Consistent with quenched potential for $r \gtrsim 0.2$ fm

Deviations at small $r$ $\longrightarrow$ enhancement of the running coupling

Fit: $V(r) = -0.2822(28)/r + \sigma\, r$

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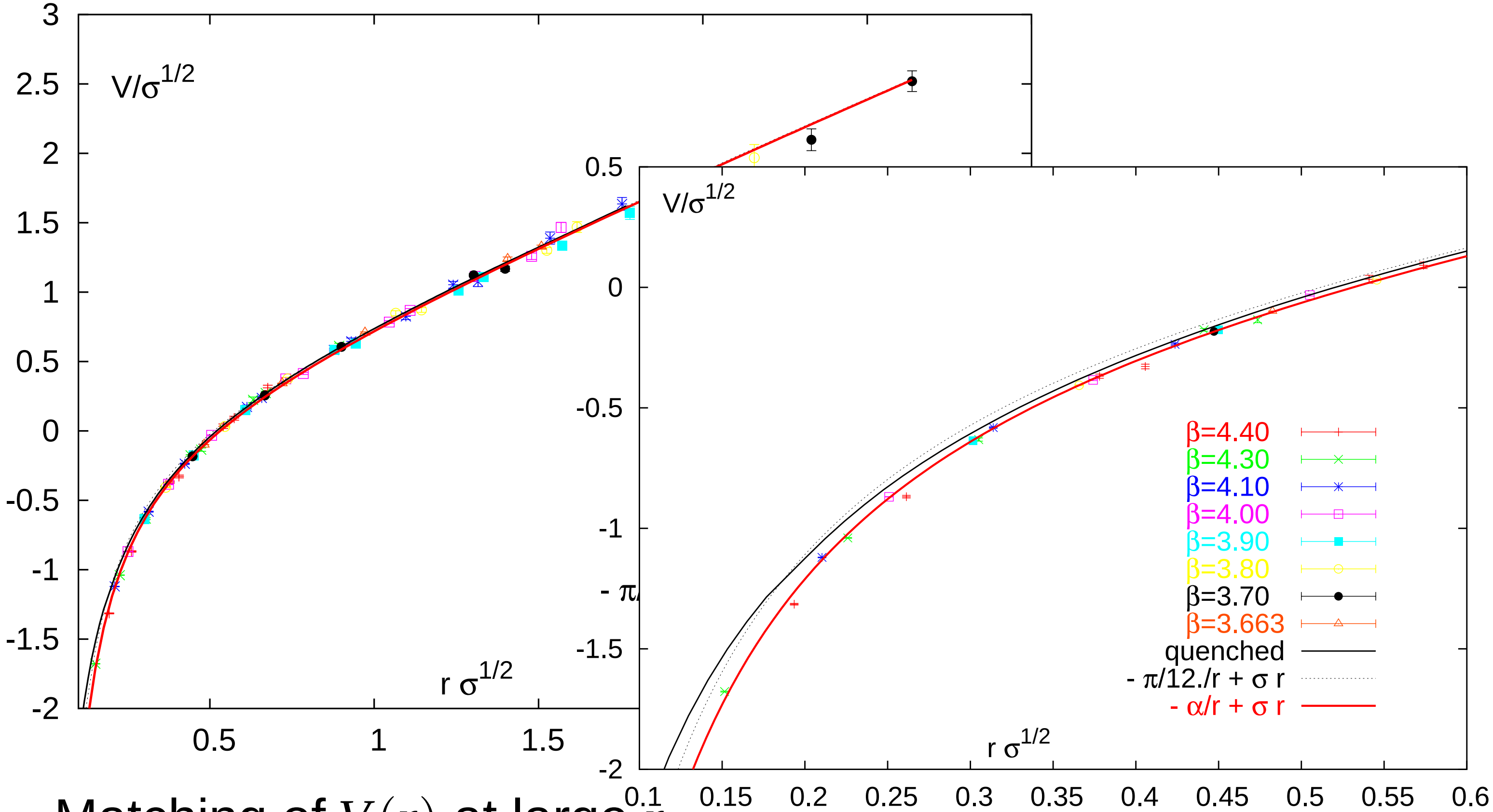


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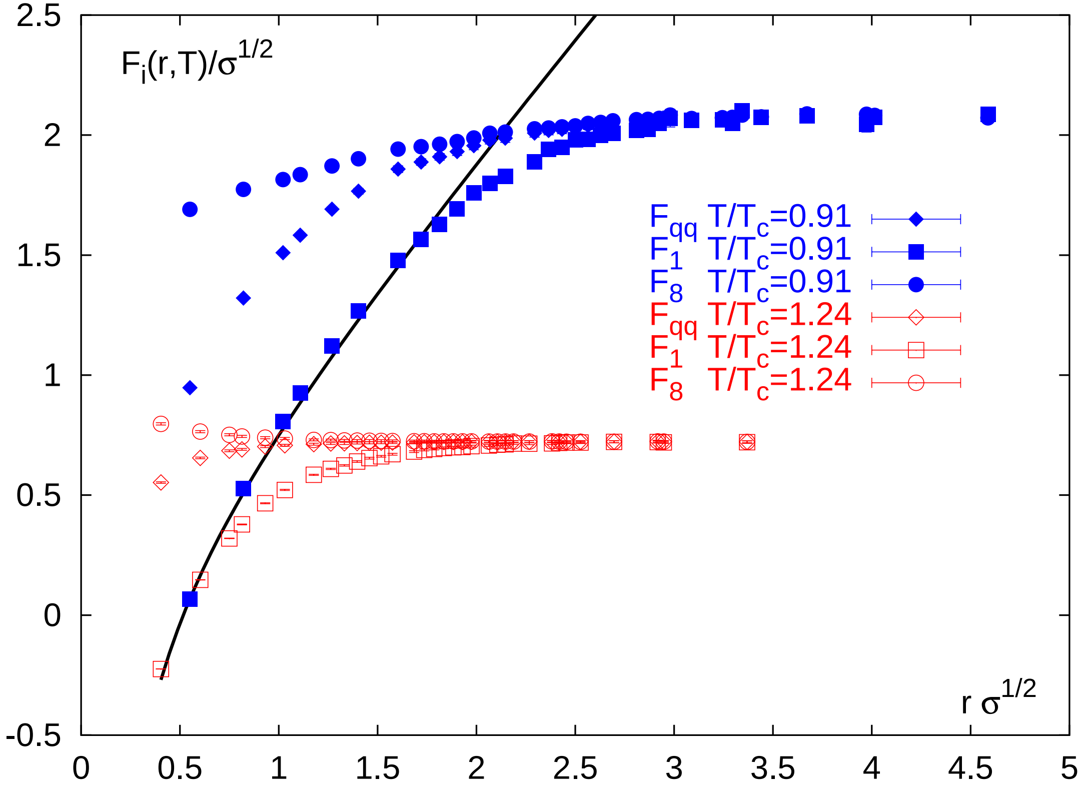
# Behaviour in the different color channels




$F_1$ is $T$-independent at small distances and coincide with $T$=0-potential

$F_1$ is attractive and $F_8$ repulsive at short separation

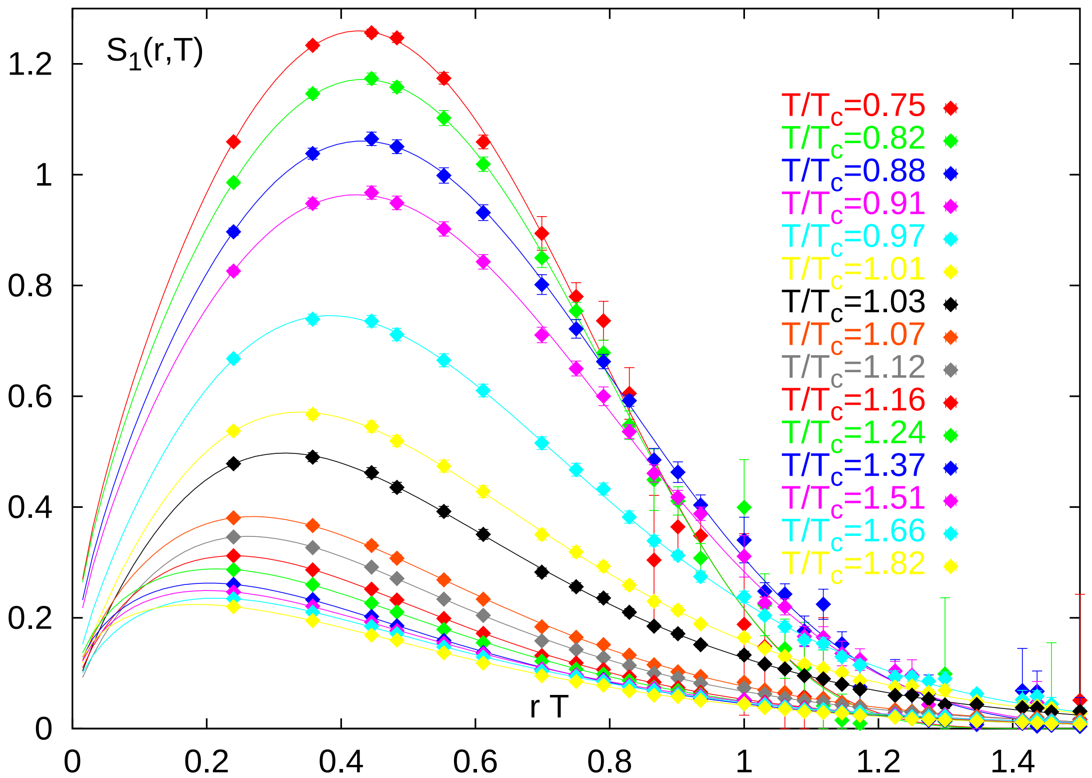
String breaking in all colour channels

Free energies coincide at large distance

# Short vs. long distance behaviour

**Screening function**

$$S_1(r,T) = -\frac{3}{4} r (F_1(r,T) - F_1(\infty,T))$$




solid lines are fits to

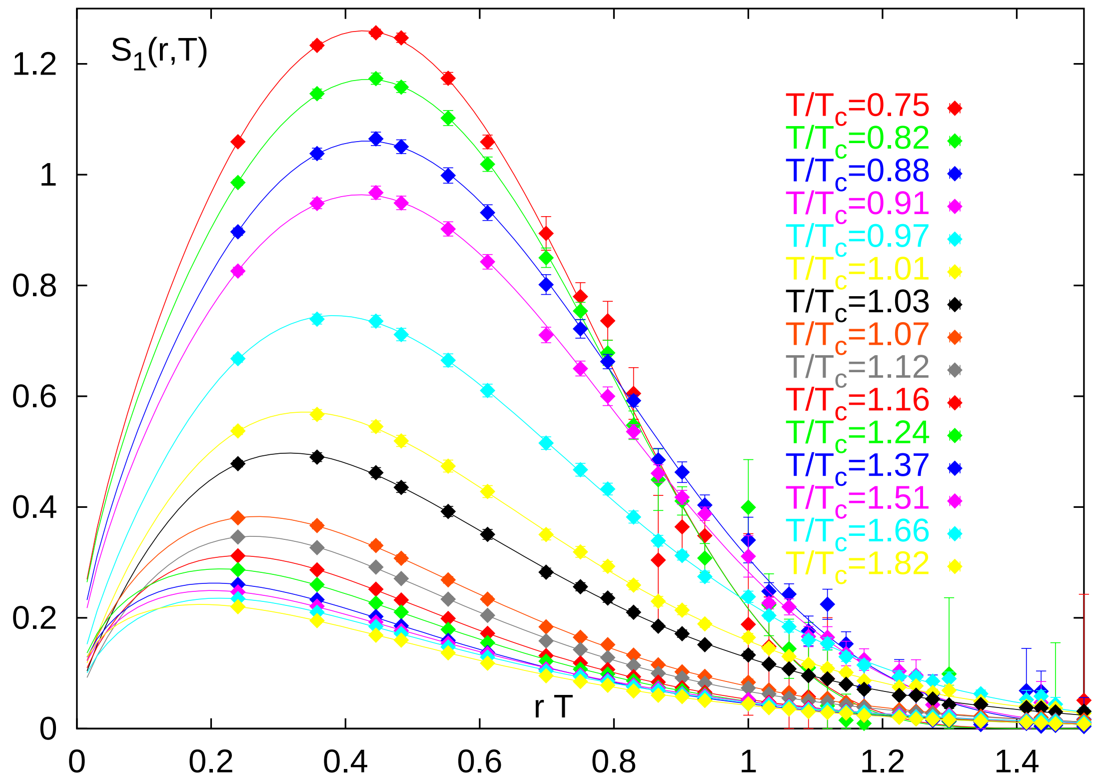
$$\frac{1}{2b_0 \log\left(\frac{1}{r\Lambda_{QCD}} + c\frac{T}{\Lambda_{QCD}}\right)} e^{-mr}$$

# Short vs. long distance behaviour

**Screening function**

$$S_1(r,T) = -\frac{3}{4} r (F_1(r,T) - F_1(\infty,T))$$

$rT \lesssim 0.5$:
dominated by $g^2(r)$
logarithmic decreasing



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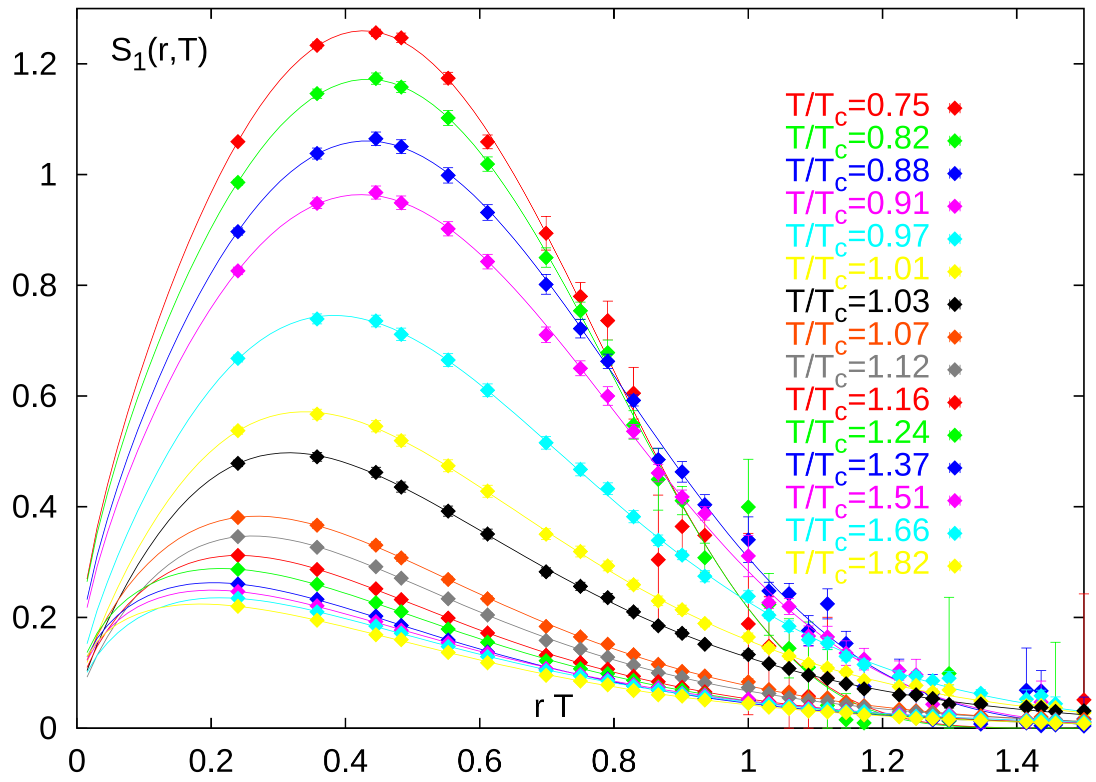

# Short vs. long distance behaviour

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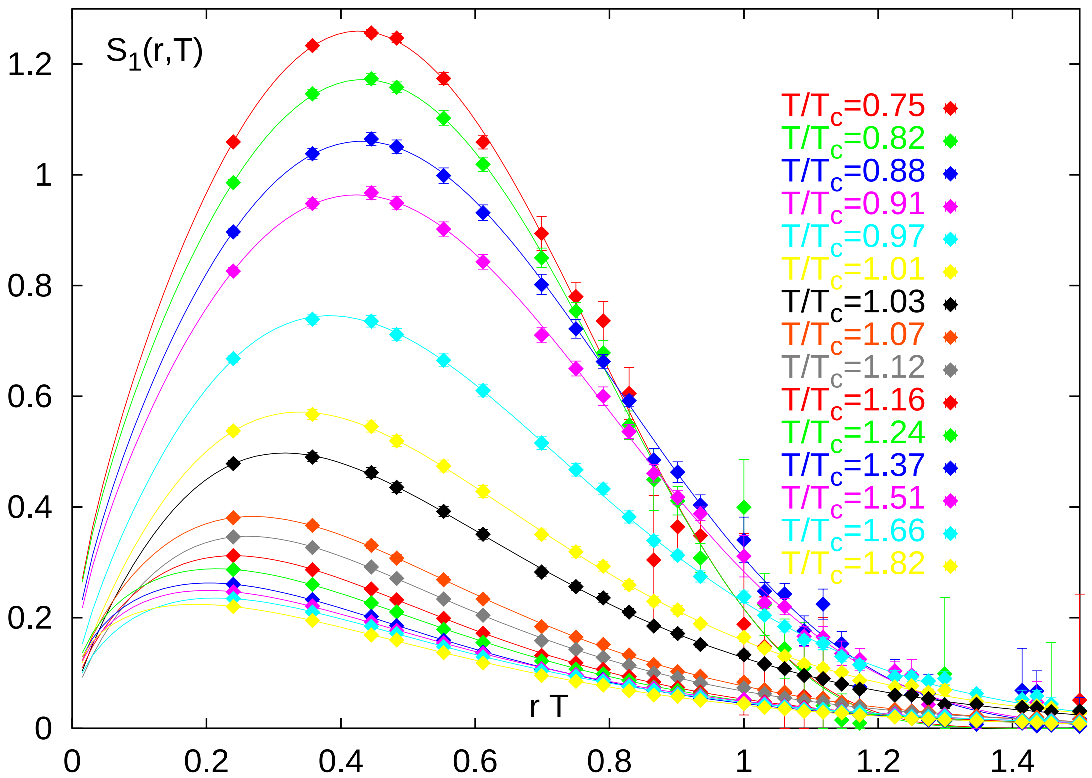

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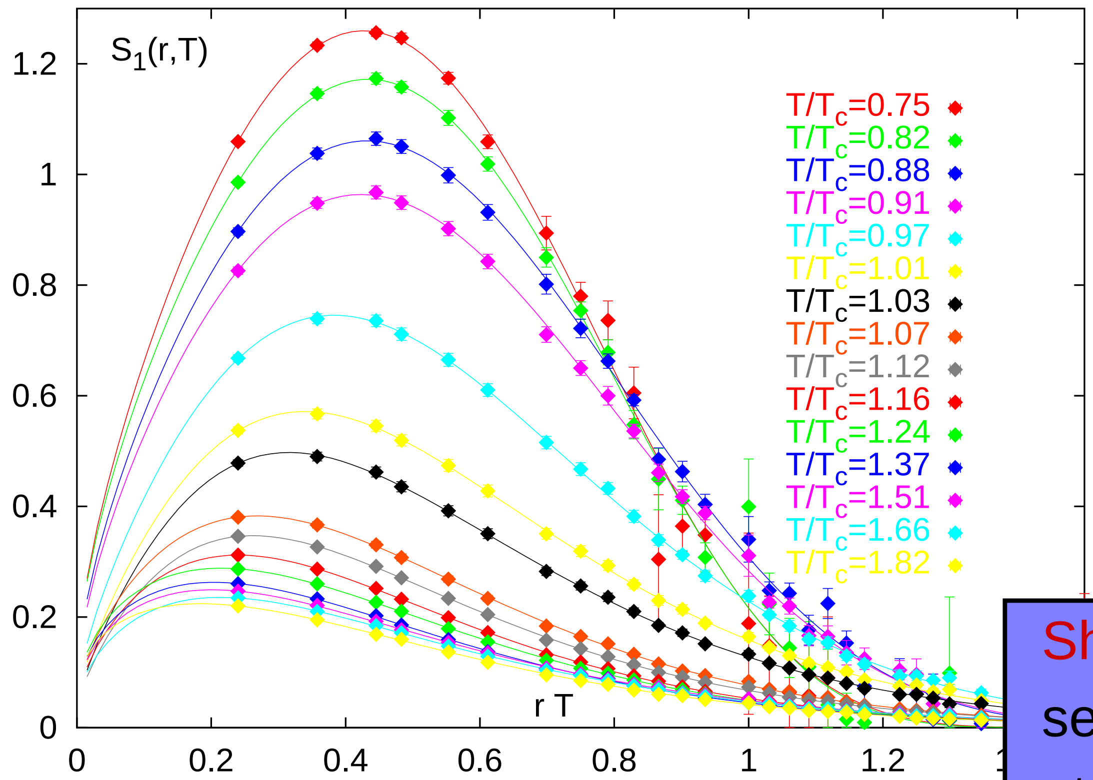
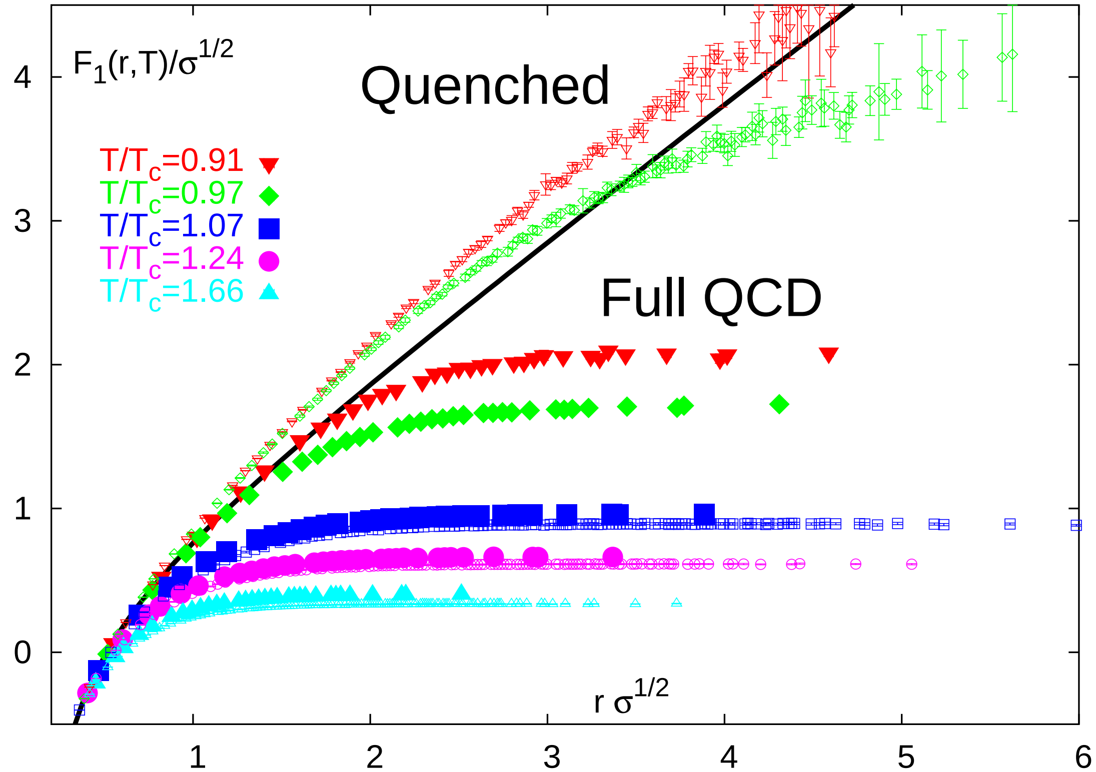



**Short vs. long distance behaviour**
seperated at $rT \simeq 0.3 - 0.5$
at smaller $r$ with increasing $T$

# Quenched vs. Full QCD




$T$=0-behaviour at short separations comparable

Large distance behavior

String tension in quenched theory

String breaking in full QCD

High temperatures and large distances

Screening properties comparable [ PT: $\left(\frac{m}{T}\right)^2 = \left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2$ ]