Jets and Data

Resummations and Interjet Radiation

George Sterman: For Hard Probes 2004, in absentia

- pQCD of hard processes
- Inclusive jets ($A \leq 1$)
- Jet particle flow
- Jet energy flow: the jet shape, event shapes
- Perturbative resummations
- Interjet radiation
- Nonperturbative corrections in Event Shapes & 1PI cross sections

⋆ pQCD of Hard Processes

- **Infrared safety & asymptotic freedom**:

$$Q^2\, \hat{\sigma}_{\rm SD}(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2)\, {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

$$= \sum_n c_n(1)\, {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

  - PT improves as Q increases
  - $\mathrm{e}^+\mathrm{e}^-$ total; jets
  - Basic requirement: group together states that differ by soft emissions/collinear rearragnements

- **Generalization: to IS hadron(s): factorization**

$$Q^2\sigma_{\rm phys}(Q,m) = \omega_{\rm SD}(Q/\mu,\alpha_s(\mu)) \otimes f_{\rm LD}(\mu,m) + \mathcal{O}\left(1/Q^p\right)$$

- $\mu =$ **factorization scale**; $m=$ **IR scale** (m *may* be perturbative)
  - **New physics** in $\omega_{\rm SD}$; $f_{\rm LD}$ "universal"
  - Deep-inelastic $(p=2)$, ${\rm p\bar{p}} \to Q\bar{Q}$ . . .
  - Exclusive decays: $B \to \pi\pi$
  - Exclusive limits: ${\rm e^+e^-} \to$ JJ as $m_J \to 0$

- **Whenever there is factorization, there is evolution**

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q,m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu}$$

- **Wherever there is evolution there is resummation**

$$\sigma_{\rm phys}(Q,m) = \omega(1,\alpha_s(Q))\, f(q,m)\, \exp\left\{ \int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right) \right\}$$

- Coherent branchings: "mini-factorizations"

⋆ Inclusive Jets

- Factorized Cross Sections (e.g. $A + B \to J(p_J) + X$)

$$p_J^4 \frac{d\sigma_{\rm phys}(p_J, m)}{dp_J^2} \;=\; f_{\rm LD,A}(\mu, m) \otimes \omega_{\rm SD}\left(\frac{p_J^2}{\hat{s}}, \frac{\hat{s}}{\mu^2}, \alpha_s(p_J)\right) \otimes f_{\rm LD,B}(\mu, m)$$

- But what's a jet? $\leftrightarrow$ define "X" and calculate ω

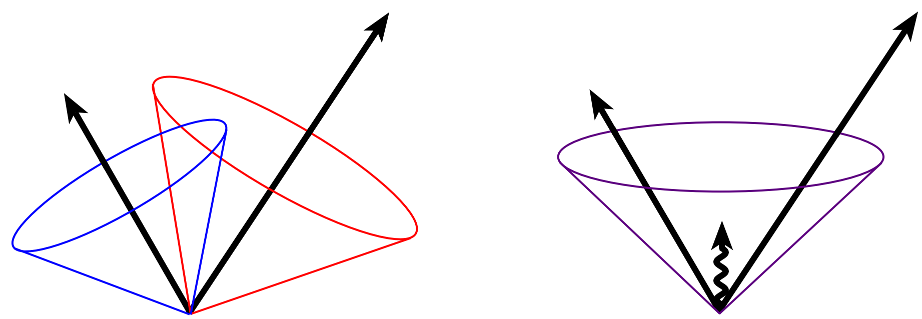
- **Need to construct jets from final states: algorithms**
  G. Blazey et al., *Run II Jet Physics* hep-ph/0005012

- Cones algorithms: towers $\rightarrow$ protojets $\rightarrow$ jets
  - Calorimeter tower mta. (directions y_i, ϕ_i)
  - Cluster within cones

$$i \subset C \quad : \quad \sqrt{(y^i - y^C)^2 + (\phi^i - \phi^C)^2} \leq R.$$

  - Task I: to identify "centers" y_C, ϕ_C
    (high-p_T towers as "seeds" (but IR safety problematic))
  - Result: "protojets"
- Task II: interpret overlapping protojets: "merge/split"
- Naive interpretation is to find jets that "really" come from a single parton, but this is not a well-defined concept.
- For single jet inclusive, a cleaner method would
  be to scan all possible protojets, identify largest p_T

– The problem with some iterative algorithms (seeds and merge/split) sensitivity to soft emissions: lose infrared safety at NNLO



– mid-point soft emission changes merging procedure discontinuously

– Corrected in modified Tevatron Run II algorithms: by testing more cones ("scans enough")

- The k_T algorithm: preclusters $\longrightarrow$ jets
- Starts with measurements in calorimeter "towers" p_i
- "For each precluster i in the list, define

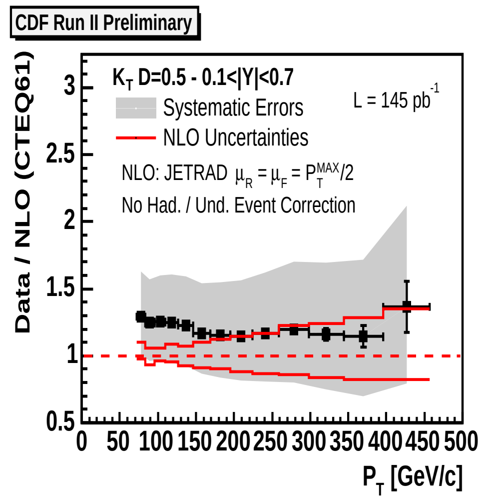
$$d_i = p_{T,i}^2$$

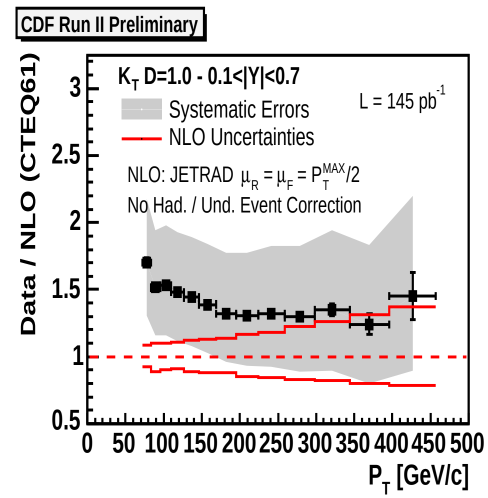
- For each pair (i,j) of preclusters ($i \neq j$), define

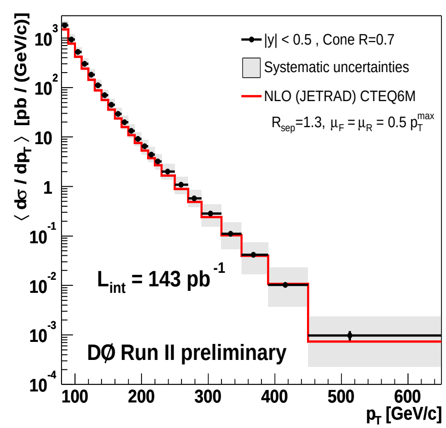
$$d_{ij} = \min\left(p_{T,i}^2, p_{T,j}^2\right) \frac{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}{D^2} \text{"}$$

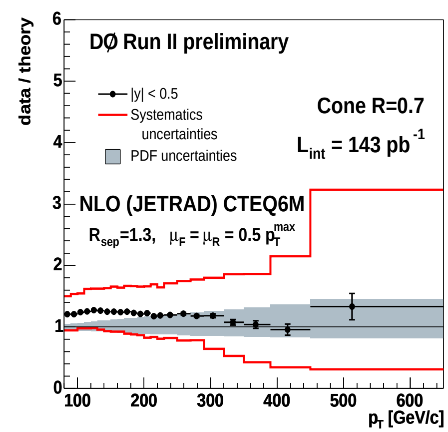
- Find $d_{\min}$ among all d_i, d_{ij}
- if $d_{\min}$ is a d_i: identify as "jet"
- if $d_{\min}$ is a d_{ij}: combine into new precluster $p_{ij} = p_i + p_j$
- Repeat (leaving out "jets")
- End result: list of "jets" (most with small d_i)

⋆ Tevatron Run II Jets









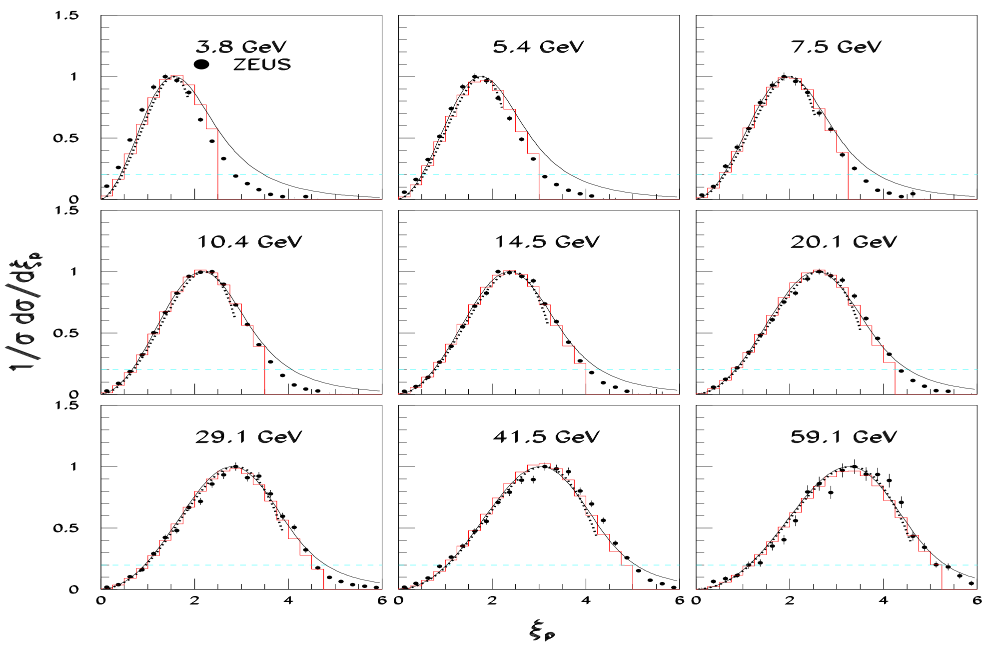
- What do we learn so far?
  - Extraordinary tracking of predicted shape to highest energies
  - Energy uncertainty remains large
    but will decrease with more statistics
  - Poorly-understood excess towards lower p_T
  - CDF k_T algorithm shows excess at largest p_T
  - But algorithms may evolve
  - Remaining discrepancies probably due to still
    incomplete understanding of particle and energy flow

⋆ Jet Particle Flow

– Low-z spectrum at Zeus; from Khoze/Ochs hep-ph/0110295

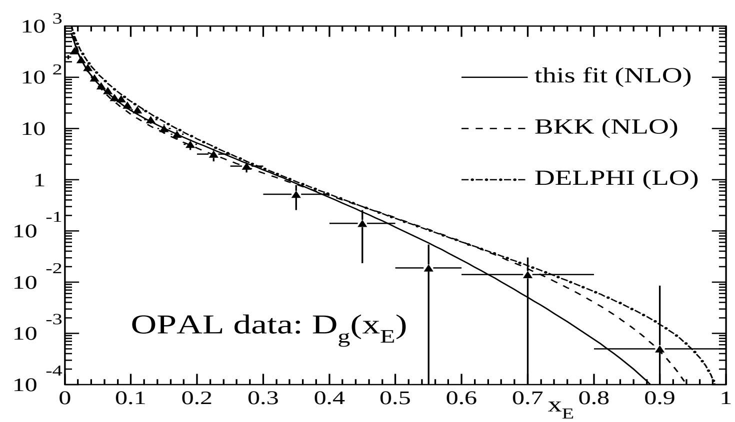
$$\xi = \ln\left(\frac{E_J}{E_{\text{particle}}}\right)$$

Angular ordering at branching → suppression at large ξ; Gaussian-like shape



– Large-z fragmentation function fit; from Kretzer hep-ph/0003177

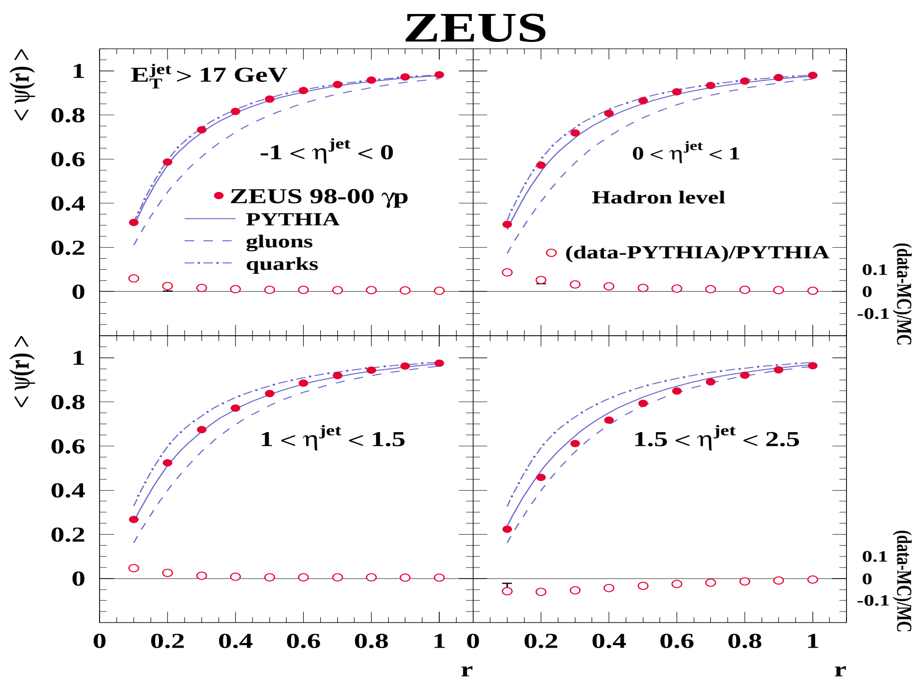
$$\frac{d\sigma^h_{P=T,L}}{dz} = \sum_{i=q,\bar{q},g} \int_z^1 \frac{d\zeta}{\zeta} C^i_P\left(\zeta, Q^2, \mu^2_{F,R}\right) D^h_i\left(\frac{z}{\zeta}, \mu^2_F\right)$$



⋆ Jet Energy Flow

- The "Jet Shape"

$$\psi(r) = \frac{E_T(r)}{E_{T,\text{jet}}},$$

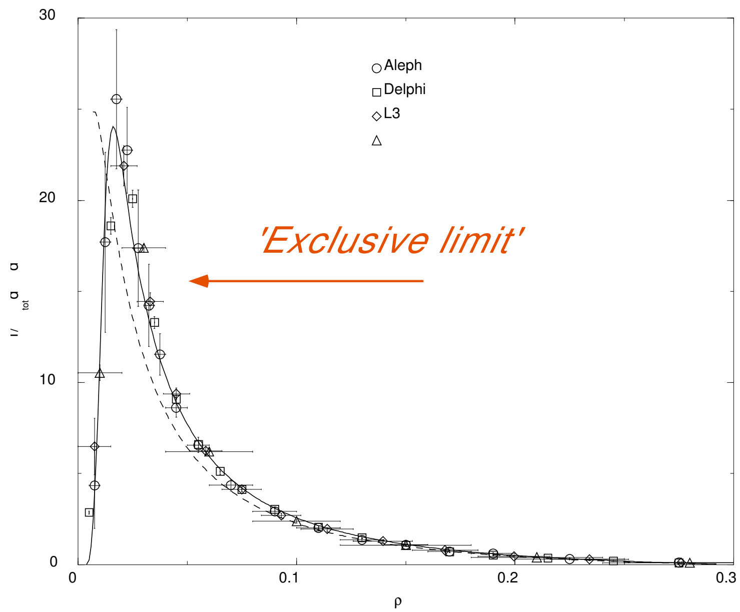


- Jets in more detail: Event Shapes

- Flexible event shapes (C.F. Berger, Kúcs, GS (2003), Berger, Magnea (2004))

$$\tau_a = \frac{1}{Q} \sum_{i \,\mathrm{in}\, N} E_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$$

  - θ_i angle to thrust ($a = 0$) axis
  - broadening: $a = 1$; inclusive limit $a \to \infty$
  - collectively: "angularities"
  - Example: Heavy jet distribution at the Z pole ($\sim \tau_0$) (Korchemsky and Tafat (2000))



∗ Dashed line: NLL resummed; solid line: NP "shape function" fit

– Jet shapes in DIS similar <u>if</u> overall final state limited (global)

Dasgupta and Salam (2000, 2002)

– Semi-numerical resummation (flexibility)

& new hadron-hadron event shapes

Banfi, Salam, Zanderighi (2002,2004)

⋆ **Perturbative resummations: Why? When? How?**

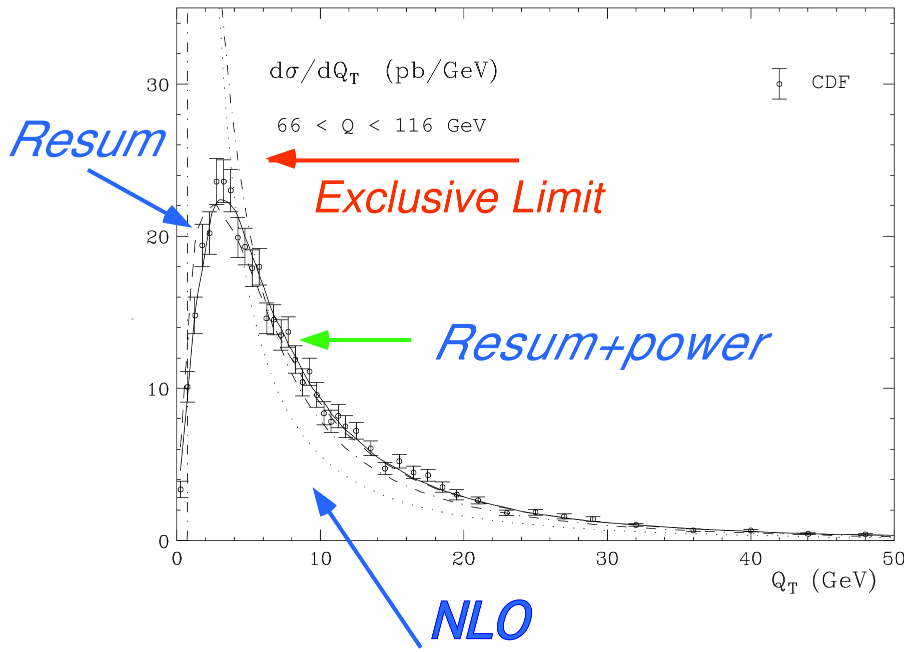
Every final state in hard scattering carries the imprint of QCD dynamics on all distance scales

- Logarithmic corrections
- Structure of IR/CO singularities
- Window to power corrections
- Exploration of gauge theory

Explicit Logs: Event shapes, p_T distributions

$$\frac{d\sigma(Q)}{dQ_1} \propto \frac{1}{Q_1} \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad \Lambda \ll Q_1 \ll Q$$

Event shapes: $Q_1 = e_a Q$



(from Kulesza, G.S., Vogelsang (2002))

– maximum then decrease near “exclusive” limit (parton model kinematics) replaces divergence

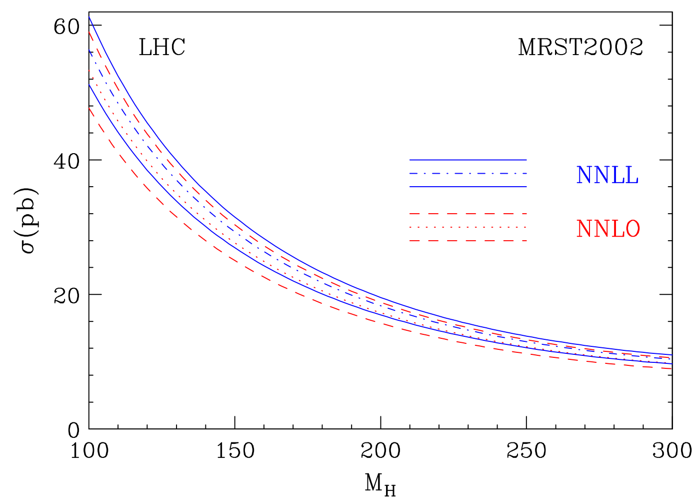
– soft but perturbative radiation broadens distribution

– typically NP correction necessary for quantitative description of data

– recover fixed order away from exclusive limit

Implicit logs: threshold resummations, 1PI high-p_T

$$\sigma(Q) \propto \int \frac{dQ_1}{Q_1} F(Q_1) \sum_n C_n \alpha_s^n \ln^{an+b}\left(\frac{Q}{Q_1}\right) \quad F(0) = 0$$



(from Catani, de Florian, Grazzini, Nason (2003))

– Modest change, scale improvement $\leftrightarrow$ increased confidence

$\star$ When Can We Resum?

$\star$ Factorization Structure and Proofs

- (1) $\omega_{\rm SD}$ **incoherent** with LD dynamics
- (2) mutual incoherence when $v_{\rm rel} = c$
- For large $Q \sim s$: long-distance logs from

$$\frac{d\sigma(Q, a+b \to N_{\rm jets})}{dQ} = \int dx_a dx_b \, H(x_a p_a, x_b p_b, Q)_{a'b' \to c_1 \ldots c_{N_{\rm jets}}}$$
$$\times \mathcal{P}_{a'/a}(x_a p, X_a) \, \mathcal{P}_{b'/b}(x_b p, X_b)$$
$$\otimes_{\rm soft} \prod_{i=1}^{N_{\rm jets}} J_{c_i}(X_i) \otimes_{\rm soft} S_{a'b' \to c_1 \ldots c_{N_{\rm jets}}}(X_{\rm soft})$$

$$\frac{d\sigma(Q, a+b \to N_{\text{jets}})}{dQ} = H \times \prod_c \mathcal{P}_c \otimes_{\text{soft}} \prod_i J_i \otimes_{\text{soft}} S$$

– A story with only these pieces:
  * Evolved incoming partons $\mathcal{P}_{a'/a}$, $\mathcal{P}_{b'/b}$ collide at H;
  * $X_{a,b}$ "fragments" to produce
  * Outgoing jets J_{c_i} and coherent soft emission S.
  * Holds to any fixed α_s^n, all $\ln^a \mu/Q$ to $\sim E_{\text{soft}}/E_{\text{jet}}$.

– W, Z, H in pp: $H \times \mathcal{P}_a \otimes_{\text{soft}} \mathcal{P}_b \otimes_{\text{soft}} S$

– $\mathrm{e}^+\mathrm{e}^- \to 2\mathrm{J}$: $H \times J_q \, J_{\bar{q}} \otimes_{\text{soft}} S$

– DIS F_i near $x = 1$: $H \times \mathcal{P}_a \otimes_{\text{soft}} J_q \otimes_{\text{soft}} S$

⋆ Application: "angularities" $\mathrm{e}^+\mathrm{e}^-$

– NLL resummed cross section

$$\sigma\,(\tau_a, Q, a) \;=\; \sigma_{\rm tot}\,\int_C d\nu\,\mathrm{e}^{\nu\,\tau_a}\;\left[\,J_i(\nu, p_{Ji})\,\right]^2$$

– At NLL can define $S_{c\bar{c}} = 1$: indepenent jet evolution

(Catani, Turnock, Trentadue, Webber (1990-92))

– The jet in transform space

$$J_i(\nu, p_{Ji}) = \int_0 d\tau_a \, \mathrm{e}^{-\nu \tau_{Ji}} \, J_i(\tau_{Ji}, p_{Ji}) = \mathrm{e}^{\frac{1}{2}E(\nu,Q,a)}$$

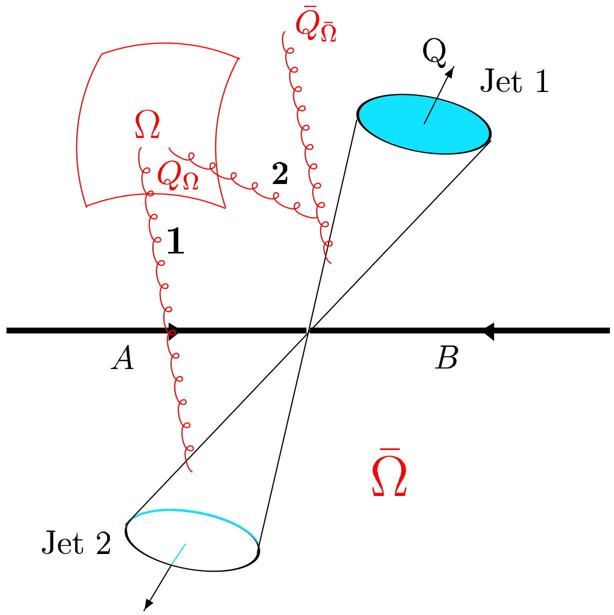
$$E(\nu, Q, a) = 2 \int_0^1 \frac{du}{u} \Bigg[\int_{u^2Q^2}^{uQ^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u^{1-a}\nu (p_T/Q)^a} - 1 \right) + \frac{1}{2} B\left(\alpha_s(\sqrt{u}Q)\right) \left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1 \right) \Bigg]$$

Enter: nonperturbative scales in resummmed PT

Can be avoided to NLL accuracy (Catani et al. 1996)

⋆ **Interjet Radiation**

- Non-global logs: color and energy flow
  (Dasgupta & Salam (2001))



  – Simplest cases: 2 jets. Measure distribution $\Sigma_\Omega(E)$

– Choices for Cross Section:

– a) Inclusive in $\bar{\Omega}$ $\longrightarrow$ Number of jets not fixed!

– b) Correlation with event shape $\tau_a \ldots$:
fixes number of jets $\longrightarrow$ factorization
(C.F. Berger, Kúcs, GS (2003), Dokshitzer, Marchesini (2003))

– for a): Number of jets not fixed: nonlinear evolution

(Banfi, Marchesini, Smye (2002)) LL in E/Q, large-N_c (all $\Sigma = \Sigma(E)$)

$$\partial_\Delta \Sigma_{ab} = -\partial_\Delta R_{ab}\, \Sigma_{ab} + \int_{k \text{ not in } \Omega} dN_{ab\to k}\, \left(\Sigma_{ak}\Sigma_{kb} - \Sigma_{ab}\right)$$

$$dN_{ab\to k} = \frac{d\Omega_k}{4\pi}\, \frac{\beta_a \cdot \beta_b}{\beta_k \cdot \beta_b\, \beta_k \cdot \beta_a} \qquad R_{ab} = \int_E^Q \frac{dE'}{E'} \int_\Omega dN_{ab\to k}$$

– Origin of the nonlinearity

* $\partial_\Delta = E\partial_E$
* ∂_E requires a "hard" gluon k
* New hard gluon acts as new, recoil-less source
* Large-N limit: $\bar{q}(a)G(k)q(b)$ sources $\to \bar{q}(a)q(k) \oplus \bar{q}(k)q(a)$

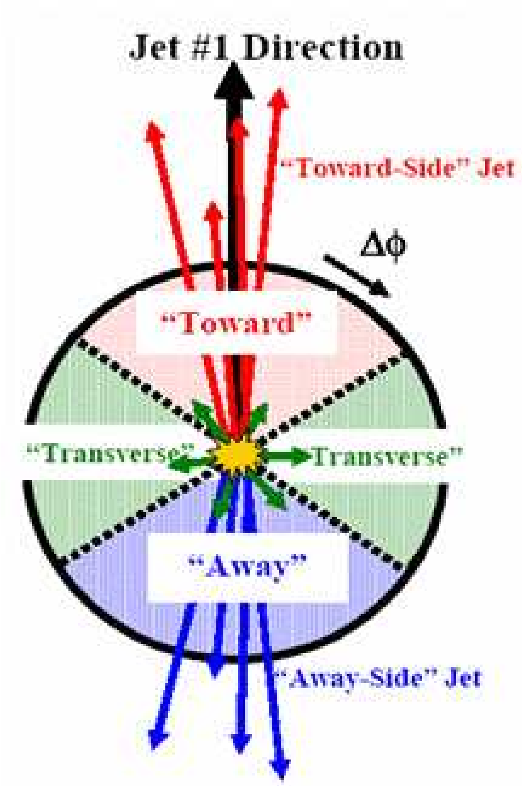
– Intriguing relation with approach to small-x saturation
(Balitsky (1995), Kovchegov (1998), Weigert (2003))

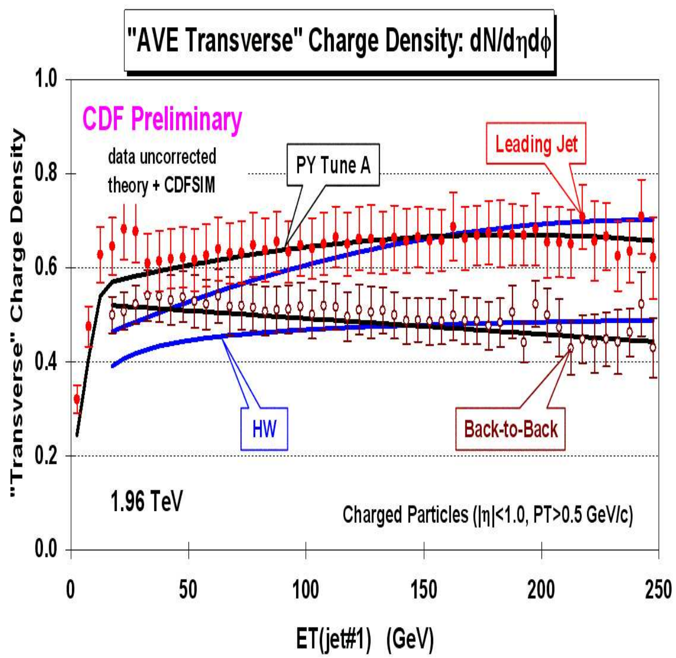
- For b) Correlation with event shape τ_a . . . :
  fixes number of jets
  - Keep $\tau_a Q \sim E_\Omega$ (BKS), Resum as above:

$$\frac{d\sigma}{dE_\Omega d\tau_a} \sim S(E_\Omega/\tau_a Q)\,\frac{d\sigma_{\rm resum}}{d\tau_a}$$

  - Limit $E_\Omega/\tau_a Q \to 0$ (DM): use nonlinear evolution for S
  - Influence of color flow on energy flow at wide angles
    (Dokshitzer, Khoze, Troyan, Mueller . . .)
  - Applications to rapidity gaps
    (Oderda, GS (1999) ; Appleby, Seymour (2003))

- Interjet multiplicity studies at CDF: slow increase with jet energy





- Energy flow studies will be interesting

- Radiation from hard scattering vs. spectator interactions

$\star$ NP corrections in Event Shapes & 1PI cross sections

- From Resummed PT to NP QCD
- How to interpret expressions like

$$
\begin{aligned}
E(\nu, Q, a) \;\; &= \;\; 2 \int_0^1 \frac{du}{u} \left[\int_{u^2 Q^2}^{u Q^2} \frac{dp_T^2}{p_T^2} A\left(\alpha_s(p_T)\right) \left(\mathrm{e}^{-u^{1-a} \nu (p_T/Q)^a} - 1 \right) \right. \\
&\left. + \frac{1}{2} B\left(\alpha_s(\sqrt{u} Q)\right) \left(\mathrm{e}^{-u(\nu/2)^{2/(2-a)}} - 1 \right) \right]
\end{aligned}
$$

- Argument of α_s vanishes but expansion in $\alpha_s(Q)$ finite at all orders

- Shape function approach for angularities
  - $p_T > \kappa$, PT
  - $p_T < \kappa$, expand exponentials
  - Low p_T replaced by $f_{\rm NP}$ "shape function"

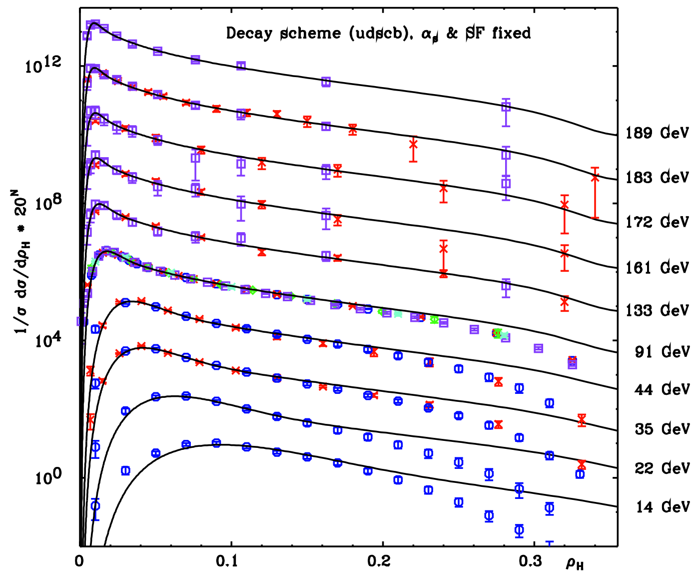
$$
\begin{aligned}
E(\nu, Q, a) &= E_{\rm PT}(\nu, Q, \kappa, a) \\
&\quad + \frac{2}{1-a} \sum_{n=1}^{\infty} \frac{1}{n\, n!} \left(-\frac{\nu}{Q}\right)^n \int_0^{\kappa^2} \frac{dp_T^2}{p_T^2}\, p_T^n\, A\left(\alpha_s(p_T)\right) + \ldots \\
&\equiv E_{\rm PT}(\nu, Q, \kappa, a) + \ln \tilde{f}_{a,{\rm NP}}\left(\frac{\nu}{Q}, \kappa\right)
\end{aligned}
$$

- Shape function factorizes in moments $\longrightarrow$ convolution

$$\sigma(\tau_a, Q) = \int d\xi f_{a,\mathrm{NP}}(\xi)\ \sigma_{\mathrm{PT}}(\tau_a - \xi, Q)$$

- Fit at $Q = M_{\mathrm{Z}}$ $\Rightarrow$predictions for all Q

- Shape function phenomenology for thrust



Strategy: $f_{\rm NP}(\epsilon)$ at Z pole; predict other Q

(Korchemsky,GS, Belitsky; Gardi Rathsman,Magnea (1998 . . .))

First pass: $f_{0,\rm NP}(\rho) = \text{const}\ \rho^{a-1}\, \mathrm{e}^{-b\rho^2}$:

$a :\sim \langle \text{no. particles} \,/\, \text{unit rapidity} \rangle$

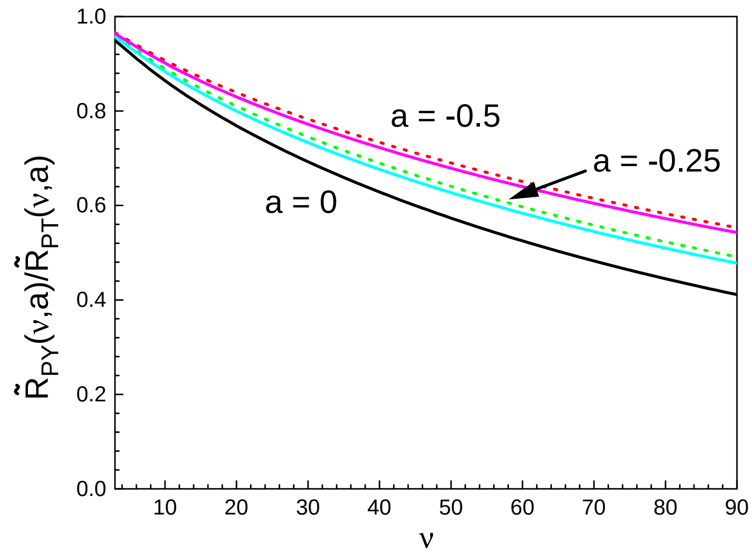
- Scaling property for τ_a event shapes
  (C.F. Berger & GS (2003) Berger and Magnea (2004)

- Test of rapidity-independence
  of NP dynamics

$$\ln \tilde{f}_{a,\mathrm{NP}}\left(\frac{\nu}{Q}, \kappa\right) = \frac{1}{1-a} \sum_{n=1}^{\infty} \lambda_n(\kappa) \left(-\frac{\nu}{Q}\right)^n$$

$$\tilde{f}_a\left(\frac{\nu}{Q}, \kappa\right) = \left[\, \tilde{f}_0\left(\frac{\nu}{Q}, \kappa\right)\right]^{\frac{1}{1-a}}$$

- What PYTHIA gives



- Most event shapes were invented for jet physics of the late 70's

- Address existing data with new analysis

- New observables to analyze final states;
  aid in searches for new physics
  (Tkachov (1995), C.F. Berger et al. (Snowmass, 2001))

$\star$ Application: power corrections for 1PI Cross Sections

- Joint Resummation (threshold $\otimes$ k_T) (Laenen,GS,Vogelsang (2001))
- Analyze transition: fixed target to collider energies
- "Implicit" logs of initial-state Q_T integrated
- Q_T integral (N imaginary) $\Rightarrow$

$$p_T^3 \frac{d\sigma_{ab}}{dp_T} \sim \int_{-i\infty}^{i\infty} dN \; \tilde{\sigma}_{ab}^{(0)}(N) \; \left(x_T^2\right)^{-N-1}$$

$$\times \; \mathrm{e}^{E\mathrm{thresh}(N,p_T)} \; \mathrm{e}^{\delta \, E\mathrm{recoil}(N,p_T)}$$

- Isolate perturbative recoil; NNLL in N:

$$\delta\, E_{\rm recoil}(N,p_T) = \delta E_{\rm PT} + \delta\, E_{{\rm N}P}$$
$$\delta E_{{\rm P}T} \propto \frac{\alpha_s(p_T^2/N^2)}{\pi}\,\frac{\zeta(2)}{2}$$

- isolate low scales $\leftrightarrow$ strong coupling

$$\delta\, E_{\rm NP} \quad = \quad \lambda_{ab}\,\frac{N^2}{p_T^2}\,\ln\frac{p_T}{N}$$

$$N \;\leftrightarrow\; \frac{1}{\ln x_T^2}$$

- Leading power suppression quadratic in $1/p_T$

$$\delta E_{\rm recoil} \;=\; {\rm PT} \,+\, {\rm const.}\, \frac{1}{p_T^2 \ln^2\left(\frac{4p_T^2}{S}\right)} \ln\left(p_T \ln\left(\frac{4p_T^2}{S}\right)\right)$$

- Also decreases with S at fixed p_T
- Insight into how NLO gets better: fixed target $\Rightarrow$ colliders

⋆ Hopeful Conclusions

- Energy flow is common language of hadronic and nuclear scattering.
- Resummations bring pQCD to the doorstep of nonperturbative field theory.
- Study of color and energy flow in hadronic scattering will shed light on the PT $\rightarrow$ NP transition.
- Eventually we will learn to translate fully the language of partons into the language of hadrons for the full range of initial conditions.