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Critical Behaviour in QCD

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The Fundamental Problems of Physics

constituents

quarks
leptons
gluons, photons
vector bosons (Z, $W^{\pm}$)
Higgs

forces

strong
e-m
weak
gravitation
unification, TOE

elementary interactions

$\Downarrow$

complex systems, critical behaviour

states of matter

solid, liquid, gas
glass, gelatine
insulator, conductor
superconductor, ferromagnet
fluid, superfluid

transitions

thermal phase transitions
percolation transitions
scaling and renormalization
critical exponents
universality classes

Complex Systems ⇒ New Direction in Physics

- **Given constituents and dynamics of elementary systems, what is the behaviour of complex systems?**
- **What are the possible states of matter and how can they be specified?**
- **How do transitions from one state of matter to another occur?**
- **Is there a general pattern of critical phenomena, independent of specific dynamics?**

Contents

1. The Physics of Complex Systems

1.1 Critical Behaviour in Thermodynamics

Phase transitions are common everyday occurrences

ice $\leftrightarrow$ water $\leftrightarrow$ steam,

melting of metals, magnetization of iron,

insulator $\leftrightarrow$ conductor, ...

But:

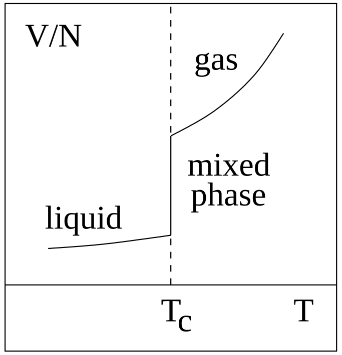
they are difficult to treat, because one cannot reduce a complex system to a sum of elementary systems;

therefore new methods of analysis are needed

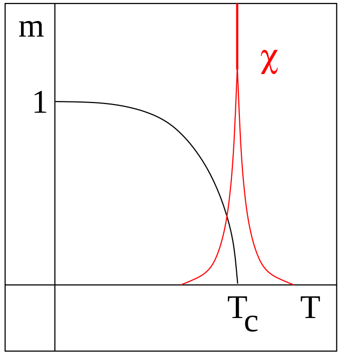
basic feature of critical phenomena:

discontinuous or singular behaviour of observables

examples: **water → steam** **magnetization**



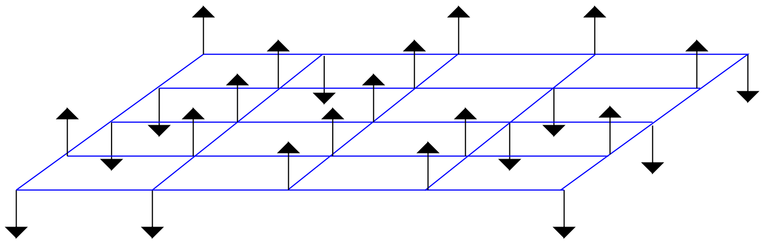
1st order transition



continuous transition

Ising model

d-dimensional lattice grid, N^d sites with spins $s_i = \pm 1\ \forall i = 1, ..., N^d$, uniform next neighbor interaction $-Js_is_{i+1}$



properties of the system are determined by the <u>partition function</u>

$$Z(T,H\!=\!0,N) = \Pi_{i=1}^{N^d} \Sigma_{s_i=\pm 1} \overbrace{\exp\{\,\beta J \sum_{i,j}^{nn} s_i s_j - \beta H \sum_i s_i\}}^{\exp -\beta\mathcal{H}}$$

temperature $T = \beta^{-1}$**, external field** H**; take** $H = 0$

$Z(T,H\!=\!0;N)$ **has <u>global symmetry</u>** (Z_2)**:**

$$s_i \to -s_i \quad \forall\ i = 1, ..., N^d$$

leaves sum over all states $Z(T,H\!=\!0;N)$ **<u>invariant</u>**

for high temperatures, system agrees:
on the average, $\exists$ **disorder, as many spins** $\uparrow$ **as** $\downarrow$

but below a certain temperature:
$\exists$ **order** $\Rightarrow$ **<u>spontaneous symmetry breaking</u>**

on the average, more $\uparrow$ **or more** $\downarrow$

Z_2 **invariance of** $Z(T,H\!=\!0;N)$**:** $\uparrow$ **and** $\downarrow$ **equally likely**

need additional measure to specify state of system: **order parameter**

$$m(T,N) = \frac{1}{Z(T,N)} \prod_{i=1}^{N^d} \sum_i \left[\frac{\sum_i s_i}{N^d}\right] \exp\{\, \beta J \sum_{i,j}^{nn} s_i s_j \}$$

under reflection $s_i \to -s_i \;\forall\; i = 1, ..., N^d$: $m(T,N) \to -m(T,N)$

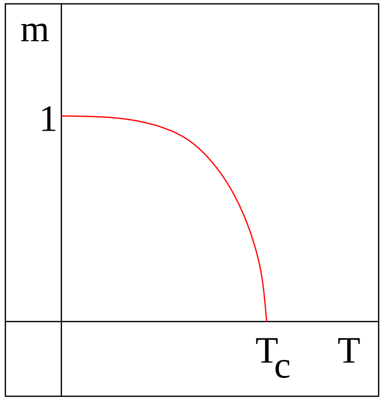
order parameter is not invariant; consequence

$$m(T) \begin{cases} \neq 0 & \text{for ordered state, broken symmetry} \\ = 0 & \text{for disordered state, symmetry} \end{cases}$$

thermodynamic limit $N \to \infty$**:**

$m(T, H = 0)$ **is not analytic ("smooth")**

$$m(T) \sim \begin{cases} (T - T_c)^{\beta} > 0 & \forall\; T < T_c \\ 0 & \forall\; T > T_c \end{cases}$$



$\Rightarrow$ **critical exponent** β

also other observables show singular behaviour:

free energy $F(T,H) = -T \log Z(T,H)$

temperature measure $t = (T - T_c)/T_c$

external field measure $h = H/T$

specific heat

$$C_H = T^2 \left(\frac{\partial^2 F}{\partial T^2} \right)_{H=0} \sim |t|^{-\alpha}$$

spontaneous magnetization

$$m(t, h=0) = -\frac{1}{N^d} \left(\frac{\partial F}{\partial H} \right)_{H=0} \sim |t|^{\beta}$$

susceptibility

$$\chi_T = \left(\frac{\partial m}{\partial H} \right)_{H=0} \sim |t|^{-\gamma}$$

magnetization on critical isotherm

$$m(t=0, h) = -\frac{1}{N^d} \left(\frac{\partial F}{\partial H} \right)_{t=0} \sim h^{1/\delta}$$

besides global also local observables diverge:

correlation function $\Gamma(r,t) \sim \langle s_i s_{i+r} \rangle \sim \exp -r/\xi$

correlation length diverges

$$\xi \sim t^{-\nu}$$

$t \neq 0$: correlation length finite, dimensional scale, given spin does not see far-away other spins

$t = 0$: correlation length diverges, no scale, all spins are correlated, the system cannot be split into independent subsystems

requires new physics: infinite correlated system

$\Rightarrow$ scaling and renormalization (Kadanoff, Wilson)

But: why is there singular behaviour?

transition $\sim$ onset of spontaneous symmetry breaking: "either-or", nothing gradual or smooth; you cannot break symmetry "a little".

rescale distances, temperature, external field

$$r \to r' = br, \quad t \to t' = b^{y_t} t, \quad h \to h' = b^{y_h} h$$

all physics must remain the same

$\Rightarrow$ **all critical exponents given in terms of** y_t, y_h

$\Rightarrow$ **critical behaviour fully specified by** y_t, y_h

$\Rightarrow$ y_t, y_h **define** **universality class**

$\Rightarrow$ **Thermodynamic Critical Behaviour*** $\Leftarrow$

- **onset of spontaneous symmetry breaking**
- **singular behaviour of thermodynamic observables**
- **critical exponents, universality class**

* continous transitions

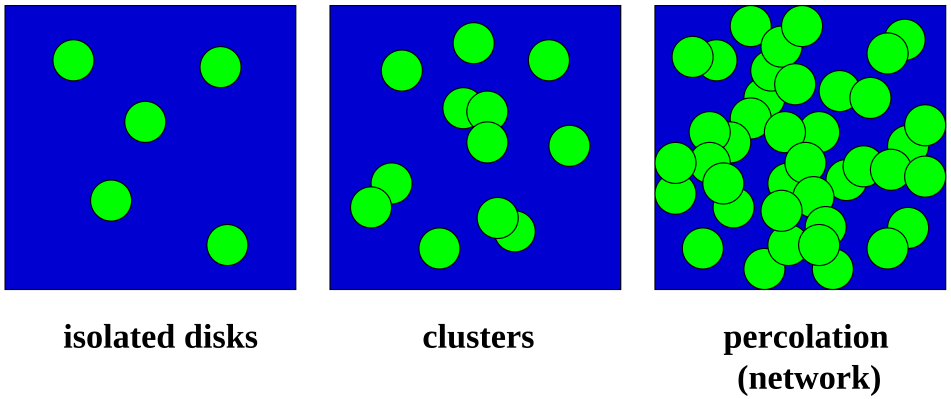
1.2 Cluster Formation and Percolation

thermal transitions, critical behaviour: dynamics ($\mathcal{H}$), symmetry

for constituents with intrinsic scale,
$\exists$ simpler, geometric form of critical behaviour:

$\Rightarrow$ formation of infinite connected clusters or networks

example: 2-d disk percolation (lilies on a pond)



distribute small disks of area $a = \pi r^2$ randomly on large area $F = L^2$, $L \gg r$, with overlap allowed: when can an ant walk across?

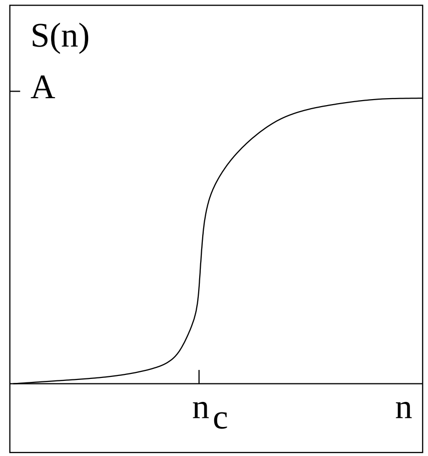
for N disks, disk density $n = N/F$

average cluster size $S(n)$ increases
with increasing density n

suddenly, for $n \to n_c$, $S(n)$ becomes
large enough to span the pond: $S \sim F$

for $N \to \infty, A \to \infty$:

$S(n_c)$ and $(dS(n)/dn)_{n=n_c}$ diverge: $\Rightarrow$ percolation



percolation as geometric critical behaviour:
large size limit $\sim$ thermodynamic limit

2 d (disks): $n_c \simeq 1.13/\pi\, r^2$, 0.68 of space covered, 0.32 empty
when an ant can cross, a ship cannot, and vice versa: 2d effect

3 d (spheres): $n_c \simeq 0.34/(4\pi/3)\, r^3$, 0.29 of space covered, 0.71 empty
both cluster and empty space connected
$n_c \simeq 1.24/(4\pi/3)\, r^3$, 0.71 of space covered, 0.29 empty
connected vacuum disappears

probability $P(n)$ that a given disk is in the infinite cluster

$$P(n) \begin{cases} = 0 & \forall \; n < n_c \\ \sim (n - n_c)^{\beta} & \text{for } n \to n_c \text{ from above} \end{cases}$$

⇒ <u>order parameter for percolation</u>

measure <u>average cluster size</u> (excluding infinite cluster)

$$\tilde{S}(n) \simeq |n - n_c|^{-\gamma}$$

∼ <u>susceptibility</u> in thermodynamic system

... other observables: again singular behaviour

→ critical exponents, universality classes

NB: here <u>random</u> distribution of disks/spheres

but distribution law not essential – can also use thermodynamic or any other form of distribution

again many everyday examples:

make pudding, boil an egg: gelatinization

conductivity in random networks, ‘ant in a labyrinth’

start a forest fire, find an oil field, ...

instead of symmetry breaking:

disconnected $\rightarrow$ connected system

$\Rightarrow$ Geometric Critical Behaviour $\Leftarrow$

- onset of infinite cluster formation
- singular behaviour of geometric observables
- critical exponents, universality class

Again, why singular behaviour?

onset of connection is “either-or”, you cannot connect “a little”.

thermodynamic vs. geometric critical behaviour?

thermodynamic transitions:

- $\exists$ **interaction dynamics for constituents**
- **equal** *a priori* **phase space probabilities**
- **state of system can spontaneously break symmetry of partition function**
- $\rightarrow$ **non-analytic partition function**

geometric transitions:

- $\exists$ **interaction range, size for constituents**
- **arbitrary distribution of constituents**
- **cluster formation, connection**
- **spontaneous onset of global connection, divergence of cluster size: percolation**

in both cases, singular behaviour

2. Critical Behaviour in Statistical QCD

2.1 Phases of Strongly Interacting Matter

What happens to strongly interacting matter at high temperature and/or density?

- hadrons have intrinsic size $r_h \simeq 1$ fm, need $V_h \simeq (4\pi/3)r_h^3$ to exist

  $\Rightarrow$ limiting density of hadronic matter

  $n_c = 1/V_h \simeq 1.5\ n_0$ [Pomeranchuk 1951]

- resonances $\rightarrow$ exponential hadron spectrum $\rho(m) \sim \exp(bm)$

  – statistical bootstrap model [Hagedorn 1968]

  – dual resonance model

  [Fubini & Veneziano 1969; Bardakçi &Mandelstam 1969]

  $\Rightarrow$ limiting temperature of hadronic matter

  $T_c = 1/b \simeq 150 - 200$ **MeV**

$\Rightarrow$ **what lies beyond n_c, T_c?** $\Leftarrow$

- **quark liberation**

  hadronic matter: colorless constituents of hadronic dimension

  $\Downarrow$

  quark-gluon plasma: pointlike colored constituents

  $\Rightarrow$ **deconfinement: insulator-conductor transition in QCD**

- **quark mass shift**

  at $T = 0$, quarks 'dress' with gluons $\rightarrow$ constituent quarks
  bare quark mass $m_q \sim 0 \rightarrow$ constituent quark mass $M_q \sim 300$ MeV

  in hot medium, dressing 'melts' $M_q \rightarrow 0$

  for $m_q = 0$, $\mathcal{L}_{QCD}$ has chiral symmetry

  $M_q \neq 0 \rightarrow$ **spontaneous chiral symmetry breaking**
  $M_q \rightarrow 0 \Rightarrow$ **chiral symmetry restoration**

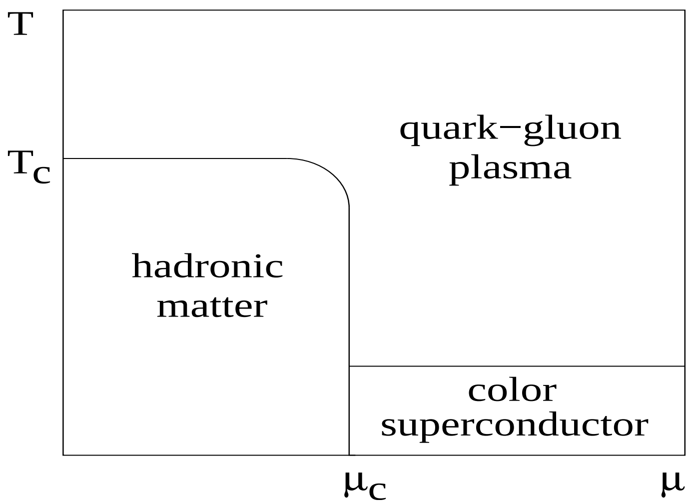
- diquark matter

  deconfined quarks $\sim$ attractive interaction

  can form colored bosonic 'diquark' pairs (QCD's Cooper pairs)

  form condensate $\Rightarrow$ color superconductor

- expected phase diagram of QCD:



baryochemical potential $\mu \sim$ baryon density.

2.2 From Hadrons to Quarks and Gluons

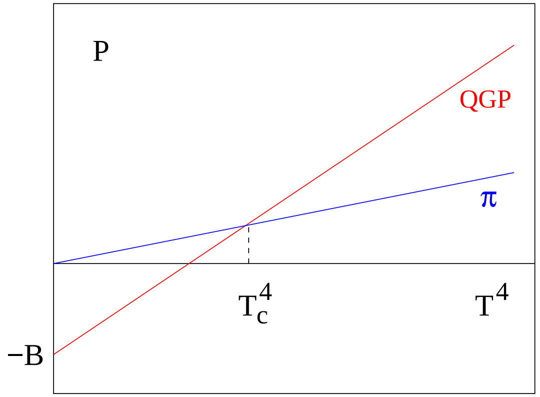
simplest confined matter: ideal pion gas $P_\pi = \frac{\pi^2}{90}\, 3\, T^4 \simeq \frac{1}{3}\, T^4$

simplest deconfined matter: ideal quark-gluon plasma

$$P_{QGP} = \frac{\pi^2}{90}\, \{ 2 \times 8 + \frac{7}{8}\, [2 \times 2 \times 2 \times 3] \}\, T^4 - B \simeq 4\, T^4 - B$$

with bag pressure B for outside/inside vacuum

$\Rightarrow$ **compare $P_\pi(T)$ and $P_{QGP}(T)$ vs. T**



phase transition from hadronic matter at low T to QGP at high T

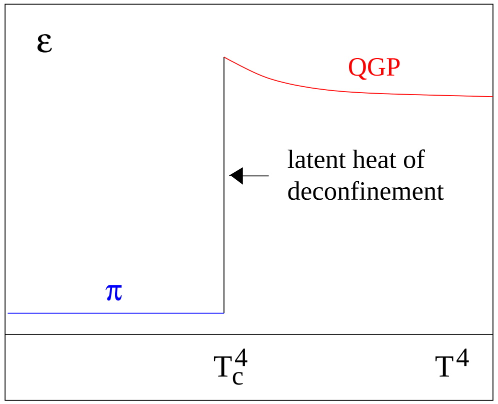
critical temperature:

$$P_\pi = P_{QGP} \rightarrow T_c^4 \simeq 0.3\ B \simeq 150\ \text{MeV}$$

with $B^{1/4} \simeq 200$ **MeV from quarkonium spectroscopy**

corresponding energy densities

$$\epsilon_\pi \simeq T^4 \rightarrow \epsilon_{QGP} \simeq 12\ T^4 + B$$



at T_c, energy density changes abruptly by latent heat of deconfinement

so far, simplistic model; real world?

2.3 Finite Temperature Lattice QCD

given QCD as dynamics input, calculate resulting thermodynamics, based on QCD partition function

⇒ lattice regularization

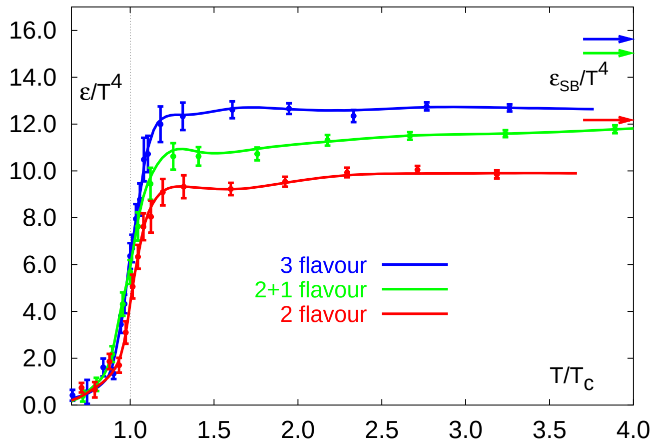
- energy density

⇒ latent heat of deconfinement

For $N_f = 2, 2+1$:

$T_c \simeq$ 175 MeV

$\epsilon(T_c) \simeq 0.5 - 1.0$ GeV/fm^3



explicit relation to deconfinement, chiral symmetry restoration?

⇒ order parameters

- **deconfinement** $\Rightarrow m_q \to \infty$

Polyakov loop $L(T) \sim \exp\{-F_{Q\bar{Q}}/T\}$

$F_{Q\bar{Q}}$: **free energy of** $Q\bar{Q}$ **pair for** $r \to \infty$

$$L(T) \begin{cases} = 0 & T < T_L \text{ confinement} \\ \\ \neq 0 & T > T_L \text{ deconfinement} \end{cases}$$

variation defines deconfinement temperature T_L

- **chiral symmetry restoration** $\Rightarrow m_q \to 0$

chiral condensate $\chi(T) \equiv \langle \bar{\psi}\psi \rangle \sim M_q$

measures dynamically generated ('constituent') quark mass

$$\chi(T) \begin{cases} \neq 0 & T < T_\chi \text{ chiral symmetry broken} \\ \\ = 0 & T > T_\chi \text{ chiral symmetry restored} \end{cases}$$

variation defines chiral symmetry temperature T_χ

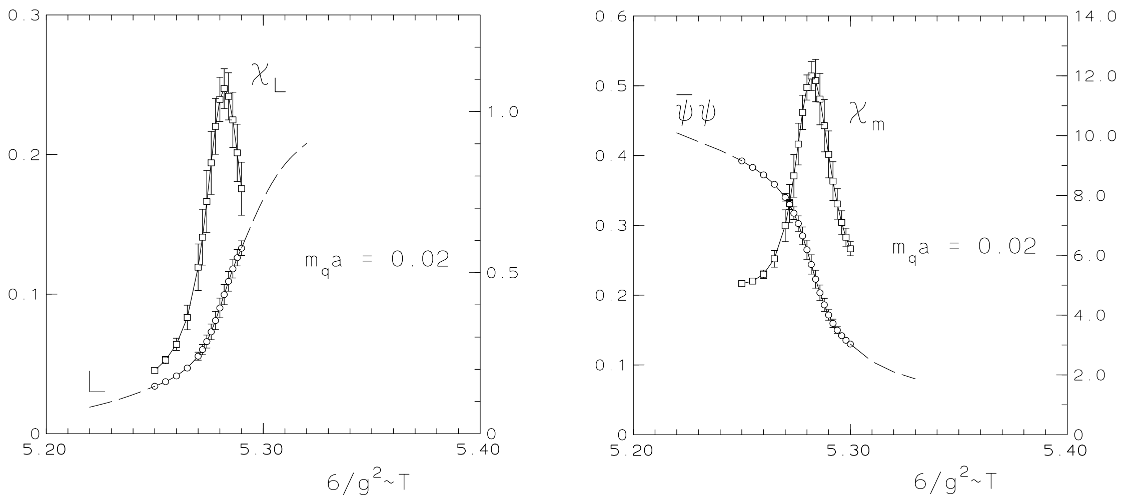
- **how are T_L and T_χ related?**

pure $SU(N)$ gauge theory: $\sim$ <u>spontaneous</u> Z_N breaking at T_L

full QCD, chiral limit: $\sim$ <u>explicit</u> Z_N breaking by $\chi(T) \to 0$ at T_χ

<u>chiral symmetry restoration $\Rightarrow$ deconfinement</u>

lattice results



Polyakov loop & chiral condensate vs. temperature

at $\mu = 0$, $\exists$ <u>one transition</u> hadronic matter $\rightarrow$ QGP

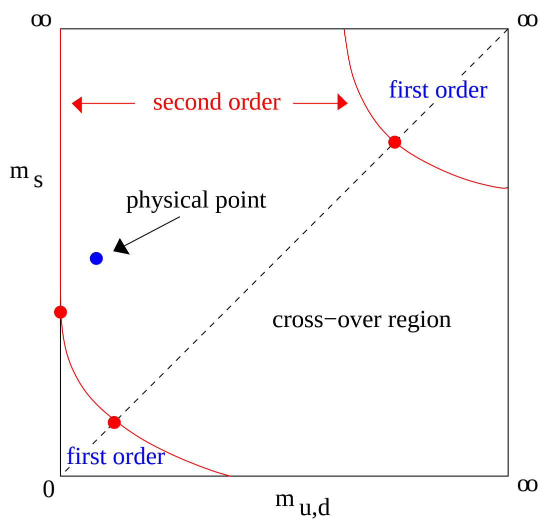
for $N_f = 2, m_q \to 0$ at $T_c = T_L = T_\chi \simeq 175$ MeV

- <u>**nature of transition**</u> **at** $\mu = 0$
  - **for** $m_q \to \infty$ **(pure gauge theory)** **spontaneous** Z_N **breaking** $\to$ **deconfinement transition**
  - **for** $m_q \to 0$**, spontaneous chiral symmetry breaking** $\to$ **chiral transition**
  - **for finite quark masses, no spontaneous symmetry breaking or restoration, hence in general no singular behaviour**
  - **both** $L(T)$ **and** $\chi(T)$ **vary sharply for all** m_q**, define common transition point** T_c
  - **what kind of transition?**

depends on N_f **and** m_q**:**

continuous, first order

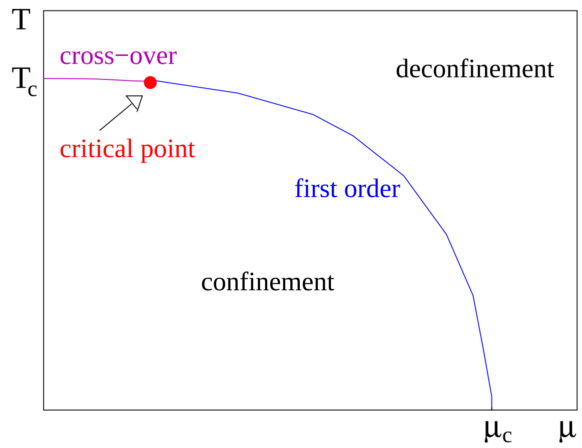
"rapid" cross-over



- <u>non-zero net baryon density</u>

  $(\mu \neq 0, N_b > N_{\bar{b}},\ \ N_f = 2+1)$

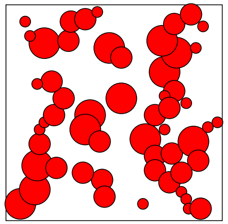
  computer algorithms break down:
  reweighting, analytic continuation,
  power series...; expect:

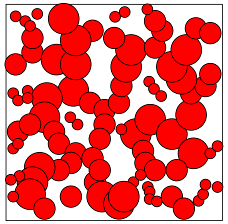


  <u>critical point</u> in $T - \mu$ plane depends on position of <u>physical point</u> in $m_s - m_{u,d}$ plane

- <u>cross-over</u> region (the real world): <u>enigmatic</u>
  - no thermal singularity, no thermal phase transition
  - so what does it mean: new state of matter?
  - observables change rapidly
  - clear transition in entire region: why?
  - what is the transition mechanism?

- **hadronic matter is formed when connected cluster is possible, deconfinement occurs when connected vacuum "disappears"**





$$n_c = \frac{0.34}{(4\pi)\, r_h^3} \qquad \bar{n}_c = \frac{1.24}{(4\pi)\, r_h^3}$$

end of hadronic state at $\mu \simeq 0$: interacting medium of hadrons

resonance domination $\Rightarrow$ ideal gas of hadrons/resonances;

at what T is $n_h(T) = n_c$?

$$\boxed{T_c \simeq \ 170 \text{ MeV}}$$

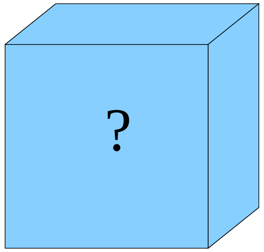
deconfinement as percolation:

when a hadronic medium becomes so dense that only isolated vacuum bubbles survive, then it becomes a quark-gluon plasma

3. Probing Matter in Statistical QCD

given a box of strongly interacting matter in thermal equilibrium,
how can *theorists* determine its state through QCD calculations?

NB:
equilibrium thermodynamics, no collision dynamics, time dependence, equilibration, expansion, cooling, etc.



3.1 Interaction Range and Colour Screening

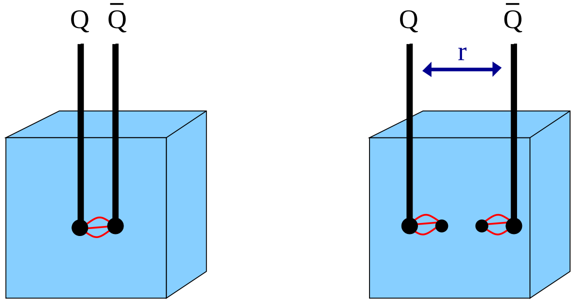
static quark/antiquark in medium: interaction vs. separation?

at $T = 0$, confining "string" potential

$$V(r) \sim \sigma\, r$$

string breaks for $V(r) \geq 2M_q$

$\Rightarrow$ two light-heavy mesons $(Q\bar{q})$, $(\bar{Q}q)$

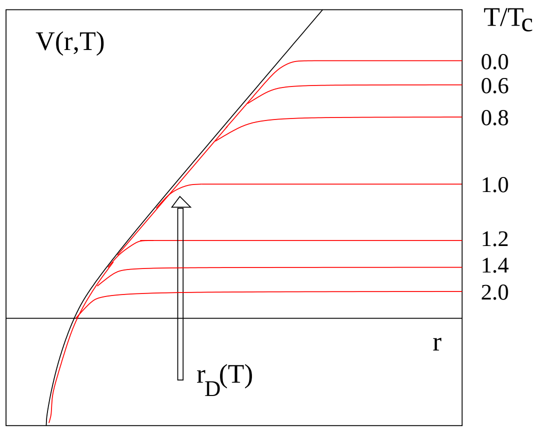


with increasing temperature,

potential strength and
range reduced (from LL^{+} correlations)

string breaks earlier

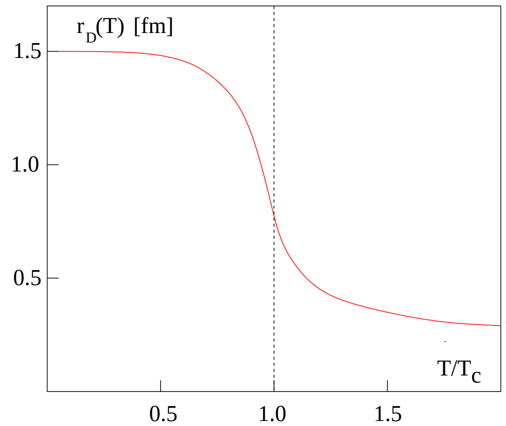
$\Rightarrow$ colour screening



(**Bielefeld**, $16^3 \times 4$, $N_f = 2$, $m_q/T = 0.4$)

screening radius $\sim$ interaction range

drop sharply as $T \to T_c$



string breaking point falls

from $r \simeq 1.5$ fm to $r \simeq 0.3$ fm

for $T/T_c = 0$ to $T/T_c = 2$

3.2 Light Hadron Spectroscopy

look at mass spectrum of virtual photons emitted from box

$$\gamma^* \to e^+e^-$$

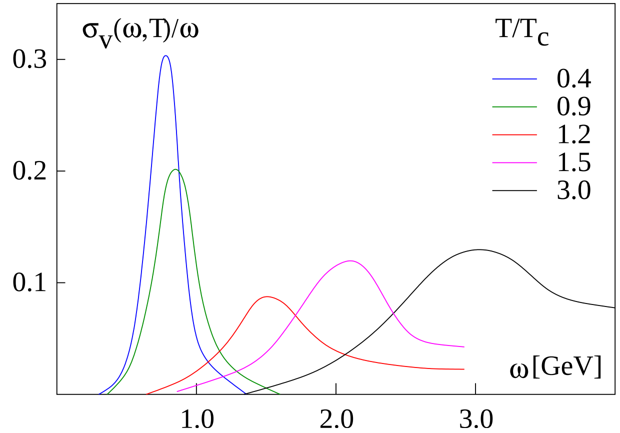
expect:

in hadronic phase $\rho \to \gamma^* \to e^+e^-$

so that $M(\gamma^*) \simeq M(\rho)$

in QGP phase $q\bar{q} \to \gamma^* \to e^+e^-$

so that $M(\gamma^*) \sim T$



lattice calculations:

(Bielefeld, quenched QCD, $64^3 \times 16$**)**

confined state: hadronic scale, peak at ρ **mass**

position $\sim$ **temperature-independent**

deconfined state: temperature scale, broad peak at position $\sim T$

3.2 Charmonium Spectroscopy

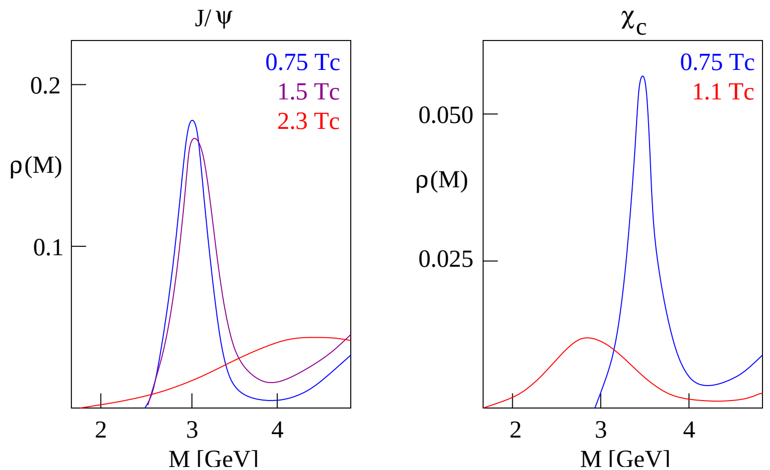
existence of heavy quark-antiquark bound states (J/ψ, χ_c, ψ',...) as indicator of nature and temperature of medium

lattice calculations for spectral functions of $c\bar{c}$ systems
(Bielefeld, quenched QCD, $48^3 \times 10 - 24$)

J/ψ **persists up to** $2.3\ T_c > T \geq 1.5\ T_c$

χ_c **is dissociated for** $T \geq 1.1\ T_c$

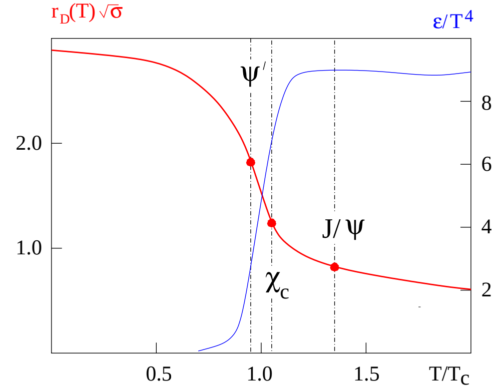
widths?



cross check:

compare to interaction range, potential models (Schrödinger equ'n)

χ_c **and** ψ' **analyze deconfinement transition**



Critical Behaviour in QCD

for $\mu \simeq 0$ and all values of m_q, N_f

$\exists$ a well-defined transition temperature T_c at which

- **deconfinement sets in**
- **chiral symmetry is restored**
- **latent heat of deconfinement increases energy density**
- **colour screening decreases interaction range**
- **dilepton spectra go from hadron decay to thermal annihilation**
- **charmonium dissociation analyzes transition region**

To study critical behaviour, you must find the transition point

and determine how the system and its observables

change from one side to the other.

To illustrate relation thermal vs. geometric, again see Ising model

- $H=0$: **use Ising dynamics for cluster definition**

  $\Rightarrow$ percolation $\equiv$ magnetization transition

  equivalent formulations of same phenomenon

- $H\neq 0$, **no thermodynamic transition**

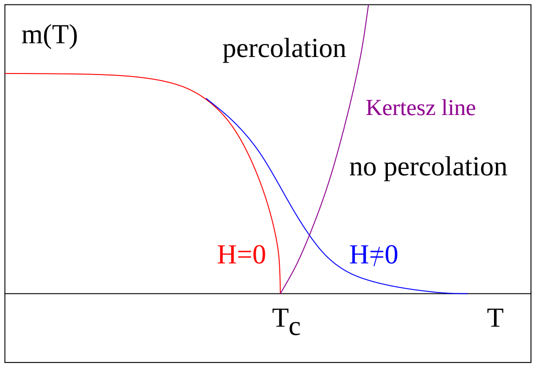
  partition function is analytic

  symmetry is always broken

  percolation persists: $\Rightarrow$ "Kertesz line"

- **thermodynamic $\sim$ geometric**

  geometric $\not\sim$ thermodynamic



percolation can occur even when partition function is analytic – cluster observables still diverge

> **...there are more critical phenomena in nature than the partition function knows of...**

So what does happen along the "Kertesz line"?

2-d Ising model, external field $H \uparrow$

consider average number $n(S)$ **of clusters of size** S **of** $\downarrow$ **spins:**

$$n(S) \sim \frac{\exp\{-hS - \Gamma(T)S^{1/2}\}}{S^\tau} \sim \frac{\exp\{-hS\,[1-(\Gamma(T)/h)S^{-1/2}\,]\,\}}{S^\tau}$$

with "bulk" term $h\,S$ **and "surface" term** $\Gamma\,S^{1/2}$

surface pressure $\Gamma(T)$ **is order parameter for percolation**

$$\Gamma(T) \sim \begin{cases} (T-T_k)^{\beta_k} > 0 & \forall\ T < T_k \\ 0 & \forall\ T > T_k \end{cases}$$

defines Kertesz line, is singular even for analytic partition function

in thermodynamic limit $S \to \infty$**, surface term does not contribute**

percolation $\sim$ NLO critical behaviour