Dilepton and photon production:
perturbation theory vs lattice QCD

Hard Probes 2004

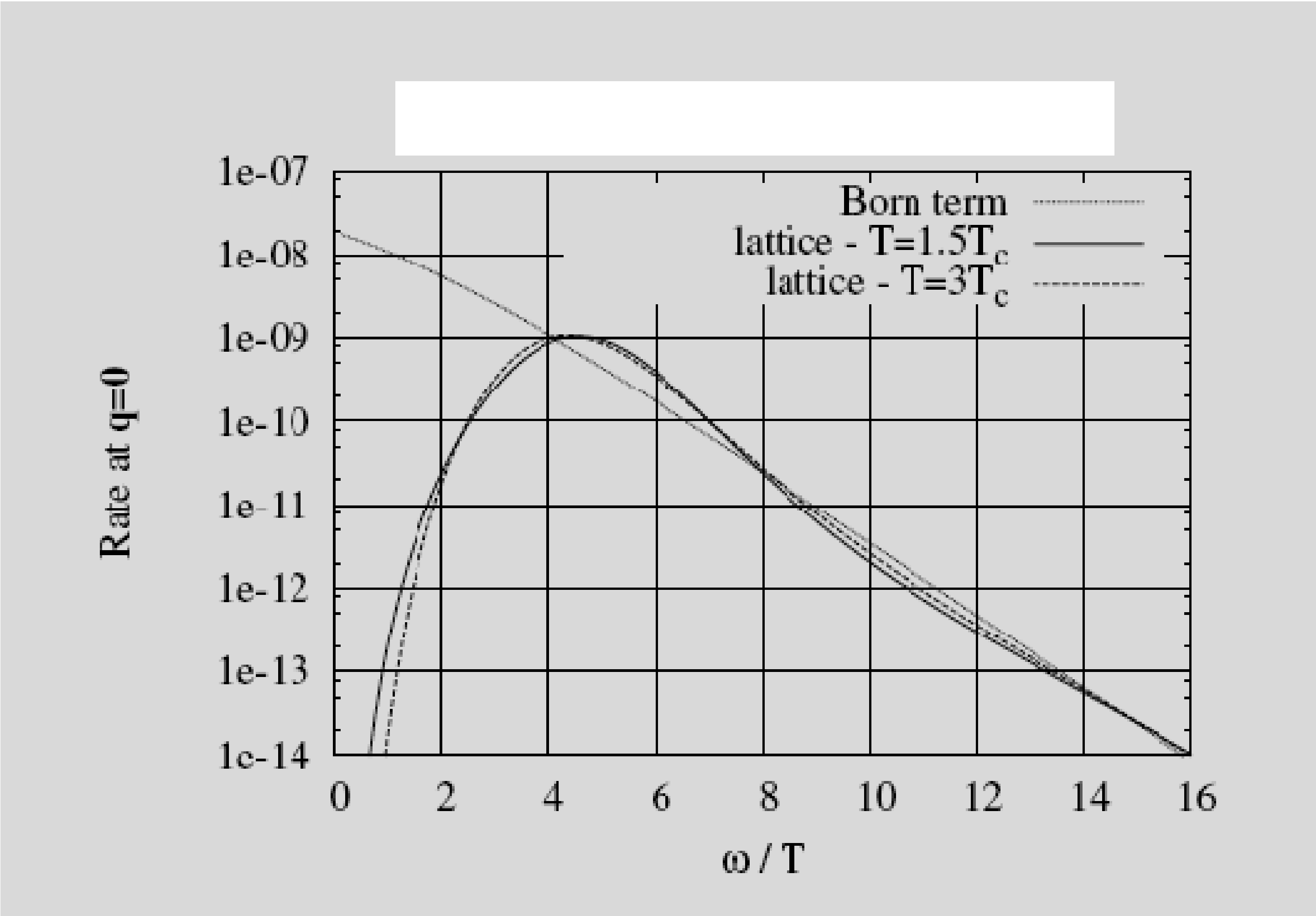
Ericeira, Portugal

November 9, 2004

Jean-Paul Blaizot, CNRS and ECT*

François Gelis, SPhT-CEA Saclay

Rate of dilepton production (p=0)

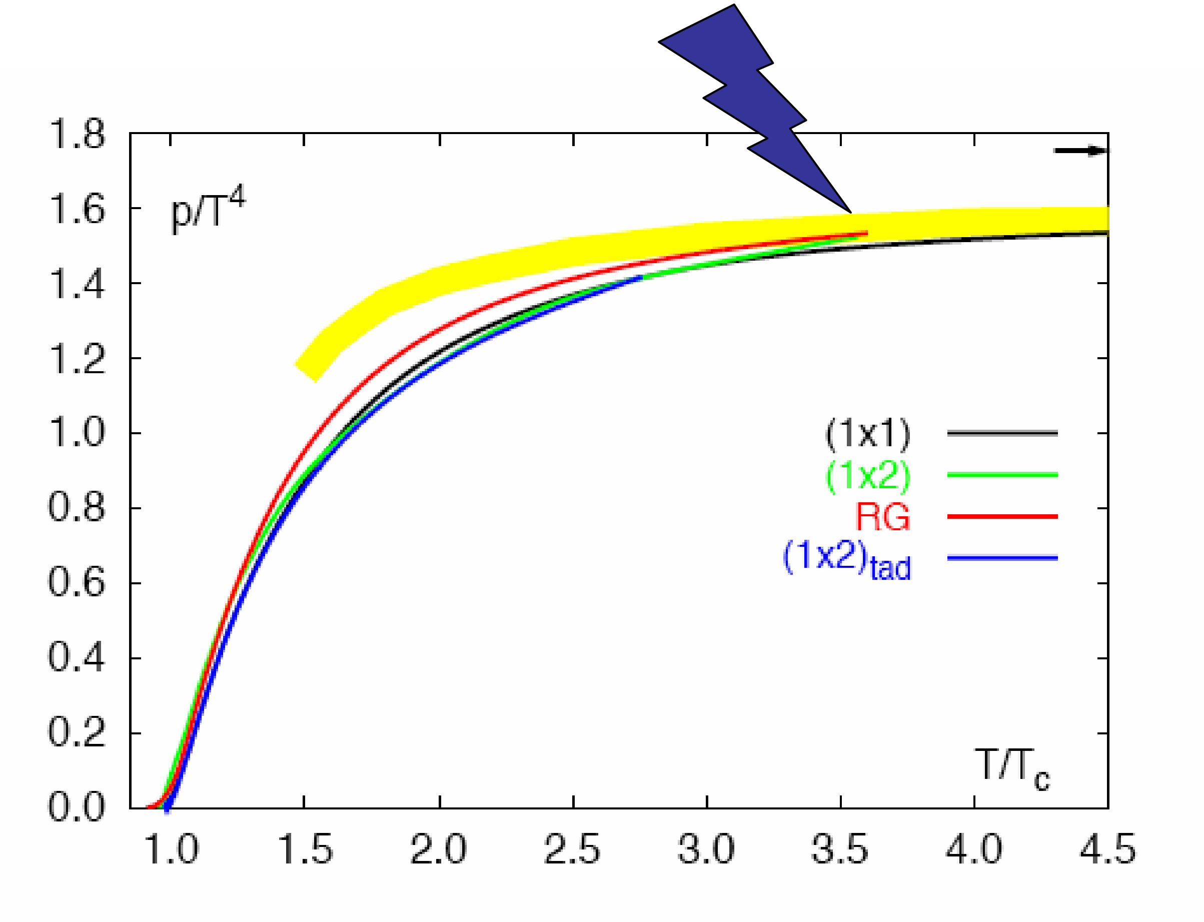


Karsch et al, hep-lat/0110208

Plot from F. Gelis, hep-ph/0209072

Weak coupling calculations

Weakly interacting quasiparticles



(from F. Karsch, hep-lat/0106019)

J.-P. Blaizot, E. Iancu and A. Rebhan, Phys. Lett. B470, 181 (1999)

Scales and degrees of freedom in the weakly coupled quark-gluon plasma

$$g \ll 1$$

Hard modes = plasma particles

$$k \sim T$$

Soft modes = collective excitations

$$k \sim gT$$ (coupled to hard modes)

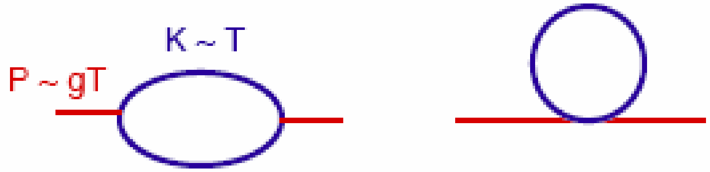
Ultra Soft modes (« magnetic sector »)

$$k \sim g^2 T$$

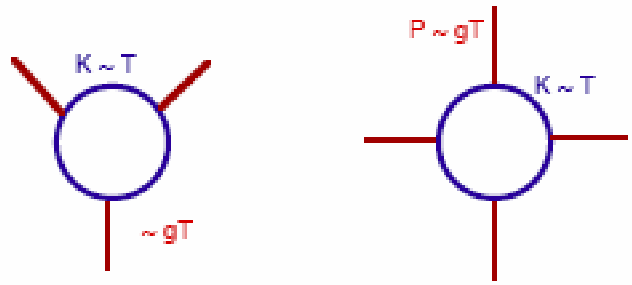
HARD THERMAL LOOPS (1)

[Braaten and Pisarski (90), Frenkel and Taylor (90)]

- **Large thermal contributions to the dynamics of the soft fields.**



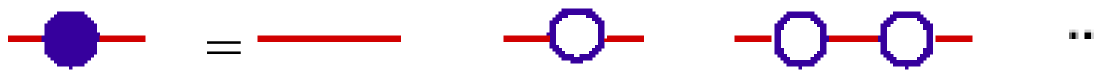
$$\Pi(P) \sim g^2 T^2 \sim P^2 \qquad \text{for} \qquad P \sim gT$$



$$g^3 \frac{T^2}{P} \sim gP \qquad\qquad g^4 \frac{T^2}{P^2} \sim g^2$$

HARD THERMAL LOOPS (2)

- **Resumed propagator** $D(\omega, q)$



$$D_0 = \frac{1}{\omega^2 - p^2} \longrightarrow D = \frac{1}{\omega^2 - p^2 - \Pi(\omega, p)}$$

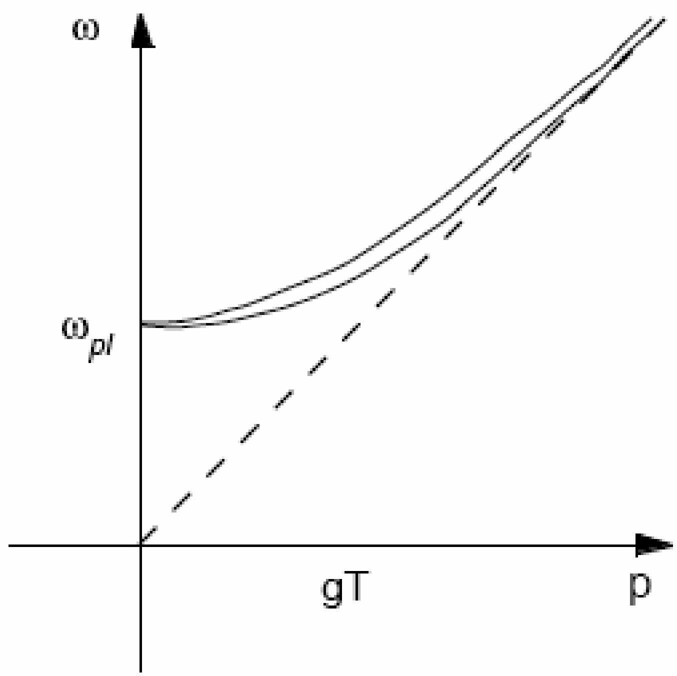
- **Debye screening:**

$$\Pi_{el}(\omega \ll p) \simeq m_D^2 \Longrightarrow D_{el}(\omega \ll p) \simeq \frac{1}{p^2 + m_D^2}$$

- **Dynamical . screening:**

$$D_{mag}(\omega \ll p) \simeq \frac{1}{p^2 - i\frac{\omega}{p}\, m_D^2}$$

Gluonic collective modes

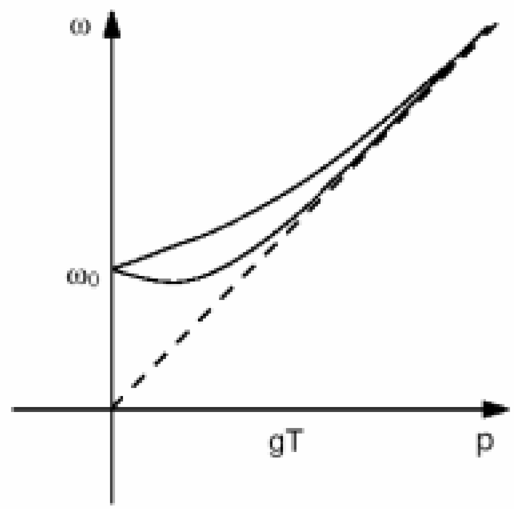


Dispersion relations for the modes $\omega_L(p)$ and $\omega_T(p)$

$$p^2 + \Pi_L(\omega_L, p) = 0, \qquad \omega_T^2 = p^2 + \Pi_T(\omega_T, p), \qquad \omega_{pl} \equiv m_D/\sqrt{3}$$

"asymptotic mass": $$m_\infty^2 \equiv \Pi_T^{1-loop}(\omega^2 = p^2) = \frac{m_D^2}{2}$$

Collective fermionic excitations



$$p \gg \omega_0 \qquad \omega_+^2(p) \simeq p^2 + M_\infty^2 \,, \qquad M_\infty^2 \equiv 2\omega_0^2$$

$$p \ll \omega_0 \qquad \omega_+(p) \simeq \omega_0 + \frac{p}{3} + \cdots \,, \qquad \omega_-(p) \simeq \omega_0 - \frac{p}{3} + \cdots \,,$$

Effect of collisions
Width of quasiparticles

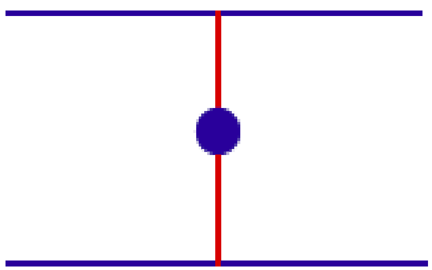
$$\gamma = n\sigma \qquad n \sim T^3$$

$$\sigma = \int \mathrm{d}q^2 (\mathrm{d}\sigma/\mathrm{d}q^2) \qquad \mathrm{d}\sigma/\mathrm{d}q^2 \sim g^4/q^4$$

$$\gamma \sim g^4 T^3 \int \mathrm{d}q^2 \frac{1}{q^4}$$

Damping is anomalously large

$$\gamma \sim g^4 T^3 \frac{1}{m_D^2} \sim g^2 T$$



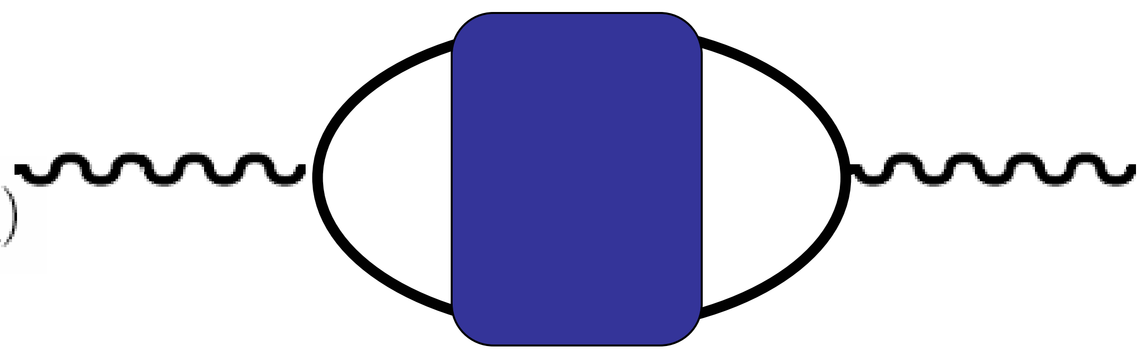
Rate of photon production

$$\omega \frac{dN_\gamma}{d^4x d^3q} = -\frac{e^2 g^{\mu\nu}}{2(2\pi)^3} \Pi^<_{\mu\nu}(\omega, q) = \frac{e^2}{(2\pi)^3} \frac{g_{\mu\nu}}{e^{\omega/T} - 1} \, \mathrm{Im}\, \Pi^{\mu\nu}_{\mathrm{ret}}(\omega, q)$$

$$\Pi^<_{\mu\nu}(\omega, q) = \int \mathrm{d}^4x \, \mathrm{e}^{iQ\cdot x} \, \langle j_\mu(0) j_\nu(x) \rangle$$

$$j_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$$

$Q = (\omega, \mathbf{q})$



Rate of dilepton production

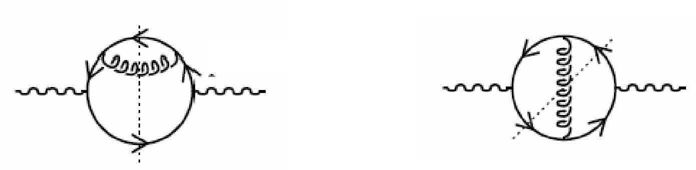
$$\frac{dN_{ll}}{d^4x d^4Q} = \frac{e^4}{3(2\pi)^4 Q^2} \frac{B g_{\mu\nu}}{e^{\omega/T} - 1} \, \mathrm{Im}\, \Pi^{\mu\nu}_{\mathrm{ret}}(\omega, q)$$

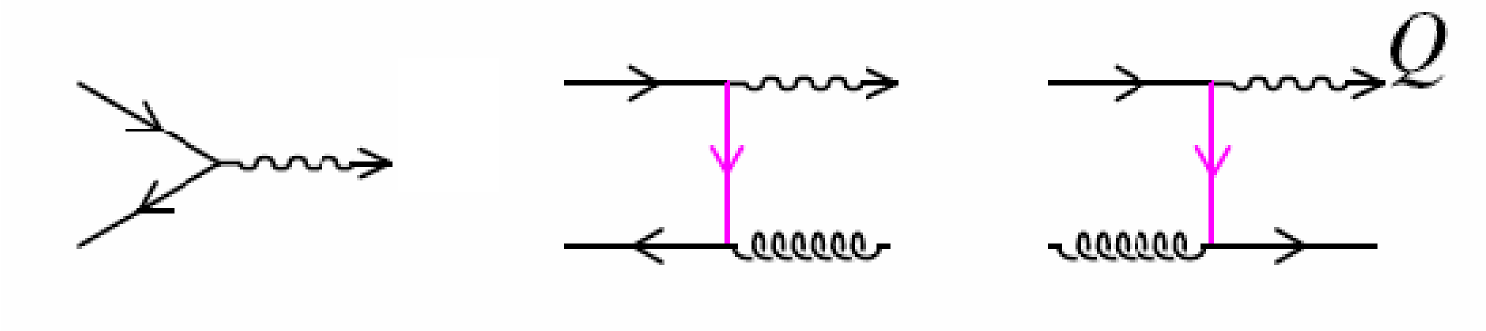
$$B = \left(1 + \frac{2m^2}{Q^2}\right) \left(1 - \frac{4m^2}{Q^2}\right)^{1/2}$$

McLerran, Tomeila (1985)
Weldon (1990)
Gale,Kapusta (1991)

Leading order calculations

McLerran, Tomeila (1985)
Baier, Pire, Schiff (1988)
Alther, Aurenche, Becherrawy (1989)

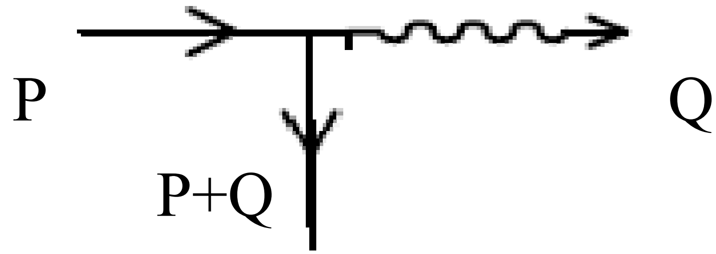




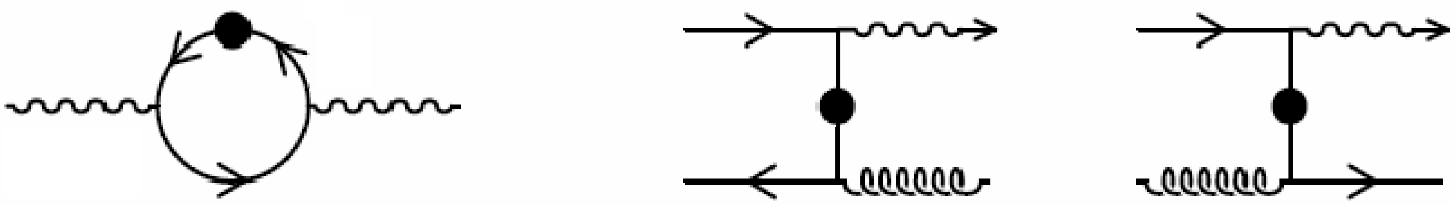
$$\mathrm{Im}\,\Pi_{\mathrm{ret}}(\omega, \vec{q}) \propto \alpha\alpha_s \ln(\omega T/Q^2)$$

Infrared divergent for real photons $(Q^2 = 0)$

HTL resummations



$$(P+Q)^2 - m^2 = 2P \cdot Q = \frac{m_\perp^2}{p_z} q \qquad m_\perp^2 = p_\perp^2 + m^2$$

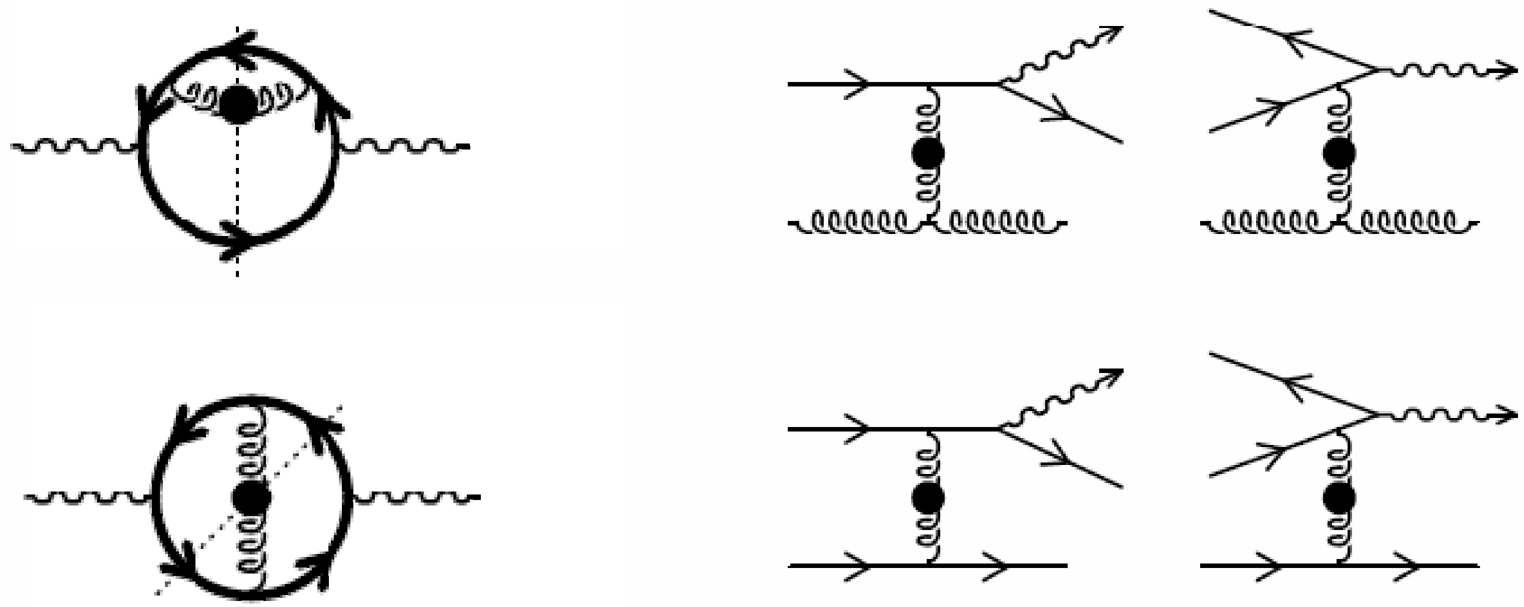


$$\mathrm{Im}\,\Pi(\omega, q) \propto \alpha\alpha_s T^2 \left[\ln\left(\frac{\omega T}{m_q^2}\right) + \mathrm{Cste} \right]$$

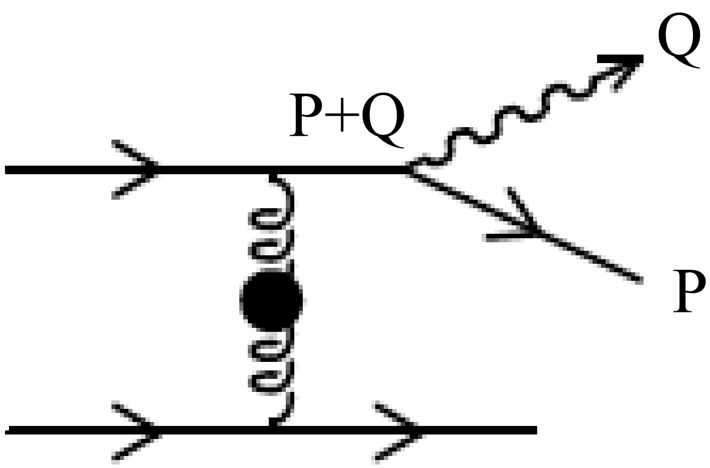
Kapusta, Lichard, Seibert (1991)
Baier, Nakkagawa, Niegawa, Redlich (1992)

Two-loop resummed calculations (1)

Aurenche, Gelis, Kobes, Petitgirard (1996)
Aurenche, Gelis, Kobes, Zaraket (1998)



Two-loop resummed calculations (2)



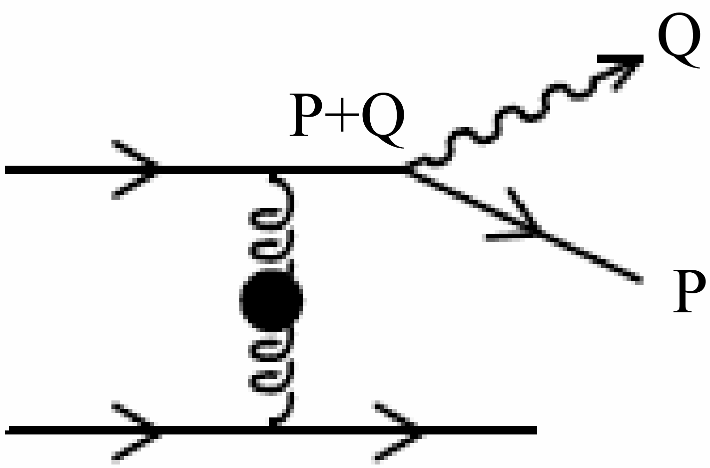
$$(P+Q)^2 - m^2 = 2P \cdot Q = \frac{m_\perp^2}{p_z} q$$

Singularity when the photon is emitted forward

$$\alpha_s^2 \frac{T^2}{m_\infty^2} \sim \alpha_s$$

$$\mathrm{Im}\,\Pi(\omega, \mathbf{q}) \propto \alpha \alpha_s \left[\pi^2 \frac{T^3}{\omega} + \omega T \right]$$

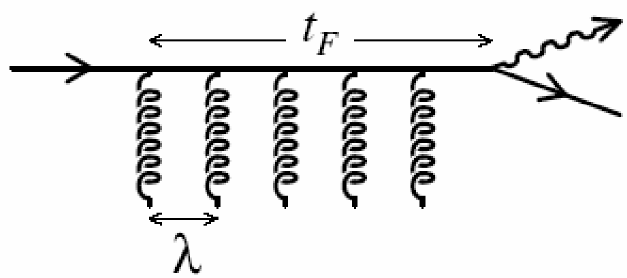
Coherence effects



$$\delta E = E_p + q - E_{p+q} \approx \frac{m_\perp^2}{2} \frac{q}{p_z(p_z+q)}$$

Formation time $t_F \sim \frac{1}{\delta E}$

Formation time comparable to mean collision time $\sim 1/g^2 T$

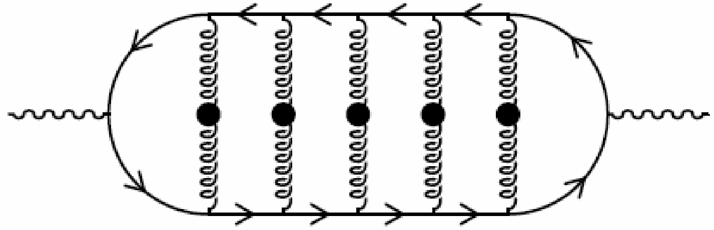


Multiple scatterings are important

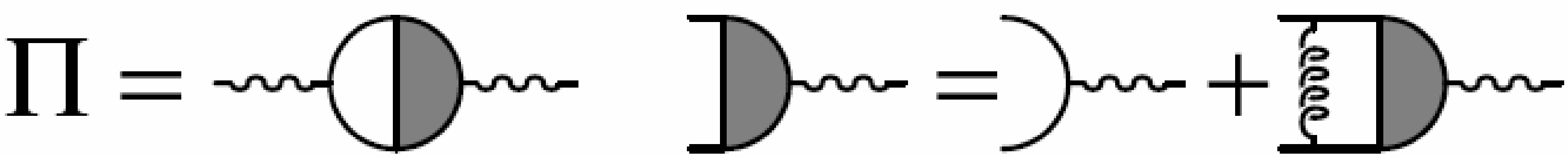
Landau, Pomeranchuk, Migdal (1953-55)

Effect of multiple collisions

Migdal (1956)
Arnold, Moore, Yaffe (2001-2002)
Aurenche, Gelis, Zaraket (2002)



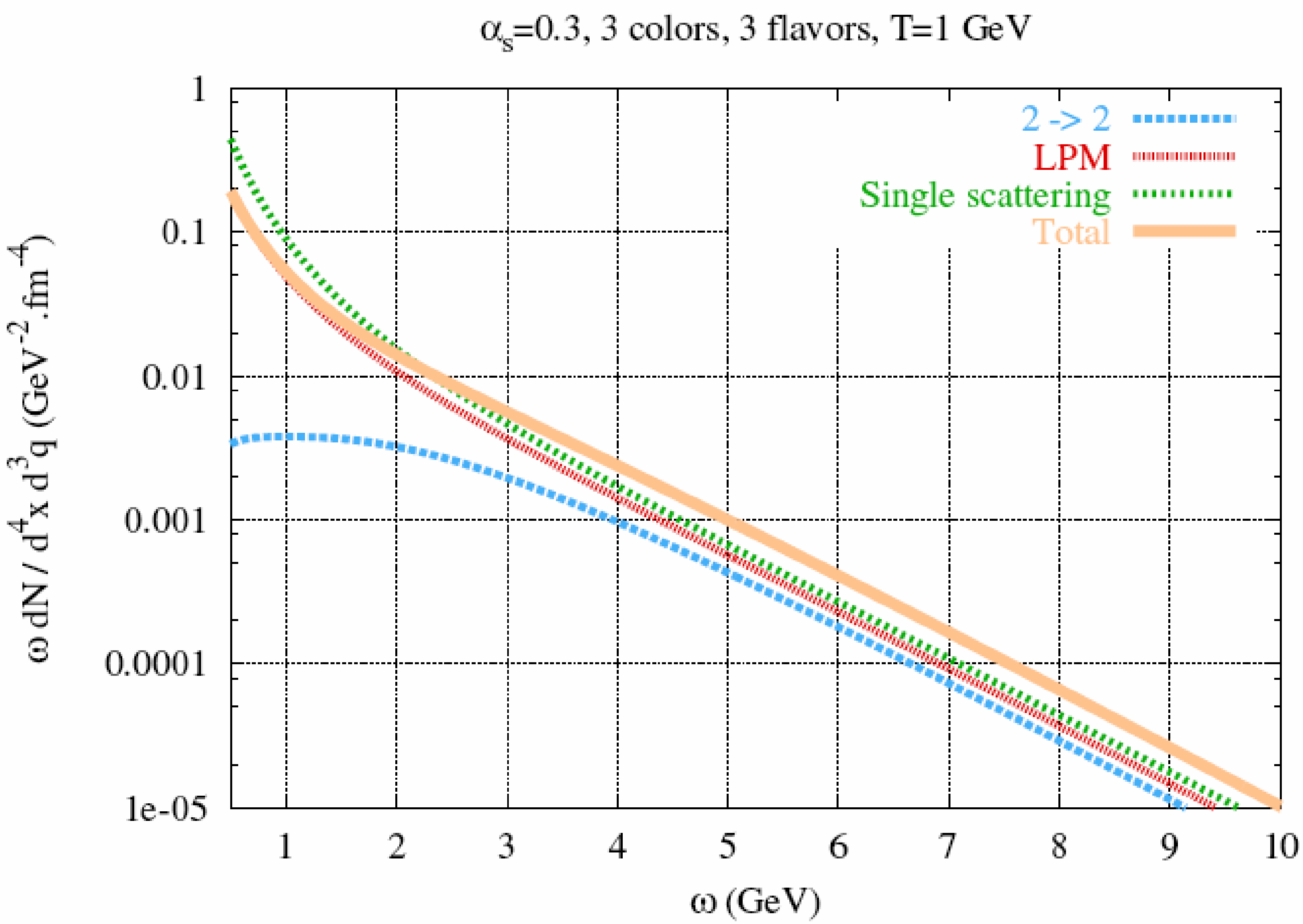
$$\mathrm{Im}\,\Pi_\mu{}^\mu(P) \approx \alpha N_c \int_{-\infty}^{+\infty} dq\, [n_F(r) - n_F(q)]\, \frac{q^2+r^2}{q^2 r^2}\, \mathrm{Re}\int \frac{d^2\mathbf{q}_T}{(2\pi)^2}\, \mathbf{q}_T \cdot \mathbf{f}(\mathbf{q}_T)$$



$$\frac{i}{2\tau_{\mathrm{F}}}\mathbf{f}(\mathbf{q}_T) = 2\mathbf{q}_T + g^2 C_F T \int \frac{d^2\mathbf{l}_T}{(2\pi)^2}\, C(\mathbf{l}_T)\, [\mathbf{f}(\mathbf{q}_T + \mathbf{l}_T) - \mathbf{f}(\mathbf{q}_T)]$$

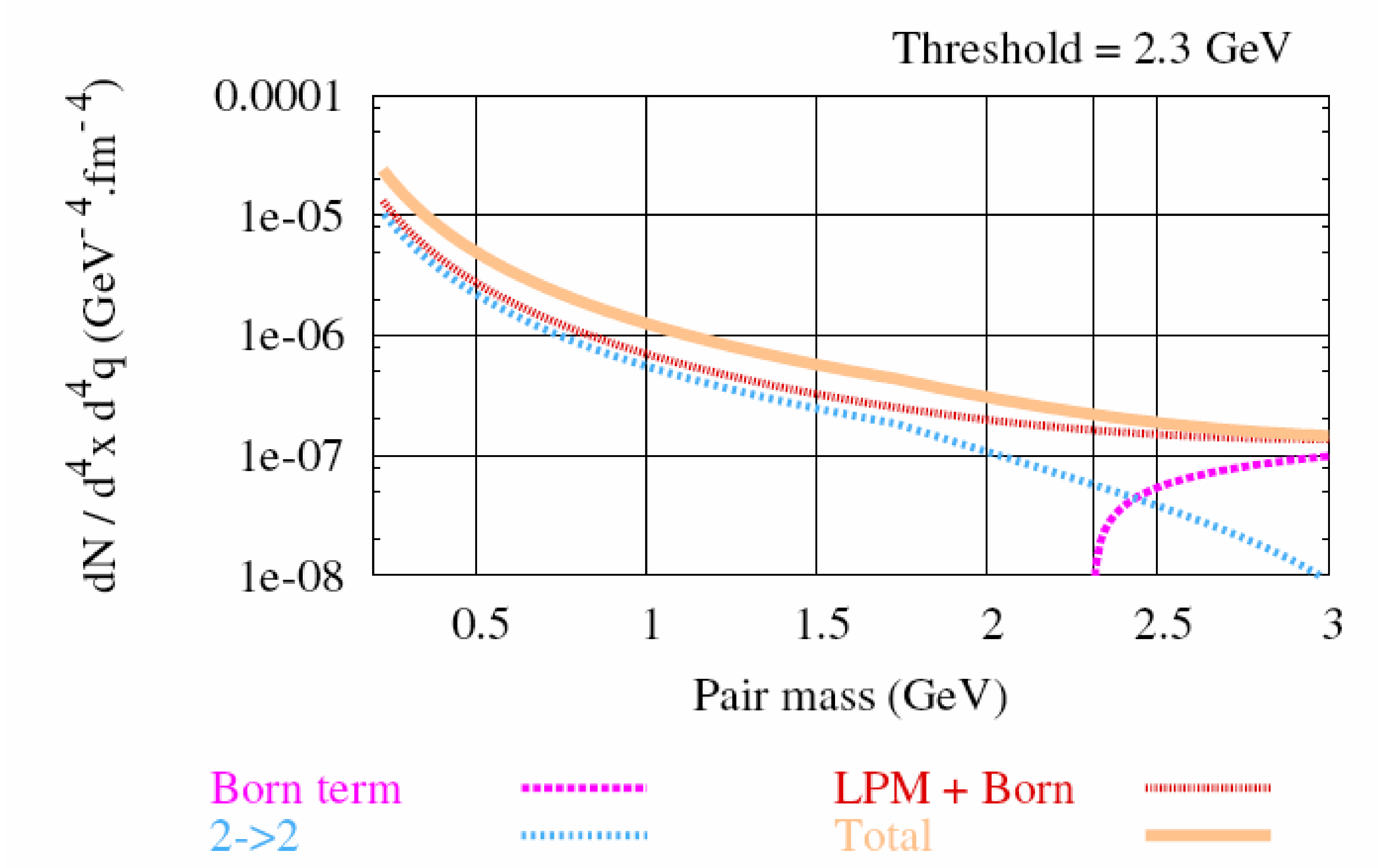
Photon rate at $\mathcal{O}(\alpha\alpha_s)$

Aurenche, Gelis, Moore, Zaraket (2002)



Dilepton rate at $\mathcal{O}(\alpha^2\alpha_s)$

Aurenche, Gelis, Moore, Zaraket (2002)



Lattice QCD calculations

Dilepton rates from lattice spectral function

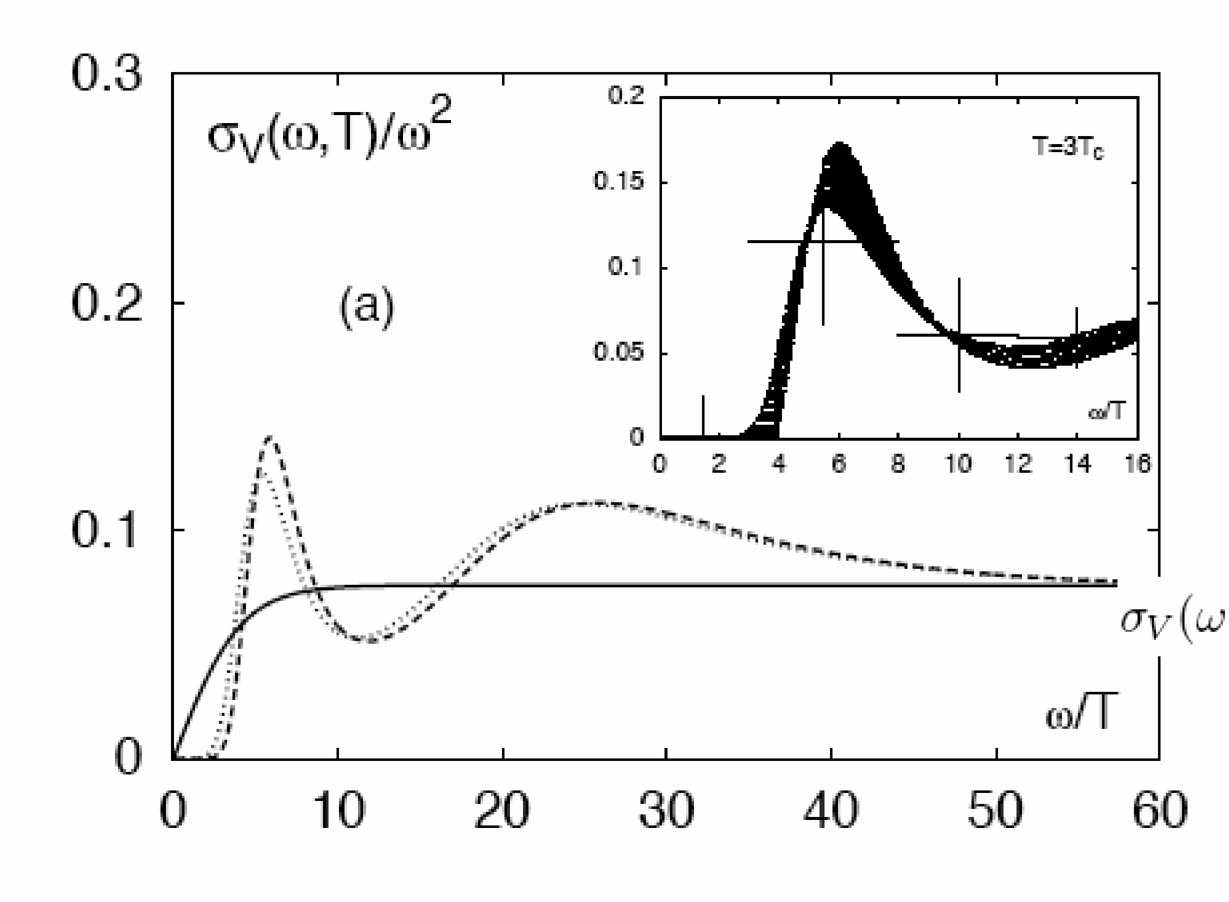
$$G_V(\tau,\vec{p},T) = \int \mathrm{d}^3x \exp(i\vec{p}\cdot\vec{x})\langle J_V^\mu(\tau,\vec{x})J_{V\mu}^\dagger(0,\vec{0})\rangle$$

$G_V(\tau,\vec{p},T)$ **is measured on the lattice**

$$G_V(\tau,\vec{p},T) = \int_0^\infty \mathrm{d}\omega\ \sigma_V(\omega,\vec{p},T)\,\frac{\mathrm{ch}(\omega(\tau-1/2T))}{\mathrm{sh}(\omega/2T)}$$

The spectral function $\sigma_V(\omega,\vec{p},T)$ **is reconstructed from the MEM**

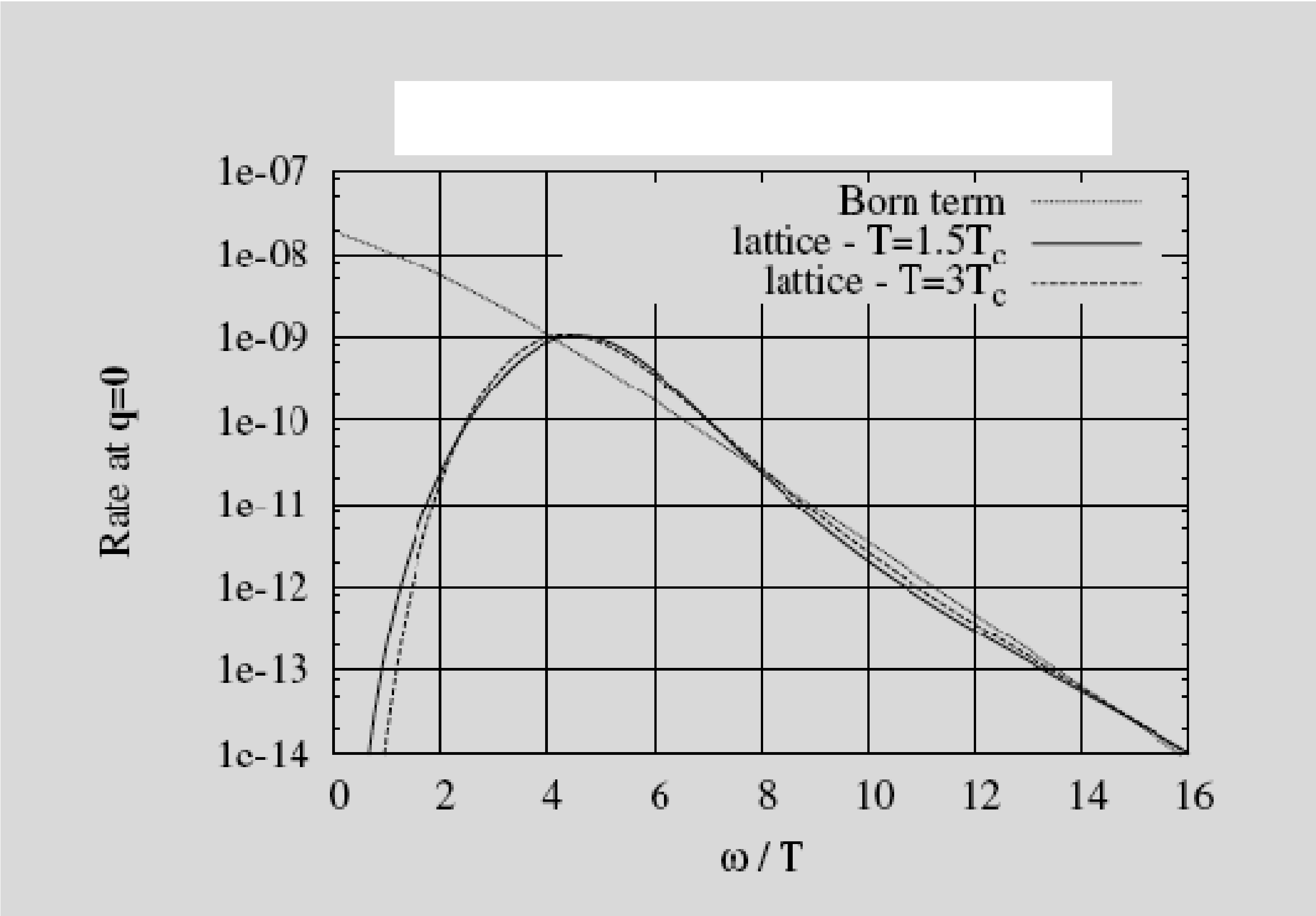
lattice spectral functions



$$\sigma_V(\omega, T) = 0.75\ \pi^{-2}\omega^2 \tanh(\omega/4T)$$

Karsch et al, hep-lat/0110208

Rate of dilepton production (p=0)



Karsch et al, hep-lat/0110208

Plot from F. Gelis, hep-ph/0209072

Puzzle at small frequency

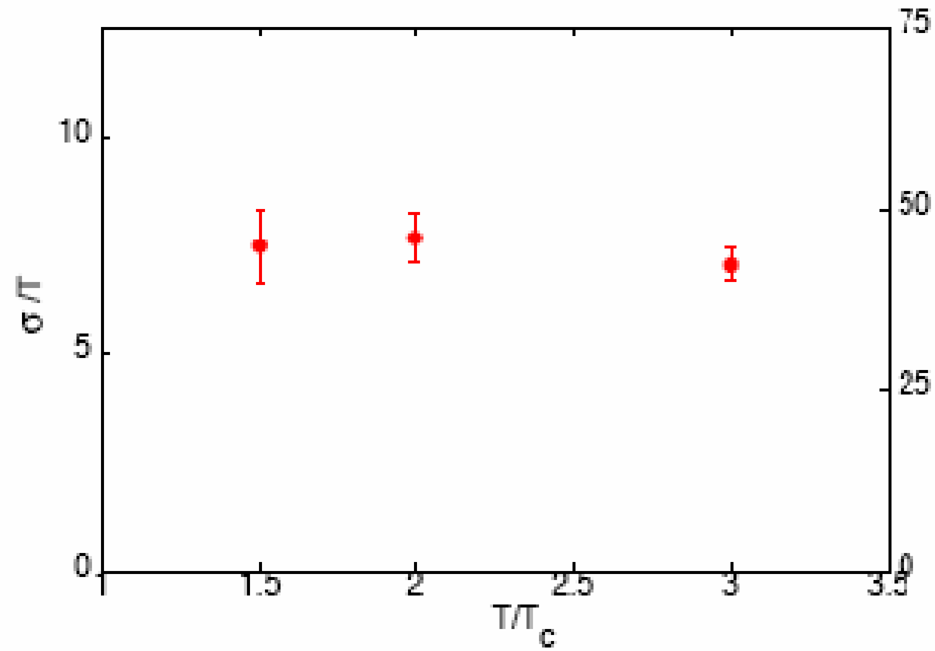
Electric conductivity

$$\sigma_{\mathrm{el}} = \lim_{\omega \to 0} \mathrm{Im}\,\Pi_{\mathrm{ret}}(\omega, 0)/6\omega$$

Simple sum rule

$$\int_0^\infty d\omega \frac{\mathrm{Im}\Pi(\omega, \mathbf{q})}{\sinh \omega/2T} = \Pi(\tau = 1/2T, \mathbf{q})$$ **(finite)**

Electric conductivity from lattice



NB. It is assumed in the MEM that the looked for spectral function leads to a finite conductivity

S. Gupta, hep-lat/0301006

Conclusions

- Leading order rates are under control
- First lattice estimates agree with perturbative ones at large frequencies, BUT differ qualitatively at small frequencies
-The low frequency domain remains to be understood