

Quark Number Susceptibilities & The Wróblewski Parameter

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** In collaboration with Sourendu Gupta, TIFR, Mumbai*

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Introduction

Quark Number Susceptibility

The Wróblewski Parameter

Summary

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- Enhancement of strangeness production as a promising signal of QGP (Rafelski-Müller, Phys. Rev. Lett '82, Phys. Rept '86..).
- Most signal considerations based on Simple Models.
 - $T_{QGP} > m_{strange}$
 - Energy Threshold for $(s\bar{s})$ in QGP $<$ in Hadron Gas.
 - Production rate : $\sigma_{QGP}(s\bar{s}) > \sigma_{HG}(s\bar{s})$.
- A variety of aspects studied and many different variations proposed.

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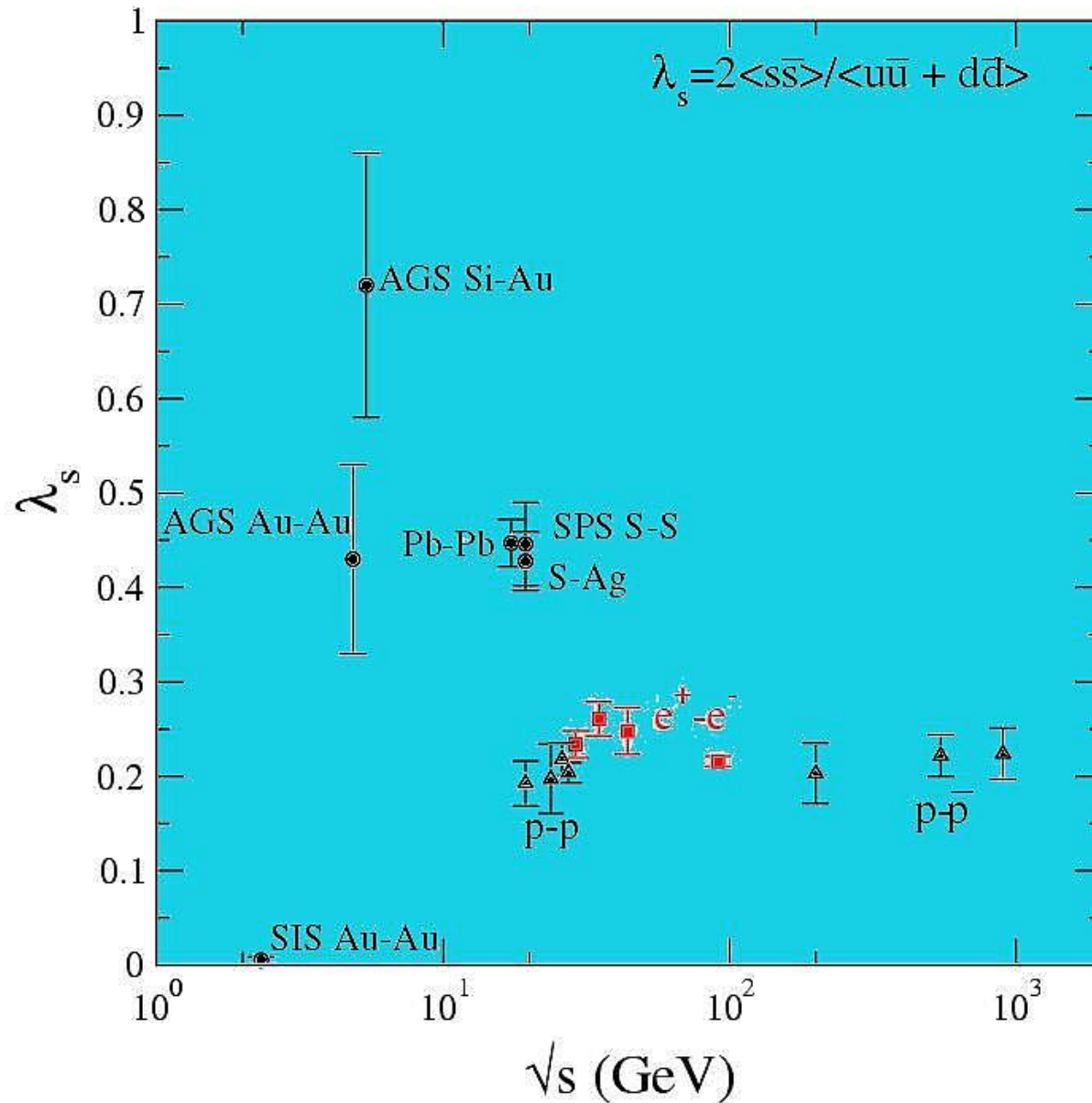
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Wroblewski Parameter



Ratio of newly created strange quarks to light quarks :

$$\lambda_s = \frac{2 \langle s\bar{s} \rangle}{\langle u\bar{u} + d\bar{d} \rangle} \quad (1)$$

Figure from Becattini et al.,
Statistical Thermal Model fit,
Phys. Rev. C 64, 024901 (2001).

Quark Number Susceptibility

Assuming three flavours, u , d , and s quarks, and denoting by μ_f the corresponding chemical potentials, the QCD partition function is

$$\mathcal{Z} = \int DU \exp(-S_G) \prod_{f=u,d,s} \text{Det } M(m_f, \mu_f) \quad . \quad (2)$$

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Defining $\mu_0 = \mu_u + \mu_d + \mu_s$ and $\mu_3 = \mu_u - \mu_d$, baryon and isospin density/susceptibilities can be obtained as :

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$$n_i = \frac{T}{V} \frac{\partial \ln \mathcal{Z}}{\partial \mu_i}, \quad \chi_{ij} = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_i \partial \mu_j}, \quad i, j = 0, 3, u, d, s$$

Higher order susceptibilities are defined by

$$\chi_{fg\cdots} = \frac{T}{V} \frac{\partial^n \log Z}{\partial \mu_f \partial \mu_g \cdots} = \frac{\partial^n P}{\partial \mu_f \partial \mu_g \cdots} . \quad (3)$$

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Setting $\mu_i = 0$, $n_i = 0$ but the diagonal χ_{ii} 's are nontrivial :

$$\chi_0 = \frac{T}{2V} [\langle \mathcal{O}_2(m_u) + \frac{1}{2} \mathcal{O}_{11}(m_u) \rangle] \quad (4)$$

$$\chi_3 = \frac{T}{2V} \langle \mathcal{O}_2(m_u) \rangle \quad (5)$$

$$\chi_s = \frac{T}{4V} [\langle \mathcal{O}_2(m_s) + \frac{1}{4} \mathcal{O}_{11}(m_s) \rangle] \quad (6)$$

Here $\mathcal{O}_2 = \text{Tr } M_u^{-1} M_u'' - \text{Tr } M_u^{-1} M_u' M_u^{-1} M_u'$, and $\mathcal{O}_{11}(m_u) = (\text{Tr } M_u^{-1} M_u')^2$, and the traces are estimated by a stochastic method:

$\text{Tr } A = \sum_{i=1}^{N_v} R_i^\dagger A R_i / 2N_v$, and $(\text{Tr } A)^2 = 2 \sum_{i>j=1}^L (\text{Tr } A)_i (\text{Tr } A)_j / L(L-1)$, where R_i is a complex vector from a set of N_v subdivided in L independent sets.

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$$\lambda_s = \frac{2\chi_s}{\chi_u + \chi_d} . \quad (7)$$

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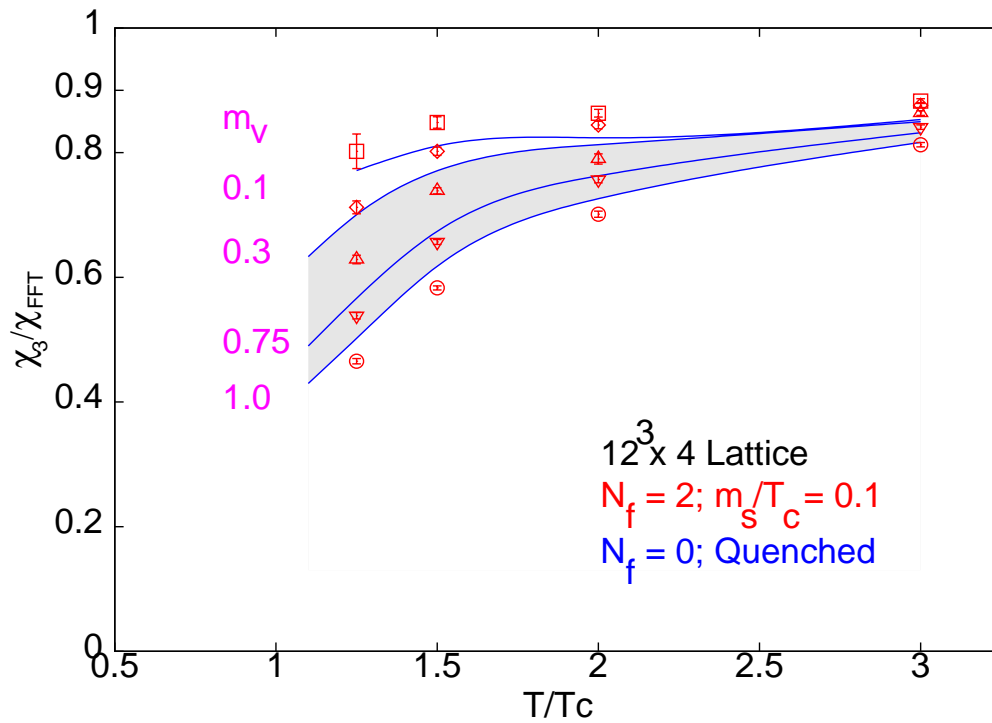
♠ Our improvement: Fixed m_q/T_c , Continuum limit...

Comparing Full and Quenched QCD

Gvai & Gupta PR D '01; Gvai, Gupta & Majumdar, PR D 2002.

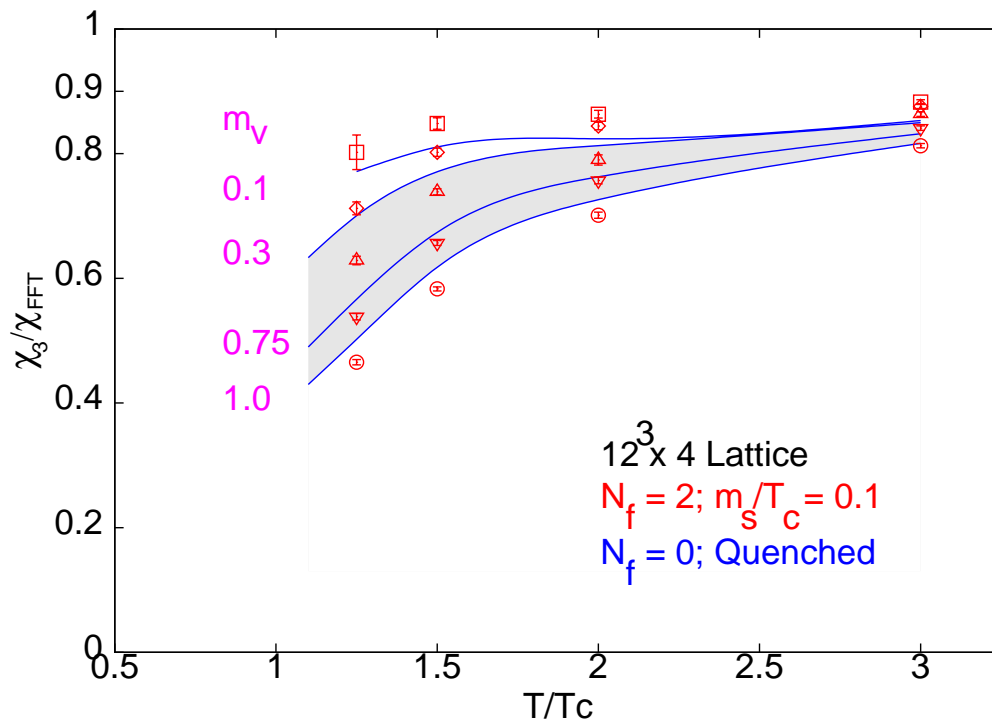
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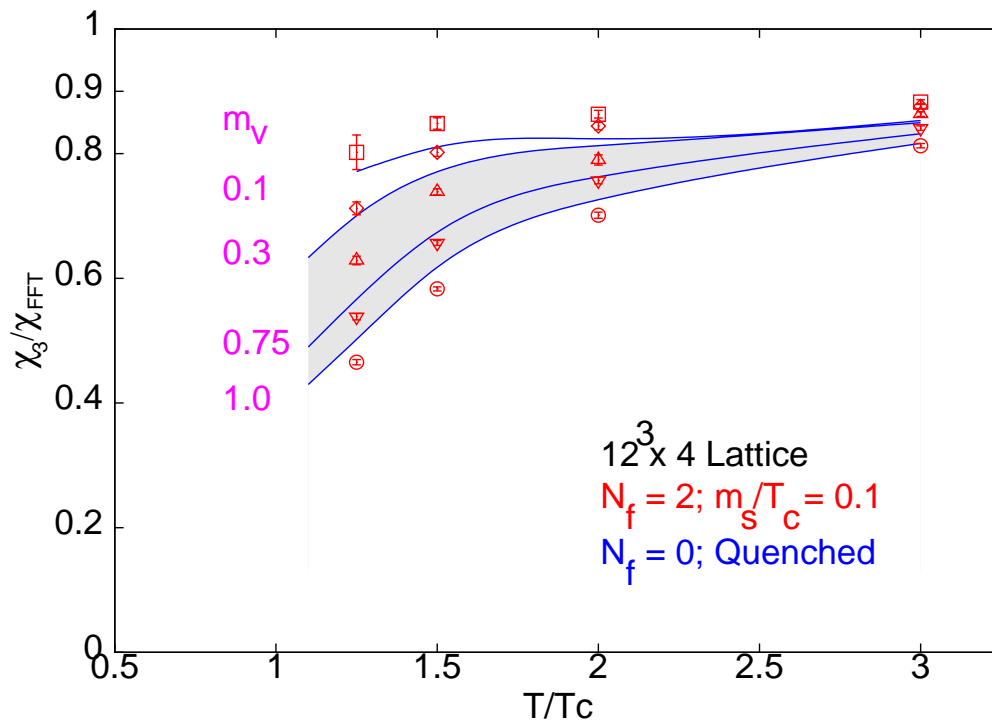


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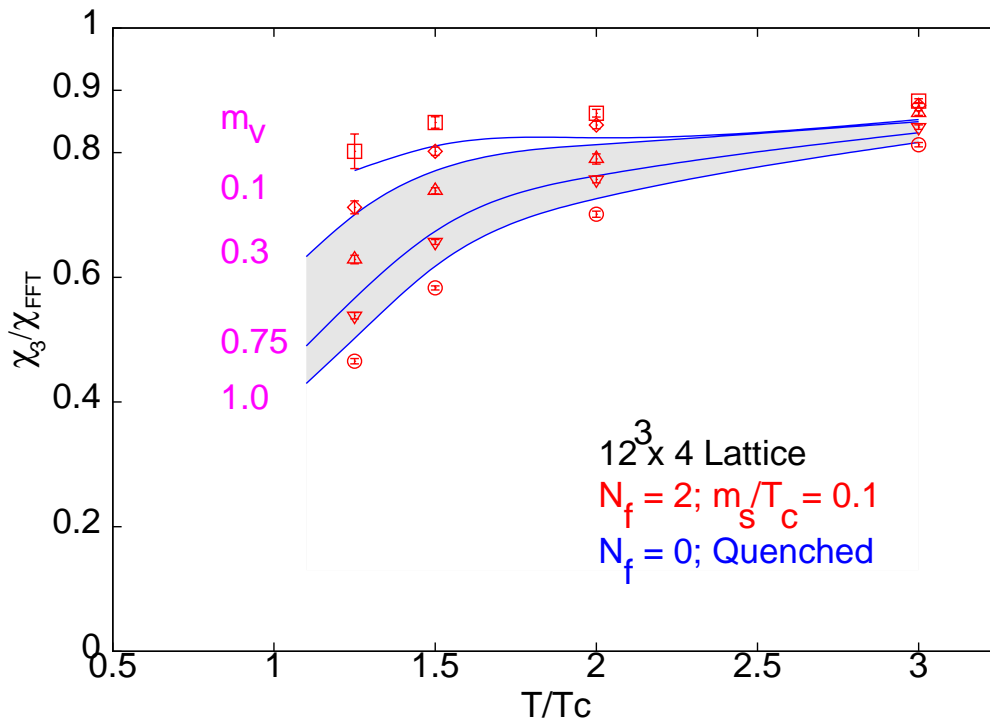
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3) PDG values for strange quark mass $\implies m_v^{strange}/T_c \simeq 0.3-0.7$ ($N_f=0$);
 $0.45-1.0$ ($N_f=2$).

Perturbation Theory

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Weak coupling expansion gives:

$$\frac{\chi}{\chi_{FFT}} = 1 - 2\left(\frac{\alpha_s}{\pi}\right) + 8\sqrt{(1 + 0.167N_f)}\left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}} - 6\left(\frac{\alpha_s}{\pi}\right)^2 \log \frac{1}{4\pi\alpha_s} + \mathcal{O}(\alpha_s^2)$$

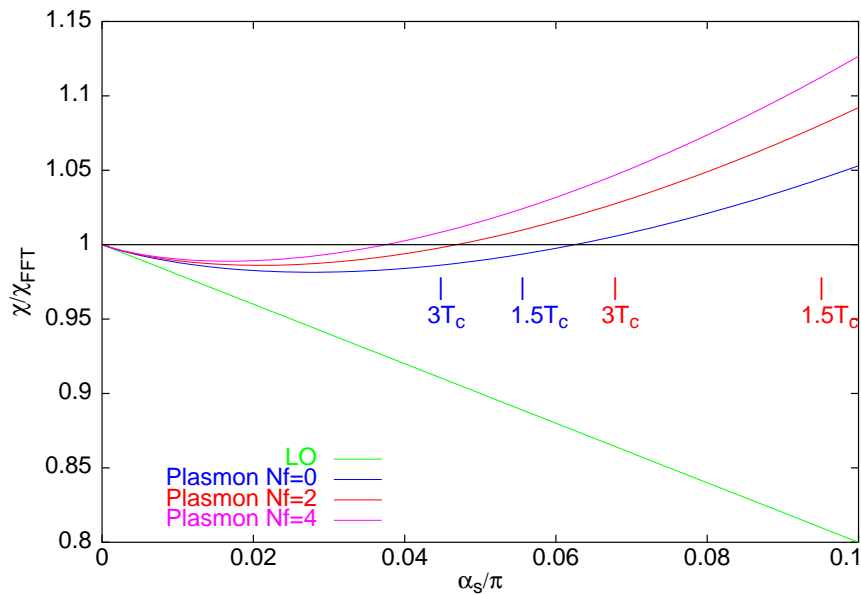
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- ♣ Minm 0.981 (0.986) at 0.03 (0.02) for $N_f = 0$ (2).
- ♣ For $1.5 \leq T/T_c \leq 3$ pert. theory \longrightarrow 0.99-0.98 (1.08=1.03) for $N_f = 0$ (2).

Resummed Perturbation Theory

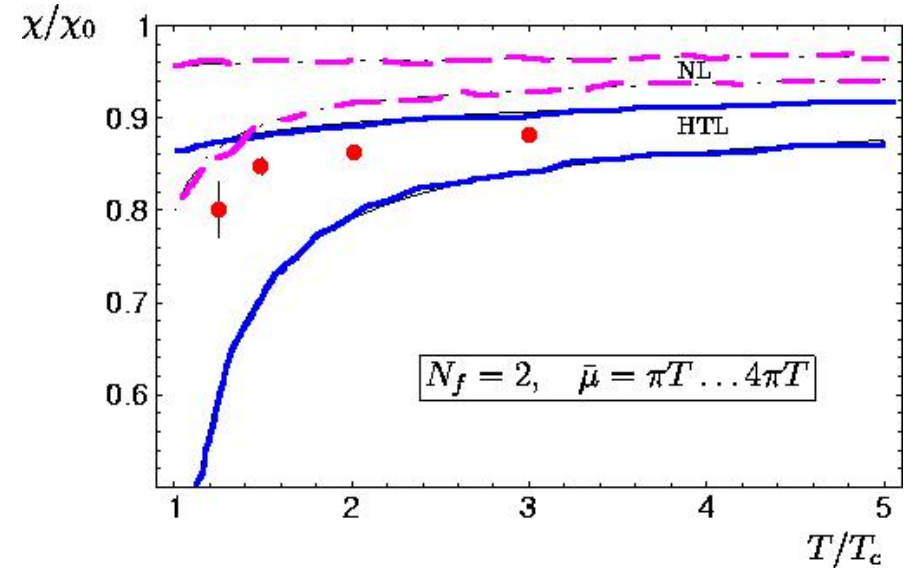
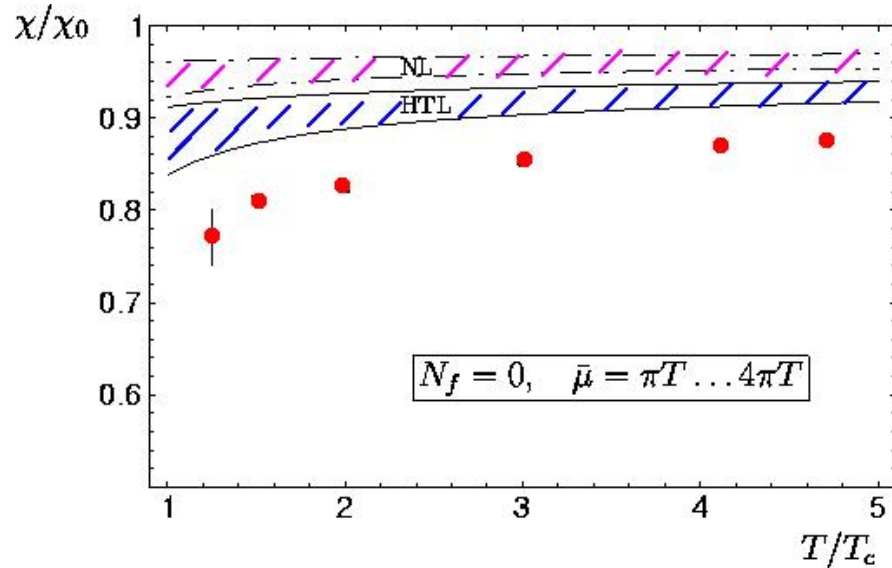
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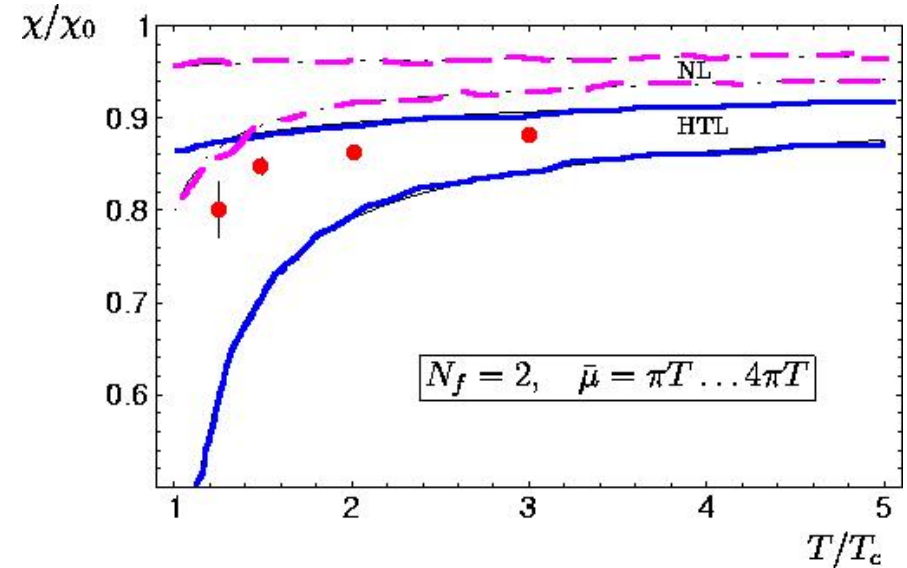
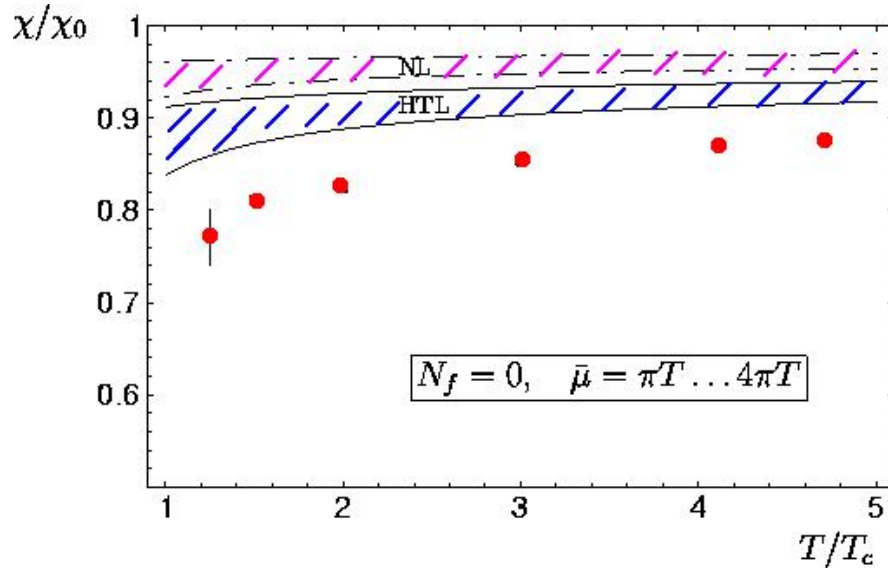
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Our results for $N_t = 4 \rightsquigarrow$ Lattice artifacts ?
Check for larger N_t and improved actions.

Taking Continuum Limit

(Gvai & Gupta, PR D '02 and PR D '03)

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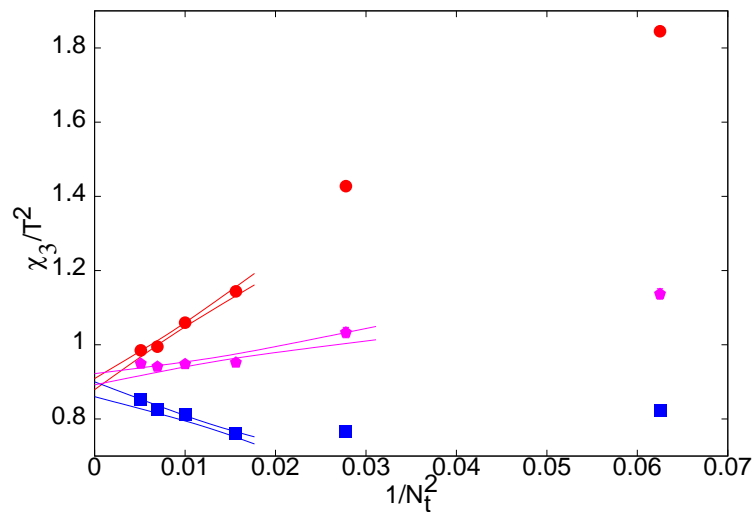
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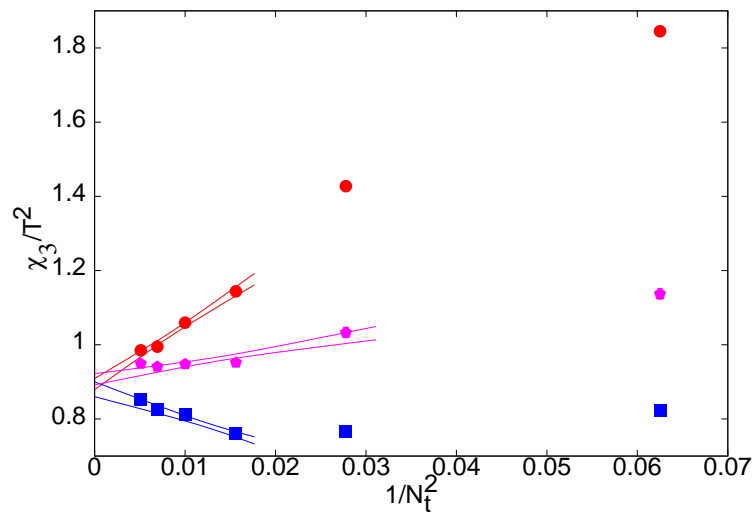
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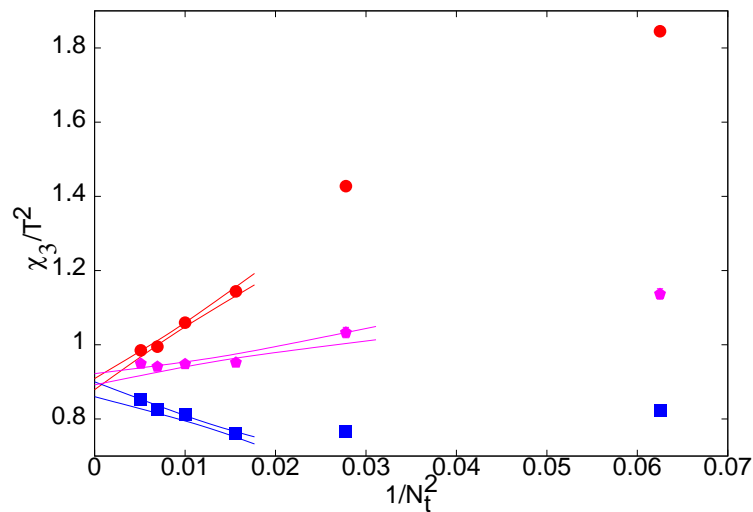
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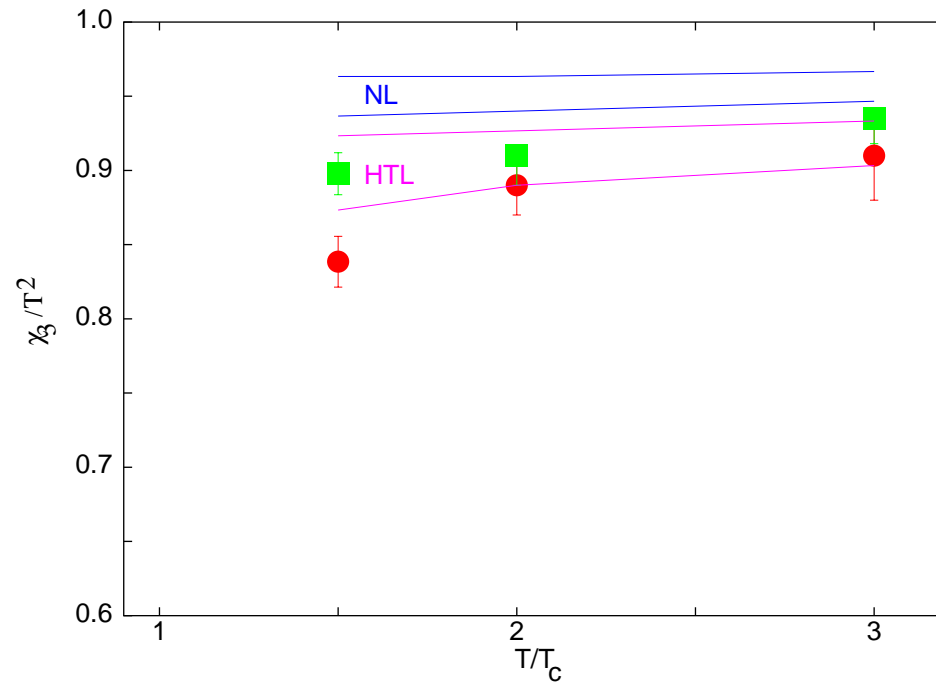
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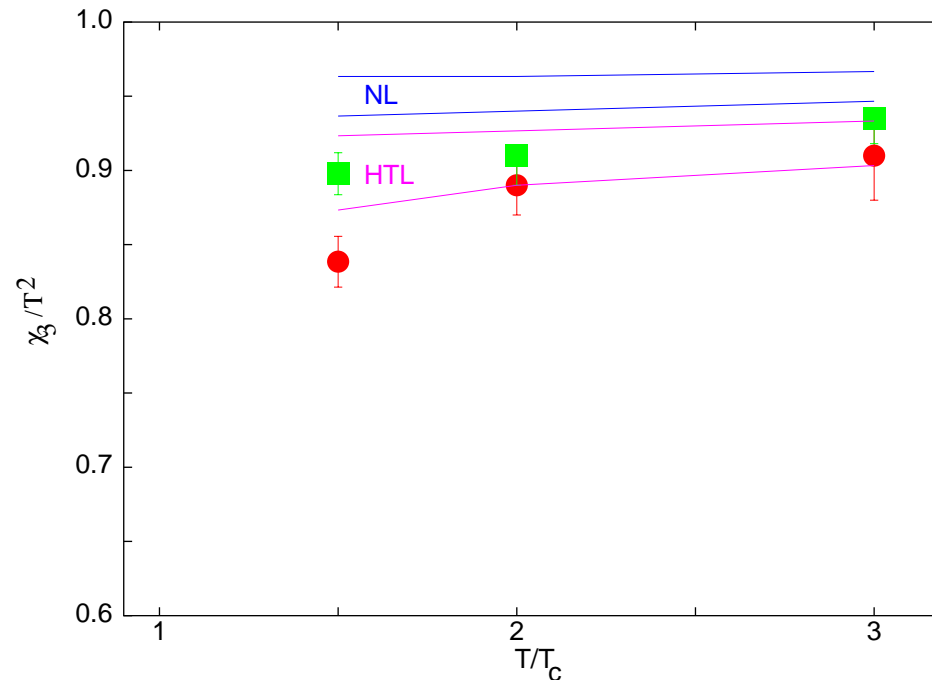
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♡ Also reproduced in dimensional reduction (1 free parameter). [Vuorinen, PR D '03.](#)

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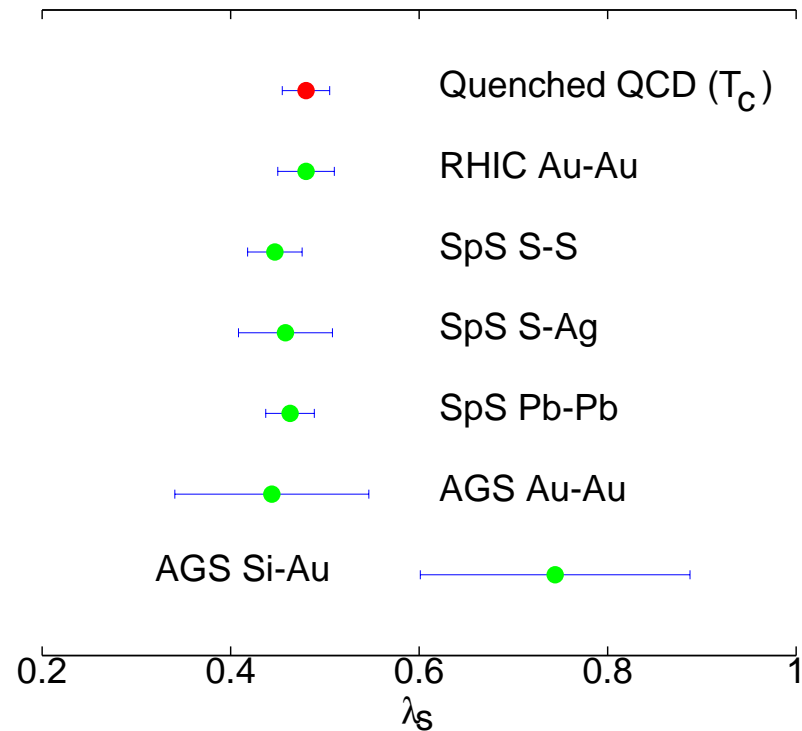
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- Using our continuum QNS, ratio χ_s/χ_u can be obtained.

We use $m/T_c = 0.03$ for u, d and $m/T_c = 1$ for s quark;
At each T , ratio of χ 's $\rightarrow \lambda_s(T)$.
Extrapolate it to T_c .

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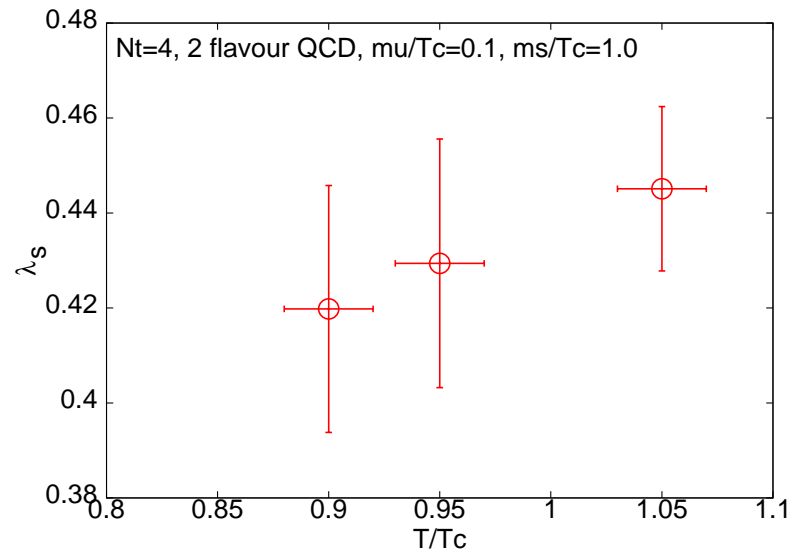
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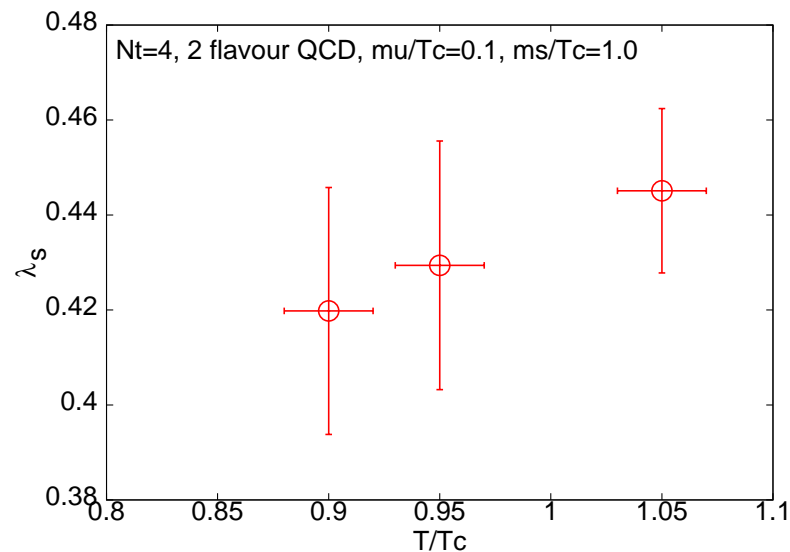
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- ♣ Large finite volume effects below T_c
- ♣ Up to 12^3 Lattices used. —Being extended to 24^3 Lattices.
- ♣ Strong dependence on m_s expected.
- ♣ Large finite a effects.

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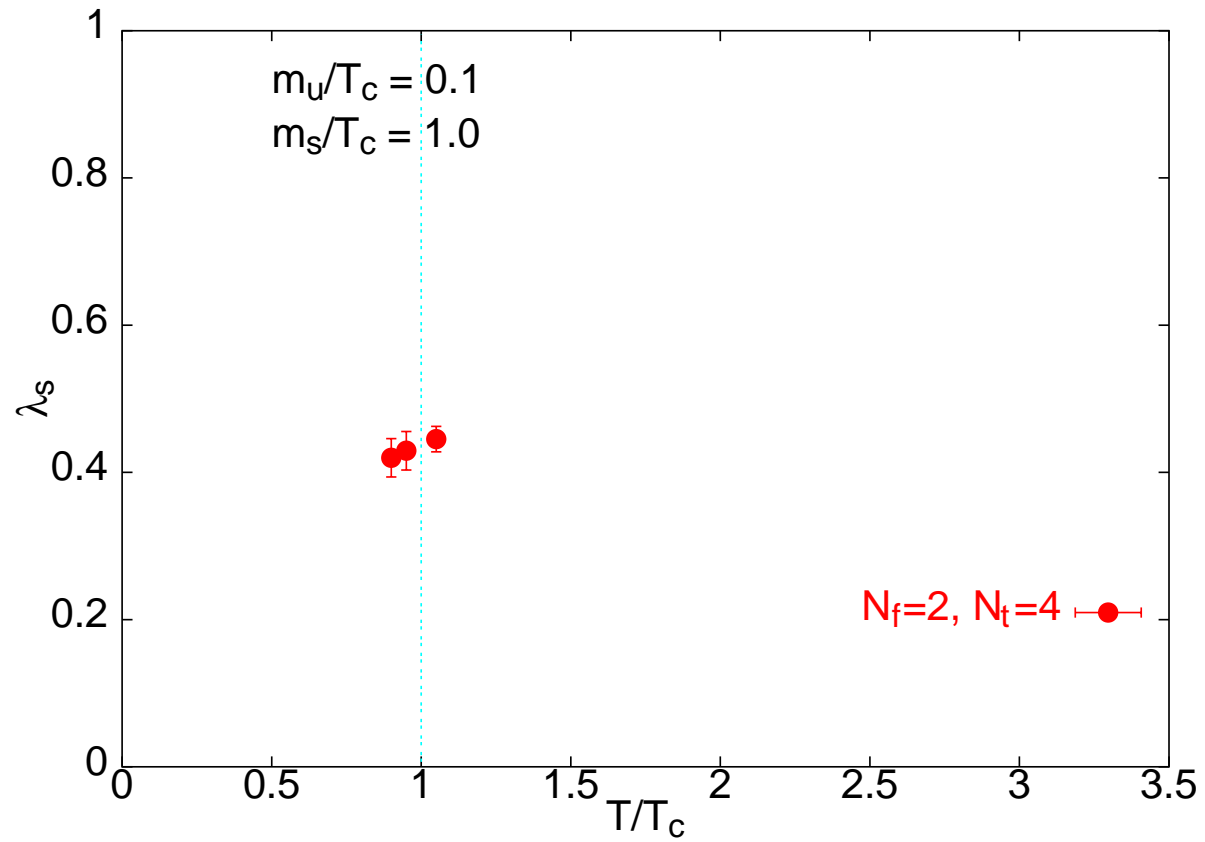
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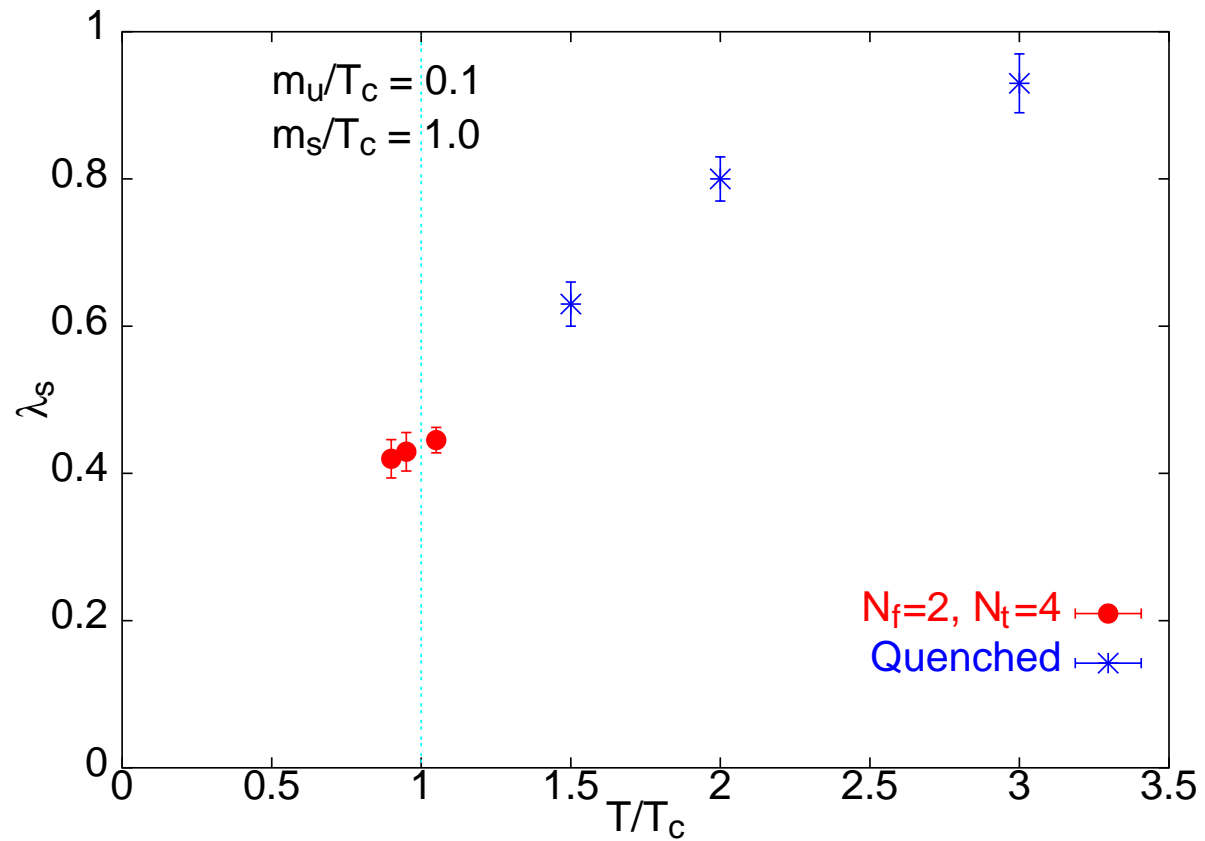
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 - Observation of spikes in photon production may falsify this.
- Assumed : Chemical equilibration in the plasma.

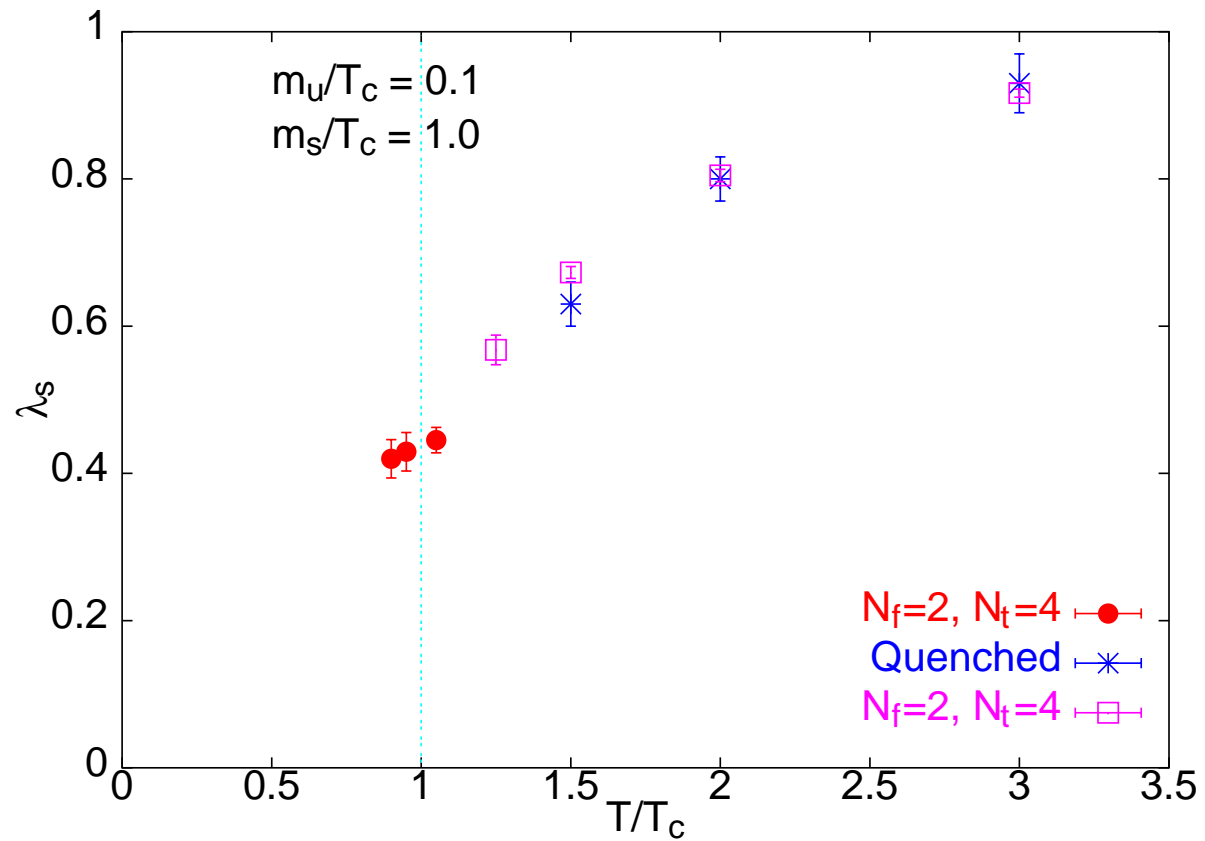
Summary



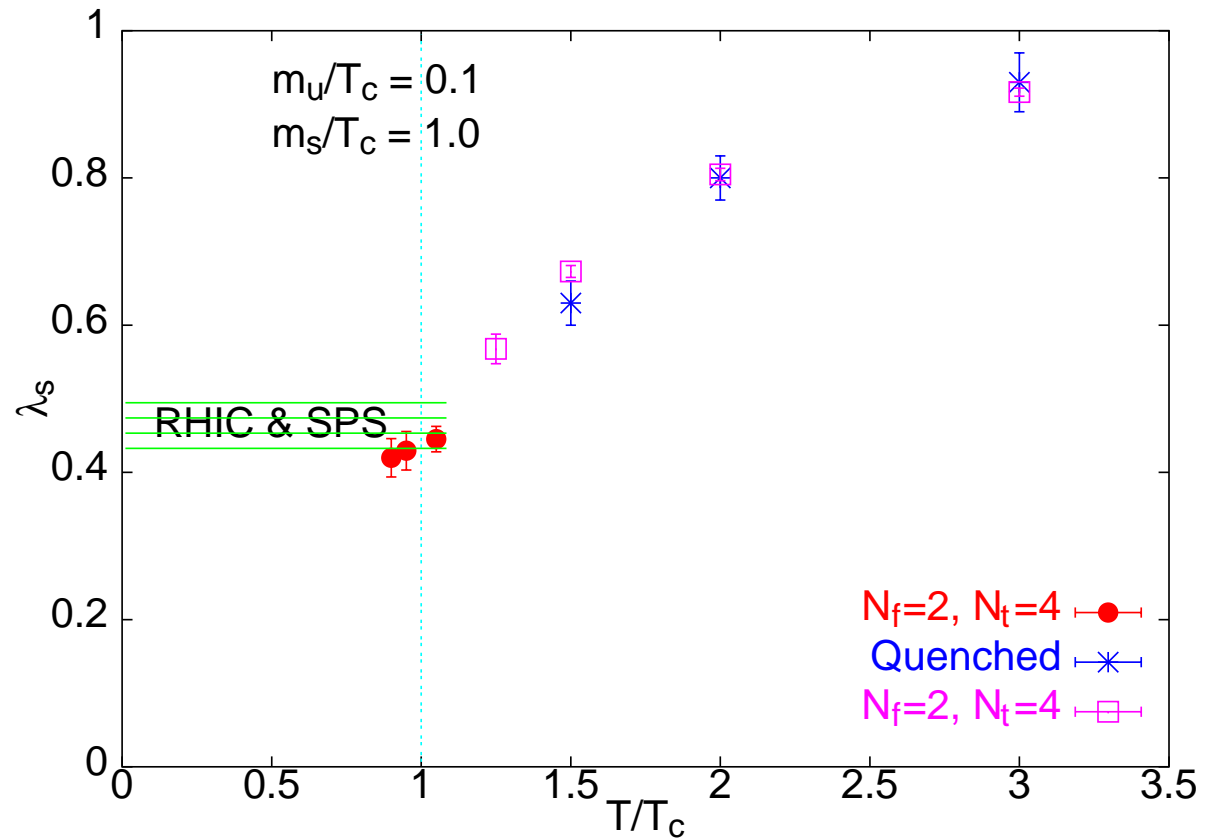
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- They offer an independent check on resummed perturbation theory which seems to do well.
- Continuum limit of χ_{uu} yields λ_s in agreement with RHIC and SPS results after extrapolation to T_c . First full QCD investigations show interesting trend.
- Higher susceptibilities up to 8th order computed. Interesting results on the QCD phase diagram soon.