The Dipole Picture in Hadronic Reactions

Jörg Raufeisen (Heidelberg U.)

HERA-LHC workshop at DESY, Nov. 16, 2004

Introduction

- The color dipole approach is known as a useful tool for the study of saturation in DIS, where the cross section can be written in the simple form

$$\sigma_{\rm tot}^{\gamma^* p} = \frac{4\pi^2\alpha_{em}}{Q^2} F_2(x,Q^2) = \int d\alpha \int d^2\rho \left|\Psi_{\gamma^*\to q\bar{q}}(\alpha,Q^2,\rho)\right|^2 \sigma_{q\bar{q}}(x,\rho).$$

- The advantage of the dipole formulation is that it is formulated in terms of interaction eigenstates. This simplifies the calculation of multiple scattering effects.
- In the target rest frame, saturation looks like multiple scattering: at high energy, the parton density grows so large that even a hard probe will scatter twice inside the target.
- Several processes can be calculated in terms of the same dipole cross section as low x DIS, *e.g.*
  - Drell-Yan dilepton production
  - Open heavy flavor hadroproduction

  even if there is no dipole present diagrammatically.
- Hence, the dipole formulation provides a link between saturation searches at HERA and in hadronic reactions at the LHC.

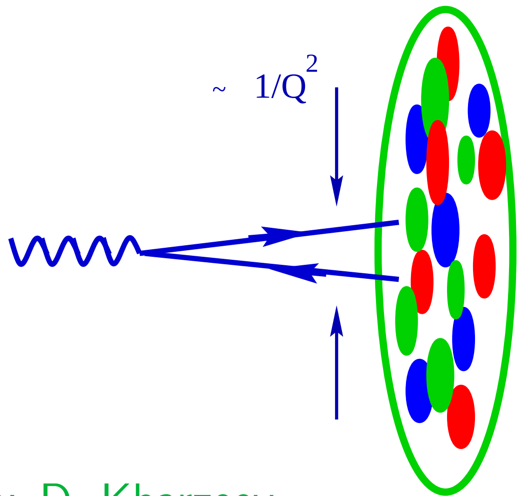
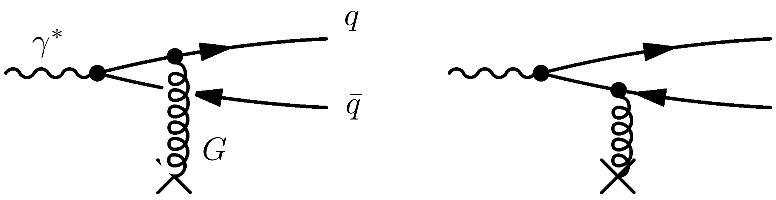


figure: D. Kharzeev

The Dipole Approach to DIS and k_T-Factorization



$$\rho = q\bar{q} \text{ transverse separation}$$
$$\alpha = p_q^+/q_{\gamma^*}^+ = \frac{1+\cos(\theta)}{2}$$
$$\varepsilon^2 = \alpha(1-\alpha)Q^2 + m_q^2$$

- At low x, photon-gluon fusion ($\gamma^* + G \to q + \bar{q}$) dominates over $\gamma^* + q \to q + G$ and the DIS cross section can be written as,

$$\frac{d\sigma_L^{\gamma^* p}}{d^2 p_T} = \frac{4\alpha_{em} e_f^2 Q^2}{\pi} \int d\alpha \alpha^2 (1-\alpha)^2 \int \frac{d^2 k_T}{k_T^4} \alpha_s \mathcal{F}(x, k_T) \left[\frac{1}{p_T^2 + \varepsilon^2} - \frac{1}{(\vec{p}_T - \vec{k}_T)^2 + \varepsilon^2}\right]^2$$
$$= \int d\alpha \int \frac{d^2\rho_1 d^2\rho_2}{2(2\pi)^2} \Psi^*(\alpha, \rho_1)\Psi(\alpha, \rho_2) e^{i\vec{p}_T\cdot(\vec{\rho}_1 - \vec{\rho}_2)} \left[\sigma_{q\bar{q}}(\rho_1) + \sigma_{q\bar{q}}(\rho_2) - \sigma_{q\bar{q}}(|\vec{\rho}_1 - \vec{\rho}_2|)\right]$$

with

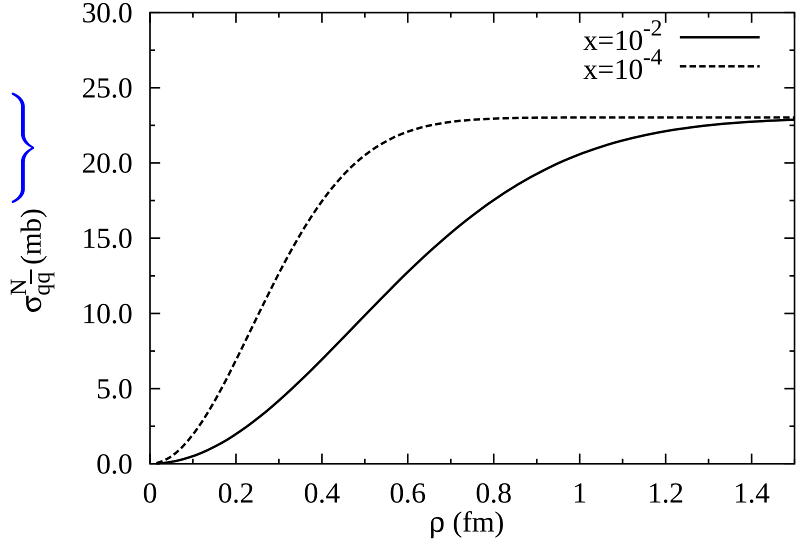
$$\sigma_{q\bar{q}}(x, \rho) = \frac{4\pi}{3} \int \frac{d^2 k_T}{k_T^4} \alpha_s \mathcal{F}(x, k_T) \left[1 - e^{i\vec{k}_T\cdot\vec{\rho}}\right]$$

- The dipole cross section $\sigma_{q\bar{q}}(x, \rho)$ carries information about the k_T dependence of the gluon distribution.

- Probably, any process that probes $\mathcal{F}(x, k_T)$ can be written in terms of $\sigma_{q\bar{q}}(x, \rho)$.

The Dipole Cross Section

- I use the DGLAP improved saturation model of Bartels, Golec-Biernat, Kowalski, Phys. Rev. D**66**: 014001, 2002 for $\sigma_{q\bar{q}}(x,\rho)$



$$\sigma_{q\bar{q}}^N(x,\rho) = \sigma_0 \left\{ 1 - \exp\left(-\frac{\pi^2\rho^2\alpha_s(\mu)xG(x,\mu)}{3\sigma_0} \right) \right\}$$

with

$$\sigma_0 = 23\,\mathrm{mb}$$
$$\mu^2 = \frac{\lambda}{\rho^2} + \mu_0^2$$

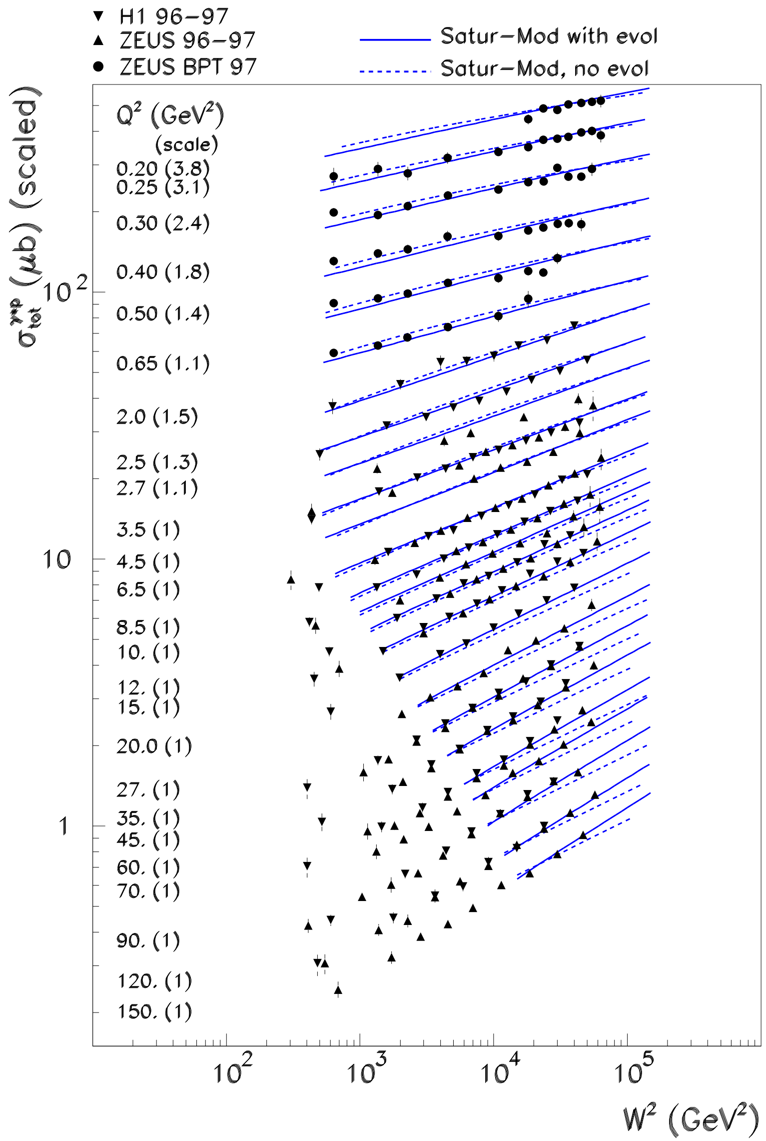
- The gluon density $xG(x,\mu)$ evolves according to DGLAP.

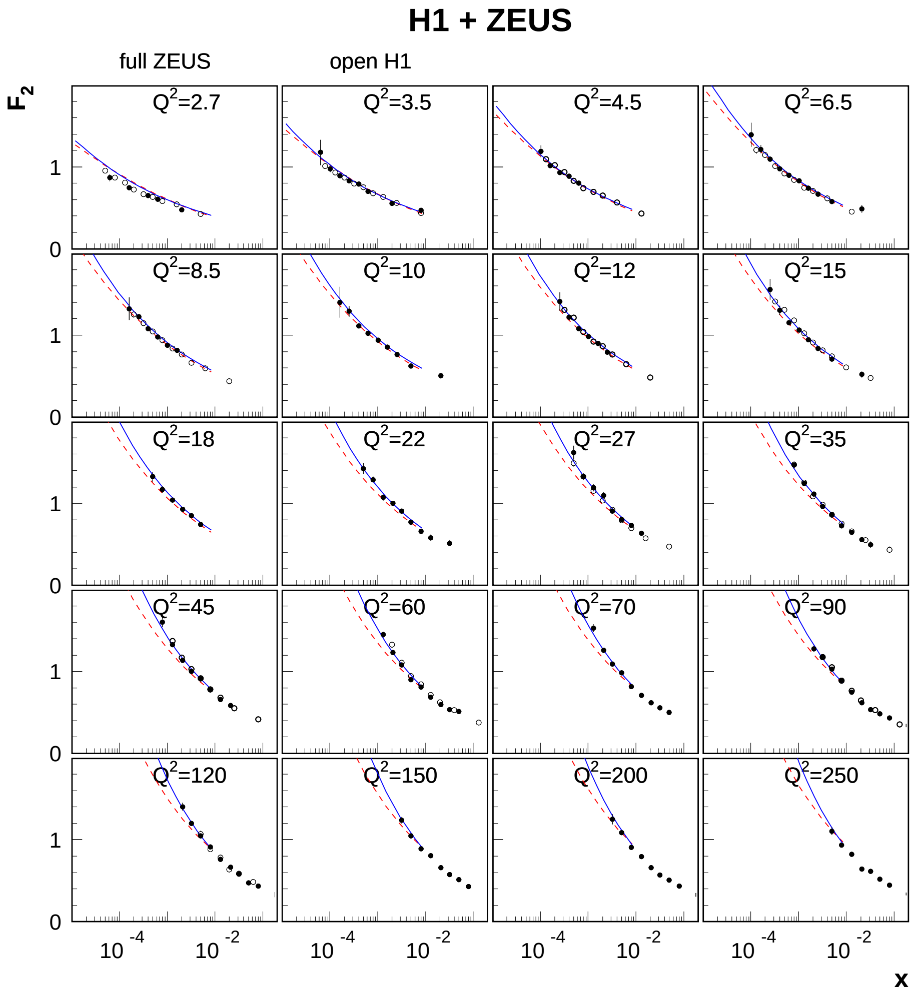
- The perturbative QCD result is recovered at small ρ:

$$\sigma_{q\bar{q}}^N(x,\rho) \to \frac{\pi^2}{3}\alpha_s(\mu)\rho^2 xG(x,\mu)$$

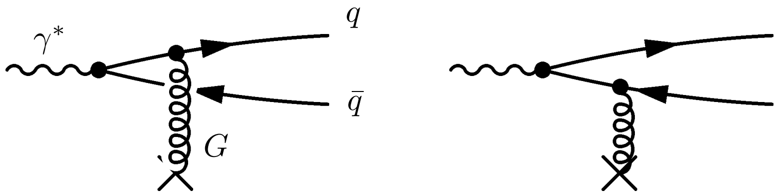
Blättel, Baym, Frankfurt, Strikman, Phys. Rev. Lett. **70**, 896, 1993.

Fit to HERA Data (Bartels et al.)





An Alternative Derivation



- Intuitive interpretation in the target rest frame:
  - The γ^* contains virtual $q\bar{q}$ fluctuations with different transverse sizes,

$$|\gamma^*\rangle = |\gamma^*_{bare}\rangle + \Psi_{q\bar{q}}(\alpha,\rho)|q\bar{q}\rangle + \Psi_{q\bar{q}G}|q\bar{q}G\rangle \ldots$$

  - The interaction with the target can set these fluctuations on mass shell, $\sigma_{q\bar{q}}$ is an eigenvalue of the diffraction amplitude operator.

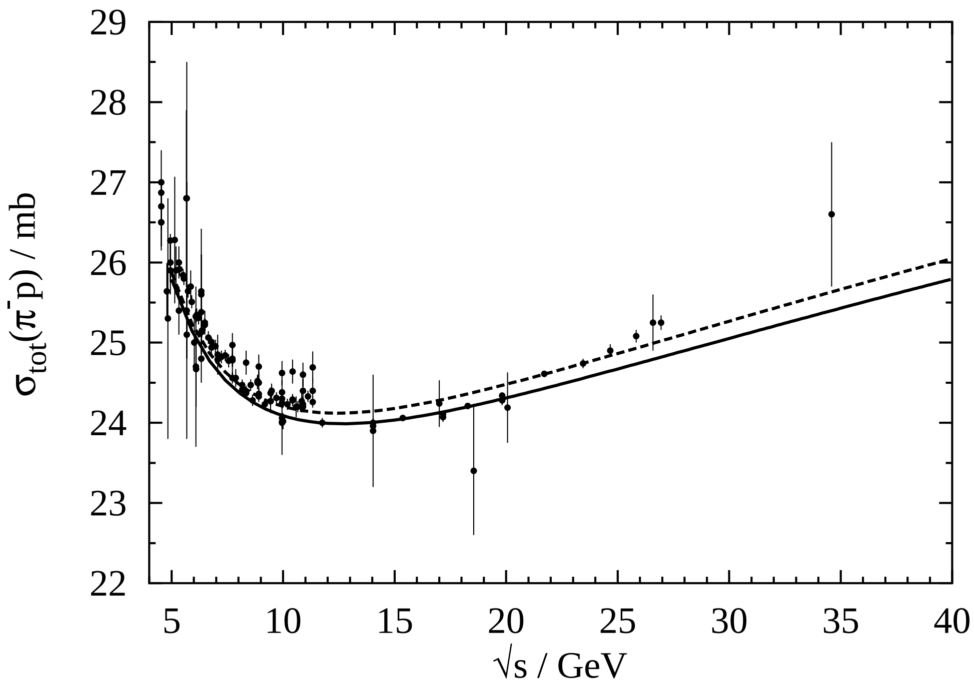
$$\sigma_{\rm tot}^{\gamma^* p} = \int d\alpha \int d^2\rho \, |\Psi_{q\bar{q}}(\alpha,\rho)|^2 \, \sigma_{q\bar{q}}(\rho) + \ldots$$

  - Higher Fock states make $\sigma_{q\bar{q}}(\rho)$ energy dependent.
- No reference to pQCD was made here.
- The impact parameter representation is advantageous for the description of multiple scattering/nuclear effects, because partonic configurations with fixed transverse separations are interaction eigenstates.

Total Hadronic Cross Sections

- Since the dipole formulation is solely based on scattering amplitudes, one can even calculate total hadronic cross sections,

$$\sigma_{\rm tot}(\pi p) = \int d^2\rho |\Psi_\pi(\rho)|^2 \sigma_{q\bar{q}}(s,\rho)$$



- The two curves were calculated with different parameterizations of $\sigma_{q\bar{q}}(s,\rho)$ (JR, PhD thesis, hep-ph/0009358)

Scattering Amplitude vs. Unintegrated Gluon Density

- Given a phenomenological parameterization of $\sigma_{q\bar{q}}(x,\rho)$, one can define the quantity

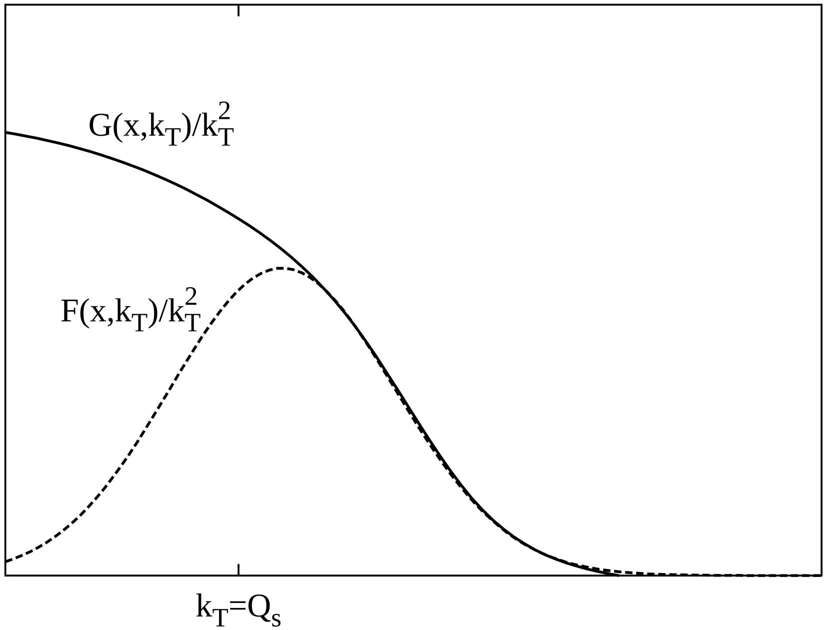
$$\mathcal{F}(x,k_T) = \frac{3k_T^4}{4\pi\alpha_s}\int\frac{d^2\rho}{(2\pi)^2}e^{i\vec{k}_T\cdot\rho}\left[\sigma_{q\bar{q}}(x,\rho\to\infty)-\sigma_{q\bar{q}}(x,\rho)\right]$$

M. Braun, Eur.Phys.J.C16:337-347,2000

- The unintegrated gluon density $\mathcal{G}(x,k_T)$ on the other hand, can be defined in a Gedankenexperiment by scattering a virtual scalar with $\mathcal{L}_I = \lambda\phi(x)G^2_{\mu\nu}(x)$ off the target.
A.H. Mueller, Nucl.Phys.B558:285-303,1999

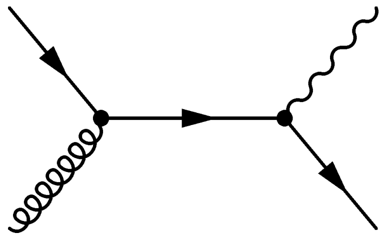
- Below the saturation scale Q_s, multiple scattering is important and there is no simple relation between the dipole cross section and the unintegrated gluon density any more.

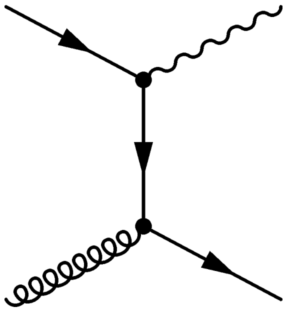
- At very low x, $Q_s(x) \gg \Lambda_{QCD}$, and QCD evolution becomes nonlinear.



The Dipole Formulation of Drell-Yan Dilepton Production

- Drell-Yan dilepton (and prompt photon) production can be expressed in terms of the same color dipole cross section as low-x DIS. (Kopeliovich, Brodsky, Hebecker, JR)
- s-channel propagator: $(\vec{l}_T = \vec{p}_{fT} - (1-\alpha)\vec{q}_T/\alpha,\ \eta^2 = (1-\alpha)M^2 + \alpha^2 m_f^2)$





$$\frac{1}{(p_f+q)^2 - m_f^2} = \frac{\alpha(1-\alpha)}{\alpha^2 l_T^2 + \eta^2},$$

u-channel propagator:

$$\frac{1}{(p_i-q)^2 - m_f^2} = -\frac{\alpha}{\alpha^2(\vec{l}_T + \vec{k}_T)^2 + \eta^2}.$$

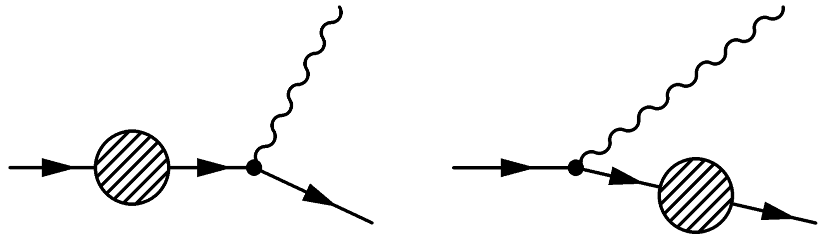
- Fourier transform: $\vec{b} \leftrightarrow \vec{k}_T,\ \vec{\rho} \leftrightarrow \alpha\vec{l}_T$
- The photon q_T distribution:

$$\frac{d\sigma}{d\ln\alpha d^2 q_T} = \int \frac{d^2\rho_1 d^2\rho_2}{(2\pi)^2}\Psi^*(\alpha,\rho_1)\Psi(\alpha,\rho_2)e^{i\vec{q}_T\cdot(\vec{\rho}_1-\vec{\rho}_2)}\frac{1}{2}\left[\sigma_{q\bar{q}}(\alpha\rho_1) + \sigma_{q\bar{q}}(\alpha\rho_2) - \sigma_{q\bar{q}}(\alpha|\vec{\rho}_1 - \vec{\rho}_2|)\right]$$

- with the same $\sigma_{q\bar{q}}(\rho)$ as in DIS:

$$\sigma_{q\bar{q}}(\rho) = \frac{4\pi}{3}\int\frac{d^2k_T}{k_T^4}\alpha_s\mathcal{F}(k_T)\left[1 - e^{-\mathrm{i}\vec{k}_T\cdot\vec{\rho}}\right]$$

An Alternative Derivation



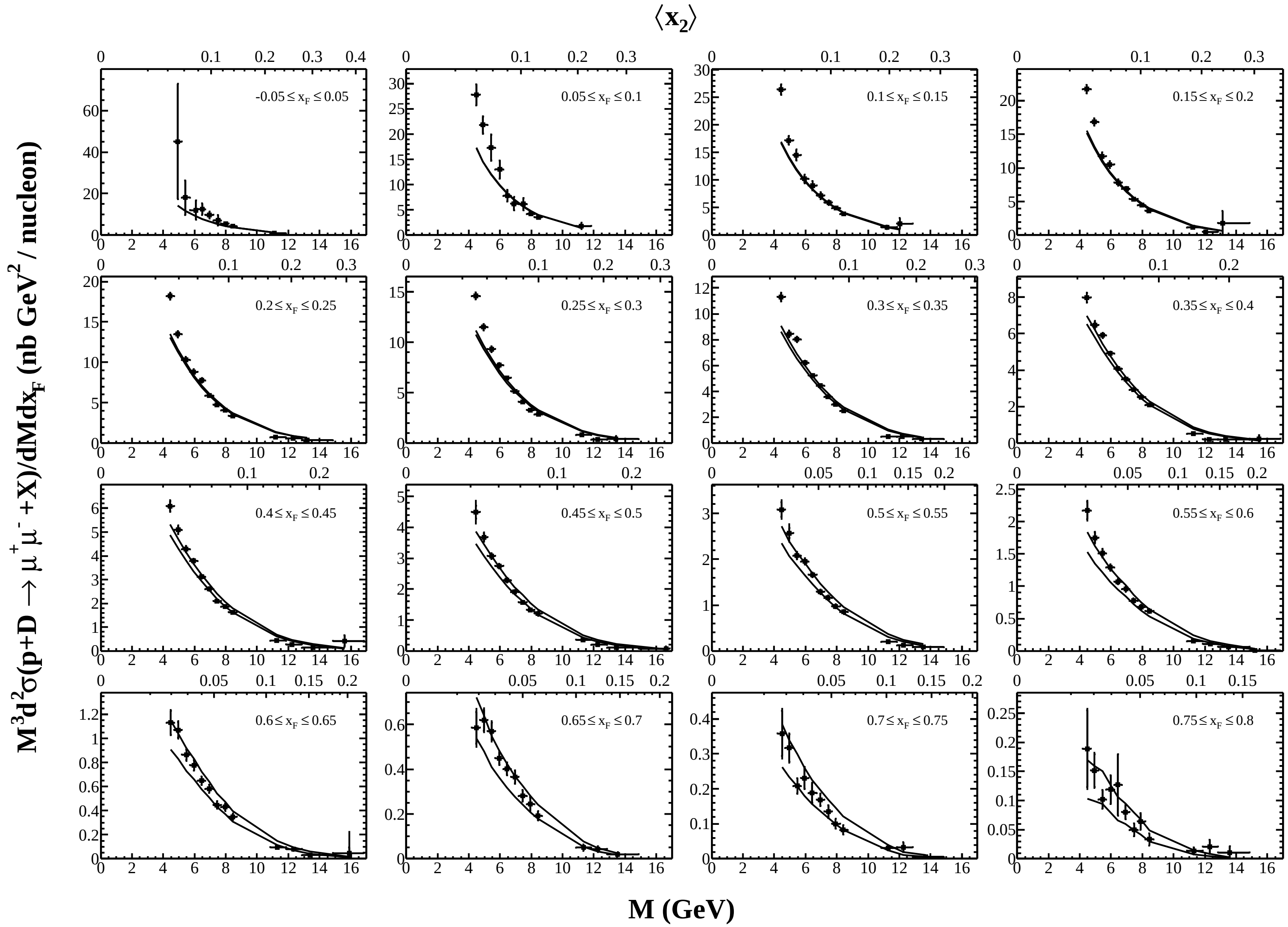
- Intuitive interpretation in the target rest frame:
  - The projectile quark q contains virtual $q\gamma^*$ fluctuations with different transverse sizes ρ,

$$|q\rangle = |q_{bare}\rangle + \Psi_{q\gamma^*}(\alpha, \rho)|q\gamma^*\rangle + \ldots$$

  - The interaction with the target can set these fluctuations on mass shell.
  - Radiation of the γ^* leads to a displacement $\alpha\rho$ of the quark in impact parameter space.
  - The dipole cross section originates from interference of the two diagrams,

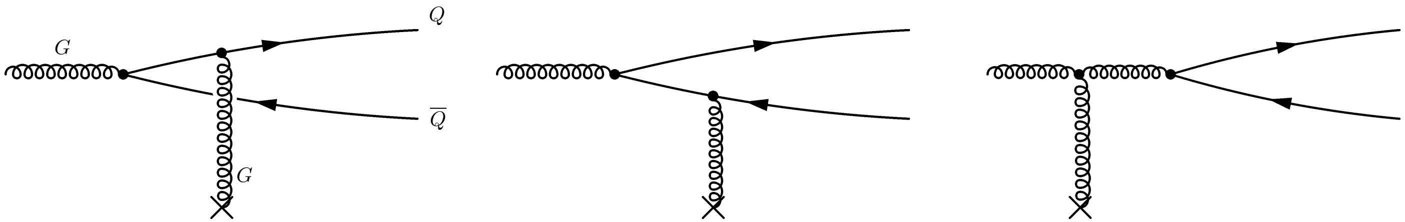
$$\frac{d\sigma(qp \to \gamma^* X)}{d\ln\alpha} = \int d^2\rho \, |\Psi_{q\gamma^*}(\alpha, \rho)|^2 \, \sigma_{q\bar{q}}(x_2, \alpha\rho)$$

Comparison to E866 Drell-Yan Data



Heavy Quark Production at High Energies

At high energies, heavy quark pairs ($Q\overline{Q}$) are predominantly produced through gluon-gluon fusion:



The amplitude reads (Kopeliovich, Tarasov, NPA710:180,2002)

$$
\begin{aligned}
\mathcal{A}_{ij}^{a}(\alpha,\vec{p}_T,\vec{k}_T) \; = \; \int d^2r d^2b e^{i\vec{p}_T\cdot\vec{\rho}-i\vec{k}_T\cdot\vec{b}}\Psi(\alpha,\rho)\Big\{\delta_{ae}\delta_{ij}\left[\gamma^e(\vec{b}-\alpha\vec{\rho})-\gamma^e(\vec{b}+(1-\alpha)\vec{\rho})\right] \\
+\frac{1}{2}d_{aeg}T^g_{ij}\left[\gamma^e(\vec{b}-\alpha\vec{\rho})-\gamma^e(\vec{b}+(1-\alpha)\vec{\rho})\right] \\
+\frac{i}{2}f_{aeg}T^g_{ij}\left[\gamma^e(\vec{b}-\alpha\vec{\rho})+\gamma^e(\vec{b}+(1-\alpha)\vec{\rho})-2\gamma^e(\vec{b})\right]\Big\}
\end{aligned}
$$

with the profile function

$$
\gamma^e(\vec{b})=\frac{\sqrt{\alpha_s}}{4\pi}\int\frac{d^2k_T}{k_T^2}e^{i\vec{k}_T\cdot\vec{b}}F^e_{GN\to X}(\vec{k}_T) \quad , \quad \sigma_{q\bar{q}}(\rho)=\int d^2b\sum_X\sum_{e=1}^{8}\left|\gamma^e(\vec{b}+\vec{\rho})-\gamma^e(\vec{b})\right|^2
$$

The Dipole Approach to Heavy Quark Production

- The final result is, (Nikolaev, Piller, Zakharov, JETP **81**, 851, 1995):

$$\frac{d\sigma(pp \to Q\overline{Q} + X)}{dy_{Q\overline{Q}}} = x_1 G(x_1, \mu_F) \int_0^1 d\alpha d^2\rho \left|\Psi_{G\to Q\overline{Q}}(\alpha,\rho)\right|^2 \sigma_{q\bar{q}G}(x_2,\alpha,\rho)$$

  - α: Light-Cone momentum fraction of the heavy quark Q
  - ρ: transverse size of the $Q\overline{Q}$ pair
  - $\left|\Psi_{G\to Q\overline{Q}}(\alpha,\rho)\right|^2 = \alpha_s(\mu_R)/(4\pi^2)\left\{\left[\alpha^2+(1-\alpha)^2\right] m_Q^2 K_1^2(m_Q\rho) + m_Q^2 K_0^2(m_Q\rho)\right\}$
  - and

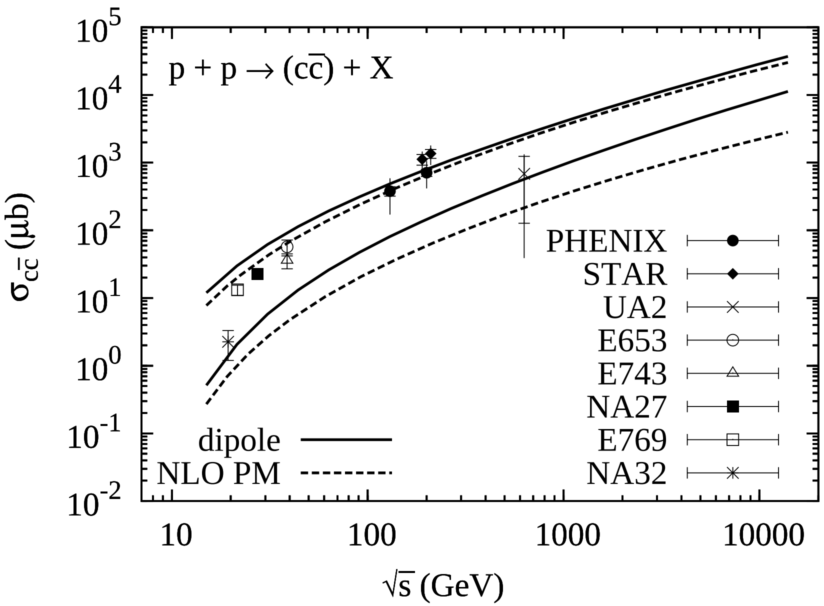
$$\sigma_{q\bar{q}G}(x_2,\alpha,\rho) = \frac{9}{8}\left[\sigma_{q\bar{q}}(x_2,\alpha\rho) + \sigma_{q\bar{q}}(x_2,(1-\alpha)\rho)\right] - \frac{1}{8}\sigma_{q\bar{q}}(x_2,\rho).$$

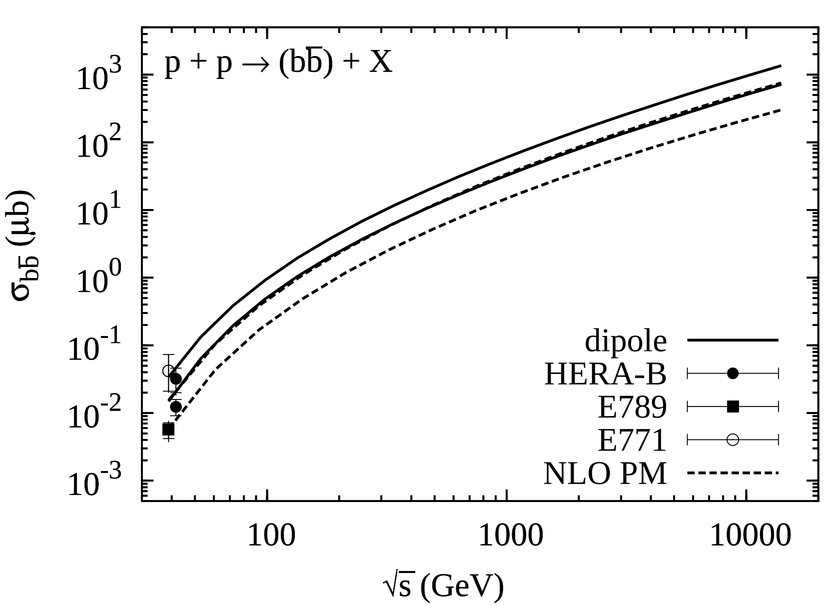
- General rule:

$$\sigma(a + N \to bcX) = \int d\Gamma \left|\Psi_{a\to bc}(\Gamma)\right|^2 \sigma^N_{bc\bar{a}}(\Gamma)$$

  - Γ: set of all internal variables of the (bc)-system
  - $\Psi_{a\to bc}$: Light-Cone wavefunction for the transition $a \to bc$
  - $\sigma^N_{bc\bar{a}}$: cross section for scattering the $bc\bar{a}$-system off a nucleon

Theoretical Uncertainties





JR, J.C. Peng, Phys. Rev. D**67**, 054008, 2003

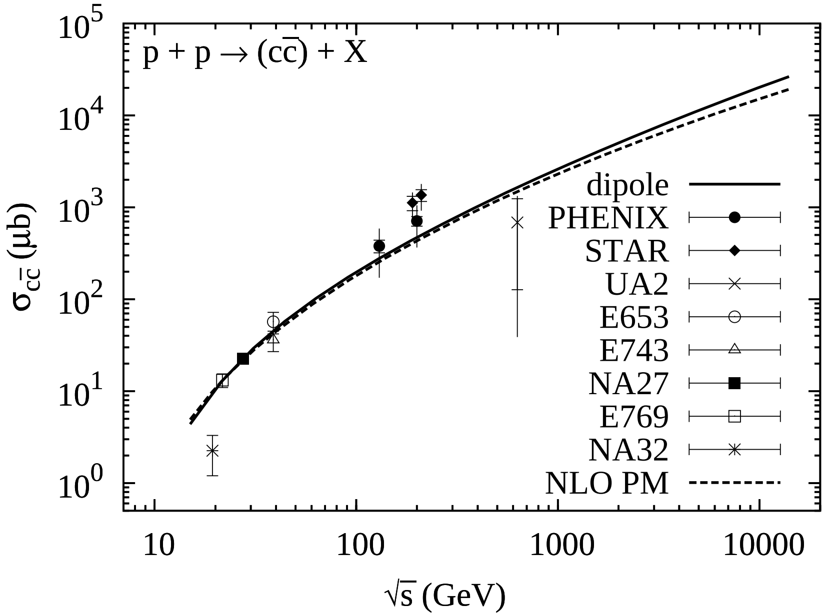
- Large uncertainties for open charm production from choice of m_c

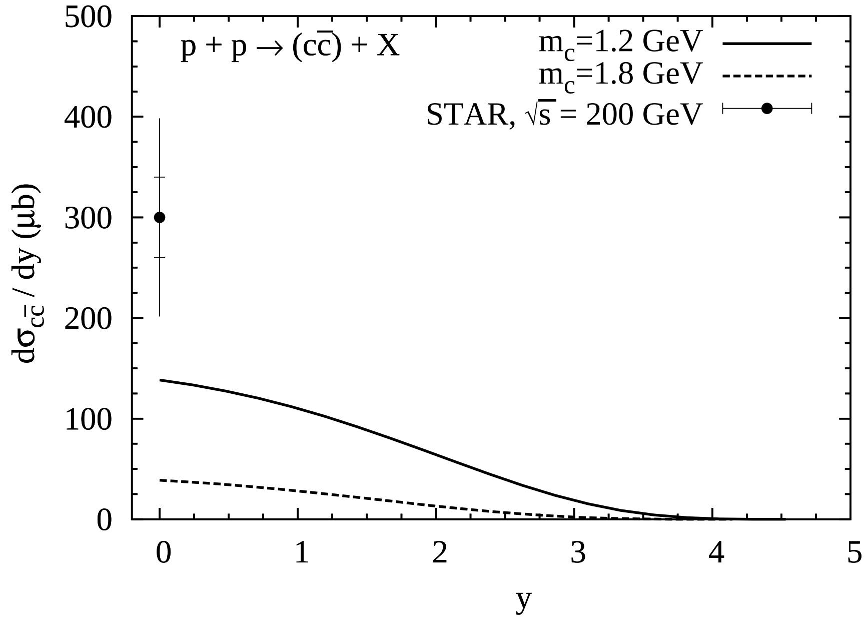
  $1.2\,\text{GeV} \leq m_c \leq 1.8\,\text{GeV}$, $m_c \leq \mu_R \leq 2m_c$, $\mu_F = 2m_c$

  $4.5\,\text{GeV} \leq m_b \leq 5.0\,\text{GeV}$, $m_b \leq \mu_R, \mu_F \leq 2m_b$

- Dipole Approach valid only at high energies (HERA-B energy too low)

Open Charm Production in pp Collisions





- Left: Curves from JR, J.C. Peng, Phys. Rev. D**67**, 054008, 2003 with parameters adjusted to data at $\sqrt{s} < 200\,$GeV. Dipole: $m_c = 1.4\,$GeV, NLO parton model $m_c = 1.2\,$GeV.

- Right: Single quark rapidity distribution in the Dipole Approach at $\sqrt{s} = 200\,$GeV.
  Large uncertainties for open charm production from choice of m_c,

  $$1.2\,\text{GeV} \le m_c \le 1.8\,\text{GeV},\ \mu_R = \mu_F = 2m_c$$

Multiple Scattering and Nuclear Effects

- When switching from a proton to a nuclear target, the profile function $\gamma_N^a(b)$ for a nucleon needs to be replaced by the profile function for a nucleus $\gamma_A^a(b)$
- Hence $\sigma_{q\bar{q}}^N(\rho) \to \sigma_{q\bar{q}}^A(\rho)$ and

$$\sigma_{q\bar{q}G}^N(\rho) = \frac{9}{8}\left[\sigma_{q\bar{q}}^N(\alpha\rho) + \sigma_{q\bar{q}}^N((1-\alpha)\rho)\right] - \frac{1}{8}\sigma_{q\bar{q}}^N(\rho)$$

$$\to \sigma_{q\bar{q}G}^A(\rho) = \frac{9}{8}\left[\sigma_{q\bar{q}}^A(\alpha\rho) + \sigma_{q\bar{q}}^A((1-\alpha)\rho)\right] - \frac{1}{8}\sigma_{q\bar{q}}^A(\rho)$$

- The advantage of the (ρ, α) representation is, that one can calculate $\sigma_{q\bar{q}}^A(\rho)$ from $\sigma_{q\bar{q}}^N(\rho)$.
- In the limit of very high energy, all partons move along straight lines and pick up only a (color) phase factor as they move through the nucleus. Averaging over the target is done as in Glauber theory,

$$\sigma_{q\bar{q}}^A(\rho) = 2\int d^2b\left\{1 - \exp\left(-\frac{\sigma_{q\bar{q}}^N(\rho)T(b)}{2}\right)\right\}.$$

- At finite energy, one has to solve the Dirac (Klein-Gordon) equation for quarks (gluons) propagating through an external color field in the (non-abelian) Furry approximation: Terms of order $1/E$ are neglected, except in phase factors. This accounts for variations of the transverse size of partonic configurations.

Shadowing in DIS vs. Heavy Quark Shadowing

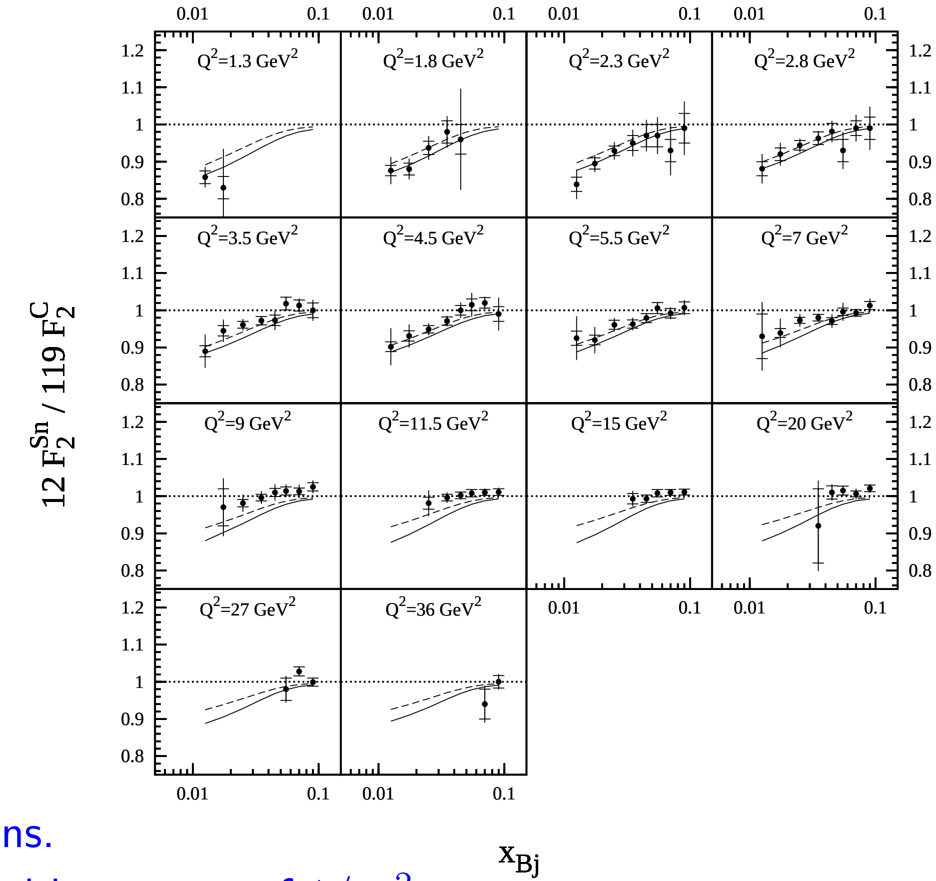
- In DIS, shadowing is caused by the aligned jet configurations, where either $\alpha \to 0$ or $\alpha \to 1$

$$\left|\Psi_{\gamma^* \to q\bar{q}}(\alpha, \rho)\right|^2 \propto \exp(-2\varepsilon\rho).$$

  Extension parameter:

$$\varepsilon^2 = \alpha(1-\alpha)Q^2 + m_q^2.$$

  These aligned jet configurations are shadowed even for $Q^2 \to \infty$.
  That is why shadowing in DIS is leading twist.



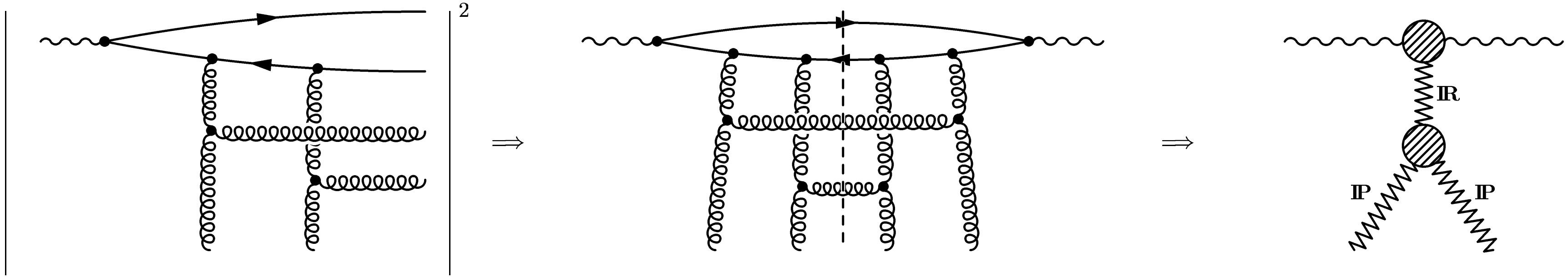
- In heavy quark production however

$$\left|\Psi_{G \to Q\overline{Q}}(\alpha, \rho)\right|^2 \propto \exp(-2m_Q\rho).$$

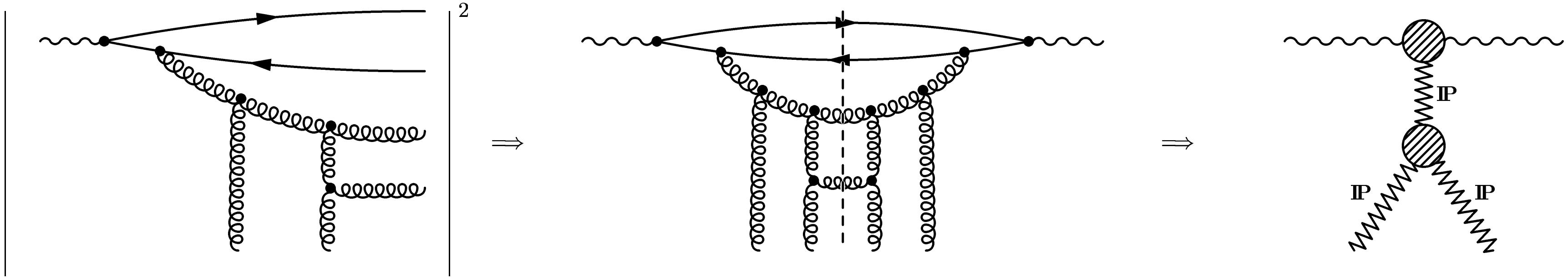
  The heavy quark mass cuts off large fluctuations.
  Multiple scattering of the $Q\overline{Q}$ pair is suppressed by powers of $1/m_Q^2$.
  Hence, eikonalization of $\sigma_{q\bar{q}}^N$ alone does not give the complete picture of heavy quark shadowing.

Mechanisms of Nuclear Suppression

- $Q\overline{Q}$ rescattering:



- $Q\overline{Q}G$ ($\approx GG$) rescattering:



Inclusion of Higher Fock States

- Higher Fock states are included in the parametrization of $\sigma_{q\bar{q}}^N(x,\rho)$.
- However, the rescattering of these higher Fock states is neglected in the eikonal approximation.
- This can be cured by the following recipe:

$$\sigma_{q\bar{q}}^A(x,\rho) = 2\int d^2b \left\{1 - \exp\left(-\frac{\sigma_{q\bar{q}}^N(x,\rho)\widetilde{T}(b)}{2}\right)\right\},$$

where

$$\widetilde{T}(b) = T(b)R_G(x,b)$$

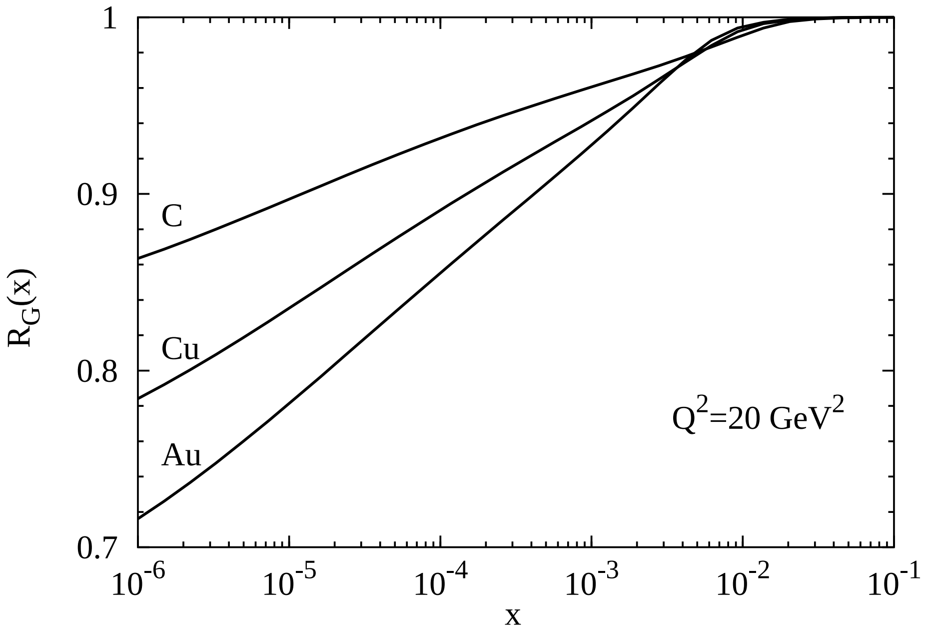
and $R_G(x,b)$ is the leading twist gluon shadowing, calculated from the propagation of a GG dipole through a nucleus.

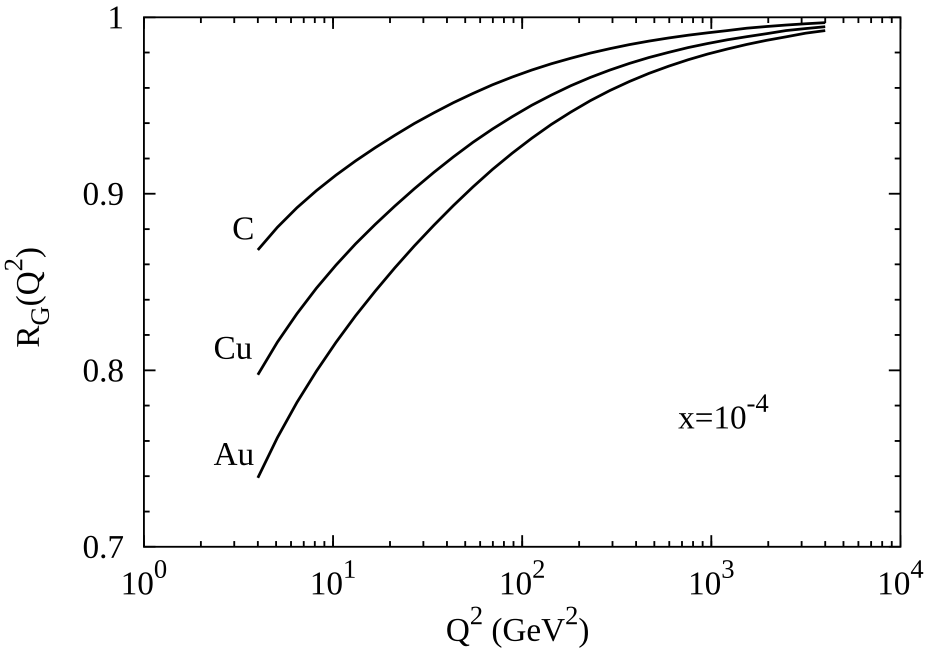
- Expansion of the nuclear dipole cross section:

$$\sigma_{q\bar{q}}^A(x,\rho) = \frac{\pi^2}{3}\alpha_s\rho^2 \int d^2bT(b)R_G(x,b)xG_N(x) - \frac{\pi^2\alpha_s^2}{36}\rho^4 \int d^2b\left[T(b)R_G(x,b)xG_N(x)\right]^2 + \dots$$

Already the single scattering term is suppressed due to gluon shadowing.

Gluon Shadowing





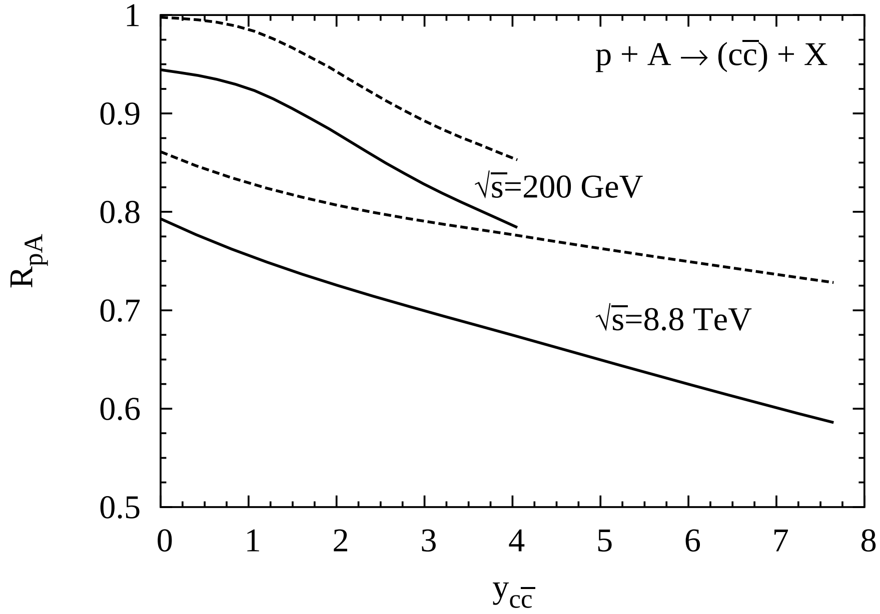
Kopeliovich, JR, Tarasov, Johnson, Phys. Rev. C**67,** 014903, 2003

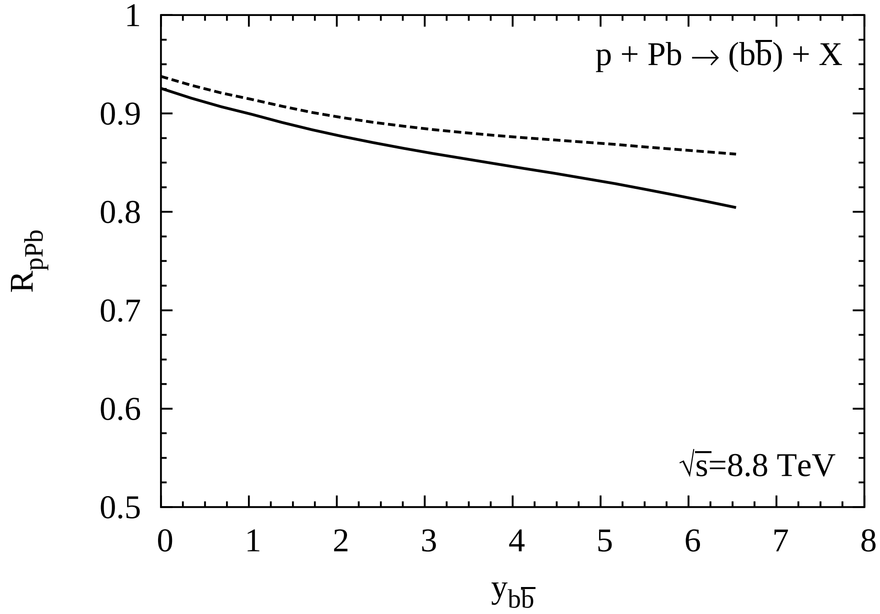
- No gluon shadowing at $x_2 > 0.01$, because of short l_c.
- The dipole approach predicts much smaller gluon shadowing than most other approaches.
- Gluon shadowing extends to large Q^2, i.e. is a leading twist effect.

Why is Gluon Shadowing So Small?

- The magnitude of gluon shadowing is unknown experimentally, but there are hints from existing data.
- The gluon can propagate only distances of order of a constituent quark radius ($\sim 0.3\,\mathrm{fm}$) from the $Q\overline{Q}$-pair. This overcompensates the color factor $9/4$ in the interaction strength.
- The smallness of the gluon correlation radius is the only known way to explain the tiny Pomeron-proton cross section ($\approx 2\,\mathrm{mb}$).
- This picture is supported by the large octet string tension, i.e. $4\,\mathrm{GeV/\,fm}$ for GG instead of $1\,\mathrm{GeV/\,fm}$.
- Similar results exist in the Instanton Liquid Model, the Stochastic Vacuum Model, QCD sum rules and in Lattice QCD.
- The smallness of the $Q\overline{Q}G$ fluctuation is incorporated into the dipole approach by a nonperturbative interaction between $Q\overline{Q}$-pair and the gluon G. Parameters are fitted to single diffraction data (in pp).

Suppression of Open Charm and Bottom in pA Collisions





JR, J. Phys. G**30**(2004)S1159

- Dashed curves: Gluon Shadowing only
- Solid curves: Total suppression (including $Q\overline{Q}$ rescattering and Gluon Shadowing)
- Gluon Shadowing reduces the probability for $Q\overline{Q}$ rescattering.

Summary

- At high energies, heavy quark production (and the DY process) can be formulated in terms of the same color dipole cross section as low-x DIS, because in all these processes the projectile scatters off the target gluon field.
- The dipole formulation is boost-invariant, but has a particularly intuitive interpretation in the target rest frame.
- The dipole cross section is an eigenvalue of the diffraction amplitude operator. At scales much larger than Q_s, it is related to the unintegrated gluon density of the proton.
- All nuclear effects are predicted: no additional free parameters.
- The dipole approach takes into account both, hard saturation effects and soft gluon shadowing.
- Gluon shadowing reduces the probability of heavy quark rescattering by making the target more dilute.
  - Both effects are comparable in magnitude at RHIC.
  - Leading twist gluon shadowing is the dominant effect at LHC.