A modified Balitsky-Kovchegov equation

Eugene Levin, Tel Aviv University

LHC-HERA WS, November 2004

The Goal: Reliable calculations for parton densities at the LHC energies based on the high parton density QCD theory

Sources:

- E. Gotsman, E. Levin, M. Lublinsky and U. Maor: "Towards a new global QCD analysis: Low x DIS data from non-linear evolution," Eur. Phys. J. C 27 (2003) 411 [arXiv:hep-ph/0209074];

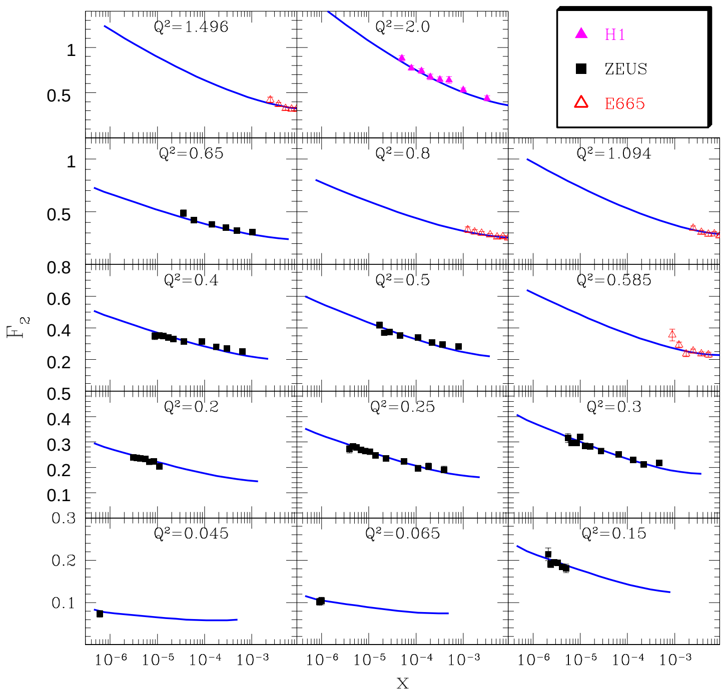
- Parameterizations for unintegrated parton densities are available at www.desy.de/$\sim$ lublinm;

- E. Gotsman, M. Kozlov, E. Levin, U. Maor and E. Naftali : "Towards a new global QCD analysis: Solution to the non-linear equation at arbitrary impact parameter," Nucl. Phys. A 742 (2004) 55 [arXiv:hep-ph/0401021];

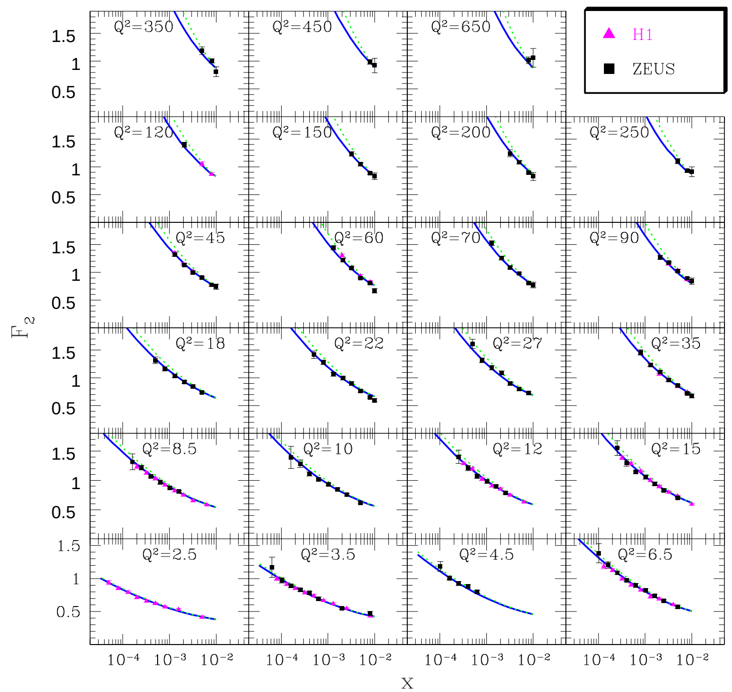
- E. Gotsman, E. Levin, U. Maor and E. Naftali : "A modified Balitsky - Kovchegov equation", (in preparation);

F_2 - HERA data :

Low Q^2:

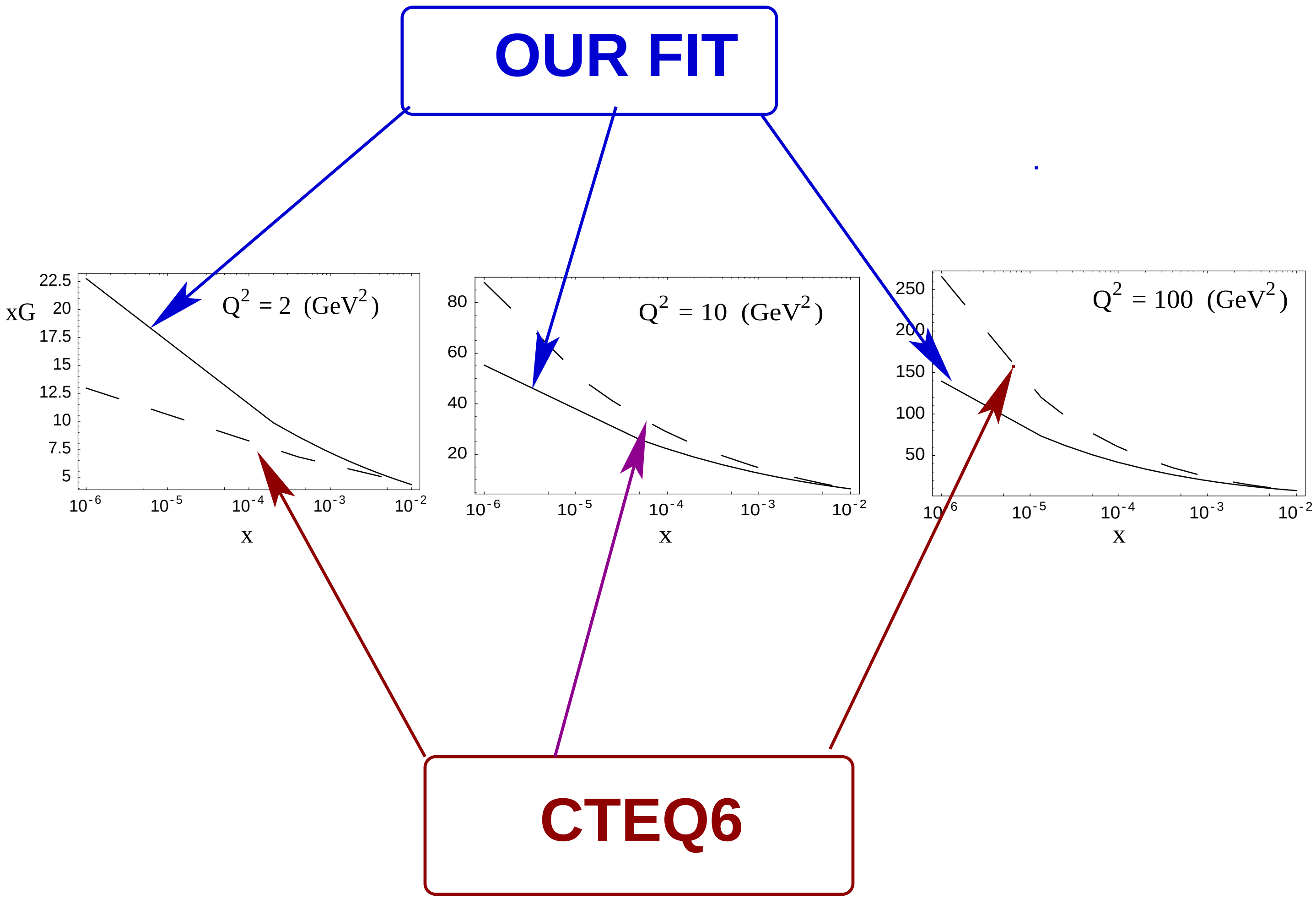


Large Q^2:



LHC predictions :

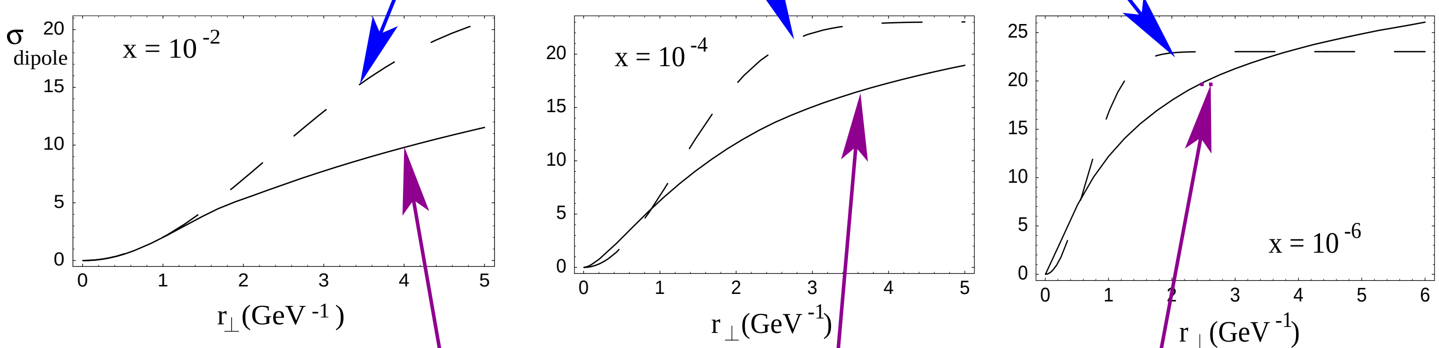
$xG(x, Q^2)$:



LHC predictions :

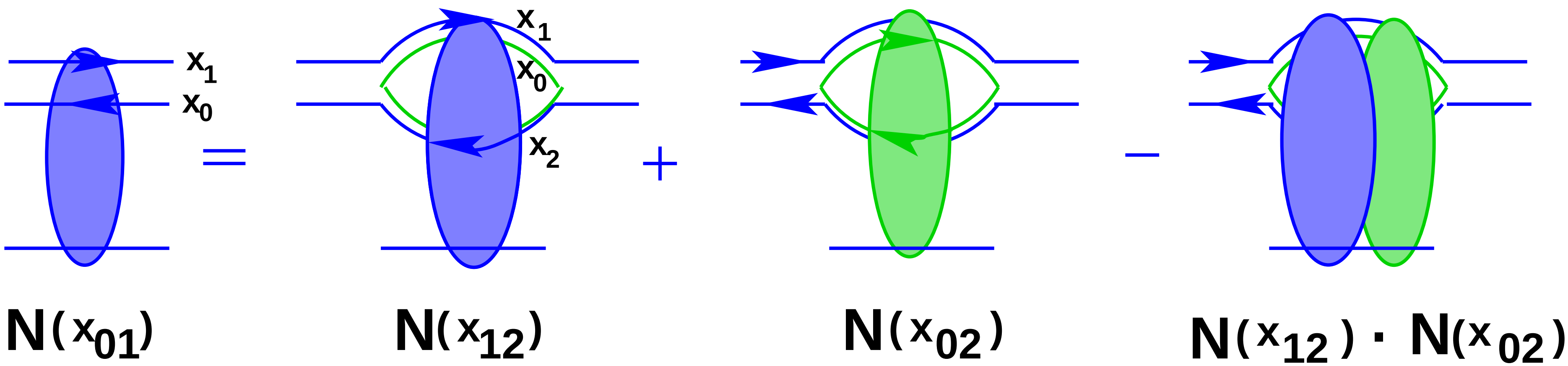
σ-dipole:

GBW MODEL



OUR PREDICTIONS

B-K non-linear equation:



$$\frac{\partial N(y,\vec{x}_{01},\vec{b})}{\partial y} = \frac{C_F \alpha_S}{2\pi^2}\int d^2 x_2 \,\frac{x_{01}^2}{x_{02}^2\; x_{12}^2}\Big(2 N(y,\vec{x}_{12},\vec{b}-\frac{1}{2}\vec{x}_{02})$$

$$- N(y,\vec{x}_{01},\vec{b}) - N(y,\vec{x}_{12},\vec{b}-\frac{1}{2}vec x_{02})\, N(y,\vec{x}_{02},\vec{b}-\frac{1}{2}\vec{x}_{12})\Big)$$

Deficiencies of B-K equation:

- **Correct only in LLA approximation of pQCD with BFKL kernel in LO;**
- **The mean field approximation to the JIMWLK equation;**
- **It is not correct in the saturation region;**
- **The region where we can neglect the non-linear corrections should be specified by conditions beyond the BK equation;**

B-K equation versus NLO BFKL:

$$N_{\text{non-linear term}} \propto \alpha_S^4 \, s^{2\Delta_{BFKL}}; \qquad N_{\text{linear term}} \propto \alpha_S^2 \, s^{\Delta_{BFKL}};$$

with $\Delta_{BFKL} = \alpha_S \, \chi_{LO\,BFKL} + \alpha_S^2 \, \chi_{NLO\,BFKL}$

Correct strategy (theory point of view):

For $1/\alpha_S > y = \ln s > 1$ $\qquad N^{LOBFKL}_{\text{linear term}}$

For $y = \ln s > (2/\alpha_S)\ \ln(1/\alpha_S)$ $\qquad N^{LOBFKL}_{\text{linear term}} + N_{\text{n-l term}}$

For $y = \ln s > (1/\alpha_S^2)$ $\qquad N^{NLOBFKL}_{\text{linear term}} + N_{\text{n-l term}}$

NLO BFKL and saturation scale $Q_s(Y)$:

- **The NLO BFKL kernel is known** (Lipatov and Fadin , Camisi and Ciafaloni 1998);
- **The needed re-summation has been performed** (Salam, Ciafaloni et al. 1998 - 2000);
- **Our procedure is based on the observation of the Durham group:**

$$\bar{\alpha}_S\,\chi_{NLO}(\gamma) \;=$$

$$\left(\frac{1+\omega\,A_1(\omega)}{\gamma}-\frac{1}{\gamma}+\frac{1+\omega\,A_1(\omega)}{1-\gamma+\omega}-\frac{1}{1-\gamma}\right)\;-\;\omega\,\chi^{HT}_{LO}(\gamma)$$

– The pole $\gamma = 0$ corresponds to the normal twist-2 DGLAP contribution with

$$Q > \dots k_{i,t} > k_{t,i+1} > \dots > Q_0$$

where Q_0 is the typical virtuality of the target;

– The pole at $\gamma = 1$ corresponds to inverse k_t ordering

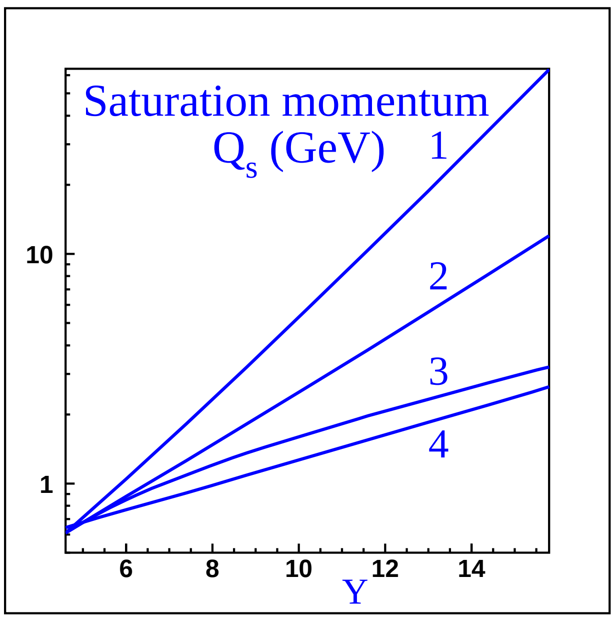
$$k_{i,t} < k_{t,i+1} < \dots Q_0$$

– The other poles, at $\gamma = -1, -2, \dots (\gamma = 2, 3, \dots)$, are the higher twists contributions due to the gluon reggeization.

Our suggestion: (Ellis, Kunszt and Levin 1994)

$$\bar{\alpha}_S\,\chi_{NLO}(\gamma) \;=\; -\,\omega\,\bar{\alpha}_S\,\chi_{LO}(\gamma)\;;$$

$$\omega(\gamma) \;=\; \bar{\alpha}_S\;(1\,-\,\omega)\;\bar{\alpha}_S\,\chi_{LO}(\gamma)\;;\quad \gamma^{DGLAP} = \bar{\alpha}_S\left(\tfrac{1}{\omega} - 1\right);$$



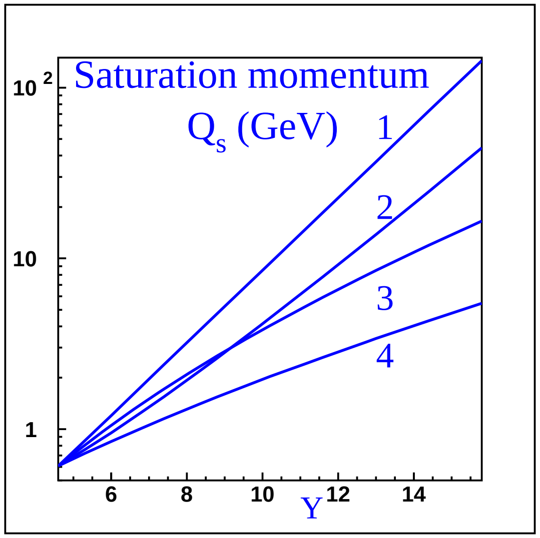
1 - LO BFKL

2 - Our kernel;

3 - NLO BFKL (Durham);

4 - BGW model;

Energy dependence of Q_s:



1 - High energy behaviour (fixed α_S);

2 - Low energy corrections (fixed α_S);

3 - High energy behaviour (running α_S);

4 - Low energy corrections (running α_S);

$$Q_s^2(Y) = Q_s^2(Y_0)\,\exp\left(\frac{\omega(\gamma_{cr})}{1-\gamma_{cr}}\,(Y - Y_0) - \frac{3}{2(1-\gamma_{cr})}\,\ln(Y/Y_0) - \right.$$

$$\left. - \frac{3}{(1-\gamma_{cr})^2}\sqrt{\frac{2\,\pi}{\omega''(\gamma_{cr})}}\,\left(\frac{1}{\sqrt{Y}} - \frac{1}{\sqrt{Y_0}}\right)\right)$$

Modified B-K equation:

- $$\frac{\partial N(r,Y;b)}{\partial Y} = \frac{C_F\,\alpha_S}{\pi^2}\int\frac{d^2r'\,r^2}{(\vec{r}-\vec{r}\,')^2\,r'^2}$$

$$\left(2N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right) - N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right)N\left(\vec{r}-\vec{r}\,',Y;b-\frac{1}{2}\vec{r}\,'\right)\right)_{B-Kterm} -$$

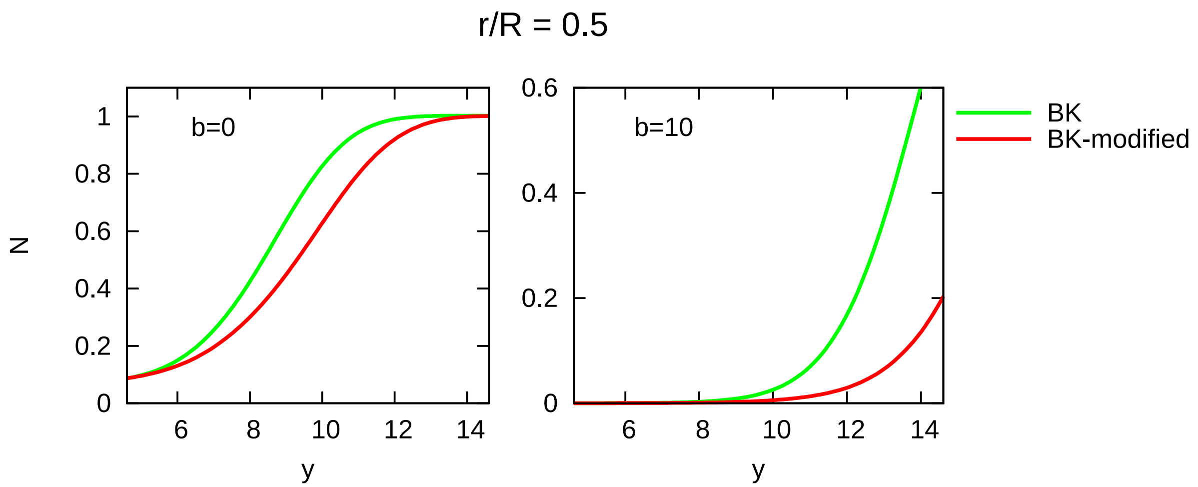
$$-\frac{\partial}{\partial Y}\left(2N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right) - N\left(r',Y;\vec{b}-\frac{1}{2}(\vec{r}-\vec{r}\,')\right)N\left(\vec{r}-\vec{r}\,',Y;b-\frac{1}{2}\vec{r}\,'\right)\right)_{new}$$

$\bar{\alpha}_S\,\omega\,\chi_{LO}(\gamma)$ has the following form in Y, r representation

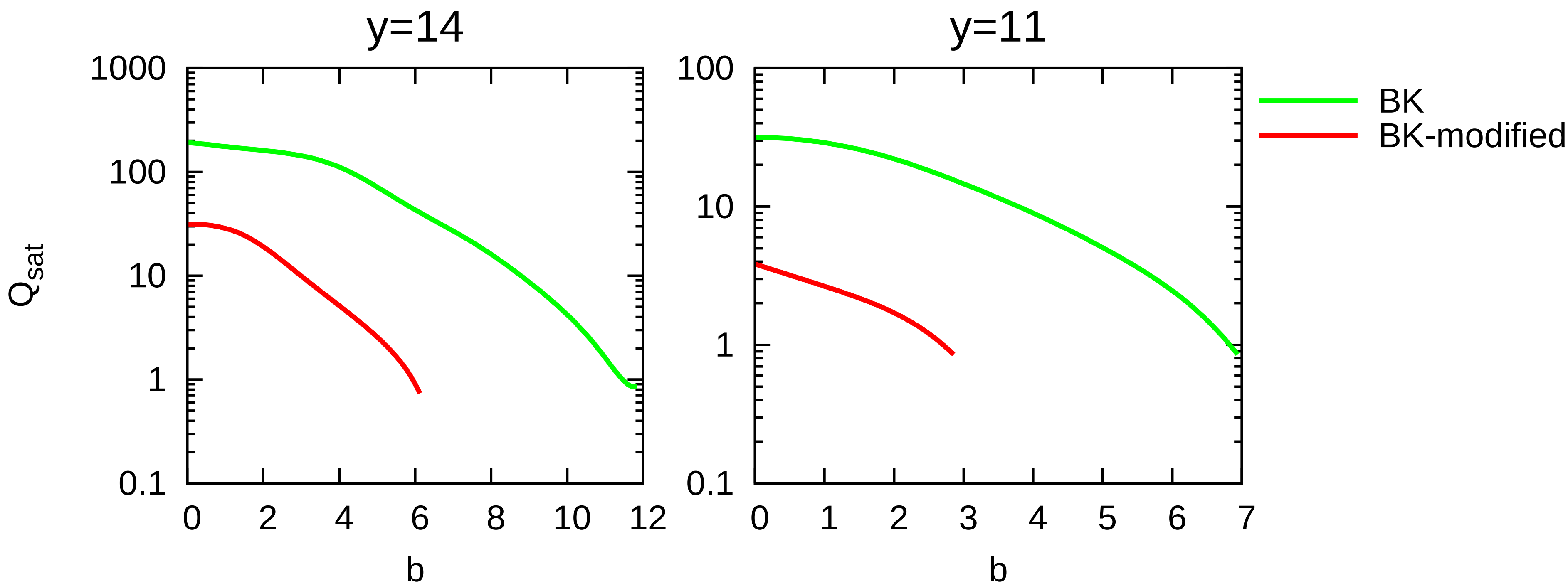
$$\bar{\alpha}_S\,\omega\,\chi_{LO}(\gamma) \;\rightarrow\; \bar{\alpha}_S\int K_{LO}(Y,r')\,d^2r'\,\frac{\partial N(Y,r')}{\partial Y}$$

Modified B-K equation that we have solved:

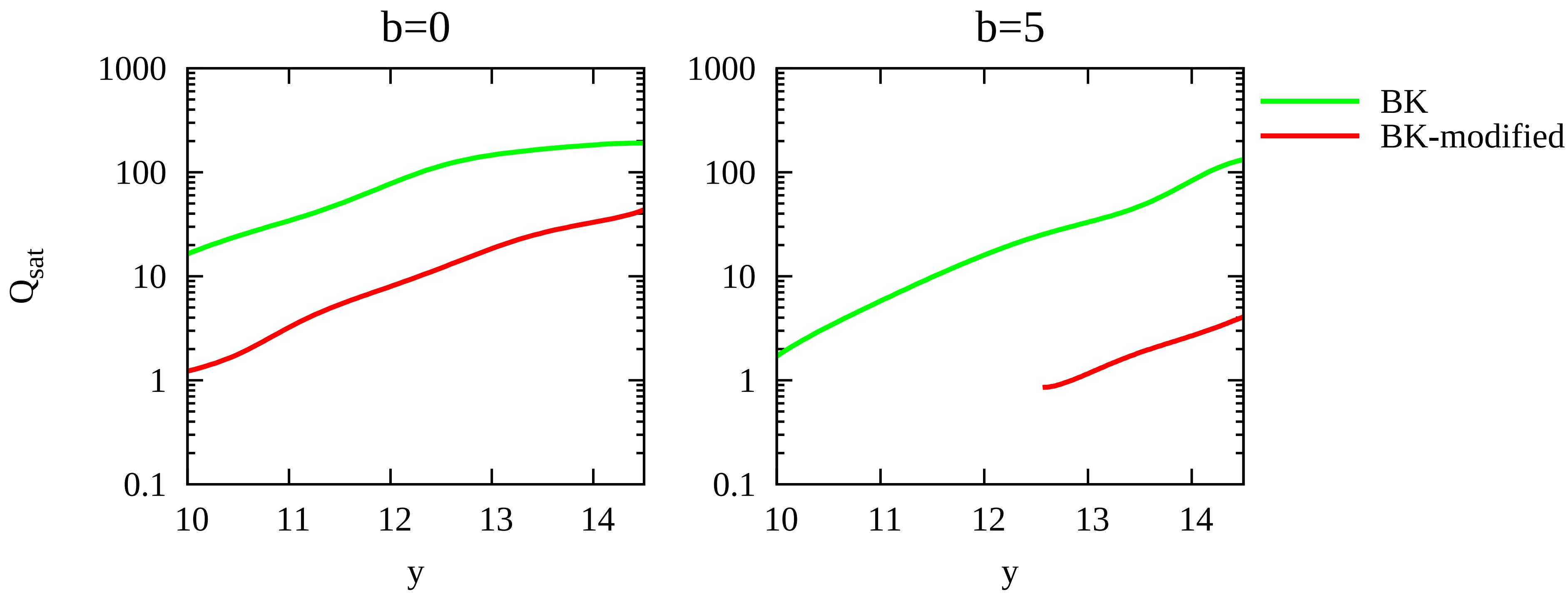
- $$\frac{\partial N\left(r,Y;b\right)}{\partial\,Y} = \frac{C_F\,\alpha_S}{\pi^2}\,\left\{-\,2r^2\int_r^R\frac{d^2r'}{r'^4}\,\frac{\partial N\left(r',Y;b\right)}{\partial\,Y}\,|_{DGLAP} + \right.$$
$$\int\frac{d^2r'\,r^2}{(\vec{r}\,-\,\vec{r'})^2\,r'^2}\,\Theta(R-r')\Theta(R-|r-r'|)\times$$
$$\left.\left[2N\left(r',Y;\vec{b}-\tfrac{1}{2}(\vec{r}-\vec{r'})\right)-N\left(r',Y;\vec{b}-\tfrac{1}{2}(\vec{r}-\vec{r'})\right)\,N\left(\vec{r}-\vec{r'},Y;b-\tfrac{1}{2}\vec{r'}\right)\right]\right\}\,.$$



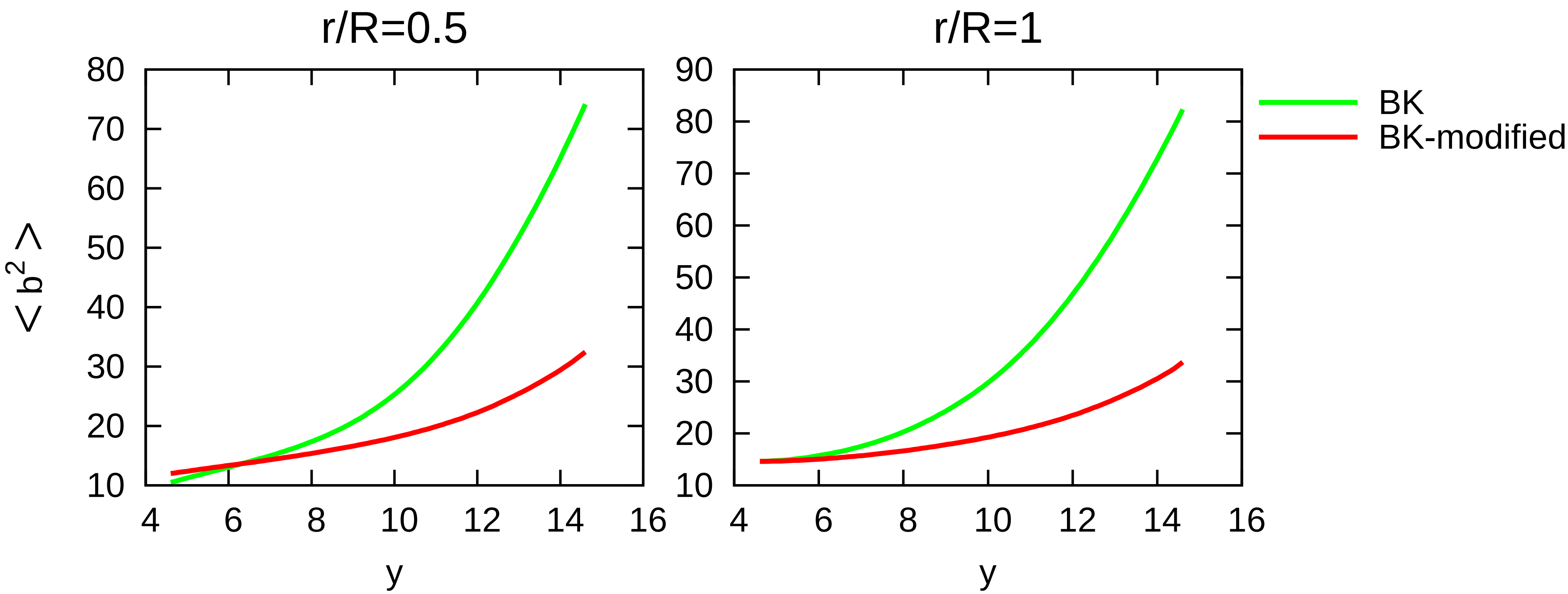
b-dependence of Q_s :



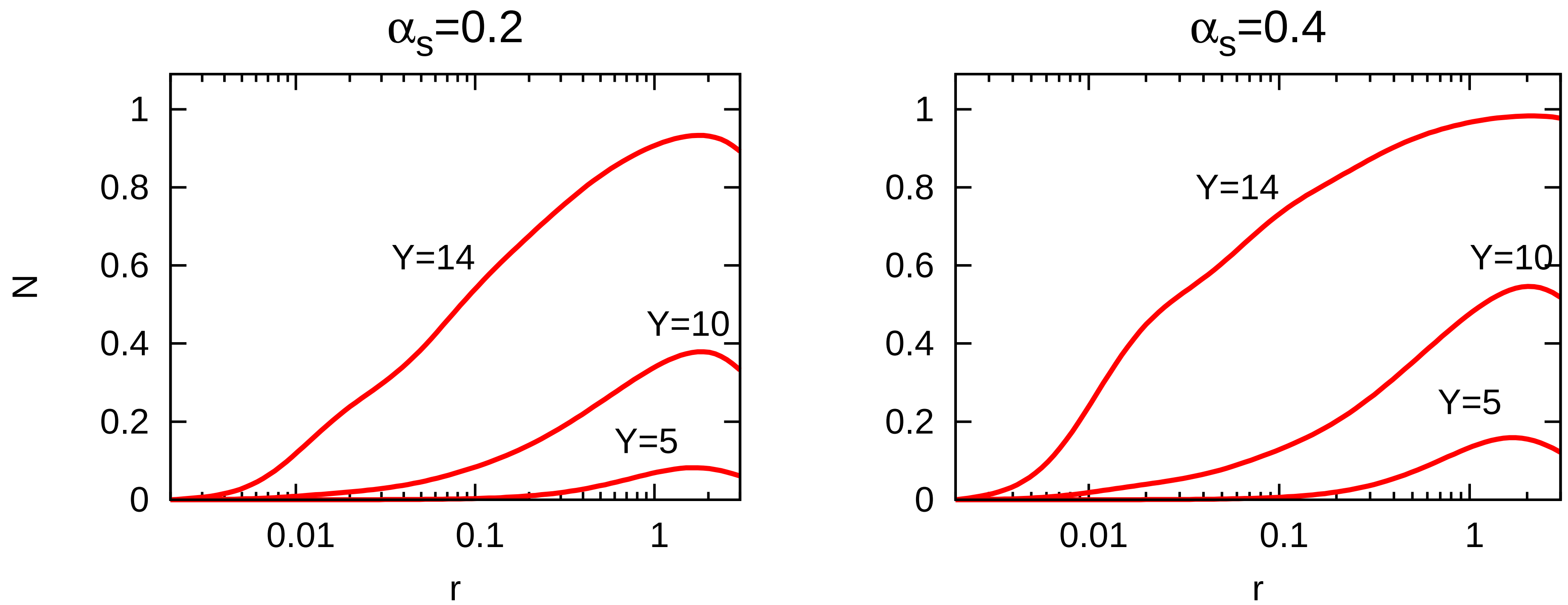
Y-dependence of Q_s :



Energy behaviour of $< b^2 >$:



α_S dependence of the dipole amplitude :



Conclusions:

- **The influence of the preasymptotic corrections, related to the full anomalous dimension of the DGLAP equation, is rather large;**
- **These corrections slow down the energy behaviour as well as the value of Q_s;**
- **The BK equation without any modification is not able to provide reliable predictions for the LHC energies;**
- **The modified BK equation has a chance to do this job;**

Problems to be solved:

- **Solution to the full modified equation ?**
- **Running α_S?**
- **Global fit of the experimental data?**
- **Reliable predictions for unintegrated parton densities for LHC energies?**