Tutorial on

Designing the Optimal Controller

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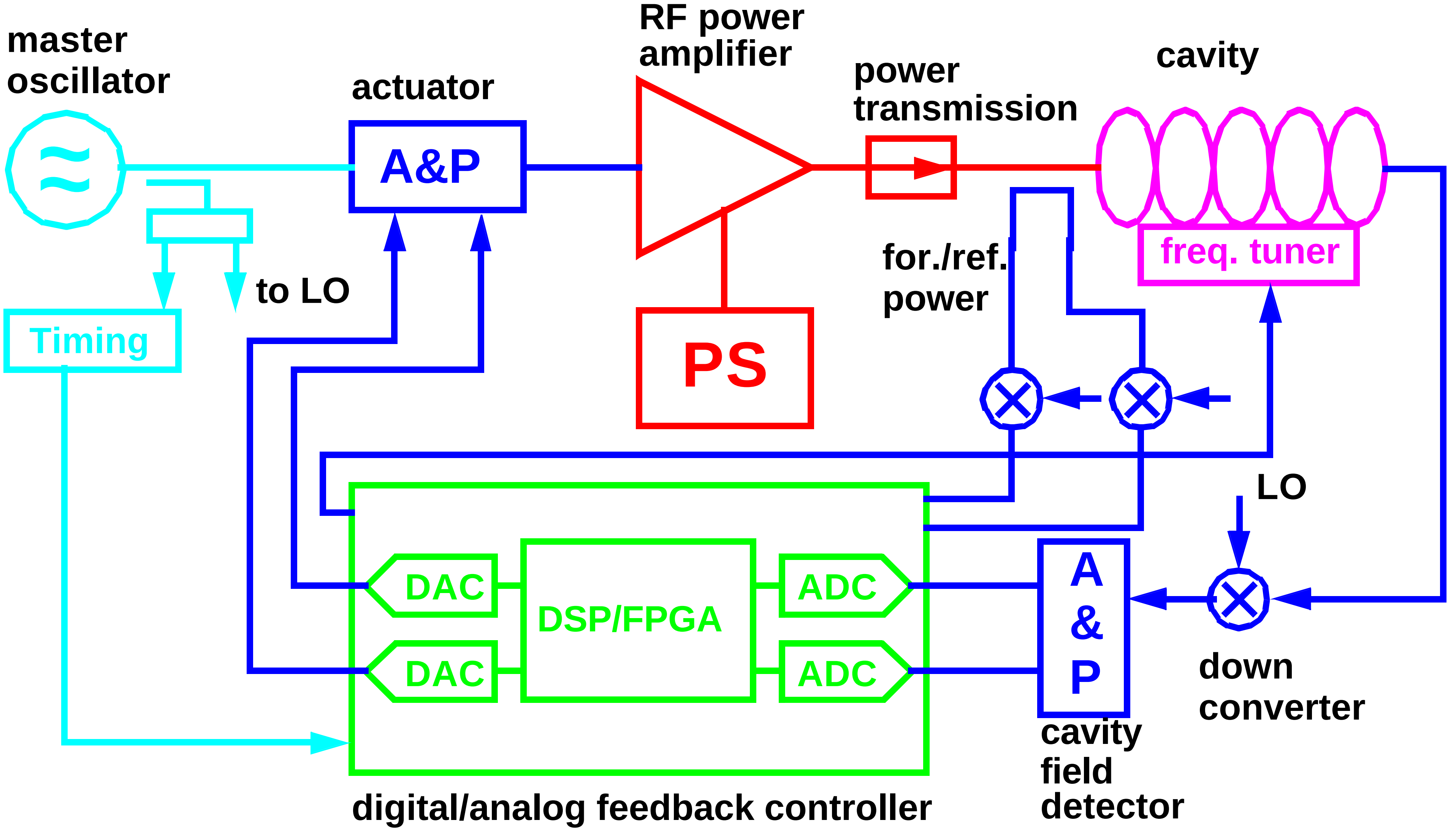
Outline

- RF Control System and Requirements
- Optimal Controller in Control Theory
- Optimal Control for LLRF Control
- Outlook

RF System Architecture (Simplified)



RF Control Requirements

- Maintain Phase and Amplitude of the accelerating field within given tolerances to accelerate a charged particle beam
  - 0.1% for amplitude and 0.1 deg.for phase

- Minimimize Power needed for control

- RF system must be reproducible, reliable, operable, and well understood.

- Other performance goals
  - build-in diagnostics for calibration of gradient and phase, cavity detuning, etc.
  - provide exception handling capabilities
  - meet performance goals over wide range of operating parameters

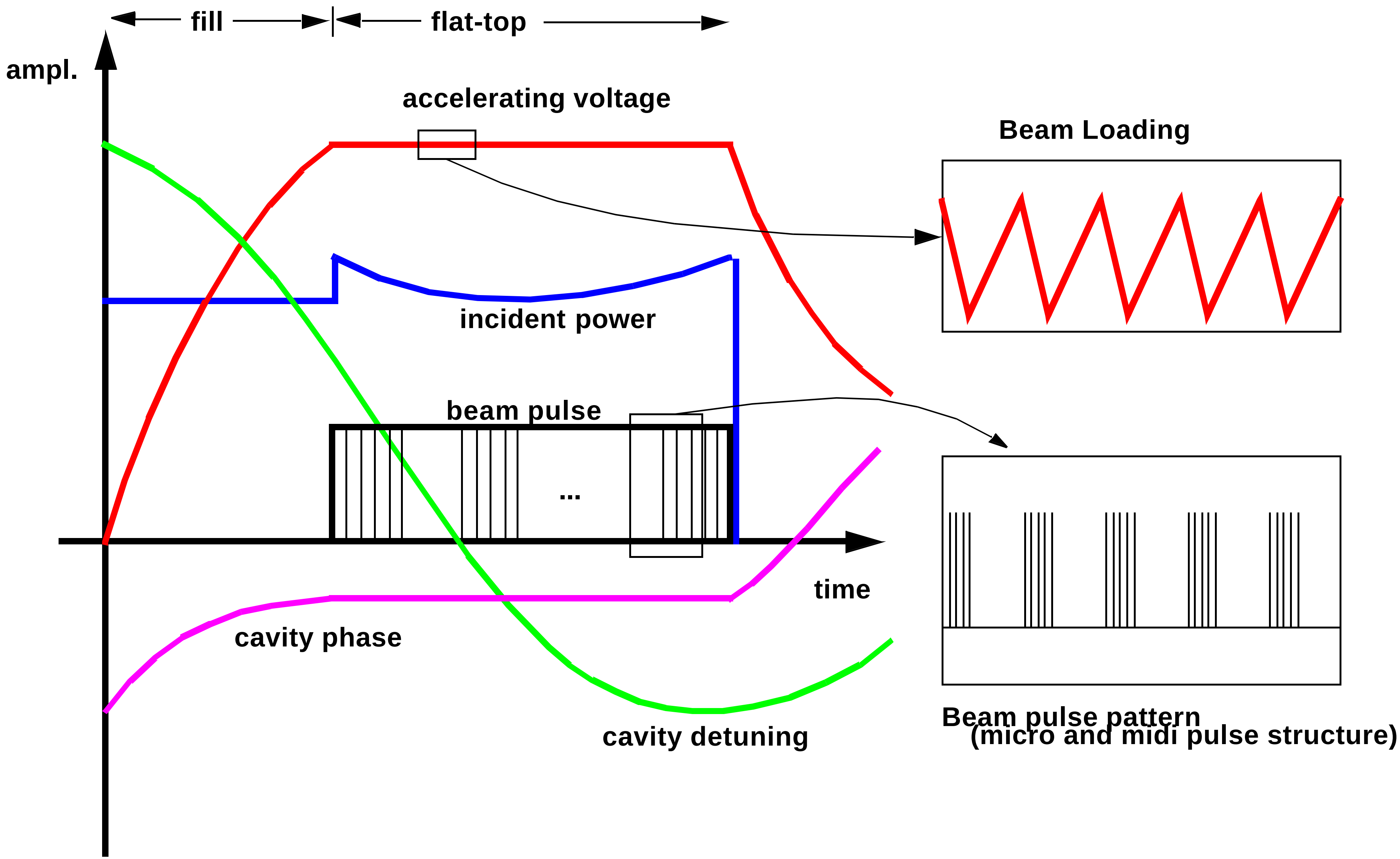
Requirements RF Control

- Derived from beam properties
  - energy spread
  - emittance
  - bunch length (bunch compressor)
  - arrival time

- Different accelerators have different requirements on field stability (approximate RMS requirements
  - 1% for amplitude and 1 deg. for phase (example: SNS)
  - 0.1% for amplitude and 0.1deg.for phase (linear collider)
  - up to 0.01% for amplitude and 0.01 deg. for phase (XFEL)

Note: Distinguish between correlated and uncorrelated error

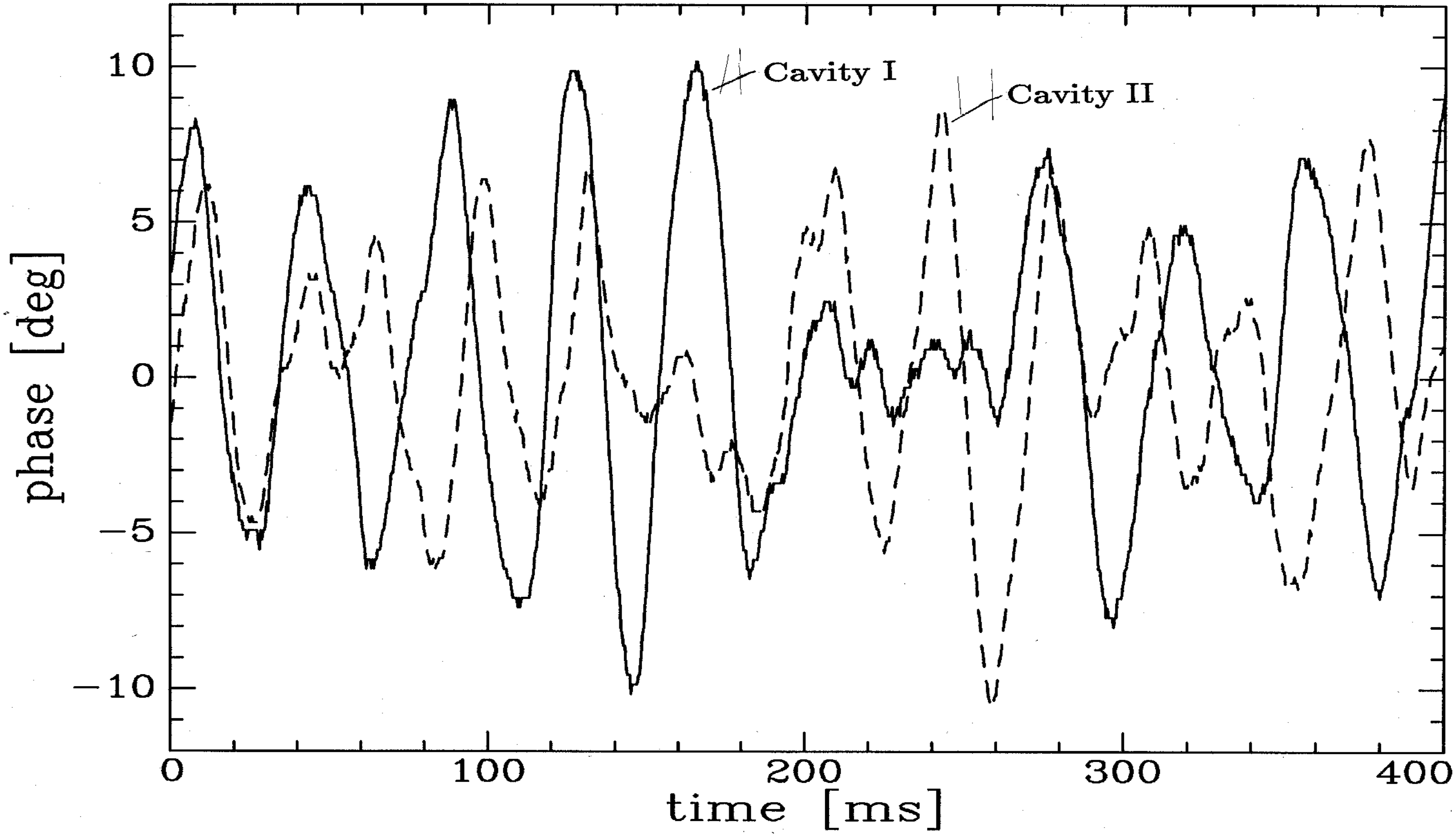
Typical Parameters in a Pulsed RF System



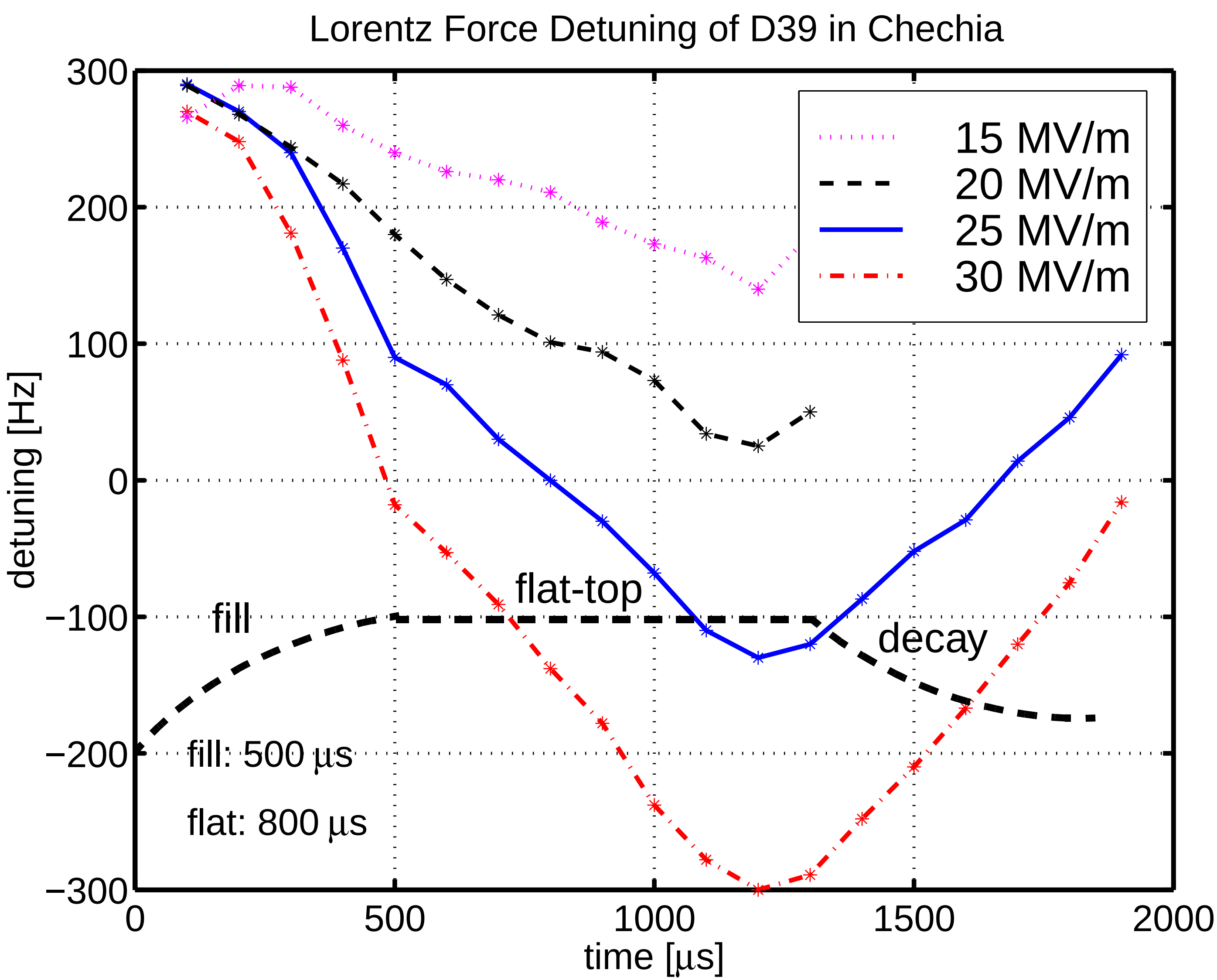
Sources of Perturbations

o Beam loading - Beam current fluctuations - Pulsed beam transients - Multipacting and field emission - Excitation of HOMs - Excitation of other passband modes - Wake fields o Cavity drive signal - HV- Pulse flatness - HV PS ripple - Phase noise from master oscillator - Timing signal jitter - Mismatch in power distribution	o Cavity dynamics - cavity filling - settling time of field o Cavity resonance frequency change - thermal effects (power dependent) - Microphonics - Lorentz force detuning o Other - Response of feedback system - Interlock trips - Thermal drifts (electronics, power amplifiers, cables, power transmission system)

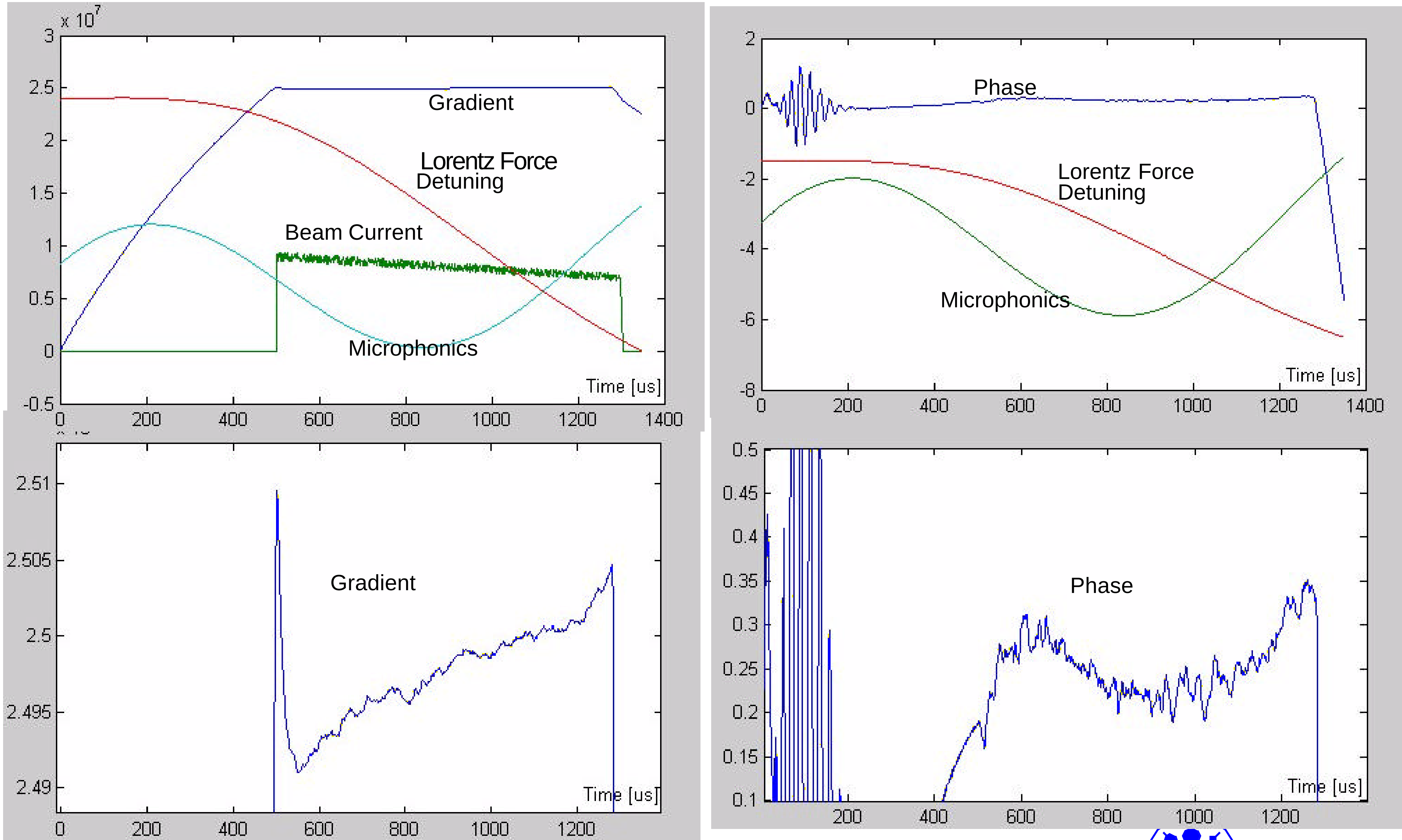
Microphonics at JLAB



Lorentz Force Detuning



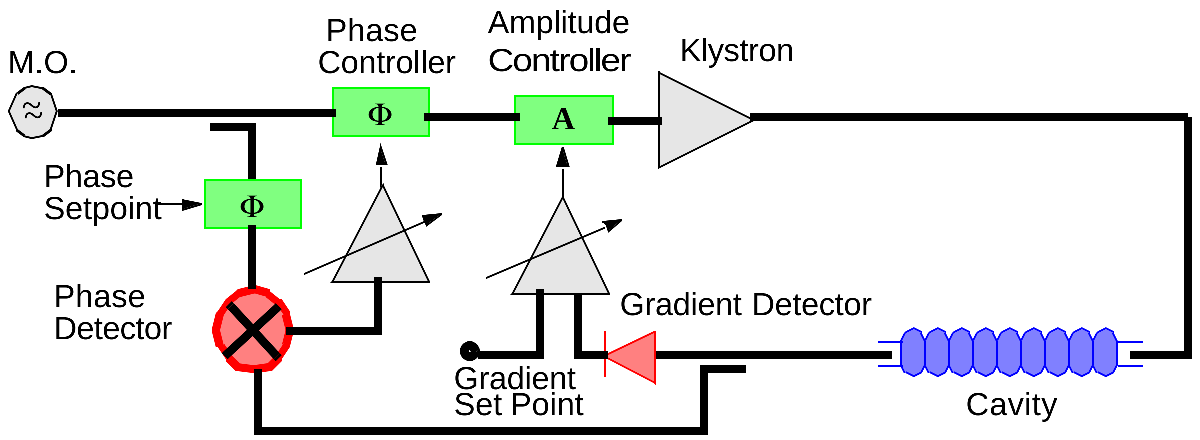
RF Regulation TESLA Cavity (Simulation)



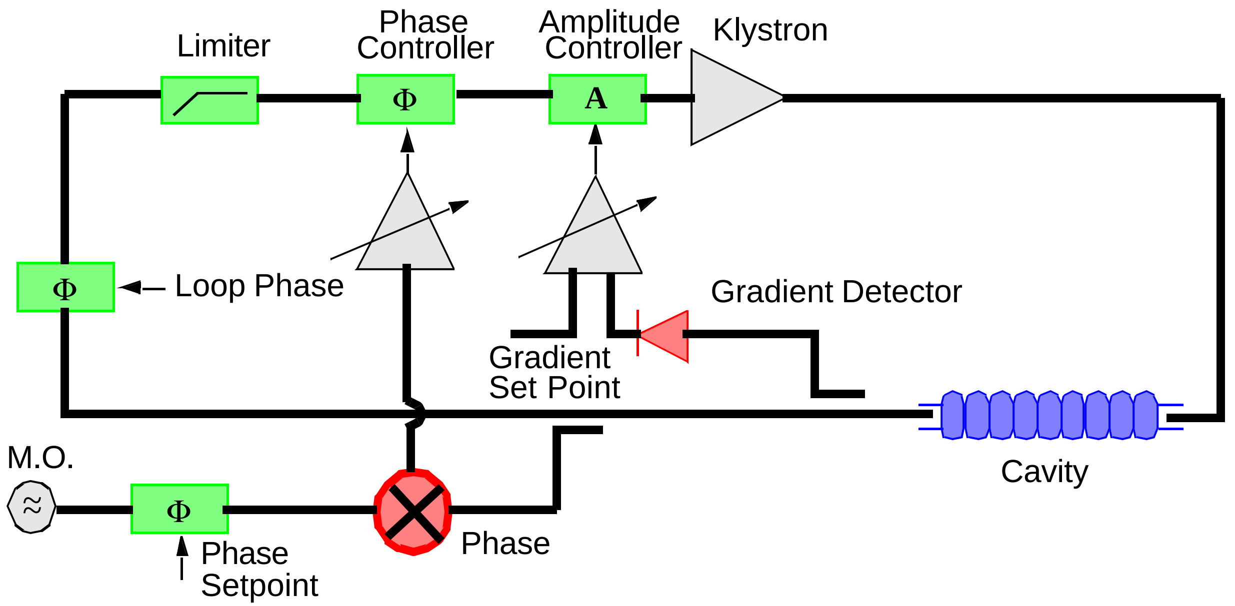
Control Choices (1)

- Self-excited Loop (SEL) vs
  Generator Driven System (GDR)
- Vector-sum (VS) vs individual cavity control
- Analog vs Digital Control Design
- Amplitude and Phase (A&P) vs
  In-phase and Quadrature (I/Q) detector and controller

Control Choices

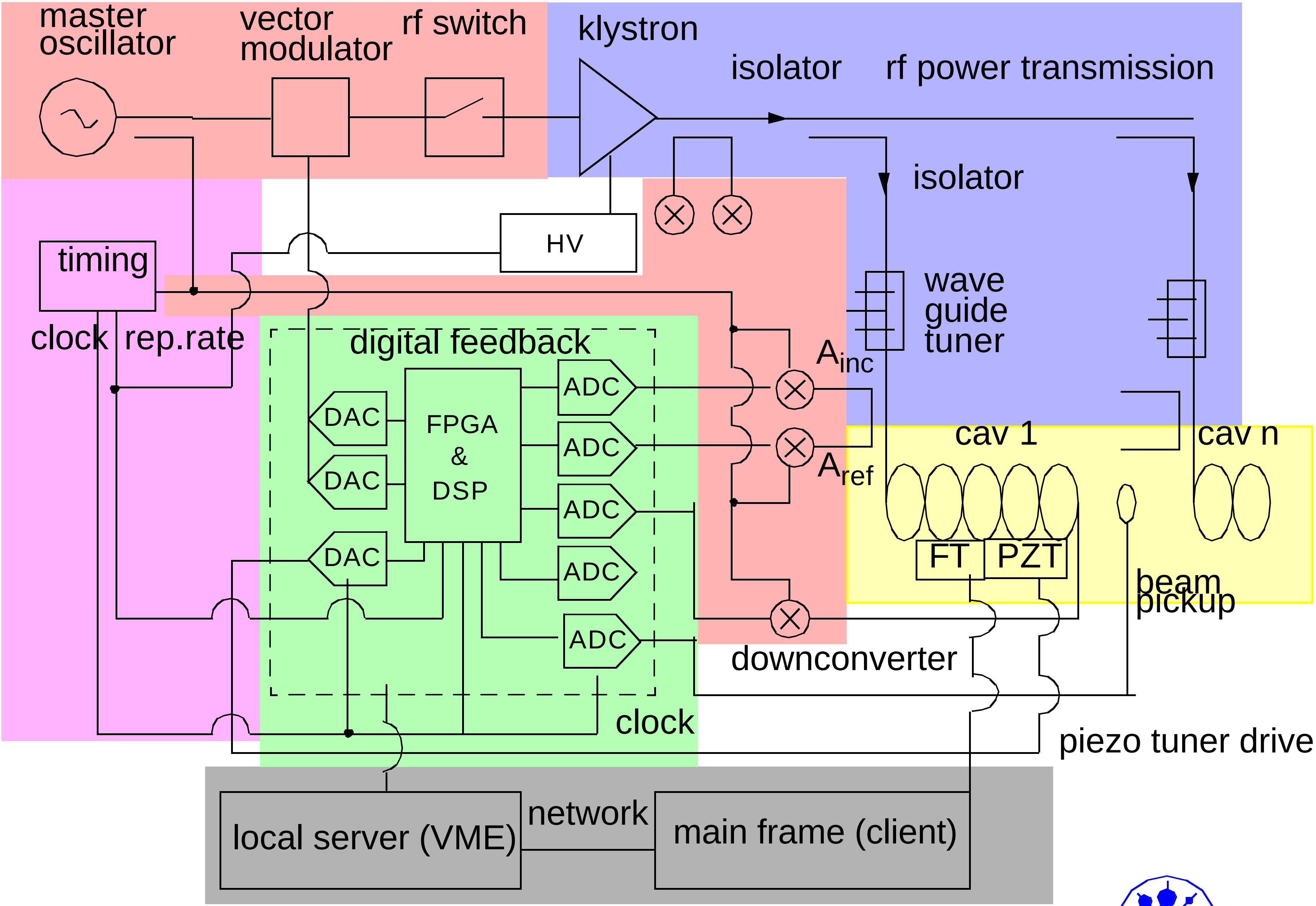


Generator Driven Resonator

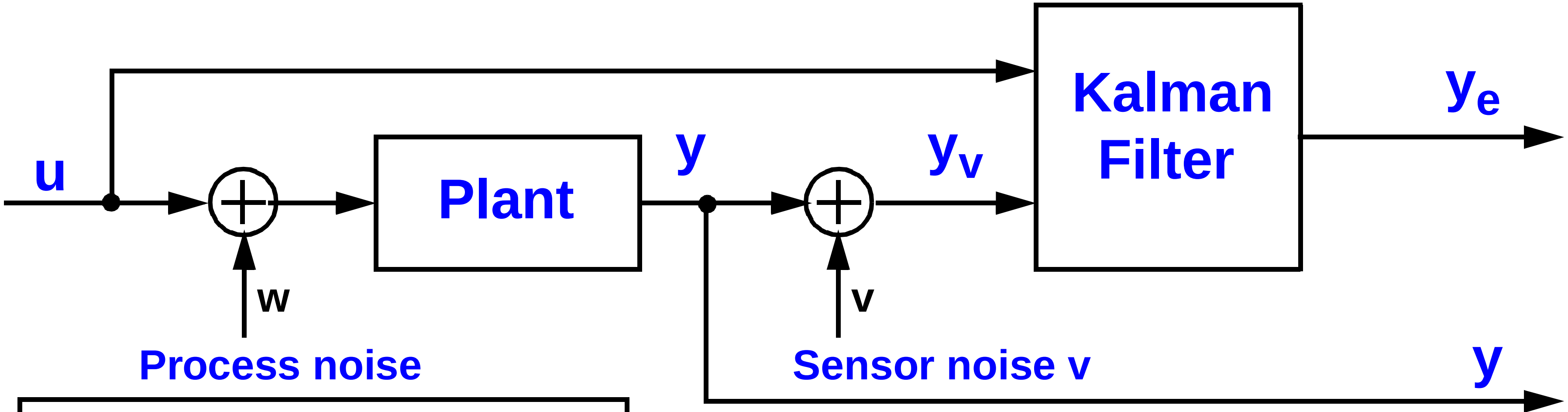


Self Excited Loop

Architecture of digital RF Control



Principle Kalman Filter (steady state)



Discrete Plant:

$x[n+1]=Ax[n]+B(u[n]+w[n])$

$y[n]=Cx[n]$

Noisy output measurement: $y_v[n]=Cx[n]+v[n]$

Measurement update:

$$\hat{x}[n|n]=\hat{x}[n|n-1]+M(y_v[n]-C\hat{x}[n|n-1])$$

Time update: $\hat{x}[n+1|n]=A\hat{x}[n|n]+Bu[n]$

The correction term is a function of the innovation, i.e. the discrepancy

$$y_v[n+1]-C\hat{x}[n+1|n]=C(x[n+1]-\hat{x}[n+1|n])$$

The innovation gain matrix M is chosen to minimize steady-state covariance of the estimation error given the noise covariances $E(w[n]w[n]^T)=Q$ and $E(v[n]v[n]^T)=R$

Kalman Filter (Cnt'd)

where M is the solution of the Riccati Equation:

$M=Q+AMA^T-AMC^T(R+CMC^T)^{-1}CMA^T$

Combining time and measurement update into state space model (the kalman filter):

$$\hat{x}[n+1|n]] = A(I-MC)\cdot\hat{x}[n|n-1]+\begin{bmatrix}B & AM\end{bmatrix}\begin{bmatrix}u[n] \\ y_v[n]]\end{bmatrix}$$

$$\hat{y}[n|n] = C(I-MC)\cdot\hat{x}[n|n-1]+CMy_v[n]$$

This filter generates and optimal estimate $\hat{y}[n|n]$ of y[n]. Note that filter state is $\hat{x}[n|n-1]$

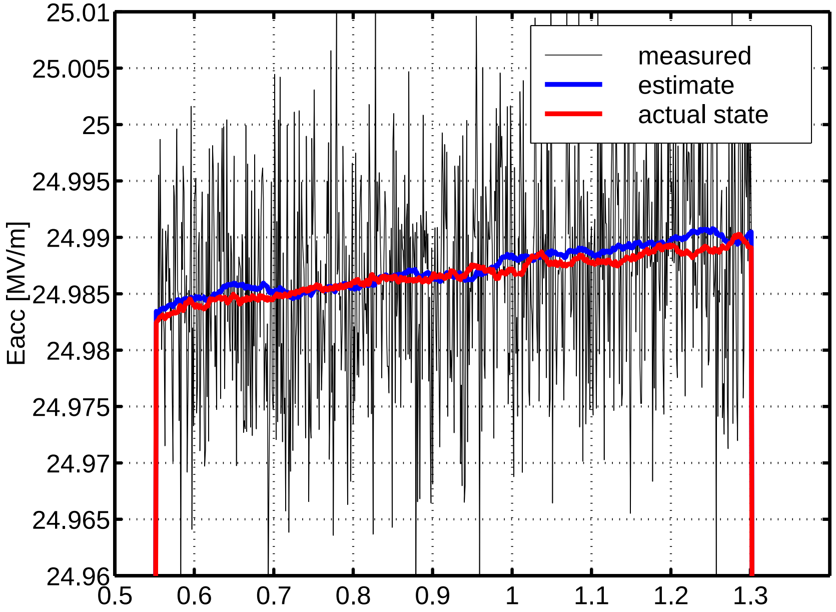
Example: TTF Cavity $Q_L = 3*10^6$

$\omega_0 = 1.3*10^9$ Hz

Beam noise : $\sigma(I_b)/I_b = 0.1$

Sensor noise : $\sigma(V_d)/V_d = 0.01$

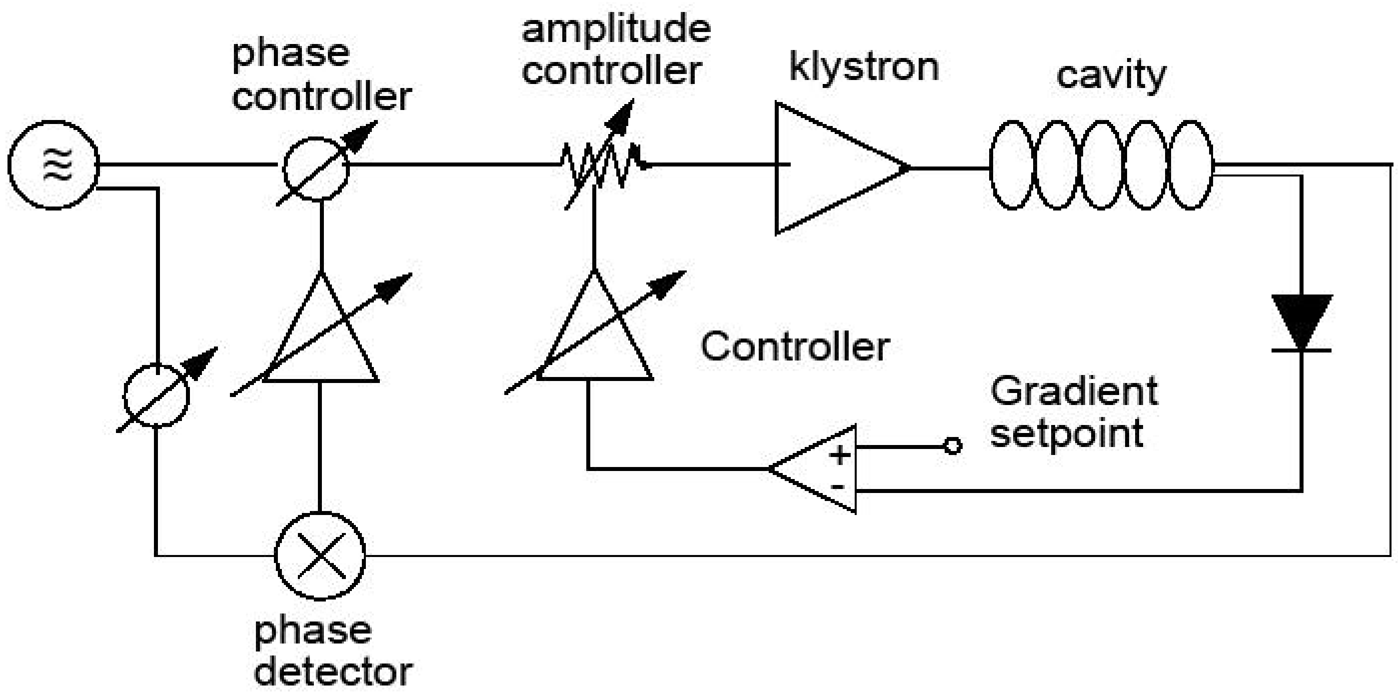
$\Longrightarrow$ $\sigma_y / y = 0.0009$



RF Control Model

Goal:
Maintain stable gradient and phase

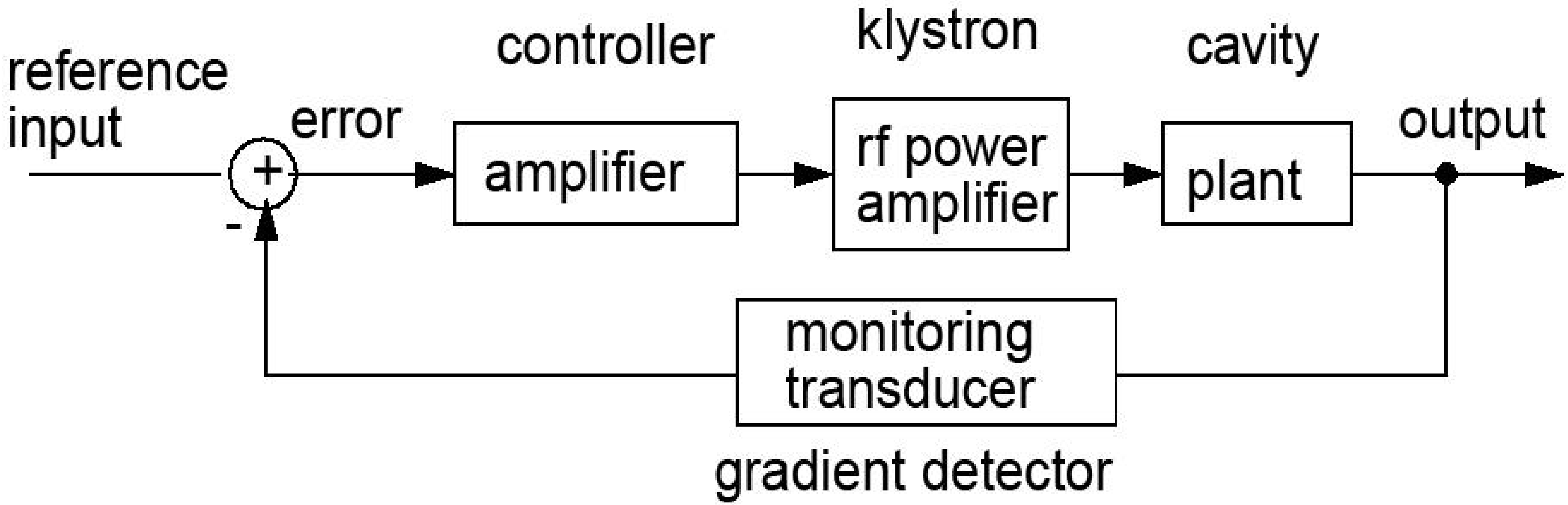
Solution:
Feedback for gradient amplitude and phase:



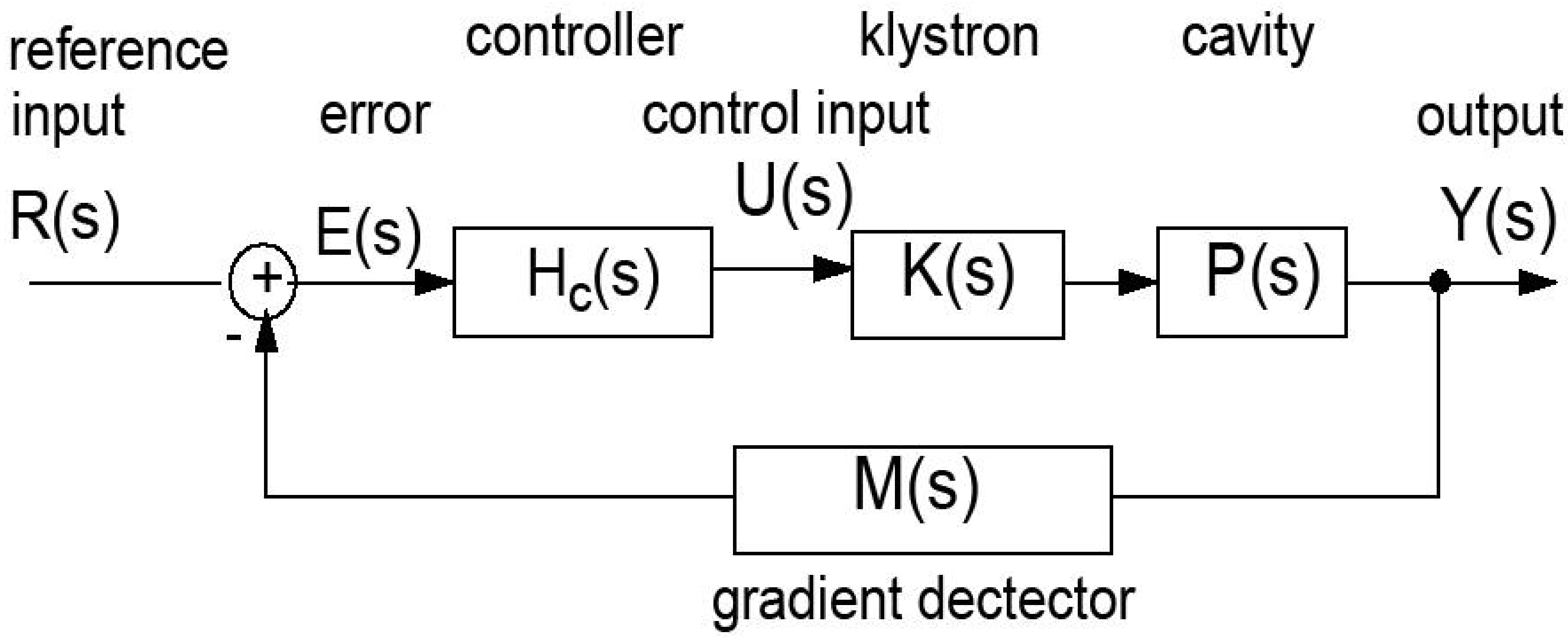
Model:

Mathematical description of input-output relation of components combined with block diagram:

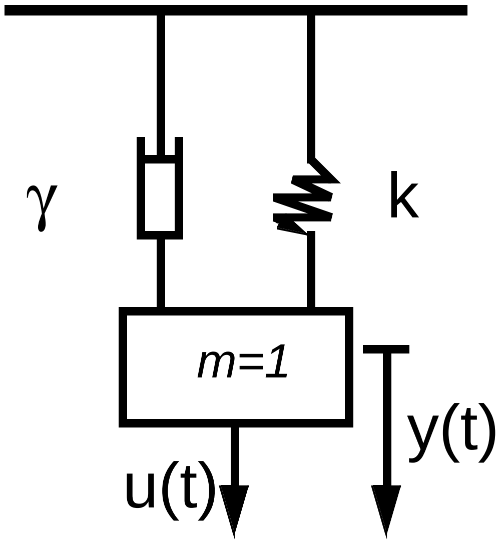
Amplitude Loop (general form):



RF Control model using “transfer functions”



Model of Dynamic Systems



Parameters:

k: spring constant

γ: damping constant

$u(t)$: force

Quantity of interest:

$y(t)$: displacement from equilibrium

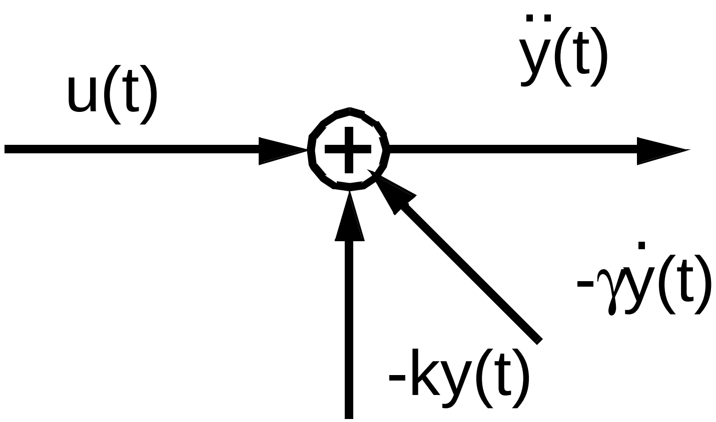
Differential equation: *Newton's third law (m=1)*

$$\ddot{y}(t) = \sum F_{ext} = -ky(t) - \gamma\dot{y}(t) + u(t)$$

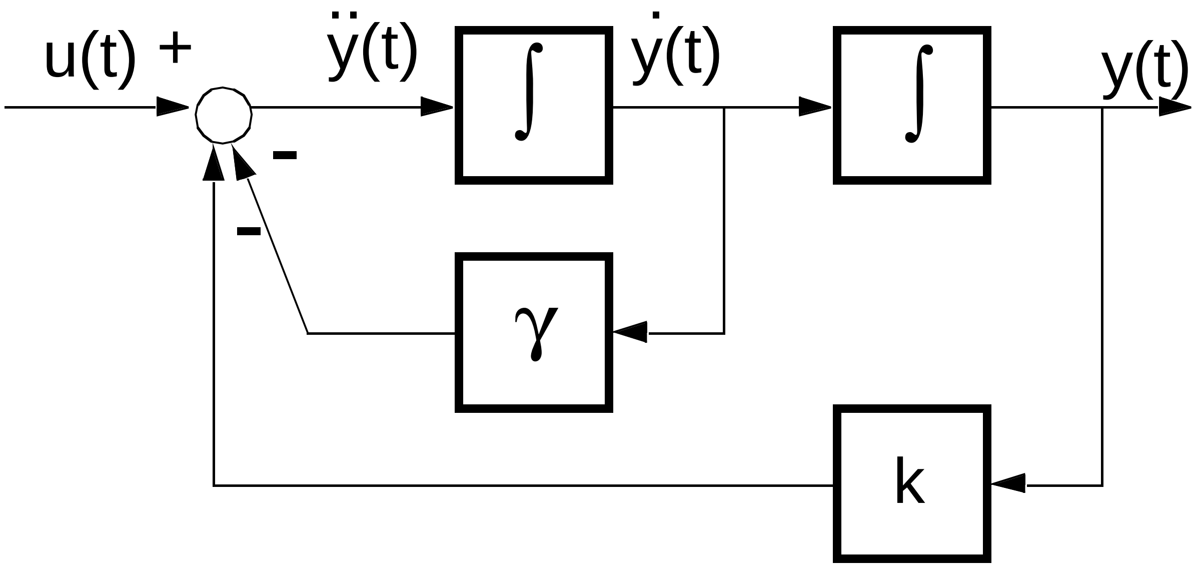
$$\ddot{y}(t) + \gamma\dot{y}(t) + ky(t) = u(t)$$

Graphical Form of Differential Equation

$$\ddot{y}(t) = -ky(t) - \gamma\dot{y}(t) + u(t)$$



Block diagram



State Space Equation

$$\ddot{y}(t) + \gamma\dot{y}(t) + ky(t) = u(t)$$

1. STEP:

Deduce set of first order differential equation in variables $x_j(t)$ (so-called states of system)

$x_1(t) \cong \text{position:} y(t)$

$x_2(t) \cong \text{velocity:} \dot{y}(t)$:

$$\begin{aligned}
\dot{x}_1(t) &= \dot{y}(t) = x_2(t) \\
\dot{x}_2(t) &= \ddot{y}(t) = -ky(t) - \gamma\dot{y}(t) + u(t) \\
&= -kx_1(t) - \gamma x_2(t) + u(t)
\end{aligned}$$

One LODE of order n transformed into n LODEs of order 1

State Space Equation (c‘tnd)

2. STEP:
Put everything together in a matrix differential equation:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k & -\gamma \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$\dot{x}(t) = Ax(t) + Bu(t)$$

State Equation

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$y(t) = Cx(t) + Du(t)$$

Measurement Equation

State Space Equation

The linear time-invariant (LTI) analog system is described via

Standard Form of the State Space Equation

$$\dot{x}(t) = Ax(t) + Bu(t) \quad \text{state equation}$$

$$y(t) = Cx(t) + Du(t) \quad \text{output equation}$$

where $\dot{x}(t)$ is the time derivative of the vector

$$x(t) = \begin{bmatrix} x_1(t) \\ \dots \\ x_n(t) \end{bmatrix}.$$

and starting conditions $x(t_0)$.

System completely decribed by state space matrixes A, B, C, D (in the most cases $D = 0$)

.

Table 1: Declaration of Variables

Variable	Dimension	Name
x(t)	n x 1	state vector
A	n x n	system matrix
B	n x r	input matrix
u(t)	r x 1	input vector
y(t)	p x 1	output vector
C	p x n	output matrix
D	p x r	matrix representing direct coupling between input and output

Solution of State Space Equation

$$x(t) = \Phi(t)x(0) + \int_0^t \Phi(\tau)Bu(t-\tau)d\tau$$

natural response + particular solution

$$y(t) = Cx(t) + Du(t)$$

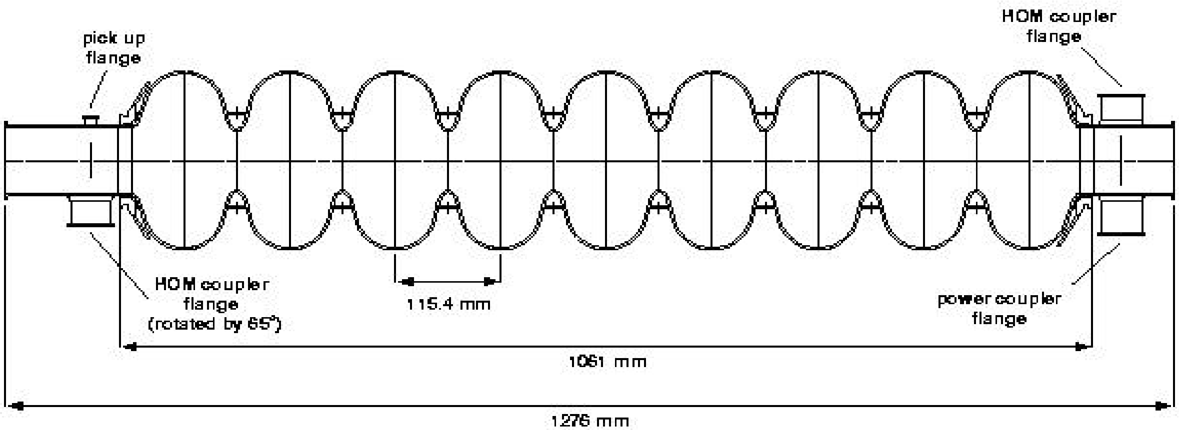
$$= C\Phi(t)x(0) + C\int_0^t \Phi(\tau)Bu(t-\tau)d\tau + Du(t)$$

with the *state transmission matrix*

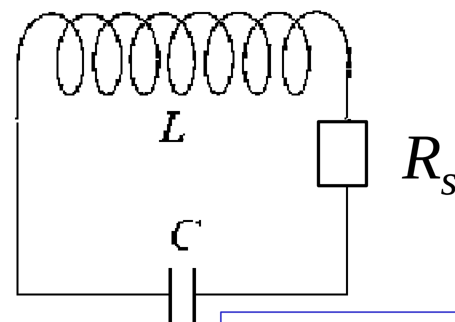
$$\Phi(t) = I + At + \frac{A^2}{2!}t^2 + \frac{A^3}{3!}t^3 + \dots = e^{At}$$

exponential series in the matrix A (time evolution operator)

Kavitäten für TESLA

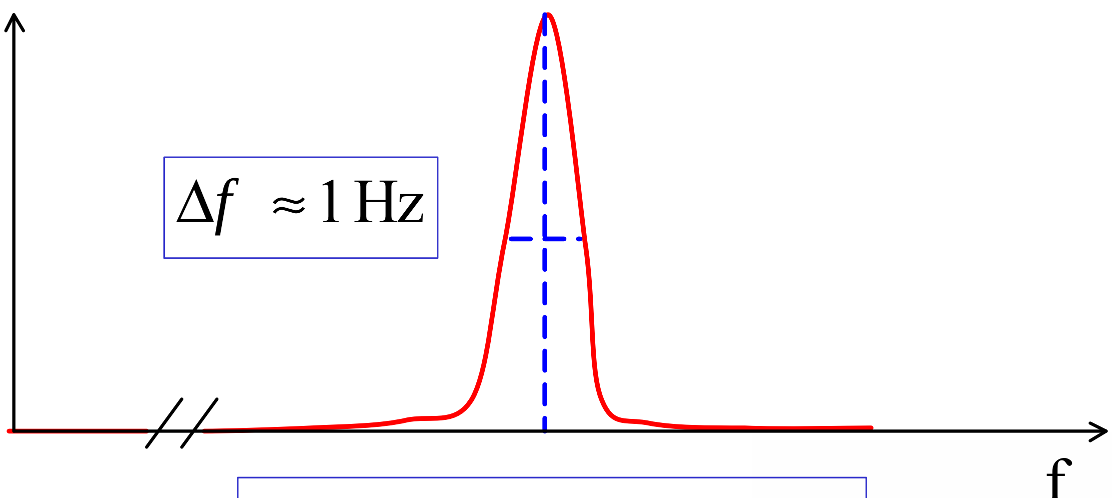


Schwingkreis:



Frequenz: $f_o = \frac{1}{2p\sqrt{LC}}$

Gütefaktor: $Q_o = \frac{f}{\Delta f} = \frac{G}{R_s}$

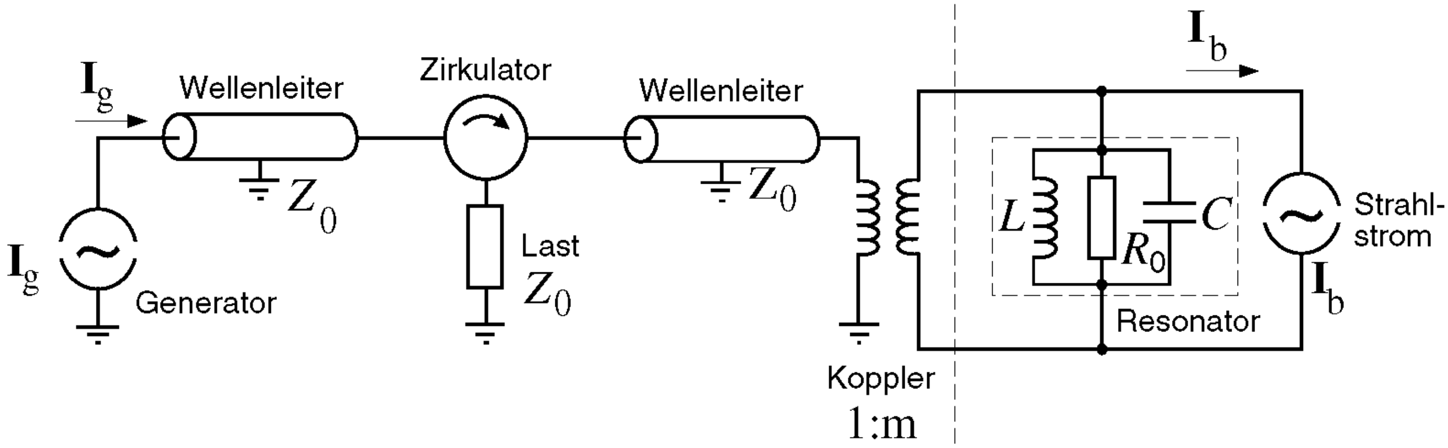


$\Delta f \approx 1\,\mathrm{Hz}$

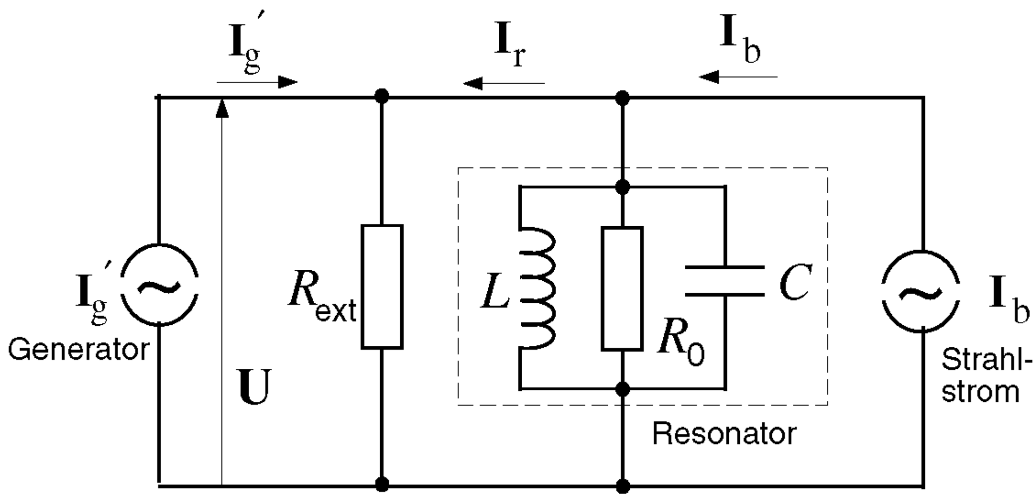
$f_o = 1.300.000.000\ \mathrm{Hz}$

Þ Q0 » 10^{9} -10^{10}

Cavity Model



Equivalent circuits



$$C \cdot \ddot{U} + \frac{1}{R_L} \cdot \dot{U} + \frac{1}{L} \cdot U = \dot{I}_g' + \dot{I}_b$$ L.O.D.E.

with $$\omega_{1/2} := \frac{1}{2R_L C} = \frac{\omega_0}{2Q_L}$$

$$\boxed{\ddot{U} + 2\omega_{1/2} \cdot \dot{U} + \omega_0^2 \cdot U = 2R_L\omega_{1/2} \cdot (\frac{2}{m}\dot{I}_g + \dot{I}_b)}$$

- Continuous Model

$$\begin{bmatrix} \dot{v}_r \\ \dot{v}_i \end{bmatrix} = \begin{bmatrix} -\omega_{1/2} & -\Delta\omega(t) \\ \Delta\omega(t) & -\omega_{1/2} \end{bmatrix} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix} + \begin{bmatrix} R \cdot \omega_{1/2} & 0 \\ 0 & R \cdot \omega_{1/2} \end{bmatrix} \cdot \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

where $\omega_{1/2} = \frac{\omega_{rf}}{2Q}$ and $\Delta\omega(t) = \omega_0(t) - \omega_{rf}$

State Space Form

$$\dot{x} = A \cdot x + B \cdot u$$

$$y = C \cdot x + D \cdot u$$

with solution

$$x(t) = e^{A \cdot t} \cdot x(0) + \int_0^t e^{A \cdot \tau} \cdot B \cdot u(t-\tau) \cdot d\tau$$

Concept of Transfer Functions or System Description in Frequency Domain

Basic tool for the analysis of time continuous systems in the frequency domain is the Laplace Transform. The Laplace Transform converts a LODE into algebraic equations (makes life easier while finding the solution!)
Laplace Transform == Generalized Fourier Transform

Fourier Transform:

$$\tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

with $\omega = 2\pi f$, f frequency in Hz.

Define in the same spirit: Laplace Transform

$$F(s) = L[f(t)] = \int_{0}^{\infty} f(t) e^{-st} dt$$

with $s = \sigma + i\omega$ complex, σ real and $\omega = 2\pi f$, f frequency in Hz.

Laplace Transform

Differentiation $\left(\frac{d}{dt}\right)$ Þ Multiplication (ˣ s)

$\frac{d}{dt}\left(te^{at}\right)$ → Differentiation → $(1+at)e^{at}$

TRANSFORM

Inverse-TRANSFORM

s Þ $\frac{1}{(s-a)^2}$ | Algebraic operation | $\frac{s}{(s-a)^2}$

Integration $\left(\int dt\right)$ Þ Division (1 ˣ s)

$\int t\,dt$ → Integration → $\frac{t^2}{2}$

TRANSFORM

Inverse-TRANSFORM

$\frac{1}{s}$ Þ $\frac{1}{s^2}$ → Algebraic operation → $\frac{1}{s^3}$

Common Laplace Transforms

Table 1: Common Laplace Transform

f(t)	Laplace Transform F(s)
Unit pulse $\delta(t)$	1
$\delta(t-T)$ for any T> 0	e^{-sT}
Unit step function $u(t)$	$\frac{1}{s}$
ramp f(t)=t (n-th order ramp t^n, $n = 1,2, \ldots$)	$\frac{1}{s^2}$, $\left(\frac{n!}{s^{n+1}}\right)$
e^{-at}	$\frac{1}{s+a}$
Cosine $\cos(\omega t)$ (damped cosine: $e^{-at}\cos(\omega t)$)	$\frac{s}{s^2+\omega^2}$, $\left(\frac{s+a}{(s+a)^2+\omega^2}\right)$

Transfer Function

Continuous-time state space model

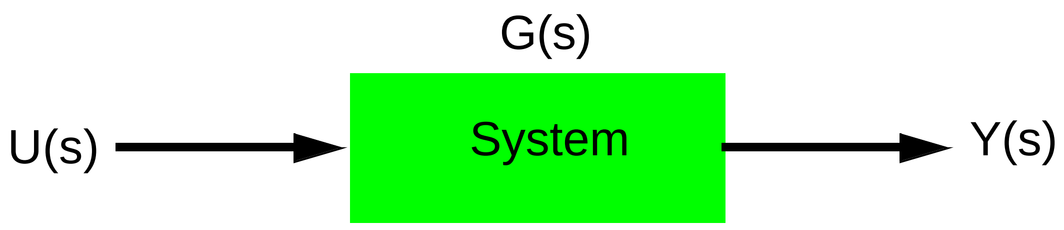
$$\dot{x}(t) = Ax(t) + Bu(t)$$

state equation

$$y(t) = Cx(t) + Du(t)$$

measurement equation

Transfer function describes input-output relation of system.



$$sX(s) - x(0) = AX(s) + BU(s)$$

Transfer Function (C'tnd)

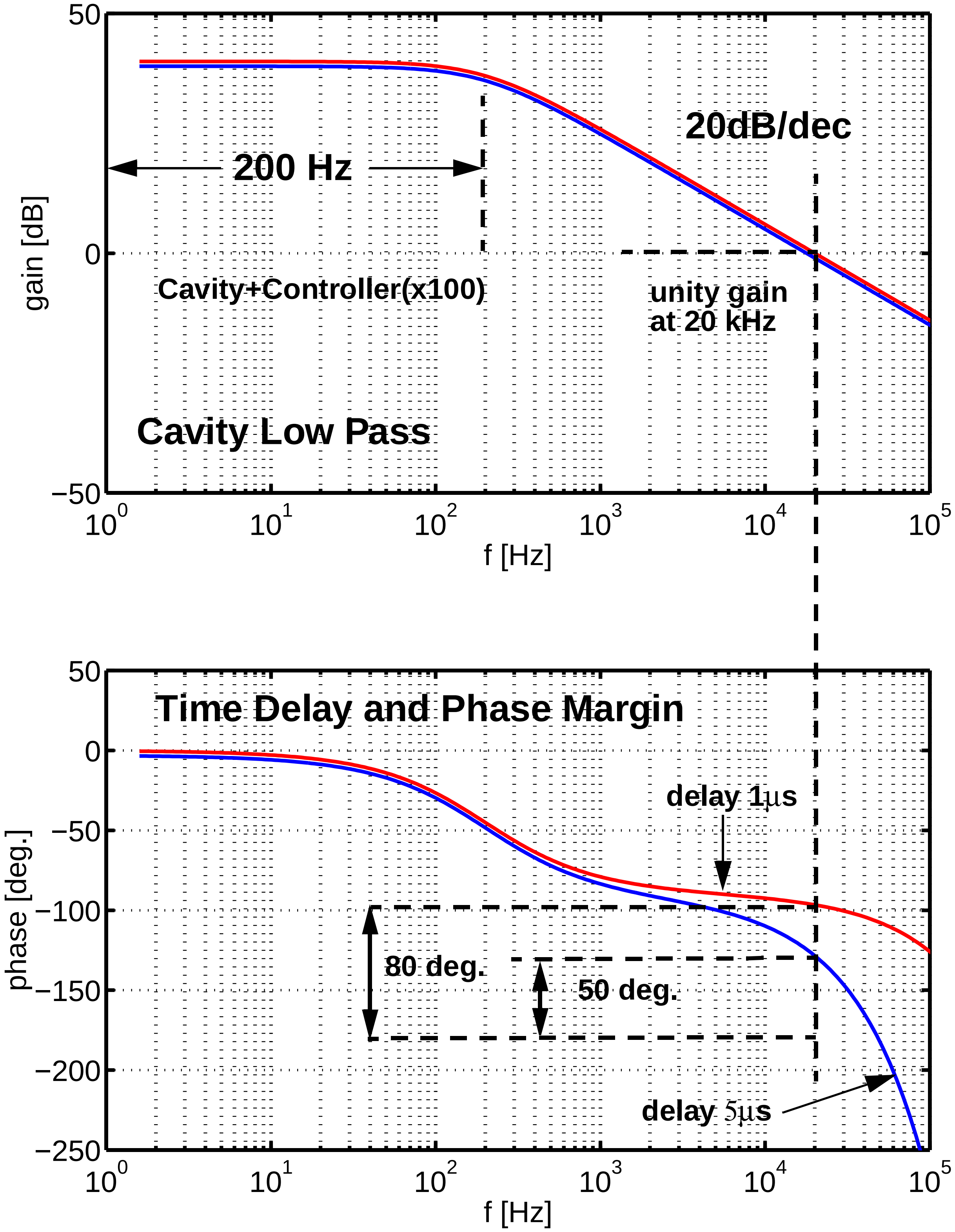
$$
\begin{aligned}
X(s) &= (sI-A)^{-1}x(0) + (sI-A)^{-1}BU(s) \\
&= \Phi(s)x(0) + \Phi(s)BU(s)
\end{aligned}
$$

$$
\begin{aligned}
Y(s) &= CX(s) + DU(s) \\
&= C[(sI-A)^{-1}]x(0) \\
&\quad + [C(sI-A)^{-1}B + D]U(s) \\
&= C\Phi(s)x(0) + C\Phi(s)BU(s) + DU(s)
\end{aligned}
$$

Transfer function G(s) (pxr) (case: x(0)=0):

$$
G(s) = C(sI-A)^{-1}B + D = C\Phi(s)B + D
$$

Cavity Transfer Function



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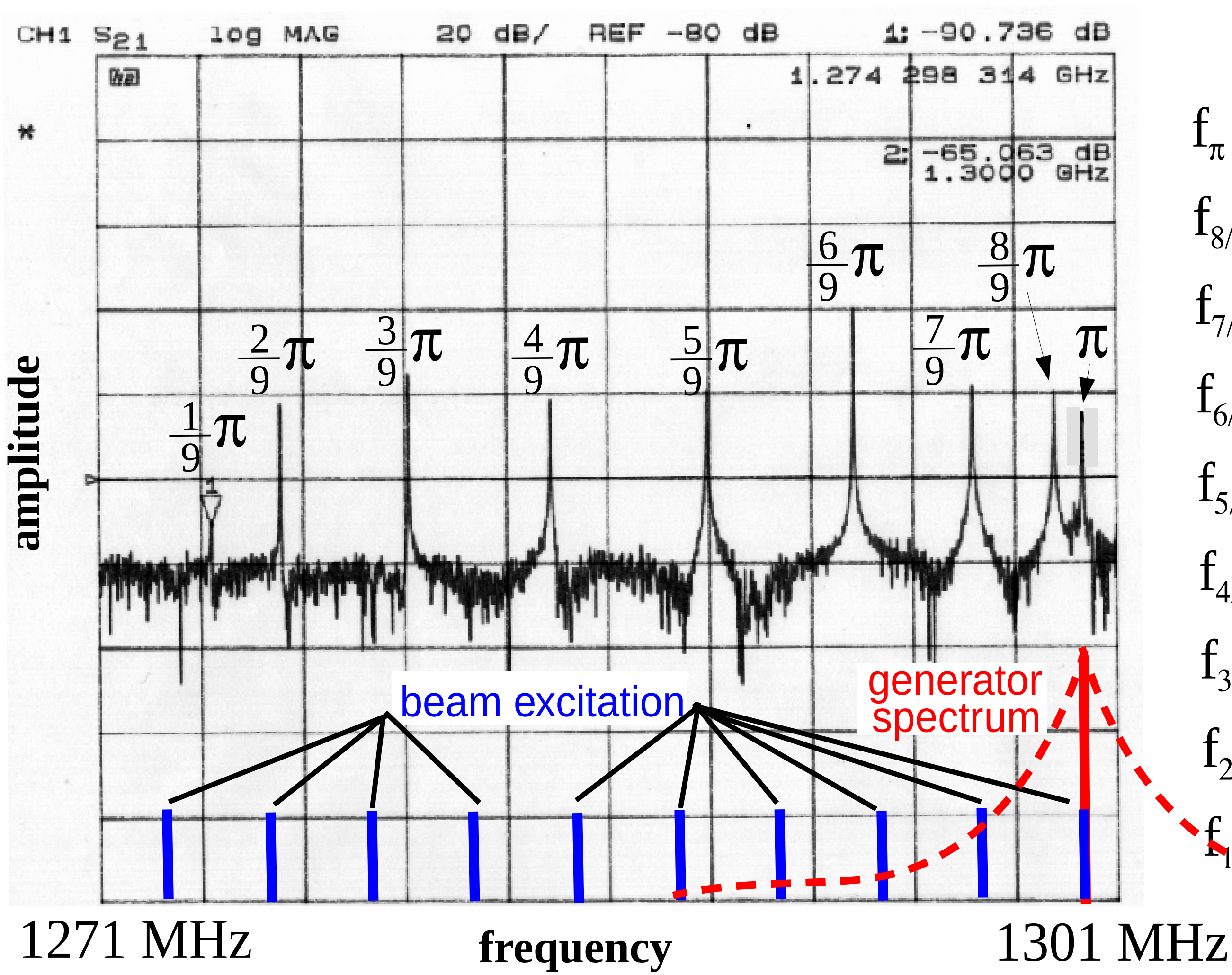
Excitation of other Passband Modes

Example: TESLA 9-cell cavity



f_π = 1300.091 MHz

$f_{8/9\pi}$= 1299.260 MHz

$f_{7/9\pi}$= 1296.861 MHz

$f_{6/9\pi}$= 1293.345 MHz

$f_{5/9\pi}$= 1289.022 MHz

$f_{4/9\pi}$= 1284.409 MHz

$f_{3/9\pi}$= 1280.206 MHz

$f_{2/9\pi}$= 1276.435 MHz

$f_{1/9\pi}$= 1274.387 MHz

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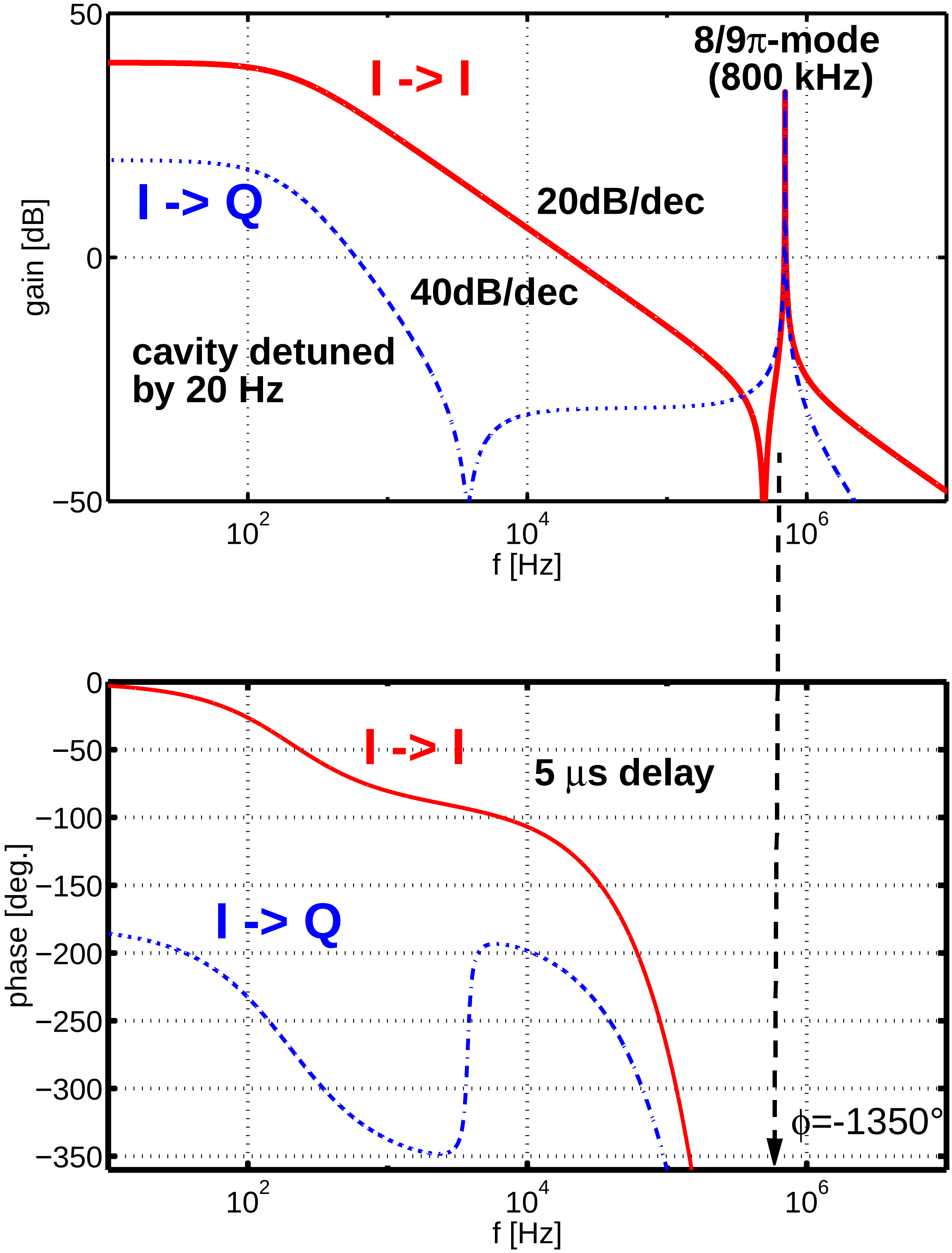
Cavity Transfer Function



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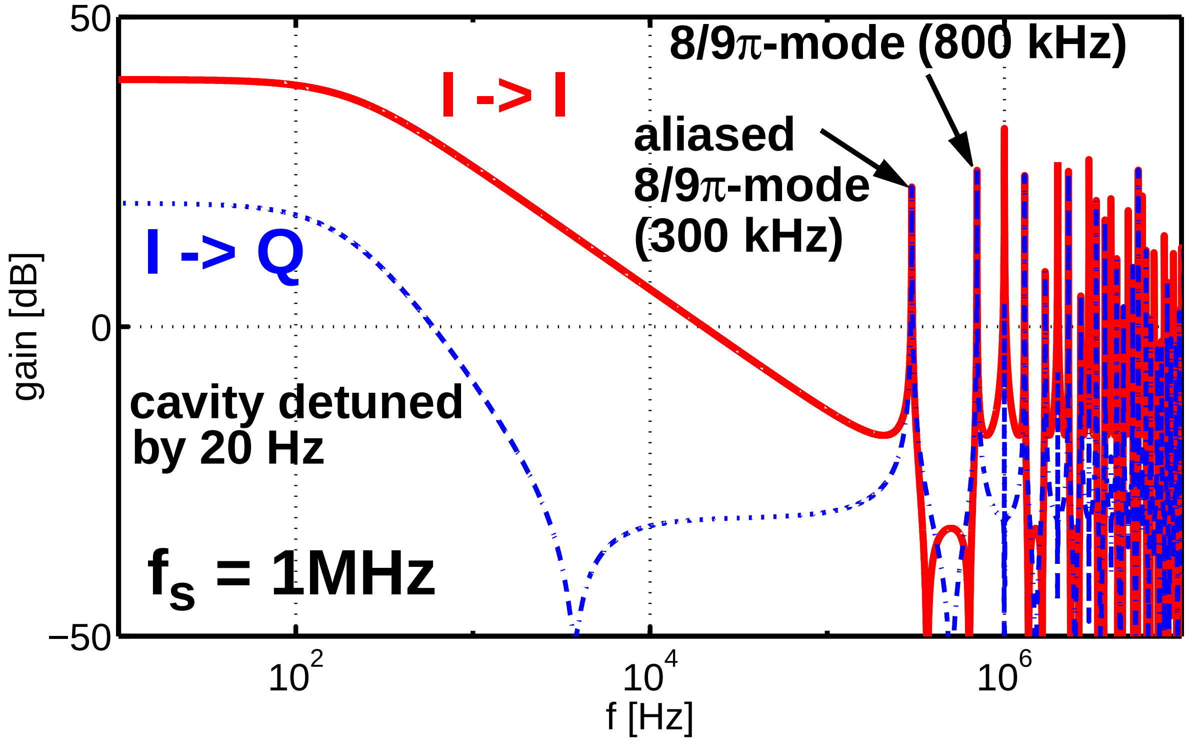
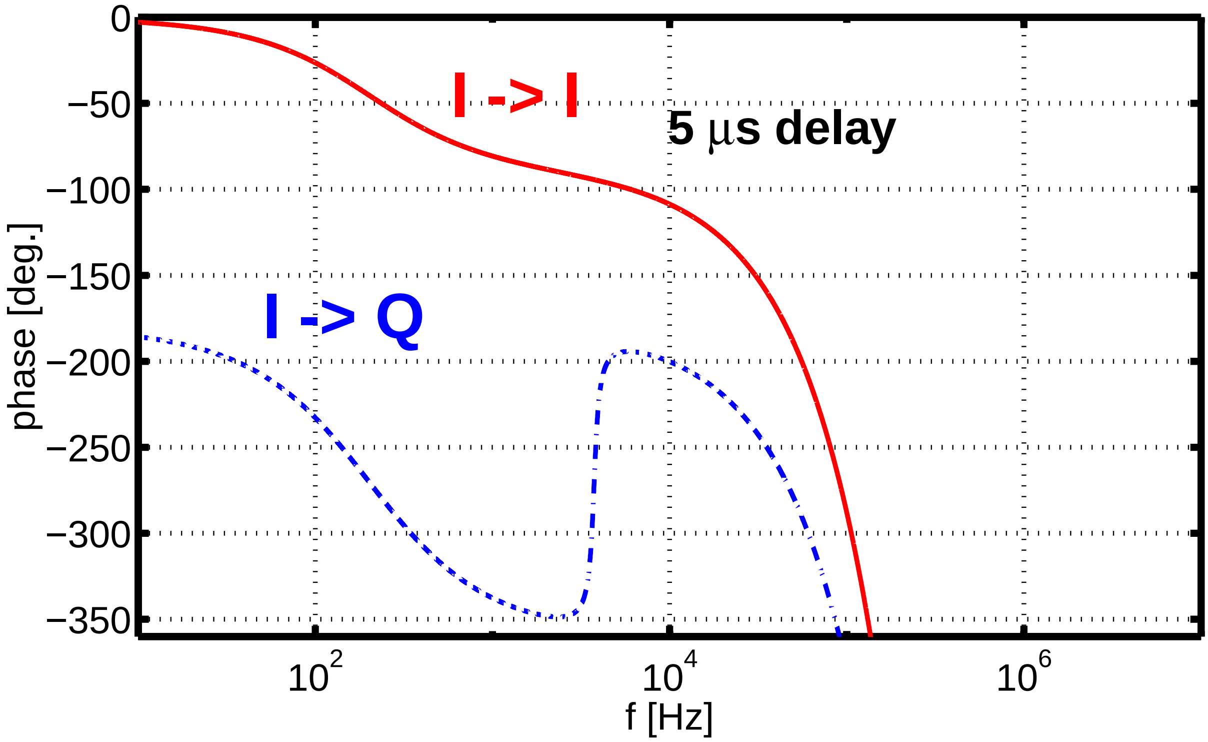
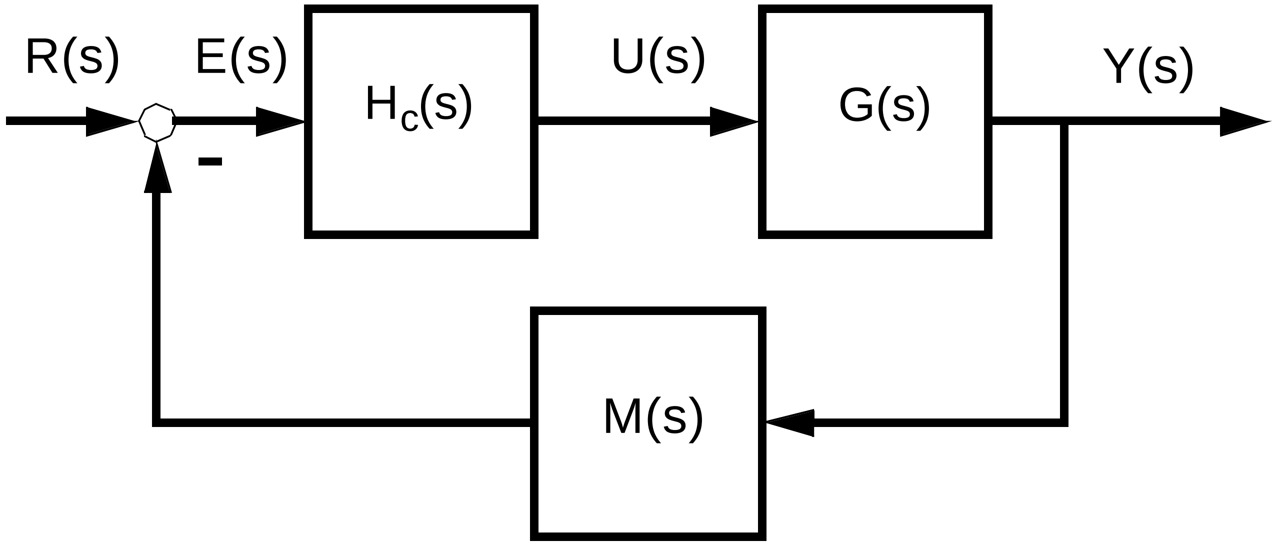
Discrete Cavity TF





Closed Loop Transfer Function



We can deduce for the output of the system:

$$
\begin{aligned}
Y(s) &= G(s)U(s) = G(s)H_c(s)E(s) \\
&= G(s)H_c(s)[R(s) - M(s)Y(s)] \\
&= L(s)R(s) - L(s)M(s)Y(s)
\end{aligned}
$$

with $L(s)$ the transfer function of the open loop system (controller plus plant).

Poles and Zeroes

Can stability be determined if we know the TF of a system?

$$G(s) = C\Phi(s)B + D = C\frac{[sI - A]_{adj}}{det(sI - A)}B + D$$

coefficients of transfer function $G(s)$ are rational functions in the complex variable s.

$$g_{ij}(s) = a \times \frac{\prod_{k=1}^{m}(s - z_k)}{\prod_{l=1}^{n}(s - p_l)} = \frac{N_{ij}(s)}{D_{ij}(s)}$$

z_k zeroes, p_l poles, α real constant, and it is $m \le n$.

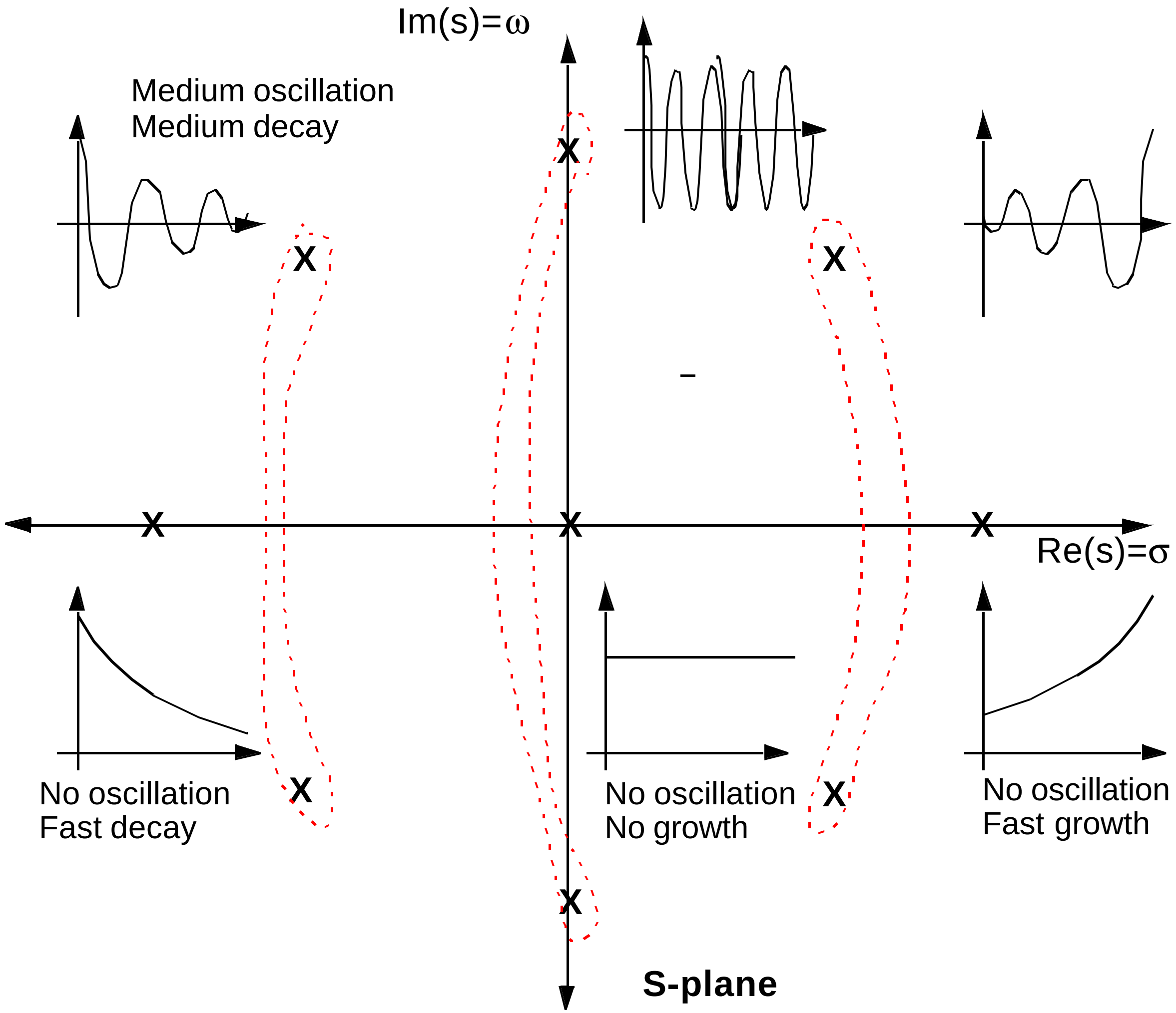
When is the BIBO stability condition $\int_0^\infty |g(t)|\,dt < \infty$ valid

The farther left the poles in the s-plane, the faster the decay, the better the stability! THUS:

The system is BIBO-stable

if $Re(p_l) < 0$ with p_l pole

Pole Locations and Stability



- Complex pole pair: oscillation with growth or decay.

- Real pole: exponential growth or decay.

- Poles are the Eigenvalues of the matrix A.

- Position of zeros goes into the size of c_j ..

Stability directly from State Space

Recall : $H(s) = C(sI - A)^{-1}B + D$

Assuming D=0 (D could change zeros but not poles)

$$H(s) = \frac{Cadj(sI - A)B}{det(sI - A)} = \frac{b(s)}{a(s)}$$

Assuming there are no common factors between the poly $Cadj(sI - A)B$ and $det(sI - A)$ i.e no pole-zero cancellations (usually true, system called “minimal”) then we can identify

$$b(s) = Cadj(sI - A)B$$

and

$$a(s) = det(sI - A)$$

i.e. poles are root of $det(sI - A)$

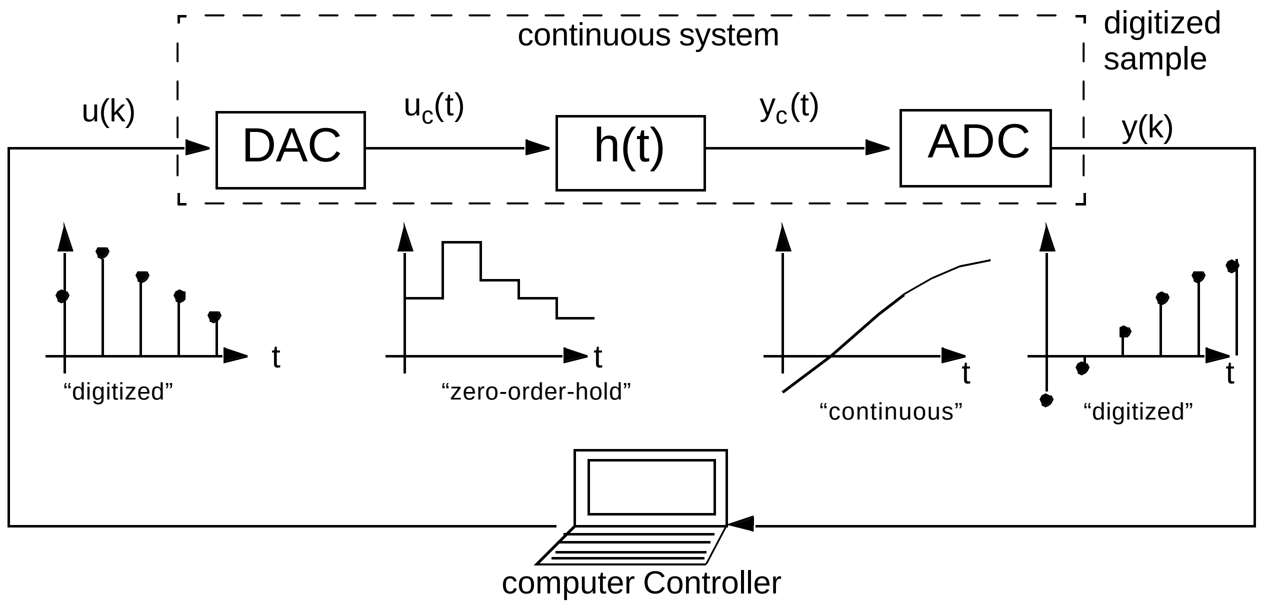
Poles are eigenvalues of A

Let λ_i be the ith eigenvalue of A

if $Re\{\lambda_i\} \leq 0$ for all i => system stable

Discrete System

Where do discrete systems arise ? Typical control engineering example:



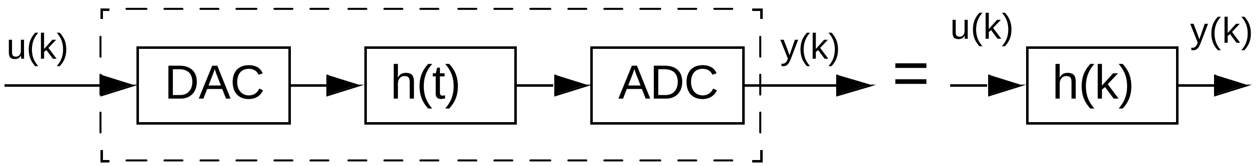
Assume that DAC + ADC are clocked at sampling period T. Then u(t) is given by:

$$u(k) \equiv u_c(t) \quad ; \; kT \leq t < (k+1)T$$

$$y(k) \equiv y_c(kT) \quad ; \; k = 0, 1, 2, \ldots$$

Discrete Systems (c'tnd)

Can we obtain direct relationship between u(k) and y(k) ? i.e. want equivalent discrete system:



Yes ! We can obtain equivalent discrete system.
Recall

$$x_c(t) = e^{At}x_c(0) + \int_0^t e^{A\tau} \cdot Bu_c(t-\tau)d\tau$$

From this

$$x_c(kT+T) = e^{AT}x_c(kT) + \int_0^T e^{A\tau} \cdot Bu_c(kT-\tau)d\tau$$

Observe that $u(kT+T-\tau) = u(kT)$ for $\tau \in [0, T]$
i.e. $u(kT+T-\tau)$ is constant $u(kT)$ over $\tau \in [0, T]$
i.e. can pull out of integral.

$$\text{==>}\quad \underbrace{x_c(kT+T)}_{} = \underbrace{e^{AT}}_{} x_c(kT) + \underbrace{\left(\int_0^T e^{A\tau}\cdot B d\tau\right)}_{} u_c(kT)$$

$$x(k+1) = A_d x(k) + B_d u(k)$$
$$y(k) = C_d x(k) + D_d u(k)$$
$$x(0) = x_c(0)$$

So

$$A_d = e^{AT}$$
$$B_d = \int_0^T e^{A\tau}\cdot B d\tau$$
$$C_d = C$$
$$D_d = D$$

So we have an exact (note: $x(k+1) = x(k) + \dot{x}(k)T + O(.)$)
discrete time equivalent to the time continuous system at sample times $t = kT$ - no numerical approximation!

Discrete Systems and Z-transform

For analyzing discrete-time systems: Z-Transform
(analogue to Laplace Transform for time-continuous system)

Given any sequence $f(k)$ the discrete Fourier transform is

$$\tilde{F}(\omega) = \sum_{k=-\infty}^{\infty} f(k) e^{-i\omega k}$$

with $\omega = 2\pi f$, $f = \frac{1}{T}$ the sampling frequency in Hz, T difference / time between two samples.

In the same spirit:

$$F(z) = Z[f(k)] = \sum_{k=0}^{\infty} f(k) z^{-k}$$

with z a complex variable.

Discrete Cavity Model

Converting the transferfunction from the continuous cavity model to the discrete model:

$$H(s) = \frac{\omega_{12}}{\Delta\omega^2 + (s+\omega_{12})^2} \begin{bmatrix} s+\omega_{12} & -\Delta\omega \\ \Delta\omega & s+\omega_{12} \end{bmatrix}$$

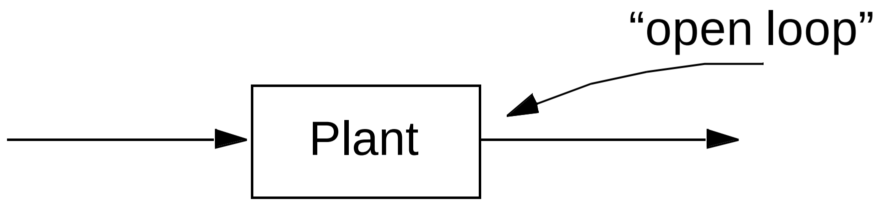
The discretization of the model is represented by the z-transform:

$$H(z) = \frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \cdot \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} - \left(\frac{\omega_{12}}{\Delta\omega^2 + \omega_{12}^2} \cdot \frac{z-1}{z^2 - 2ze^{\omega_{12}T_s} \cdot \cos(\Delta\omega T_s) + e^{2\omega_{12}T_s}} \right) \cdot$$

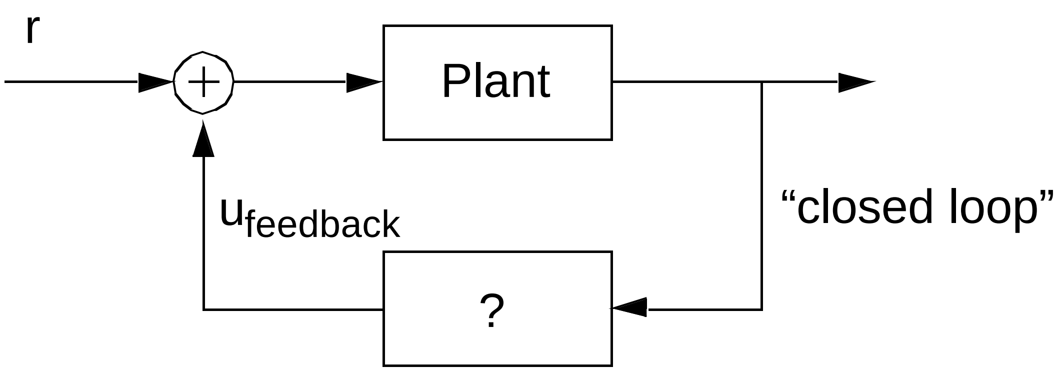
$$\left\{ \left((z - e^{\omega_{12}T_s} \cdot \cos(\Delta\omega T_s)) \cdot \begin{bmatrix} \omega_{12} & -\Delta\omega \\ \Delta\omega & \omega_{12} \end{bmatrix} \right) - e^{\omega_{12}T_s} \cdot \sin(\Delta\omega T_s) \cdot \begin{bmatrix} \Delta\omega & \omega_{12} \\ -\omega_{12} & \Delta\omega \end{bmatrix} \right\}$$

Feedback

The idea:
Suppose we have a system or "plant"



We want to improve some aspect of plant's performance by observing the output and applying a appropriate "correction" signal. This is feedback.

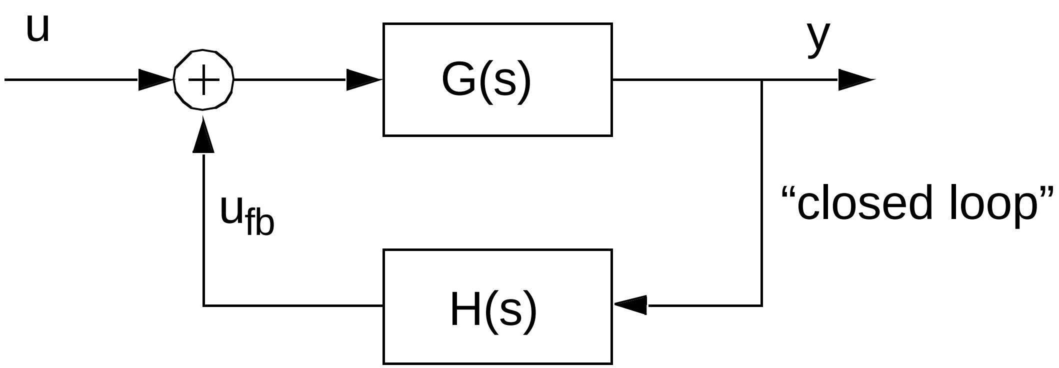


Question: What should this be ?

Open-loop gain:

$$G^{\text{O.L.}}(s) = G(s) = \left(\frac{u}{y}\right)^{-1}$$

Closed-loop gain:



$$G^{\text{C.L.}}(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Proof: $y = G(u-u_{fb})$ => $y+GHy = Gu$

$= Gu - Gu_{fb}$ => $y/u = G\,/\,(1+GH)$

$= Gu - GHy$

Full State Feedback

Suppose we have system

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

Since the state vector x(t) contain all current information about the system the most general feedback makes use of **all** the state info.

$$u = -k_1 x_1 - \ ... \ -k_n x_n$$
$$= -kx$$

where $k = \begin{bmatrix} k_1 & ... & k_n \end{bmatrix}$ (row matrix)

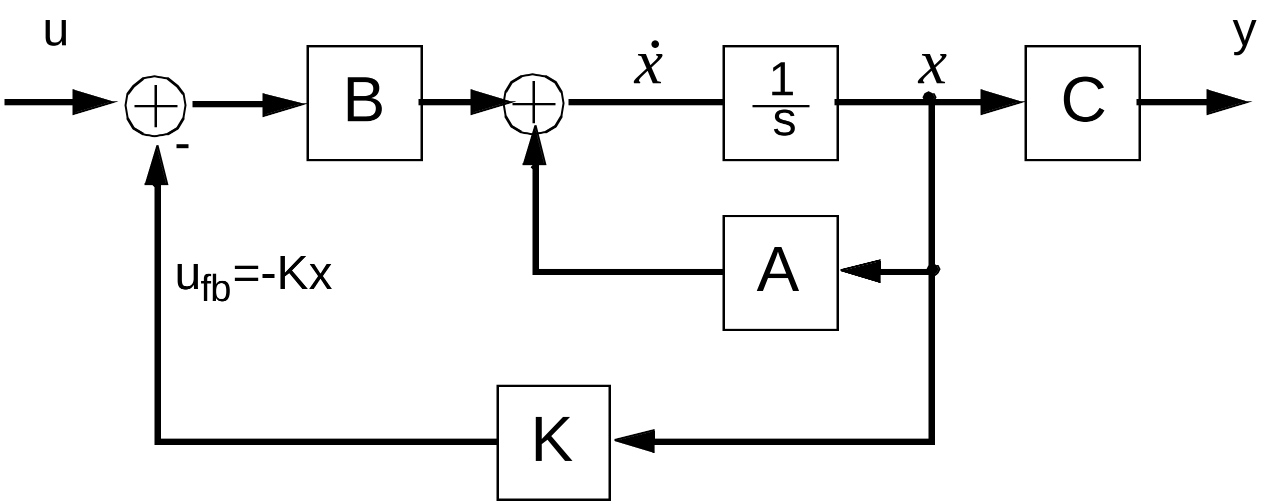
examples:

Proportional fbk : $u_P = -k_P x = -\begin{bmatrix} k_P & 0 \end{bmatrix}$

Differential fbk : $u_D = -k_D \dot{x} = -\begin{bmatrix} 0 & k_D \end{bmatrix}$

Full State Feedback

With full state feedback have (assume D=0)



$$\begin{aligned}\dot{x} &= Ax + B[u + u_{fb}] \\ &= Ax + Bu + BKu_{fb}\end{aligned}$$

$$\dot{x} = (A - BK)x + Bu$$

$$\begin{aligned}u_{fb} &= -Kx \\ y &= Cx\end{aligned}$$

$\therefore$ with full state feedback, get new closed loop matrix

$$A^{\text{C.L.}} = (A^{\text{O.L.}} - BK)$$

Now all stability info is now given by the eigen values of new A matrix

Pole Placement

Theorem

If there are no poles cancellations in

$$G_{\mathrm{O.L.}}(s) = \frac{b(s)}{a(s)} = C(sI-A)^{-1}B$$

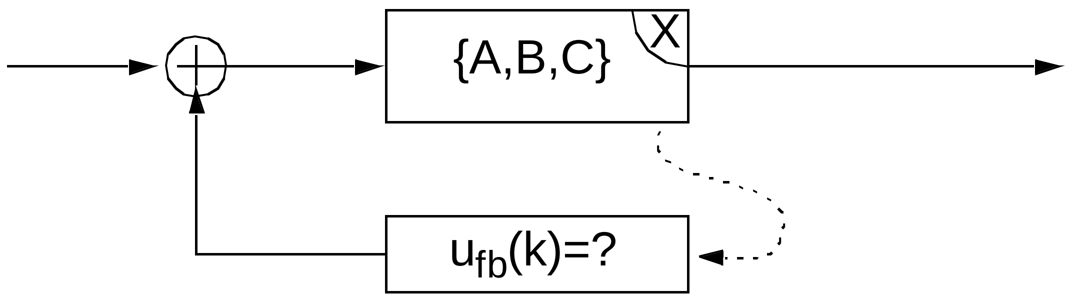
then can move eigen values of $A-BK$ anywhere we want using full state feed-back.

Linear quadratic regulator

Given: $$x(k+1) = Ax(k) + Bu(k)$$
$$z(k) = Cx(x)$$
(Assume D=0 for simplicity)

Suppose the system is <u>unstable</u> or almost unstable. We want to find $u_{fb}(k)$ which will bring $x(k)$ to zero, <u>quickly</u>, from any initial condition.

i.e.



What do we mean by “bad” damping and “cheap” control ? We now define <u>precisely</u> what we mean. Consider :

$$J \equiv \sum_{i=0}^{\infty} \left\{ x_i^T Q x_i + u_i^T R u_i \right\}$$

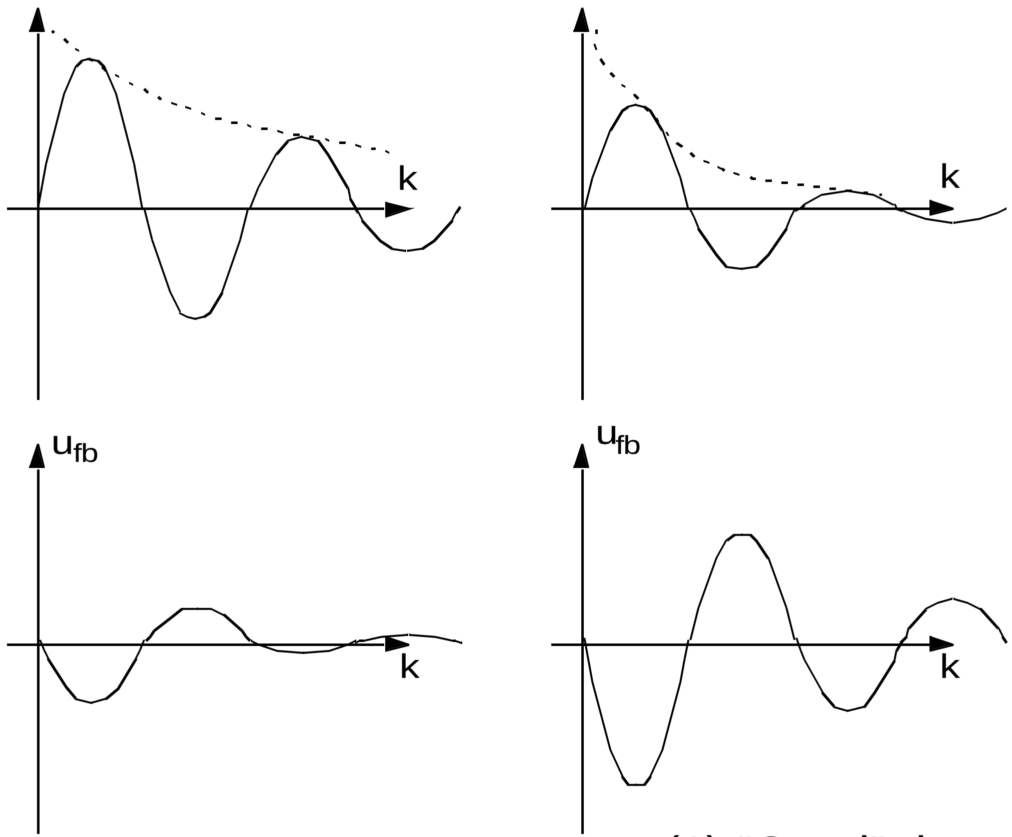
The first term penalizes large state excursions, the second penalizes large control.

$Q \geq 0 \ , \quad R > 0$

Can tradeoff between state excursions and control by varying Q and R.

Large Q => “good” damping important

Large R => actuator effort “expensive”



(1) "Bad" damping
=> <u>large</u> output
excursions

(2) But "cheap" control
i.e. u_{fb} <u>small</u>

(1) "Good" damping
=> <u>small</u> output
excursions

(2) But "expensive
control
i.e. u_{fb} <u>large</u>

Quadratic Cost for Regulator

What do we mean by "bad" damping and "cheap" control ? We now define <u>precisely</u> what we mean. Consider :

$$J \equiv \sum_{i=0}^{\infty} \left\{ x_i^T Q x_i + u_i^T R u_i \right\}$$

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Can tradeoff between state excursions and control by varying Q and R.

Large Q => "good" damping important

Large R => actuator effort "expensive"

LQR Problem Statement

(Linear quadratic regulator)

Given: $x_{i+1} = Ax_i + Bu_i \quad ; \; x_0$ given

Find control sequence $\{u_o, u_1, u_2, ...\}$ such that :

$$J = \sum_{i=0}^{\infty} \{x_i^T Q x_i + u_i^T R u_i\}$$

= minimum

Answer:

The optimal control sequence is a state feedback sequence $\{u_i\}_0^{\infty}$

$$u_i = -K_{opt} x_i$$

where $K_{opt} = (R + B^T S B)^{-1} B^T S A$

$$S = A^T S A + Q - A^T S B (R + B^T S B)^{-1} B^T S A$$

Algebraic Riccati Equation (A.R.E.) for discrete-time systems

Note:Since u_i = state feedback, it works for any initial state x_o

Stochastic LQR Problem

Suppose we had a system driven by white noise w_i

$$x_{i+1} = Ax_i + Bw_i$$
$$z_i = Cx_i$$

and want to minimize

steady state rms z while

reasonable steady state rms u

Find : $\{u_i^*\}_0^\infty$ such that

$$J = \lim_{i \to \infty} E[x_i^T Q x_i + u_i^T R u_i]$$

Solution: $u_i^* = -K_{opt} x_i$

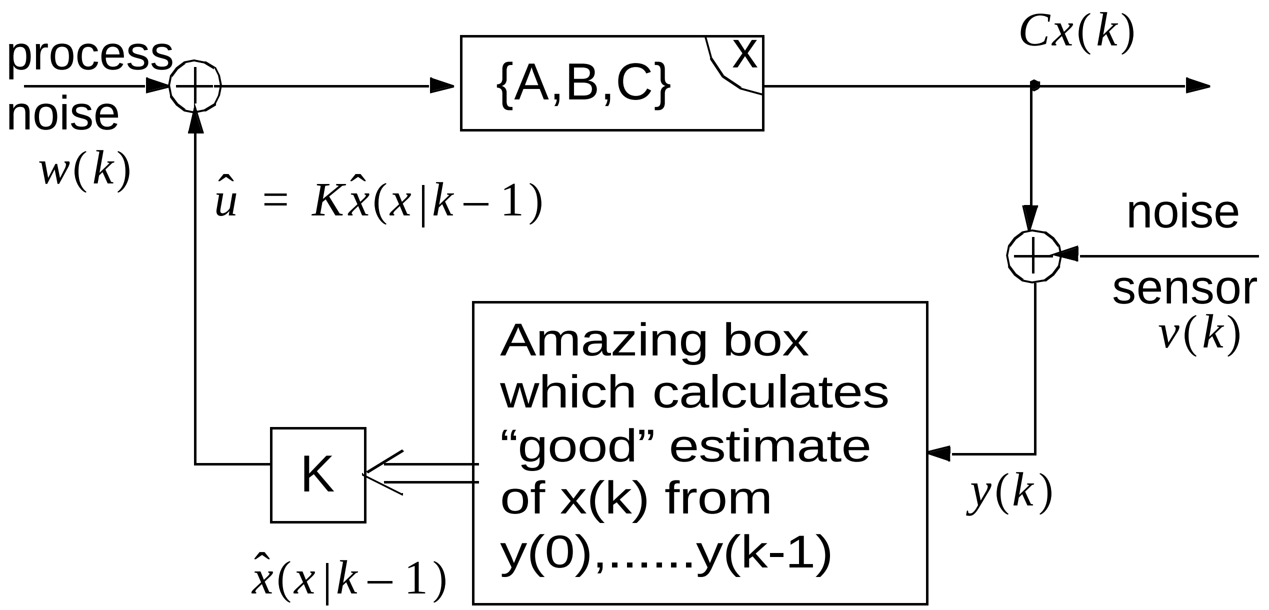
i.e. use same feedback as for deterministic problem (called “Certainty Equivalence Principle).

Optimal Linear Estimation

Our optimal control has the form $u_{opt}(k) = -K(k)x_{opt}(k)$

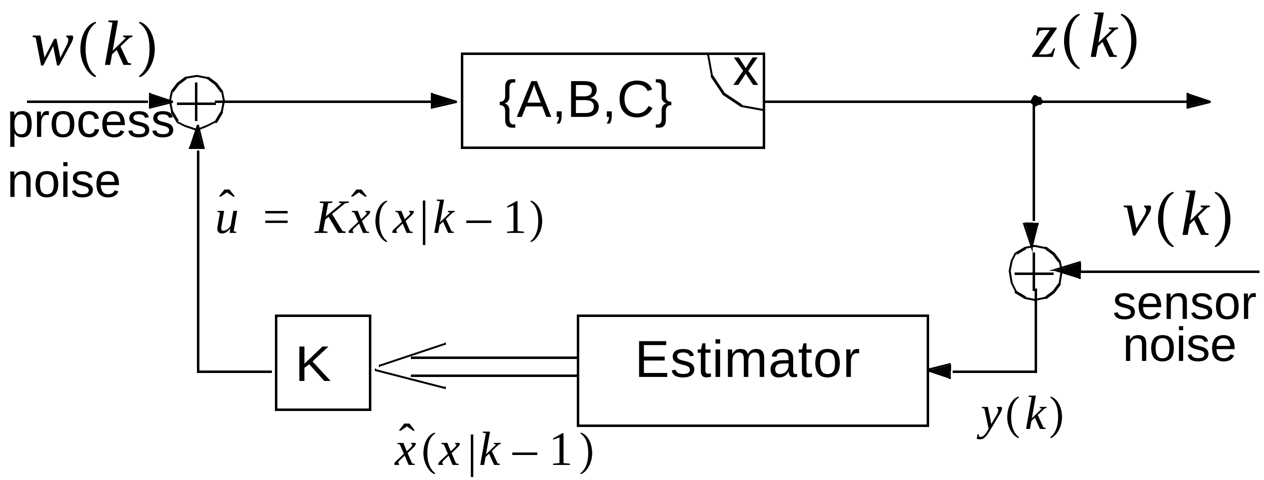
This <u>assumes</u> that we have complete state information $x_{opt}(k)$ - not actually true !.

How can be obtain “good” estimates of the velocity state from just observing the position state ? Furthermore the sensors may be noisy and the plant itself maybe subject to outside disturbances (process noise) i.e. we are looking for this:



Problem Statement

Given:
$$\begin{aligned} x(k+1) &= Ax(k) + Bw(k) \\ z(k) &= Cx(x) \\ y(k) &= Cx(k) + v(k) \end{aligned}$$



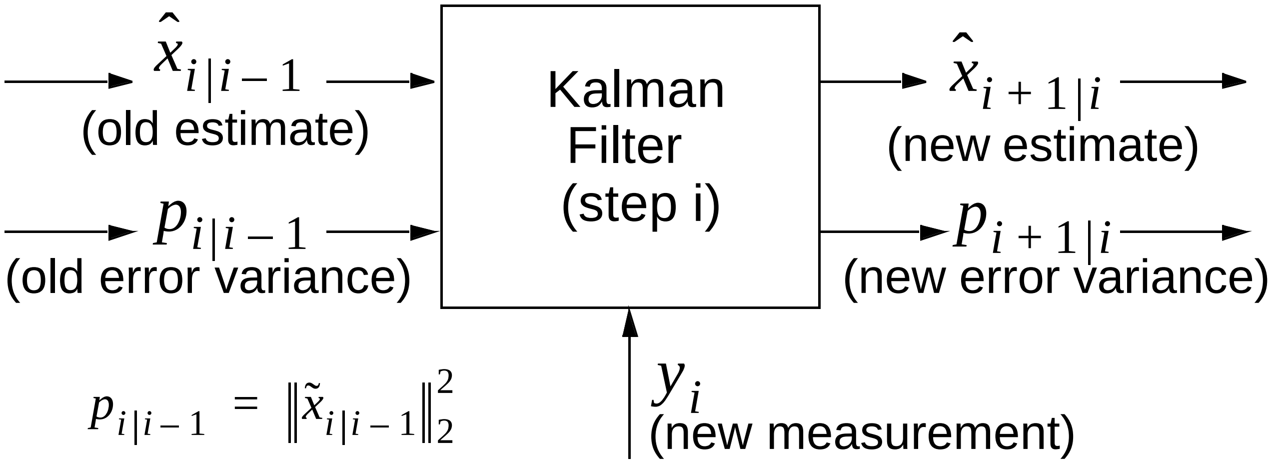
Assume also x(0) is Random & Gaussian and that x(k), w(k)+v(k) are <u>all mutually independent</u> for all k. Find: $\hat{x}(k|k-1)$ optimal estimate of x(k) given y_0, ... , y_{k-1}

such that "mean squared error"

$$E[\|x(k) - \hat{x}(k|k-1)\|_2^2] = \text{minimal}$$

Kalman Filter

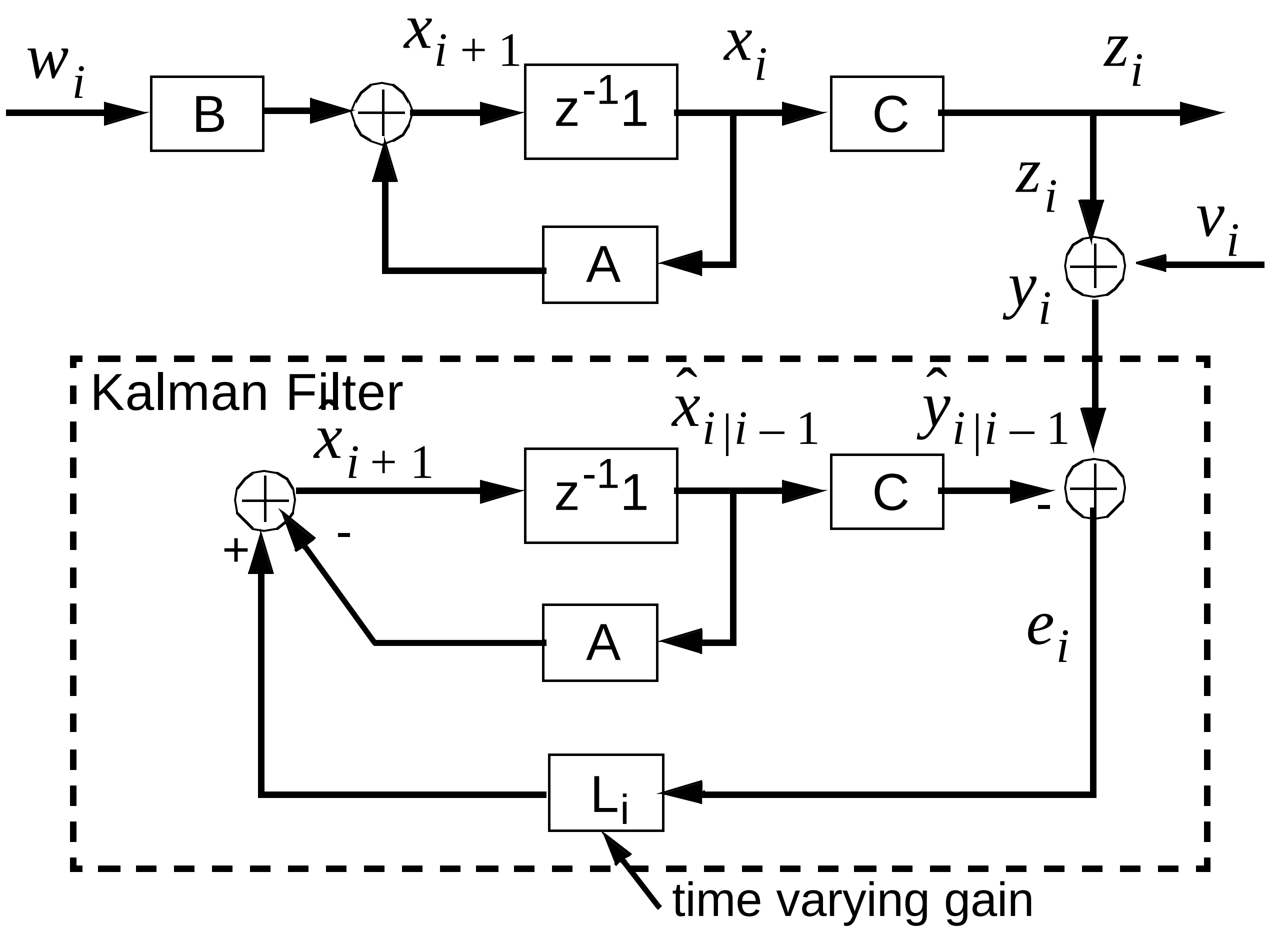
The Kalman filter is an efficient algorithm that computes the new $\hat{x}_{i+1|i}$ (the linear-least-mean square estimate) of the system state vector x_{i+1}, given $\{y_0, \ldots, y_i\}$, by updating the old estimate $\hat{x}_{i|i-1}$ and old $\tilde{x}_{i|i-1}$ (error).



$$p_{i|i-1} = \|\tilde{x}_{i|i-1}\|_2^2$$

The Kalman Filter produces $\hat{x}_{i+1|i}$ from $\hat{x}_{i|i-1}$ (rather than $\hat{x}_{i|i}$), because it “tracks” the system “dynamics”. By the time we compute $\hat{x}_{i|i}$ from $\hat{x}_{i|i-1}$, the system state has changed from x_i to $x_{i+1} = Ax_i + Bw_i$

Picture of Kalman Filter



Plant Equations:

$$x_{i+1} = Ax_i + Bu_i$$

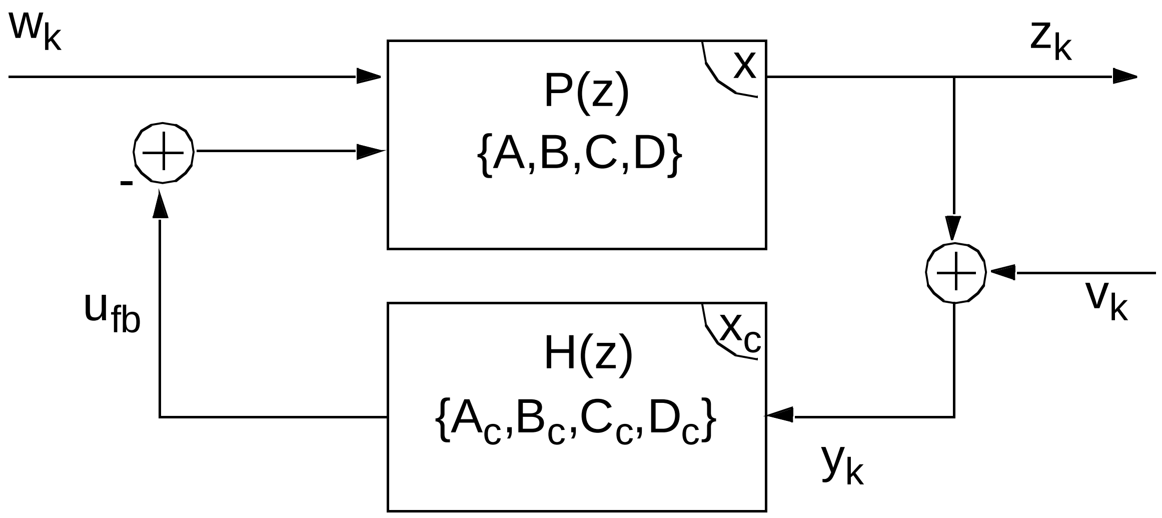
$$y_i = Cx_i + v_i$$

Kalman Filter:

$$\hat{x}_{i+1|i} = A\hat{x}_{i|i-1} + L_i(y_i - \hat{y}_{i|i-1})$$

$$\hat{y}_{i|i-1} = C\hat{x}_{i|i-1}$$

LQG Problem



Problem Statement

Want a controller which takes as input noisy measurements, y, and produces as output a feedback signal, u, which will minimize excursions of the regulated plant outputs (if no pole-zero cancellation, then this is equivalent to minimizing state excursions.)

Also want to achieve "regulation" with as little actuator effort , u, as possible.

Solution of LQG Problem

The separation principle states that the LGQ optimal controller is obtained by:

(1) Using Kalman filter to obtain least squares optimal estimate of the plant state, i.e.: Let

$$x_c(k) = \hat{x}_{k|k-1}$$

(2) Feedback <u>estimated</u> LQR-optimal state feedback

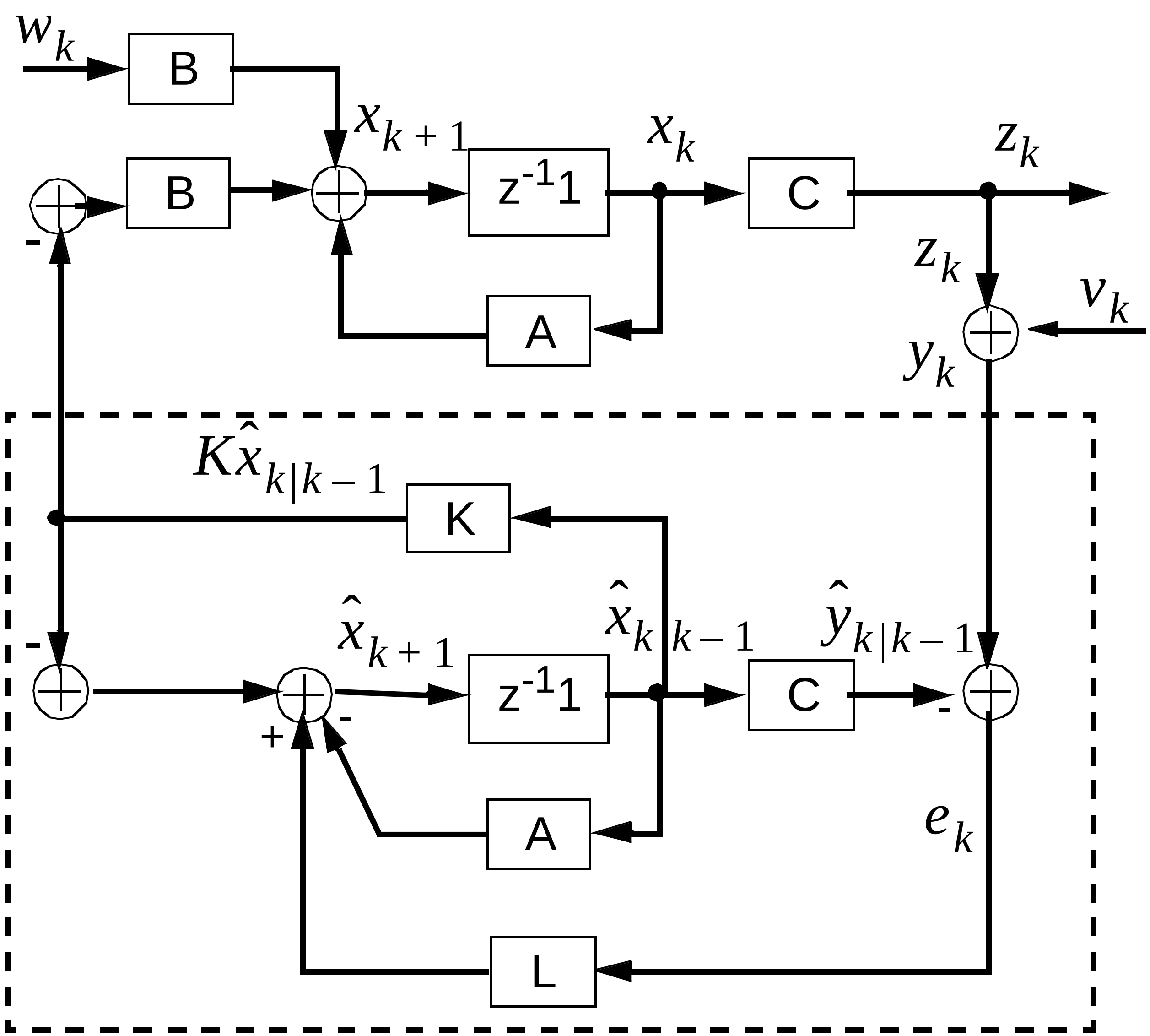
$$\begin{aligned} u(k) &= -K_{LQR} x_c(k) \\ &= -K_{LQR} \hat{x}_{k|k-1} \end{aligned}$$

i.e. can treat problems of

- optimal feedback &
- state estimate

separately.

Picture of LQG Controller



Equation of LQG Control

Plant

$$\begin{aligned} x_{k+1} &= Ax_k + (-Bu_k) + B_w w_k \\ z_k &= Cx_k \\ y_k &= Cx_k + v_k \end{aligned}$$

LQG Controller

$$\begin{aligned} \hat{x}_{k+1|k} &= A\hat{x}_{k|k-1} + Bu_k + L(y_k - C\hat{x}_{k|k-1}) \\ u_k &= -K\hat{x}_{k|k-1} \end{aligned}$$

$$K = -[R + B^T SB]^{-1} + S = A^T SA + Q - A^T SB[R + B^T SB]^{-1} B^T SA$$

$$L = APC^T[V + CPC^T]^{-1} + P$$

$$= APA^T + BWB^T - APC^T[V + CPC^T]^{-1} CPC^T$$

Procedure for LQR Controller

So design procedure is

1 - Obtain state-space model of plant

2(a) - Estimate process noise + measurement noise covariances W and V

2(b) - “Design” L for {A,B,C,D},W and V

--> estimator

3(a) - Pick weighting matrices Q and R

3(b) - “Design” K for {A,B,C,D}, Q and R

--> optimal state feedback

4 - Form {Ac,Bc,Cc}={(A-BK-LC),L,-K}

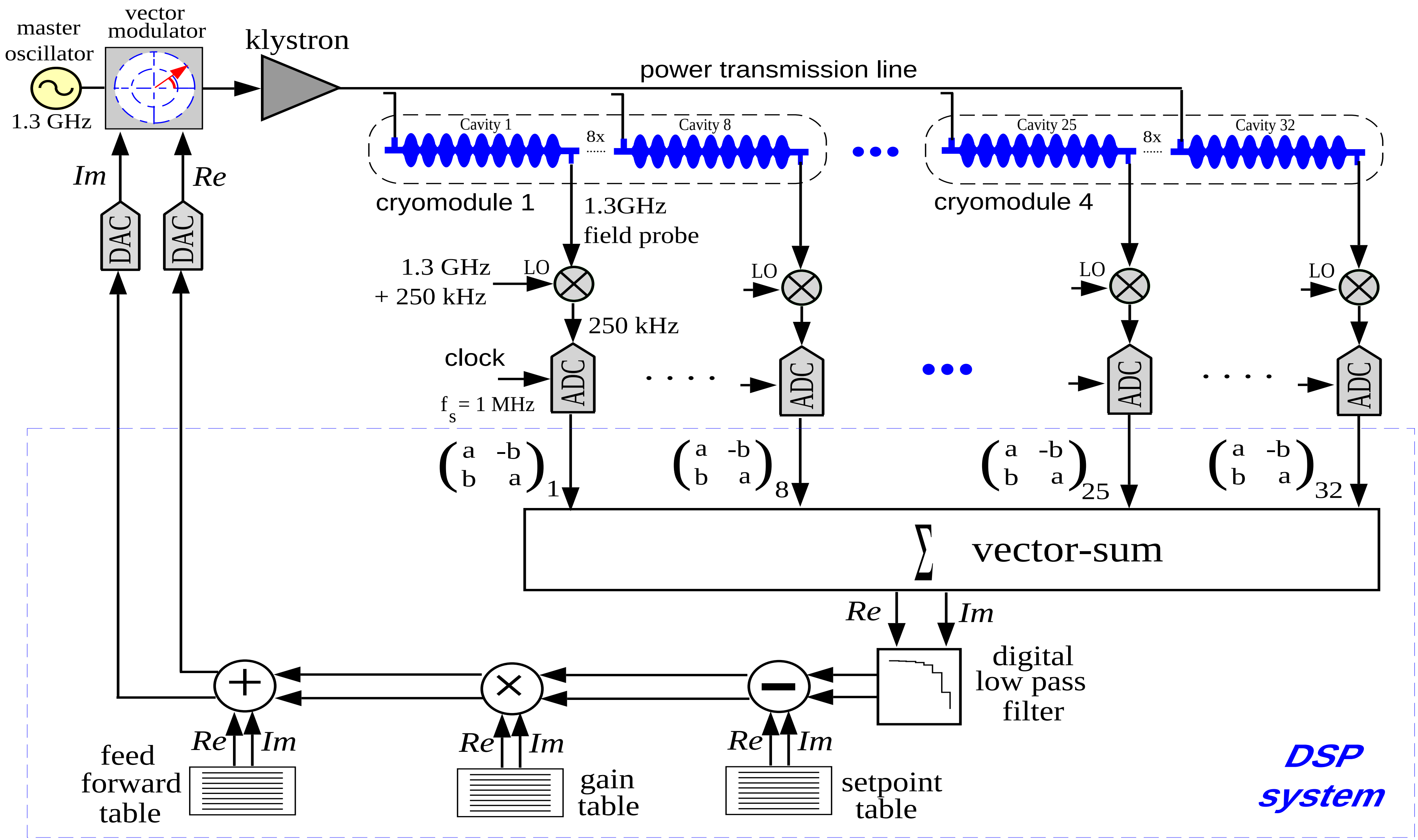
--> LQG optimal controller

(4) Once a controller has been specified by L + K from {A,B,C,D} and {W,V,Q,R} can calculate rms z and rms u

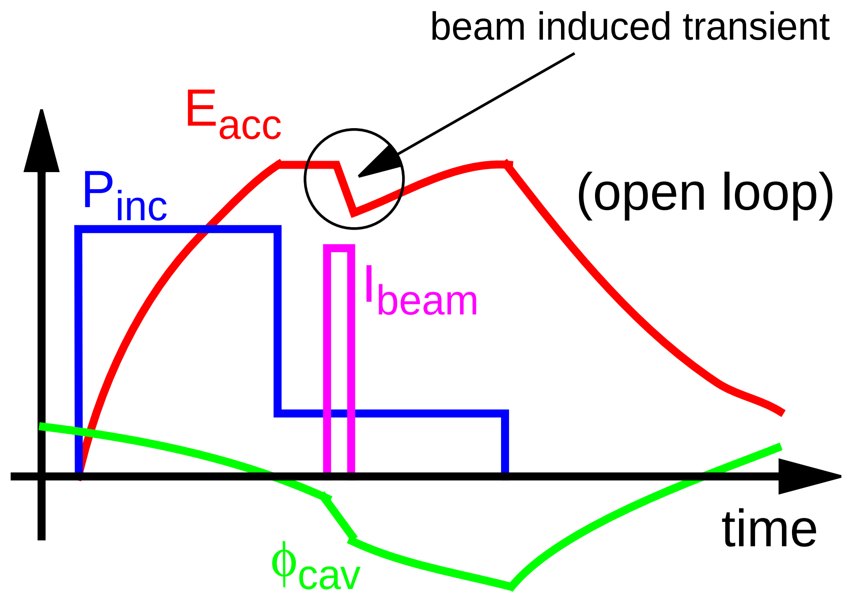
Features of Optimal Controller

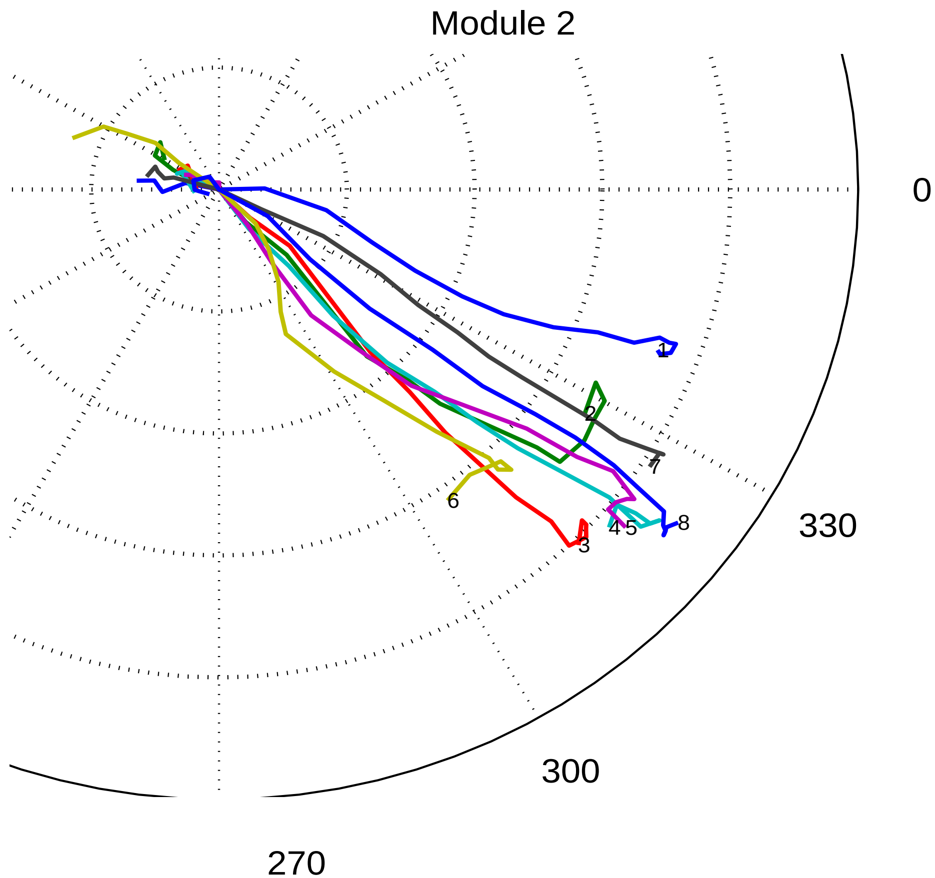
- Field regulation to required stability
- Cavity frequency control
- Robustness against parameter variations
- Operation close to the performance limit
- Maximize availability of linacs
- Support automated operation

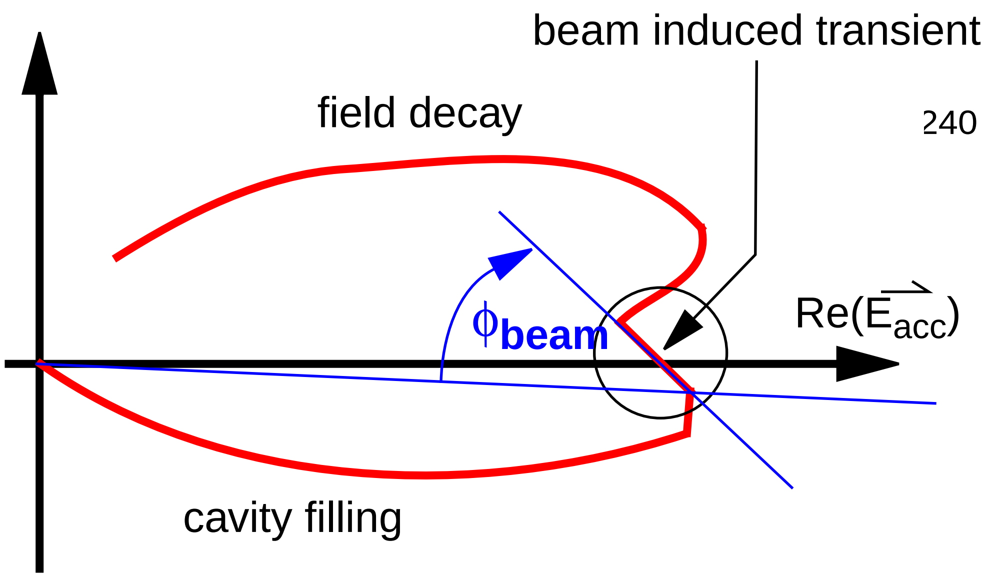
Digital Control at the TTF



Beam Transient based Phase and Gradient Calibration







for $\Delta t << \tau_{cav}$:

$$\Delta V_{ind} = I \cdot \Delta t \cdot \left(\frac{r}{Q}\right) \cdot \pi \cdot f$$

Cavity Model

Cavity Field

$$\begin{bmatrix} \dot{v}_r \\ v_i \end{bmatrix} = \begin{bmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{bmatrix} \cdot \begin{bmatrix} v_r \\ v_i \end{bmatrix} + R \cdot \omega_{12} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

Mechanical Properties

$$\left[\dot{\Delta\omega}\right] = \left[-1/\tau_m\right] \cdot \left[\Delta\omega\right] + \left[-2\pi/\tau_m K_m\right] \cdot \left[(v_r^2 + v_i^2)\right]$$

or

$$\begin{bmatrix} \dot{\Delta\omega} \\ \dot{\Delta\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_m^2 & -1/\tau_m \end{bmatrix} \cdot \begin{bmatrix} \Delta\omega \\ \dot{\Delta\omega} \end{bmatrix} + 2\pi\omega_m^2 K_m \cdot \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ (v_r^2 + v_i^2) \end{bmatrix}$$

Typical Parameters

$\Delta\omega = \omega_0 - \omega_{rf}$, $\omega_{12} = \frac{\omega_0}{2 \cdot Q_L}$, $R = \left(\frac{r}{Q}\right) \cdot Q_L$,

$\omega_0 = 2\pi \cdot 1.3 \cdot 10^9$, $Q_L = 3 \cdot 10^6$, $\left(\frac{r}{Q}\right) = 1030\ \frac{\Omega}{m}$, $K_m = -1\ \text{Hz/(MV/m)}^2$

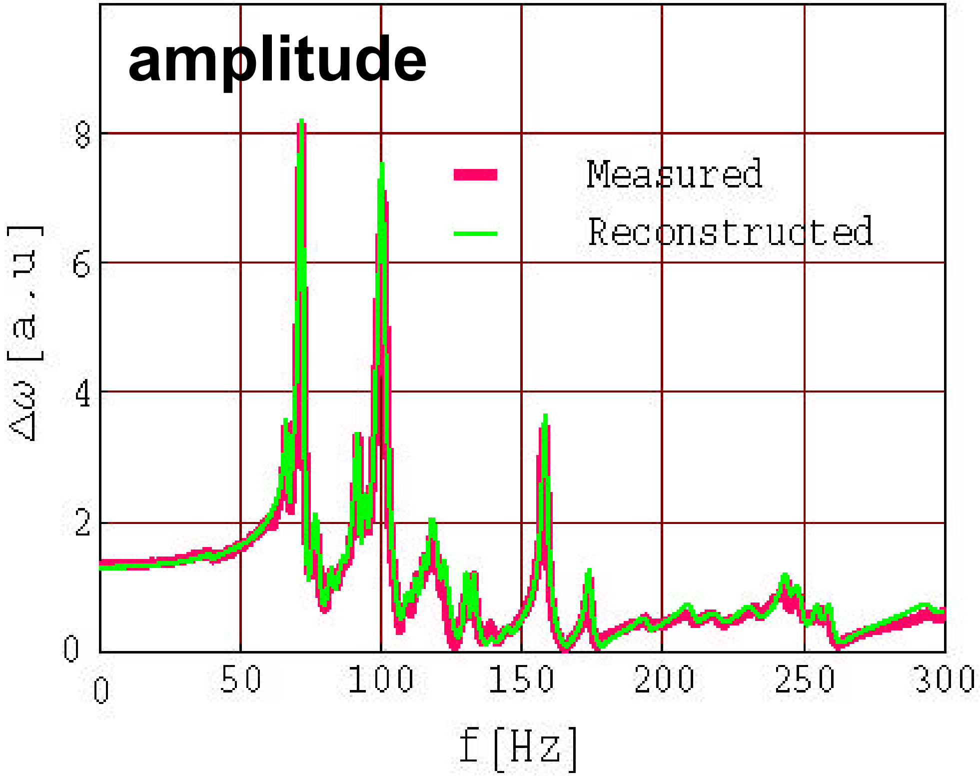
Modelling Lorentz Force Detuning

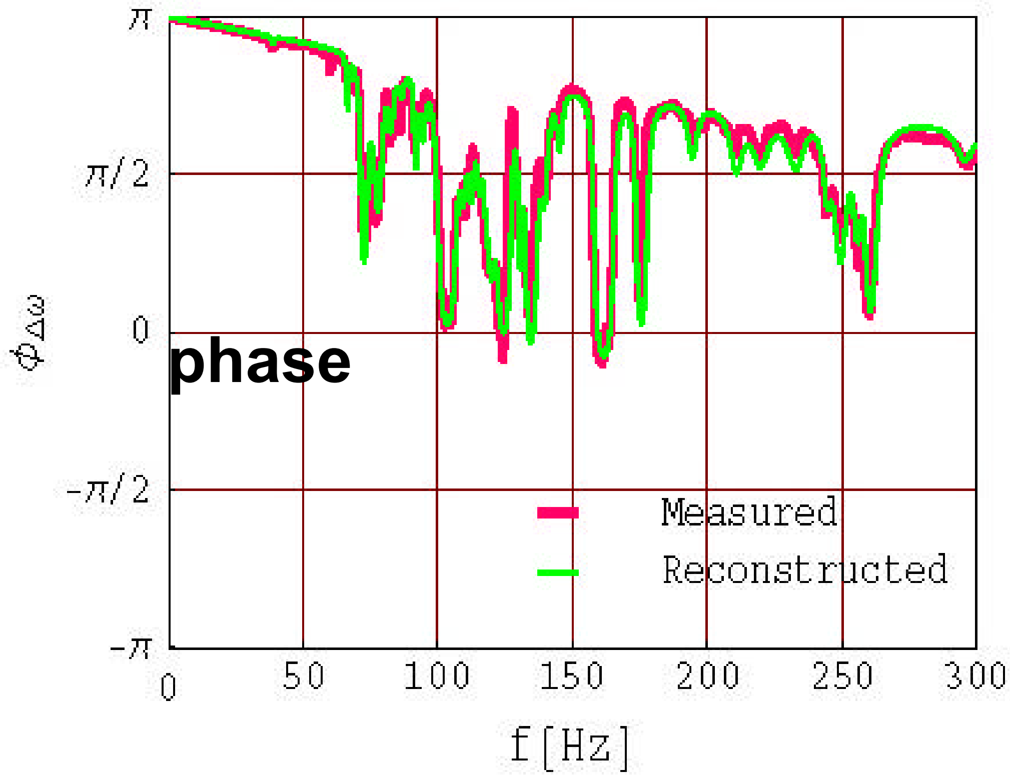
$$\begin{bmatrix} \Delta\dot{\omega}_1 \\ \Delta\ddot{\omega}_1 \\ \vdots \\ \Delta\dot{\omega}_N \\ \Delta\ddot{\omega}_N \end{bmatrix} = \begin{bmatrix} 0 & 1 & \ldots & 0 & 0 \\ -\omega_1^2 & -\frac{1}{\tau_1} & \ldots & 0 & 0 \\ & & \vdots\vdots & & \\ 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & -\omega_N^2 & -\frac{1}{\tau_N} \end{bmatrix} \cdot \begin{bmatrix} \Delta\omega_1 \\ \Delta\dot{\omega}_1 \\ \vdots \\ \Delta\omega_N \\ \Delta\dot{\omega}_N \end{bmatrix} + 2\pi \begin{bmatrix} 0 \\ -K_1\omega_1^2 \\ \vdots \\ 0 \\ -K_N\omega_N^2 \end{bmatrix} \cdot \begin{bmatrix} \mathrm{V}_{acc}^2 \end{bmatrix}$$

where $\Delta\omega_m$: detuning of mode *m*, V_{acc} : accelerating voltage, τ_m : mechanical time constant of mode *m* and K_m : Lorentz force detuning constant of mode *m*.

Transfer Function

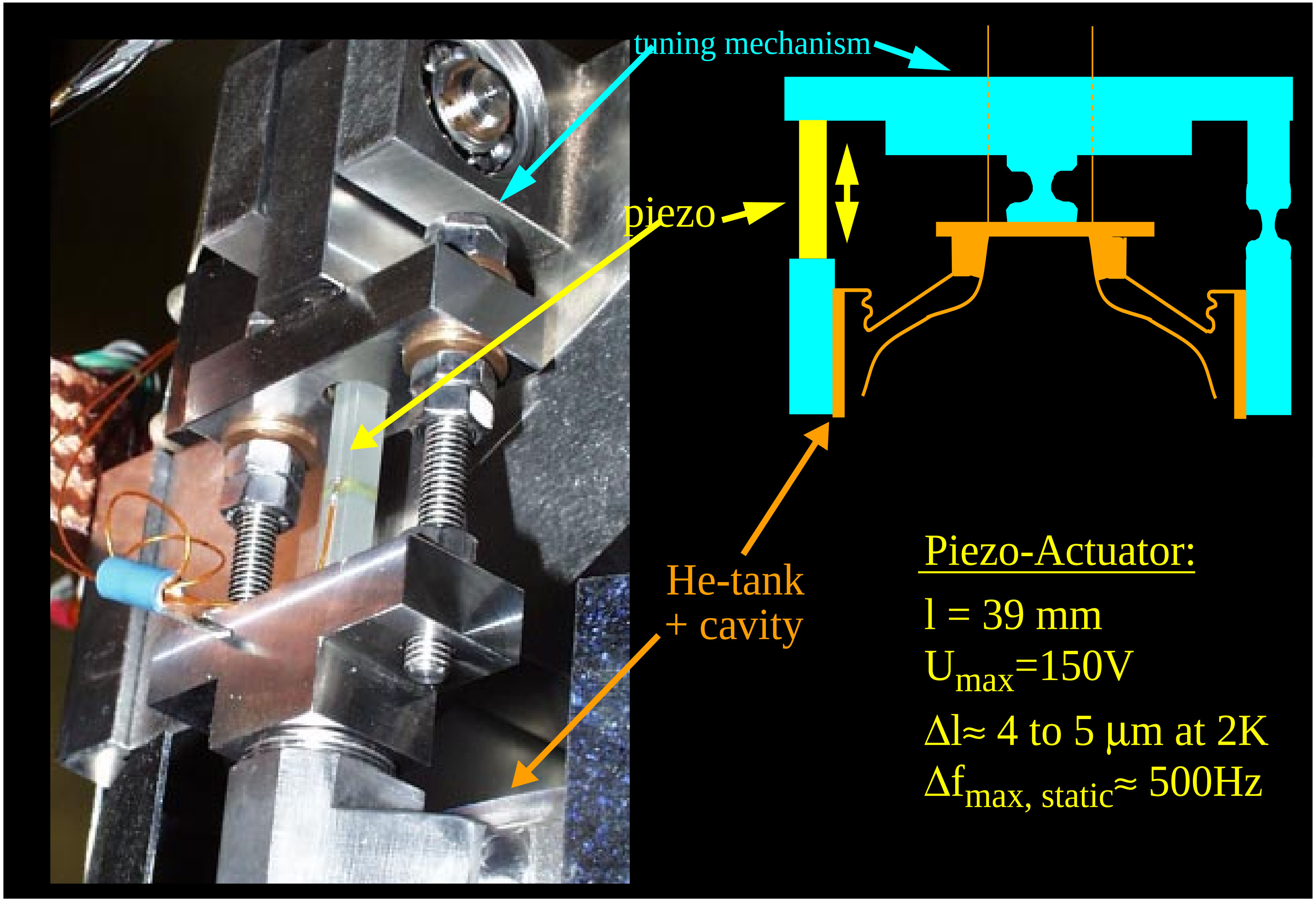
Transfer function Lorentz Force --> Detuning, SNS cavity





courtesy: J. Delayen, JLAB, M. Doleans, ORNL

Active Compensation of Lorentz Force Detuning (1)



Piezo-Actuator:

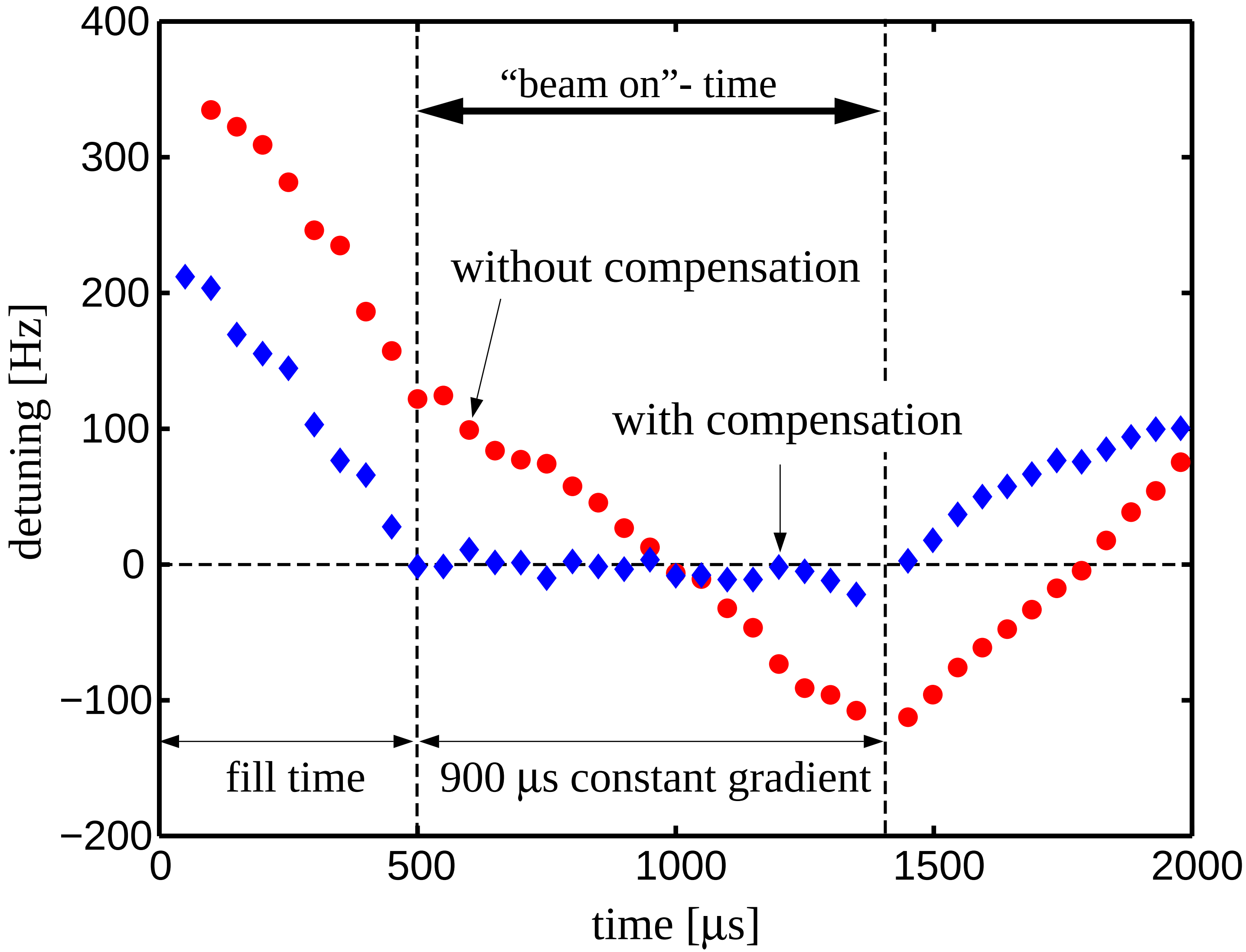
l = 39 mm

U_{max}=150V

Δl≈ 4 to 5 µm at 2K

$\Delta f_{max, static}$≈ 500Hz

Active Compensation of Lorentz Force Detuning (2)

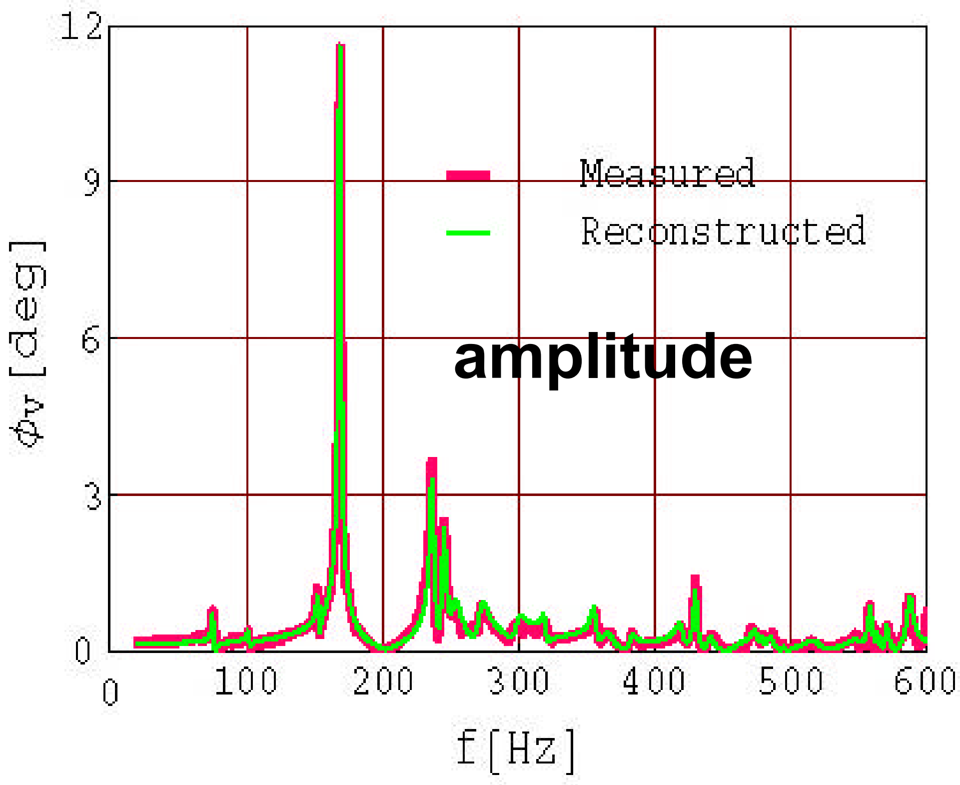


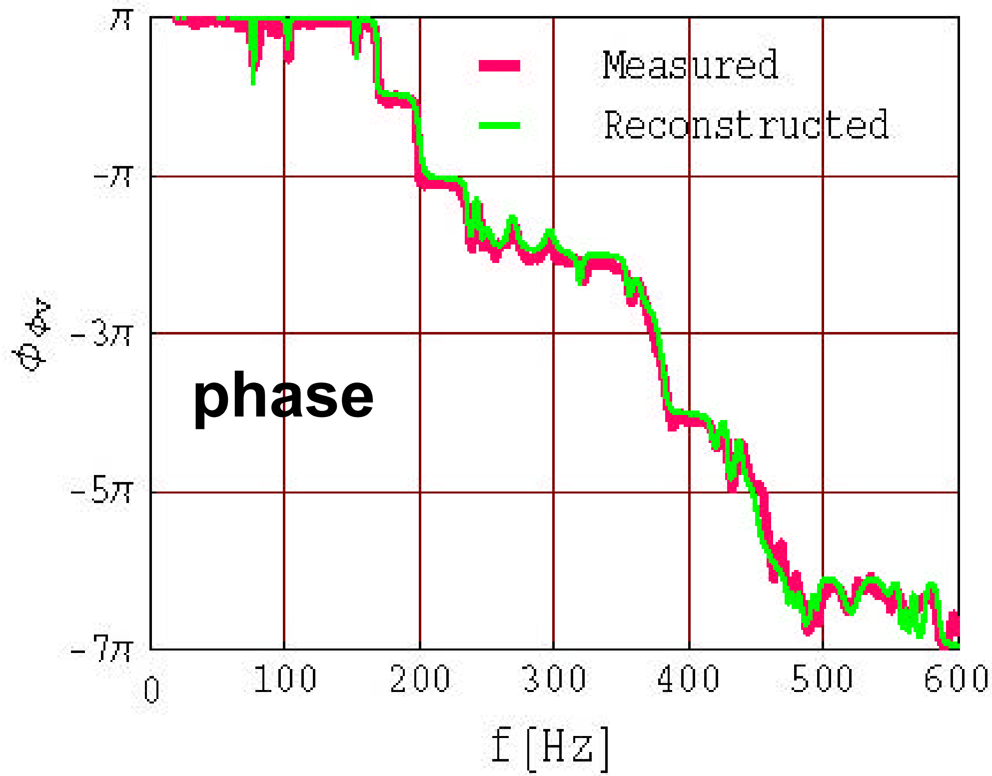
9-cell cavity operated at 23.5 MV/m

Lorentz force compensated with fast piezoelectric tuner

Transfer Function

Transfer function Piezo Tuner --> Detuning, SNS cavity





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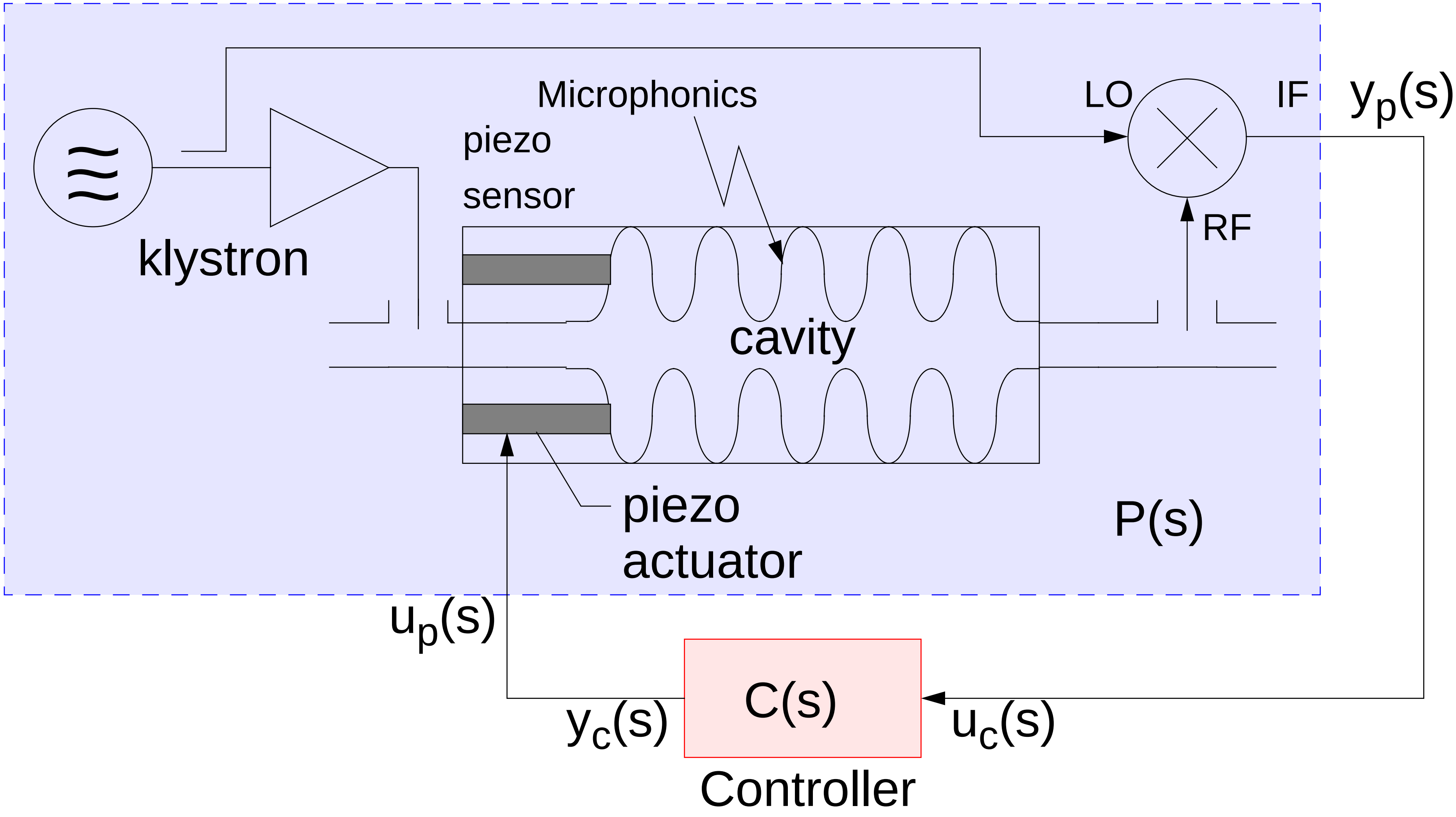
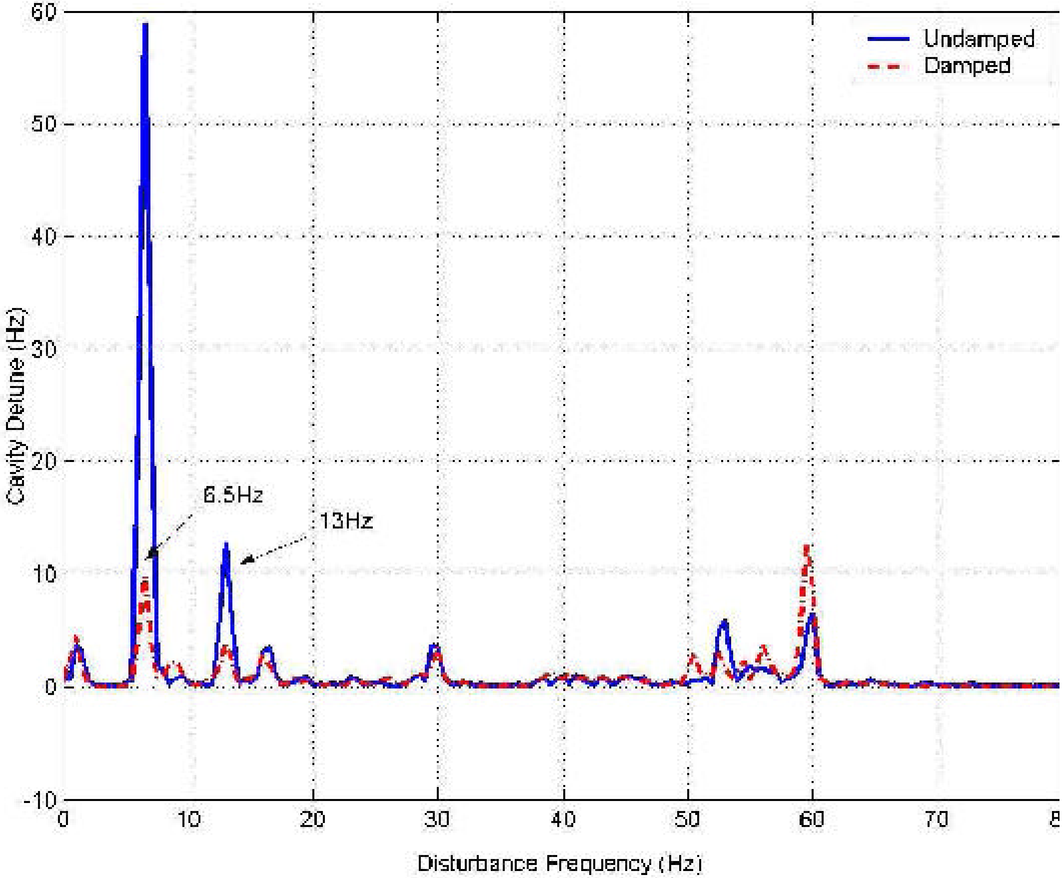
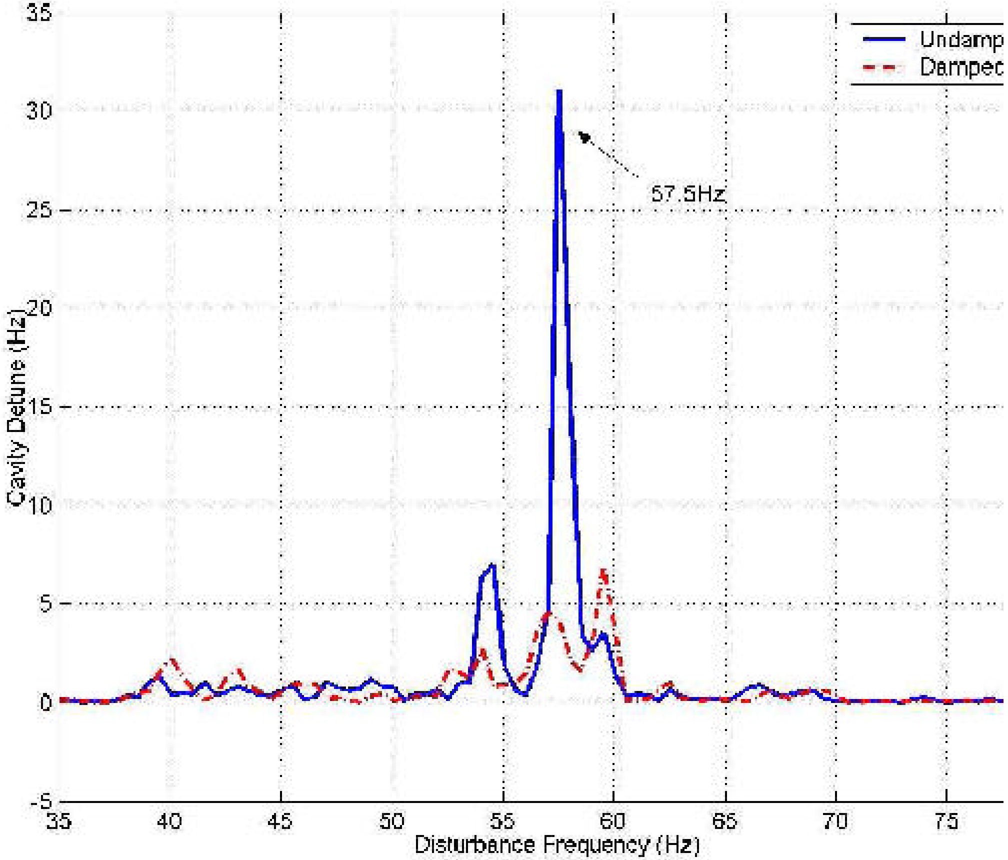
Microphonics Control



Microphonics Suppression with Feedforward



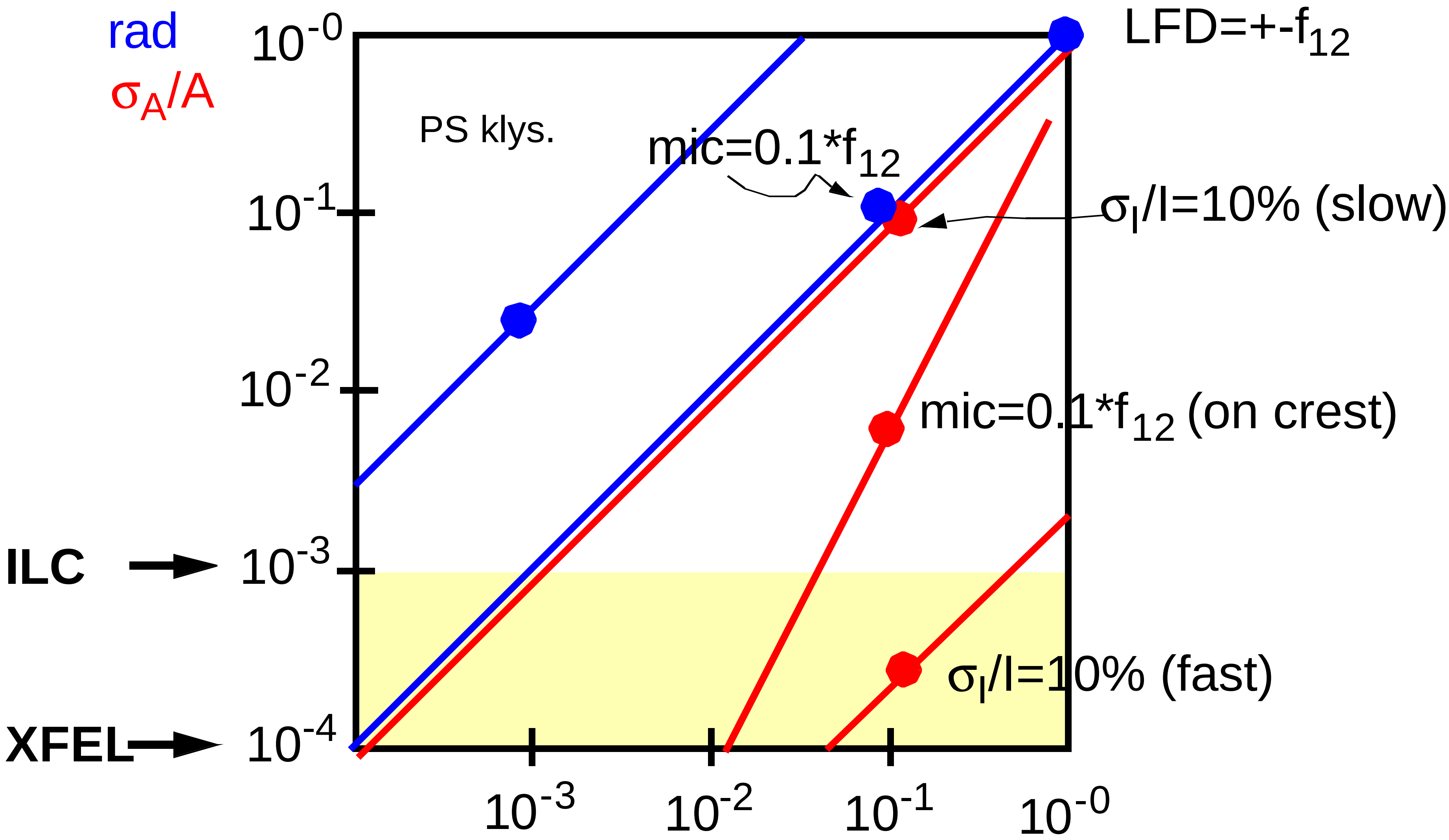
Active damping of helium oscillations at 2K.



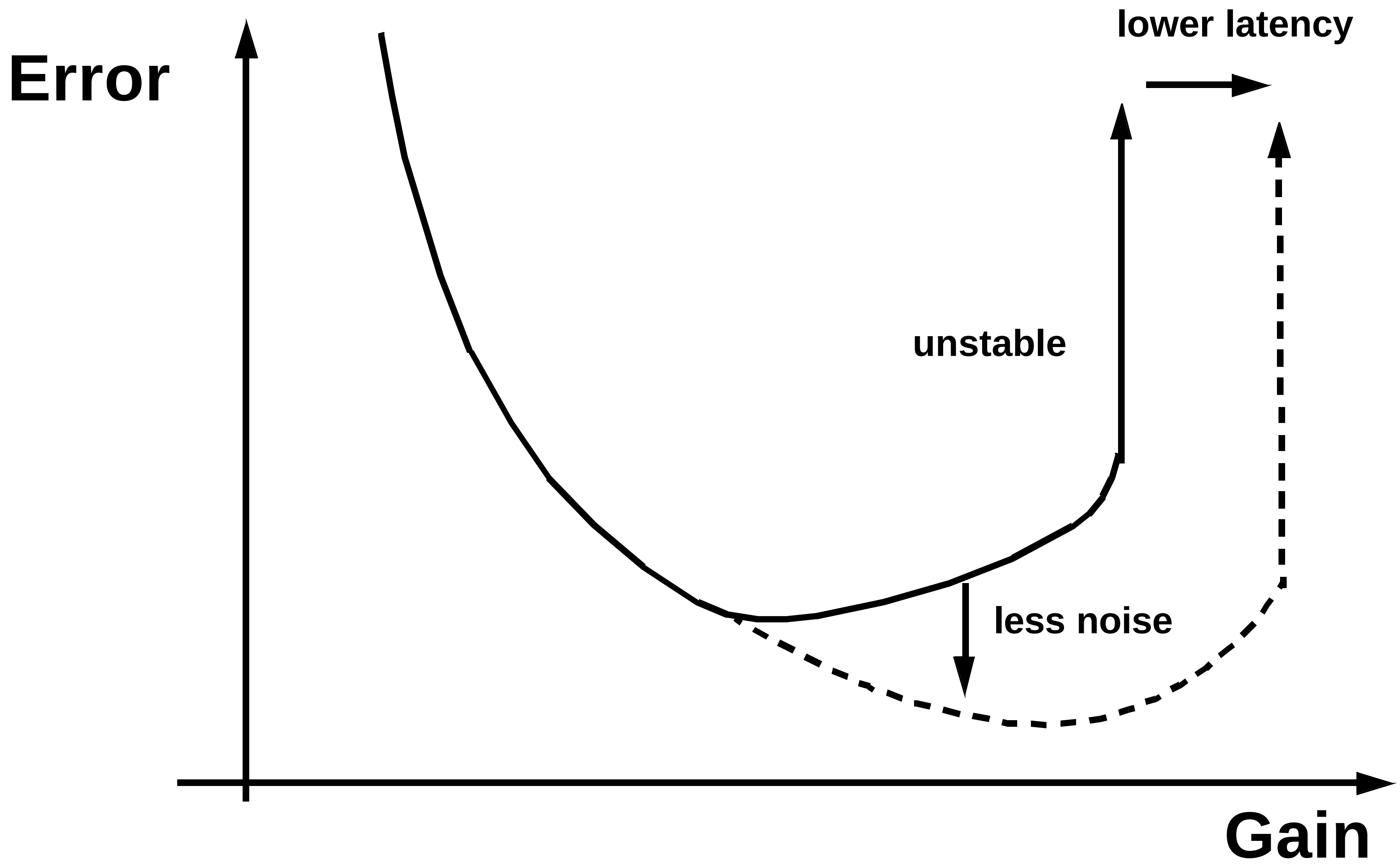
Active damping of external vibration at 2K.

T. Grimm

Error Contributions

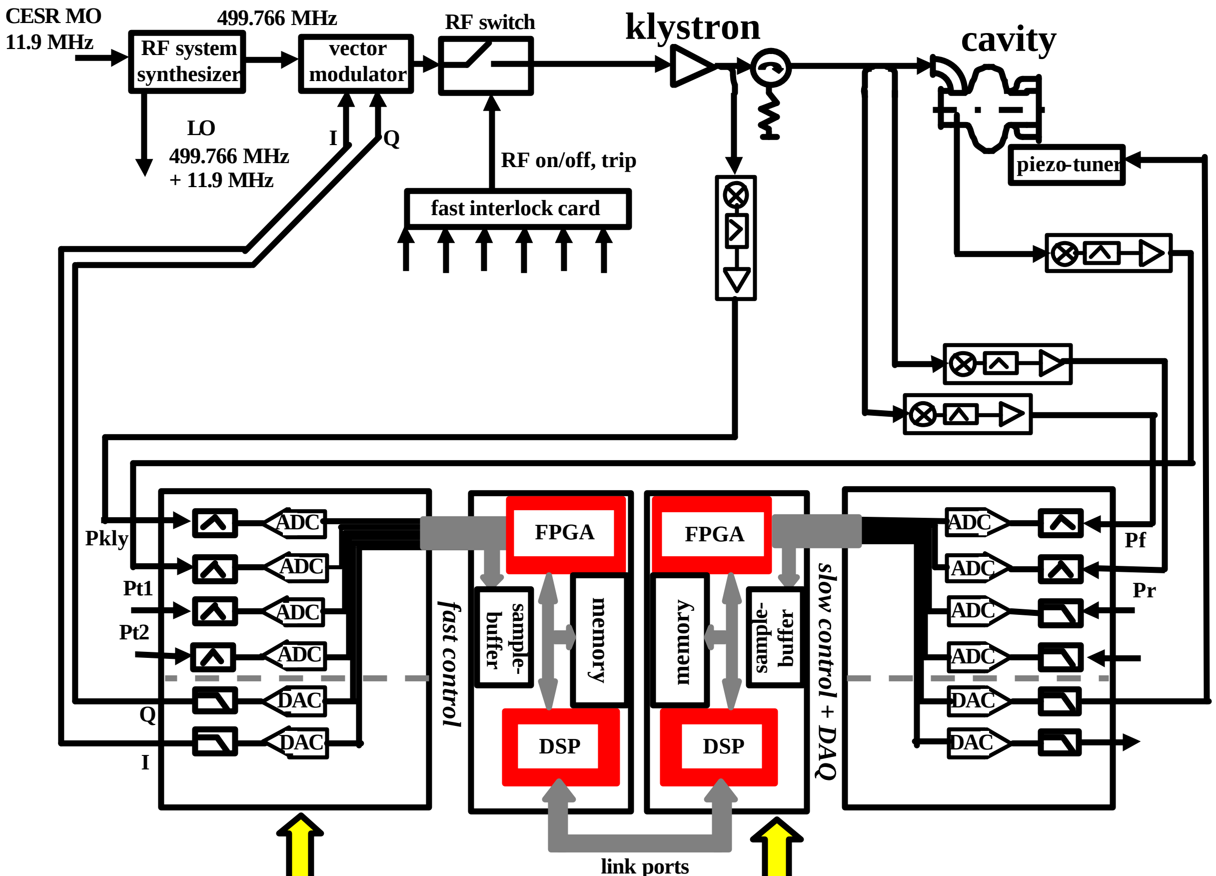


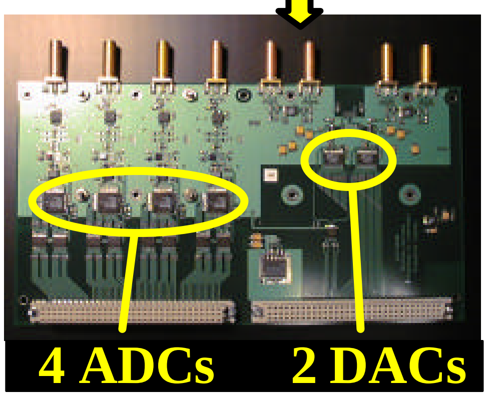
RMS Error as Function of Feedback Gain

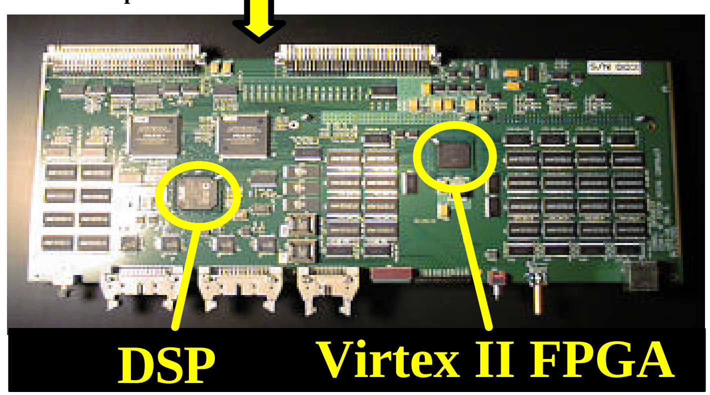


Matthias Liepe
CORNELL
UNIVERSITY

Ultra-Fast Digital RF Field Control System for CESR and ERLs







- *very low delay in the control loop (» 1 ms)*
- *Field Programmable Gate Array (FPGA) design combines the speed of an analog system and the flexibility of a digital system*
- *high computation power allows advanced control algorithms*
- *all boards have been designed in house*
- *generic design: digital boards can be used for a variety of control and data processing applications*

Multichannel Downconverter

Meeting the high field stability requirements demands for new, noise-reduced, highly linear downconverters.

New downconverters are already installed in VUV-FEL and undergo intensive testing.

Picture of 3rd generation downconverter.

- 8 in/output channels, 1 LO input
- Linearity <-50dB
- Crosstalk between channels <-50dB
- LO leakage <-50dB @ 1.3GHz
- LO stability –15dB - –5dB

Design and assembly at DESY, layouting by external company